

Post-buckling behaviours of axially restrained steel columns in fire

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Abstract. This paper presents a simplified model to study post-buckling behaviours of the axially restrained steel column at elevated temperatures in fire. The contribution of axial deformation to the curvature of column section is included in theoretical equations. The possible unloading at the convex side of the column when buckling occurs is considered in the stress-strain relationship of steel at elevated temperatures. Parameters that affect structural behaviours of the axial restrained column in fire are studied. The axial restraint cause an increase in the axial force before the column buckles; the buckling temperature of restrained columns will be lower than non-restrained steel columns. However, the axial force of a restrained column decreases after the column buckles with the elevation of temperatures, so make use of the post-buckling behaviour can increase the critical temperature of restrained columns. Columns with temperature gradient across the section will produce lower axial force at elevated temperatures.

Keywords : Axial Restraint; Fire; Post-buckling; Critical temperature; Steel Column.

1. Introduction

As part of a complete structure, the behaviour of a framed column is affected by the adjacent structural members. When the steel column subjected to a fire, the adjacent structural member will restrain the thermal expansion of the column and bring additional axial force to the column; on the other hand, when the column lose its load bearing capacity due to the high temperature of the fire, the adjacent structural may shield the load which acts on the column at ambient temperature. The influences of axial restraint on heated steel columns have been studied numerically and experimentally by many researchers.

Simms *et al.* (1995-1996) performed an experimental investigation on the structural performance of axially restrained steel columns subjected to elevated temperatures in fire. However, the restraint was assumed to be active only in the loading phase of the column, i.e., only when the column was expanding and its compressive load increasing. When the column was contracting, the restraint was assumed not to be effective. Thus if columns failure were defined as that caused by increased compressive load during the column expansion phase, results of the studies showed that, depending on the level of the restraint stiffness and the column slenderness, the column critical temperature could be

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much lower than that of the column without any restraint.

In general, the fire resistance of an element is defined as the time after which this element, when subjected to the action of a fire, ceases to be capable of performing the functions for which it is designed. Since the initial force represents the force for which the column is designed at room temperature, so the fire resistance can be defined by the instant when the continuously changing column axial force again reaches this starting value.

By defining the column failure as the point when the column axial load returns to its original value at the ambient temperature, Neves (1995) performed a parametric study on restrained steel columns in fire. Studied parameterise included axially restrained stiffness, slenderness of the column, and eccentricity of the restraining force, on the fire resistance of axially restrained steel columns. Analysis results showed that, for small load eccentricities, the critical temperature of axially restrained steel columns decreased with the increasing of the axially restrained stiffness and column slenderness. However, there was no dependence of the critical temperature on the stiffness whenever the eccentricity of the compression load was sufficiently great. Using the finite element computer program SAFIR, Franssen (2000) studied the post-buckling behaviour and critical temperature of an axially restrained column. Franssen (2000) demonstrated that when the column post-buckling behaviour was considered, effects of the axial restraint on the column critical temperature were much less severe than previously suggested. Rodrigues *et al.* (2000) carried out a large number of fire tests on axially restrained steel columns. It was observed that the axial restraint to centrally loaded columns having slenderness higher than 80 can lead to reduction in their critical temperature by up to 200°C. Nevertheless, if the loading was eccentric and the eccentricity was high, the axial restraint did not cause significant reduction in the critical temperature. To a great extent, the experimental results confirmed the collusion obtained in Neves's (1995) numerical analysis.

Wang (2004) presented a theoretically method to study the post-buckling behaviours of the axially restrained steel column at elevated temperatures in fire. Results of this study indicated that the column critical temperature could be much higher than that of the column at first buckling and the higher the column slenderness, the larger the difference between temperatures of column failure and first buckling. Nevertheless, for columns with light restraints (restraint stiffness to column stiffness less than 5%) or high load ratios (load ratio higher than 0.5), the critical temperature of the column with realistic unloading stiffness of the restraint was only slightly affected by the post-buckling behaviour.

When the fire resistance of the global structure as a whole is concerned, there might be a reason to treat the fire action differently somehow. If it can be proved that the collapse of a given structural member does not produce the collapse of the rest of the structure, then it would be a waste to neglect the residual mechanical resistance of that member after the point when its axial force has decreased to the level of the design effect of actions at ambient temperature. After that point there will be load transfer from the heated column to the cold surrounding elements. This load transfer can be accepted as long as these elements are capable of supporting it without producing the collapse of the structure. This is what happened in the Cardington fire tests (Lamont 2001), where in spite of the squashing of one heated column, no global structural collapse occurred. In most real hyper-static structures this is also will happen.

The substructure in Fig. 1(a) illustrates interactions between the heated axial restrained column and cold surrounding elements. If the beam EG in Fig. 1(c) does not collapse both at temperature T_1 and temperature T_2 (the magnitude of the load on the beam is shown in Fig. 1(c)), the post-buckling behaviour of the axially restrained steel column can be fully utilized to increase the column critical temperature. In Fig. 1, T_0 stands for the ambient temperature; and P_0 is the design load of the column at ambient temperature; P is the changing axial force of the axially restrained steel column; T_{cr} is the temperature at which the axial force returns to its initial value, and it is defined as critical temperature by Franssen *et*

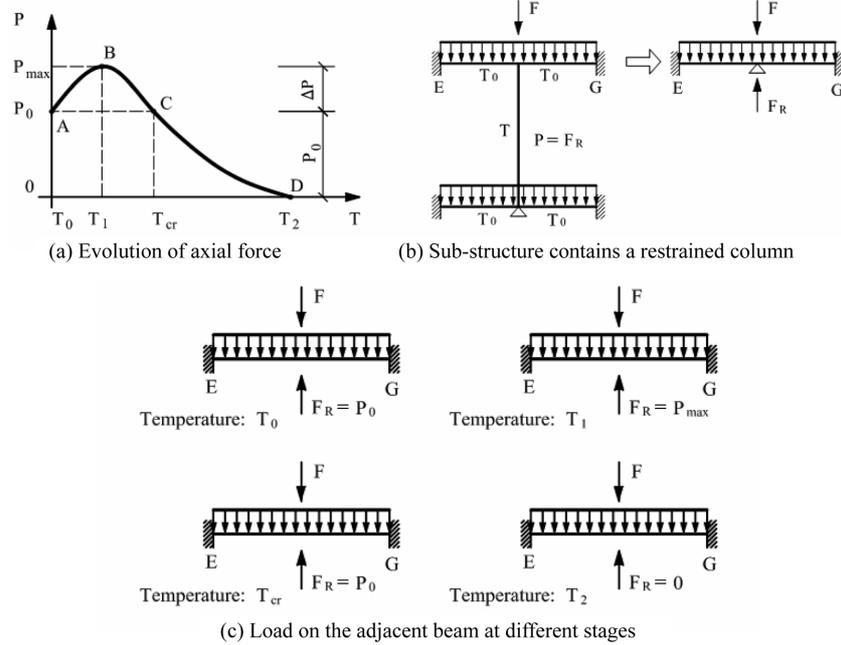


Fig. 1 Interactions between a heat column and the adjacent beam

al.(1995, 2000, 2000, 2004).

In this paper, a simply model to study the behaviours of the axially restrained steel column under axial load is presented. Parameters that affected the structural behaviours of the axially restrained column are studied. The strain reversal effect is considered for the possible unloading on the convex side of the column at elevated temperatures. Assume that the cold surrounding elements do not collapse both at temperature T_1 and temperature T_2 , as shown in Fig. 1, the post-buckling behaviour of the axially restrained steel column can be fully studied.

2. Treatment of strain reversal at elevated temperatures in fire

As refer to the inelastic buckling of steel column at ambient temperature, there exist three classical theories, which are tangent modulus theory, reduced modulus theory, and Shanley theory (Shanley 1947).

Suppose that the critical stress in a column exceeds the proportional limit of the material. Replacing the Young's modulus E in the Euler's formula with the tangent modulus E_t , the tangent modulus critical load, P_t , becomes,

$$P_t = \frac{\pi^2 E_t I}{L^2} \tag{1}$$

The tangent modulus theory tends to underestimate the strength of the column, since it uses the tangent modulus once the stress on the convex side exceeds the proportional limit while the concave

side is still below the elastic limit.

The reduced modulus theory defines a reduced Young's modulus E_r to compensate for the underestimation given by the tangent-modulus theory. For a column with rectangular cross section, the reduced modulus E_r is defined by,

$$E_r = \frac{4EE_t}{(\sqrt{E} + \sqrt{E_t})^2} \quad (2)$$

Replacing E in Euler's formula with the reduced modulus E_r , the reduced modulus critical load, P_r , becomes

$$P_r = \frac{\pi^2 E_r I}{L^2} \quad (3)$$

The reduced modulus theory tends to overestimate the strength of the column, since it is based on stiffness reversal on the convex side of the column.

Both tangent modulus theory and reduced modulus theory were accepted theories of inelastic buckling until Shanley (1947) published his logically correct paper in 1947. The critical load of inelastic buckling is in fact a function of the transverse displacement w . According to Shanley theory (Shanley 1947), the critical load is located between the critical load predicted by the tangent modulus theory (the lower bound) and the reduced modulus theory (the upper bound). When the deflection is big enough, the critical load is proceeding to the reduced modulus load.

For structural member may undergo large deformation when subjected to fire, the stress-strain relationship of steel at elevated temperatures should account for the possible strain reversal on the convex side of the column. El-Rimawi *et al.* (1996) proposed a model to account for the strain reversal in the cooling phase of a fire, as shown in Fig. 2.

In his model, the plastic strain is assumed not to be affected by temperature variations. To illustrate the model, consider a uni-axial stress state with a constant stress σ_0 , which is high enough to cause yield at a temperature T_2 . The equilibrium position on the load-deformation curve (BOB') shown in Fig. 2 at this temperature is given by point C . When temperature changes from T_2 to T_1 , the new equilibrium stress-strain state is determined by the following steps:

- (1) Unload to zero stress state at temperature T_2 . The behaviour is assumed to be linear and the unloading path CD is parallel to the initial tangent of the curve BOB' , which is denoted by $E_1^{T_2}$;
- (2) Calculate the residual strain ε_r by

$$\varepsilon_r = \varepsilon_0 - \frac{\sigma_0}{E_1^{T_2}} \quad (4)$$

- (3) Reloading at the new temperature T_1 is also linear, but this time the path is parallel to the elastic part of the curve AOA' and displaced horizontally by ε_r . The full reloading path at this new temperature is therefore $AEDFE'A'$ and the new equilibrium position, which clearly must lie on this path. The line EDF is parallel to the tangent of the skeleton curve at the origin, and the second line FE' is the tangent to the skeleton curve at the point E' where the load is equal, but opposite in sign, to that at the point E .

3. Simplified model of axially restrained column in fire

An axially restrained steel column with an initial-out-of-straightness of y_0 is shown in Fig. 3. The restraint of cold surrounding elements to the heated column is simulated by a spring with stiffness of k_s , which is supposed not to be affected by the temperature. Here, $y_0 = a_0 \sin(\pi z/L)$, $k_s = \beta E^{20} A/L$, β is the axial restraint stiffness ratio, E^{20} is the elastic modulus of steel at ambient temperature, A is the cross section area of the column. In Fig. 3, P_0 is the designed axial force of the column at ambient temperature, and F_R is the reaction force of the spring at elevated temperature, b_0 and b_1 are the mid-span deflection of the column at ambient temperature and elevated temperature respectively.

3.1 Stress-strain relationship of steel at elevated temperatures

The stress-strain curves of steel at elevated temperatures are according to EC3 part 1.2 (2001), as illustrated in Fig. 4. For easy of explanation, the stress-strain relationship at temperature T is expressed as $\sigma^T = f(T)\{\varepsilon_m^T\}$, where, σ^T and ε_m^T are the stress and mechanical strain respectively. The unloading behaviour of steel at changing temperature adopts that described by El-Rimawi model (El-Rimani 1996).

3.2 Structural analysis of the restrained steel column at ambient temperature

Suppose the deformation curve of the column is $y = b_0 \sin(\pi z/L)$ when the column is loaded by the axial force P_0 at ambient temperature. The section curvature at the mid-span is

$$\phi = -\frac{y''}{[1 + (y')^2]^{3/2}} + \frac{y_0''}{[1 + (y_0')^2]^{3/2}} = (b_0 - a_0) \frac{\pi^2}{L^2} \quad (9)$$

Denote the axial strain at the section centroid as ε_0 . The mechanical strain $\varepsilon_{m,j}$ and stress σ_j at the point j at the mid-span are

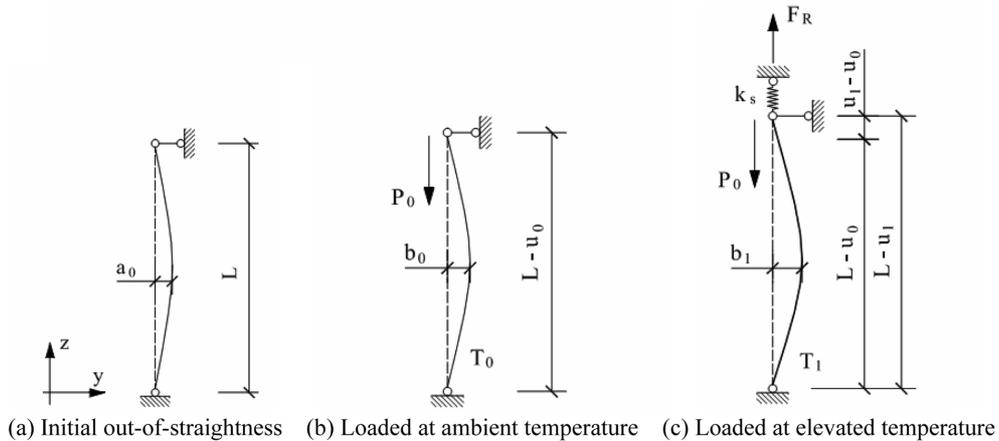


Fig. 3 Analysis Model of an Axially Restrained Steel Column

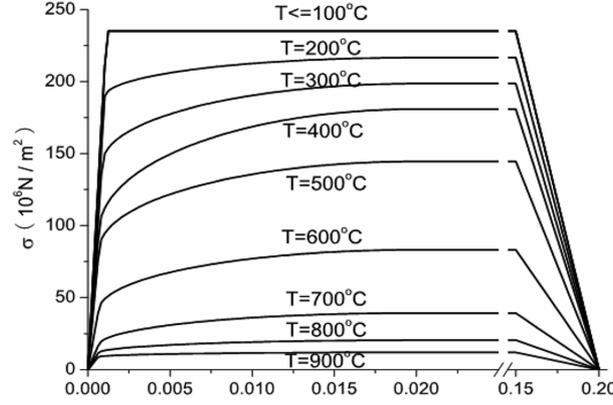


Fig. 4 Stress-Strain Relationship of Steel at Elevated Temperatures

$$\begin{aligned} \varepsilon_{m,j} &= \varepsilon_0 + \phi y_j \\ \sigma_j &= f_{(20)}(\varepsilon_{m,j}) \end{aligned} \quad (10)$$

So b_0 and ε_0 can be determined by

$$\begin{cases} P_0 - \sum_j (\sigma_j A_j) = 0 \\ P_0 b_0 - \sum_j (\sigma_j y_j A_j) = 0 \end{cases} \quad (11)$$

where, A_j is the cross section area of the element j . The axial deformation of the column at ambient temperature is u_0 , and can be obtained calculated by

$$u_0 = \varepsilon_0 L + \frac{(b_0 \pi)^2}{4L} \quad (12)$$

3.3 Structural analysis at elevated temperature

Suppose the stress and mechanical strain of the point j in the mid-span section at temperature T_1 have been obtained, which are represented by $\sigma_j^{T_1}$ and $\varepsilon_{m,j}^{T_1}$ respectively. The original tangent modulus of steel at the point is $E_{1,j}^{T_1}$. So the residual strain can be calculated by

$$\varepsilon_{r,j}^{T_1} = \varepsilon_{m,j}^{T_1} - \frac{\sigma_j^{T_1}}{E_{1,j}^{T_1}} \quad (13)$$

When temperature changes from T_1 to T_2 , assume that $y = b_1 \sin(\pi z/L)$ at temperature T_2 . The mechanical strain of point j is

$$\varepsilon_{m,j}^{T_2} = \varepsilon_0^{T_2} + \phi y_j - \varepsilon_{th,j}^{T_2} \quad (14)$$

where, $\varepsilon_0^{T_2}$ is the axial strain at the section centroid, $\phi = (b_1 - a_0)(\pi^2 / L^2)$, $\varepsilon_{th,j}^{T_2} = \alpha(T_{2,j} - 20)$, α is the coefficient of thermal strain, which takes the value of 1.4×10^{-5} . Through Eqs.(2)-(5), (10) and (11), the stresses across the section at temperature T_2 can be abtained. So b_1 and $\varepsilon_0^{T_2}$ can be determined by

$$\begin{cases} P - \sum_j (\sigma_j^{T_2} A_j) = 0 \\ P b_1 - \sum_j (\sigma_j^{T_2} y_j A_j) = 0 \end{cases} \quad (15)$$

where, $P = P_0 + k_s(u_1 - u_0)$, which is the axial force of the column at temperature T_2 , b_1 is mid-span deflection, u_1 is the axial displacement of the column, and $u_1 = \varepsilon_0^{T_2} L + (b_1 \pi)^2 / (4L)$.

4. Verification and Parametric Study

4.1. Verification

Rodrigues *et al.* (2000) seemed to be the only ones to have carried out fire resistance test on the post-buckling behaviour of the axially restrained steel column. However, it is rather difficult to use their experimental results to validate the method presented in the previous section. Their test specimens were steel bars with rectangular cross section and with four slenderness values of 80, 133, 199, and 319. Among these, the columns with the unrealistically high slenderness values of 199 and 319 were under substantial influence by bending, induced by a small amount of initial imperfection. For the columns with the lower slenderness values of 80 and 133, the return of the column axial load to the initial level at the temperature of column first buckling was generally fast, and in many cases, there was no information on further reductions in the column axial load at higher temperatures. Furthermore, temperature distributions in the longitudinal direction of the test specimens of Rodrigues *et al.* (2000) were heavily non-uniform and they used the average temperatures in their presentation of results.

In this paper, Franssen's (2000) numerical results are used to verify the proposed model. Using the finite element program SAFIR, Franssen (2000) analyzed the following restrained column:

- (1) Column section size: HEA100, buckling major axis;
- (2) Length of the member (L): 4m, slenderness (λ) is 98.6;
- (3) Elastic modulus at ambient temperature: $E^{20} = 2.1 \times 10^5$ N/mm²;
- (4) Yield strength: $f_y^{20} = 235$ N/mm², $N_u^{20} = f_y^{20} A = 498.9$ kN;
- (5) Stress-strain relationship at elevated temperature: EC3, part1.2 (2001);
- (6) Axial restraint ratio ($\beta = k_s L / (E^{20} A)$): infinite, 0.1, 0.05, 0.02, 0.01, and 0;
- (7) Initial axial load: $P_0 = 50$ kN, load ratio (η) is 0.1, where $\eta = P_0 / N_u^{20}$.

In addition, numerical results by using the finite element method (FEM) program ANSYS is presented. In the ANSYS analysis, the steel column and the axial restraint are simulated by the BEAM89 element and COMBIN14 element respectively. An initial out-of-straightness of $a_0 = L/1000$ is introduced in the model.

The comparison among the results of Franssen (2000), finite element method (FEM) analysis using ANSYS and the proposed simplified model is presented in Fig. 5. From this comparison, the following conclusions may be drawn.

The predicated post-buckling behaviour using the simplified method is quite close to the FEM analysis both by Franssen (2000) and ANSYS. Especially, the simplified method confirms Franssen's (2000) conclusion that for axially restrained column, the post-buckling temperature at which the column's load returns to its initial load is almost the same for different levels of restraint stiffness.

There exist small differences in the slopes of the axial force-temperature curves in the post-buckling phase between the proposed results of this study and the FEM results by Franssen (2000). This may be caused by that an initial out-of-straightness is introduced in the proposed model. However, in Franssen's (2000) analysis, the column is a perfect straight one.

It is noted that the applied load of 50kN on this column was rather low (about 10% of its axial load capacity at ambient temperature) and the slenderness was rather high (about 98.6). The post-buckling behaviour of columns with higher initial load and lower slenderness ratio should be studied further.

4.2. Parametric study

In this study, combinations of the following parameters were studied:

- (1) Yield Strength and elastic modulus of steel at ambient temperature: $f_y^{20} = 235 \text{ N/mm}^2$ and $E^{20} = 2.1 \times 10^5 \text{ N/mm}^2$;
- (2) Stress-strain relationship: combination of EC3 part 1.2(2001) and El-Rimawi model (El-Rimawi 1996)
- (3) Section size: HEA100, buckling major axis;
- (4) Axially restrained stiffness ratio (β): 0.10, 0.05, and 0.01;
- (5) Slenderness (λ): 30, 60, and 90, the corresponding length of the column is 1.2 m, 2.4 m, and 3.6 m;
- (6) Load ratio ($\eta = P_0/N_u^{20}$, where $N_u^{20} = 498.9 \text{ kN}$): 0.5, 0.7, and 0.9;
- (7) Initial out-of-straightness (a_0): $L/1000$, $L/500$, and $L/250$;
- (8) Temperature gradient between the two flanges (ΔT): 0°C , 100°C , and 200°C , keep constant at elevated temperatures, and the bowing direction caused by temperature gradient is the same with that of the initial out-of-straightness.

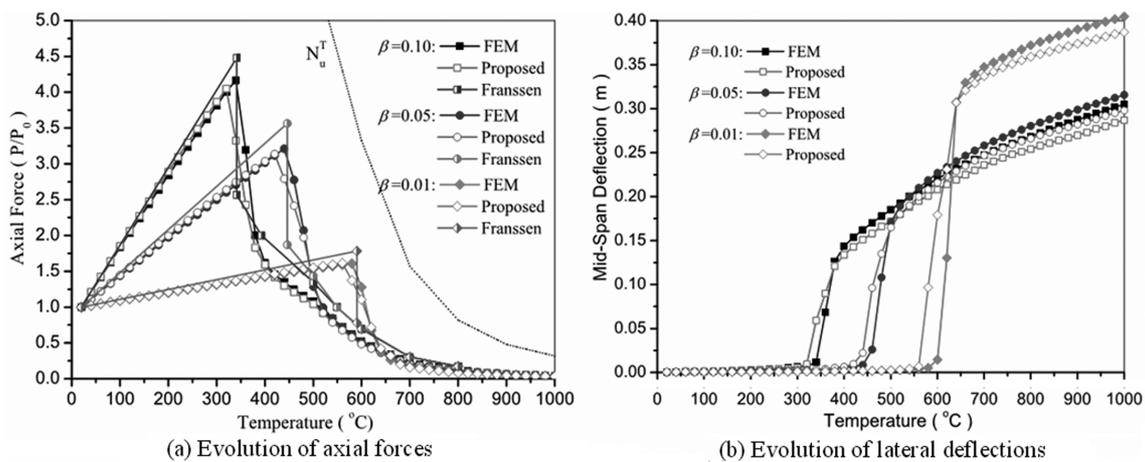


Fig. 5 Comparison of results among Franssen, ANASYS and Proposed

4.2.1 Effect of restraint stiffness

The axial restraint causes an increase of column axial force and a decrease in the column buckling temperature in fire. However, if the post-buckling behaviour is considered, the influence of the axial restraint is small. As illustrated in Fig. 5(a), when the temperature exceeds 600 °C, the evolution curves of axial force almost coincide. When the column buckles, there will be a sudden increase in the mid-span deflection, as shown in Fig. 5(b). Opposite to columns with no axial restraint, the increase of the mid-span deflection will stop soon, which makes it possible to utilize the post-buckling behaviour.

4.2.2 Effect of load ratio

Fig. 6 presents the evolution of axial force and mid-span deflection of columns with different load ratio. With the increase of η , the buckling temperature of the column decrease, and the temperature at which the axial force returns to its initial value decrease too. However, the axial force has its upper bound, which is the load capacity of the section at elevated temperatures. So columns with lower load ratio will get bigger additional axial force. As shown in Fig. 6(a), the column with $\eta = 0.5$, the additional axial force is about 50% of the initial axial load; but for the column with $\eta = 0.9$, it is only about 10%. The influence of load ratio to the buckling temperature of the column can be get from Fig. 6(b) too. The column with $\eta = 0.5$, the buckling temperature is about 250 °C. With the increase of load ratio, it decrease quickly, for column with $\eta = 0.9$, the buckling temperature is only 50 °C.

4.2.3 Effect of slenderness

With the increase of slenderness, the buckling temperature of the column decreases, as shown in Fig. 7(a), and the axial force drops quickly after buckling. If the critical temperature is defined as the temperature at which the axial force returns to its initial value, as that done by Franssen *et al.* (Neves 1995, Franssen 2000, Rodrigues *et al.* 2000, Wang 2004), there will be no increase in the fire endurance temperature of these kinds of columns, even if the post-buckling phase considered. However, for columns with small slenderness, the axial force drops slowly, the column critical temperature may be

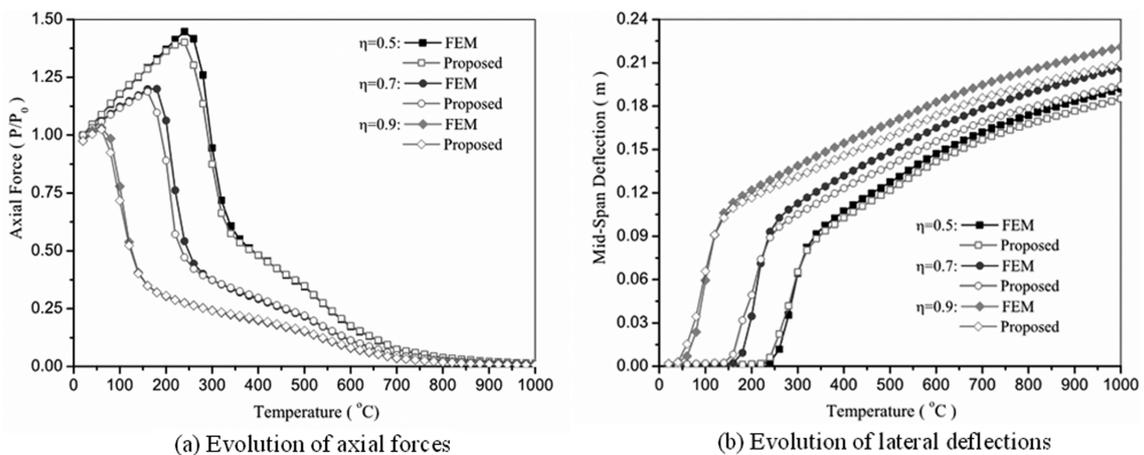


Fig. 6 Effects of load ration on the behaviours of restrained steel columns in fire ($\lambda = 60$; $\beta = 0.10$; $\Delta T = 0^{\circ}\text{C}$)

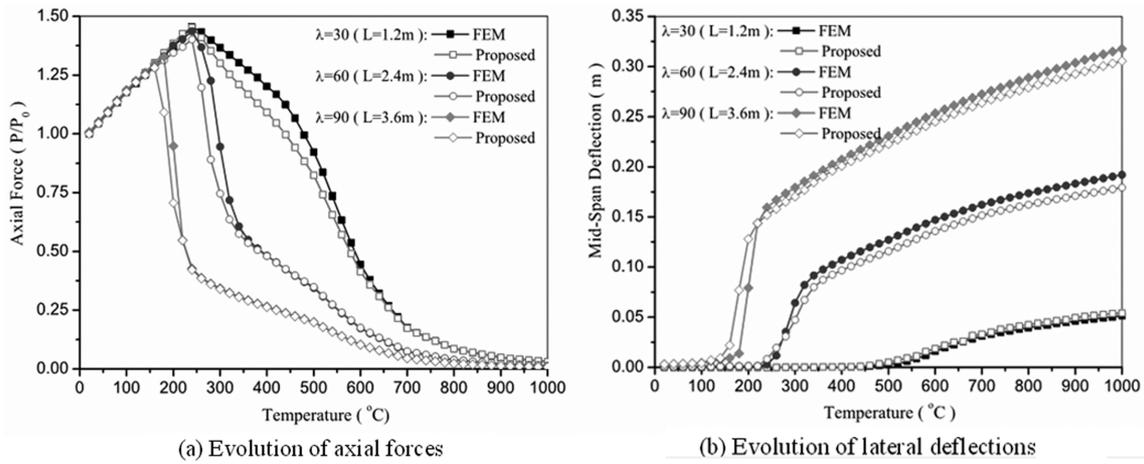


Fig. 7 Effects of slenderness on the behaviours of restrained steel columns in fire ($\eta = 0.5$; $\beta = 0.10$; $\Delta T = 0^\circ\text{C}$)

much higher than the column buckling temperature, as shown in Fig. 7(a). The mid-span deflection is greatly influenced by the slenderness too. Columns with small slenderness, the mid-span deflection is small after buckling. For example, for the column with $\lambda = 30$, the mid-span deflection is about $L/70$ when the temperature is 1000°C ; and for column with $\lambda = 90$, the mid-span deflection is as high as $L/10$, as illustrated in Fig. 7(b).

4.2.4 Effect of Initial out-of-straightness

As illustrated in Fig. 8(a), with the increase of the initial out-of-straightness, the buckling temperature and axial force of the column decrease. This phenomenon may be explained by the “bowing effect”,

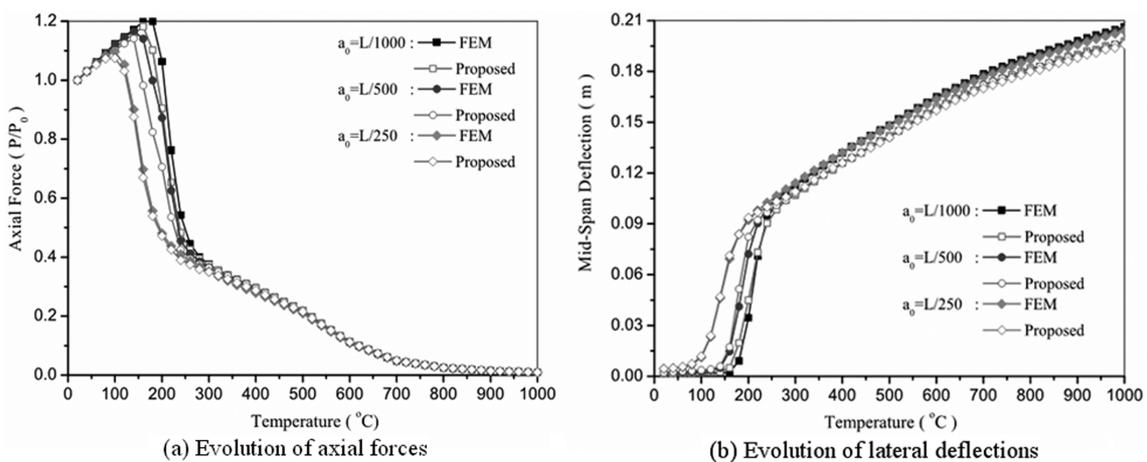


Fig. 8 Effects of initial out-of-straightness on the behaviour of restrained steel columns in fire ($\eta = 0.5$; $\beta = 0.10$; $\lambda = 60$; $\Delta T = 0^\circ\text{C}$)

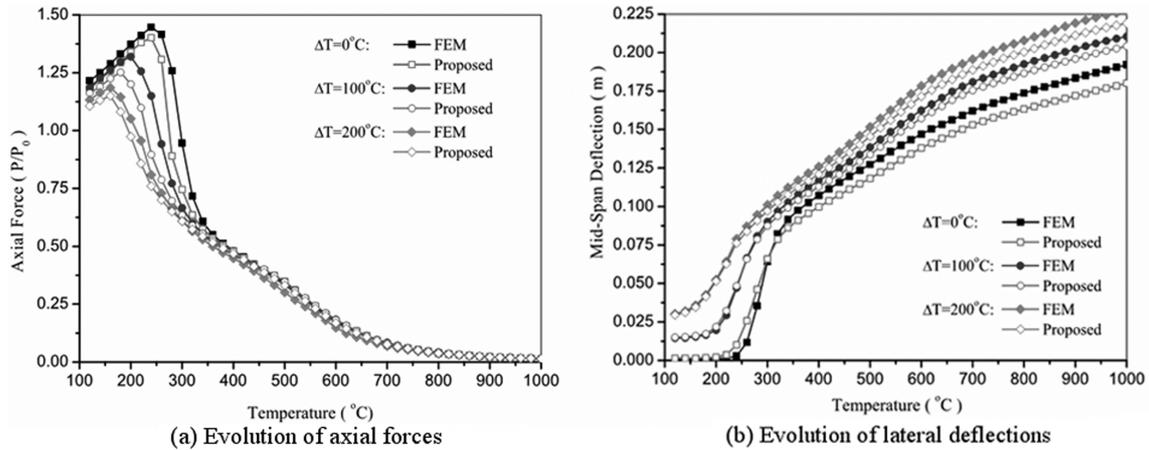


Fig. 9 Effects of temperature gradient across the section ($\eta = 0.5$; $\beta = 0.10$; $\lambda = 0^\circ\text{C}$)

that is, the increase of the axial length is used to enlarge the bowing deflection, and leads to the decrease of axial displacement, which in turn, decreases the axial force. The initial out-of-straightness has little influence on the mid-span deflection at the post-buckling phase, as shown in Fig. 8(b), the evolution curve of the mid-span deflection almost coincide for columns with different initial out-of-straightness.

4.2.5 Effect of Temperature Gradient

Three temperature gradients were studied, which were $\Delta T = 0^\circ\text{C}$, 100°C and 200°C . Assume that the temperature elevated from 120°C in the fire, to ensure the temperature in concave side is still great than 20°C for the column with $\Delta T = 200^\circ\text{C}$. In order to compare with the effect of initial out-of-straightness, suppose the bowing direction caused by the temperature gradient is the same with that of initial out-of-straightness.

The effect of temperature gradient to the column behaviours was similar to that of initial out-of-straightness; hence, some researches treated the thermal bowing caused by temperature gradient as initial-out-of-straightness (Sharples 1994). With the increase of ΔT , the buckling temperature and the axial force decrease, as shown in Fig. 8(a). But different to the effect of initial-out-of-straightness, columns with greater ΔT , the mid-span deflection is greater in the post-buckling phase, as shown in Fig. 8(b).

5. Conclusion

This paper has presented the results of a theoretical study on the post-buckling behaviour of axially restrained steel columns in a fire. The possible strain reversal on the convex side of column is taken into consideration. The proposed simplified model is verified by FEM analysis results, and they agree well with each other.

From the results of this study, the following conclusion may be drawn:

- (1) Axial restraint cause an increase in the column axial force, thereby reducing the column buckling temperature, defined as the temperature at which the column compressive load gets its maximum

value. The higher the restraint stiffness, the lower the buckling temperature. However, if the post-buckling behaviour is taken into consideration, the effect of restraint stiffness to the fire endurance temperature of the column is small;

- (2) Columns with great load ratio has small fire endurance temperature, even if the post-buckling behaviour is considered;
- (3) For columns with great slenderness, the buckling temperature decrease and axial force decreases suddenly in the post-buckling phase; however, columns with small slenderness ratio, the axial force decreases slowly in the post-buckling phase, hence, for this kind of column, the critical temperature can be increased by utilizing the post-buckling behaviour of the column;
- (4) The effect of temperature gradient is similar to that of initial-out-of-straightness, with the increase of ΔT , both the buckling temperature and the axial force decrease.

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