

Practical and efficient approaches for semi-rigid design of composite frames

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Abstract. The use of composite semi-rigid connections is not fully exploited, in spite of its great number of advantages. Composite semi-rigid connections may lead to an optimal moment distribution that will render lighter structures. Furthermore, using the appropriate semi-rigid connection design, the stability of the frames against lateral loads may entirely rely on the joint stiffness, thus avoiding bracing systems and permitting more diaphanous designs. Although modern codes, such as the Eurocode 4 (EC4), propose thorough methods of analysis they do not provide enough insight and simplicity from the design point of view. The purpose of this paper is to introduce practical and efficient methods of analysis that will facilitate the work of a structural analyst starting from the global analysis of the composite frame and ending on the final connection design. A key aspect is the definition of the stiffness and strength of the connections that will lead to an optimal moment distribution in the composite beams. Two examples are presented in order to clarify the application of the proposed methods and to demonstrate the advantages of the semi-rigid composite design with respect to the alternative pinned and rigid ones. The final aim of the paper is to stimulate and encourage the designer on the use of composite semi-rigid structures.

Keywords: composite semi-rigid connections; semi-rigid frame; component method.

1. Introduction

The structural, economical and functional enhancements that may be obtained by composite structures when compared to concrete and even steel structures are significant. Composite construction minimises the weight of steel, fire protection costs, time and complexity of execution. In addition, the stiffness and monolithic characteristics achieved in composite structures are greater than those in steel structures.

Recent research and the appearance of the new codes have brought new methods for structural analysis and design. In particular and in regard to connections, the Eurocodes have widely opened the possibility of designing them not only as rigid or pinned, but also as semi-rigid. This new design opportunity brings along with it an additional complexity in the analysis process that is only justified if an economical advantage is gained by its use, and provided that computational tools and design guides are available.

Connections in composite structures are usually designed and fabricated as pinned; however, composite semi-rigid connections may lead to an optimal moment distribution that will lead to lighter

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structures. If one takes into account the current increase in the cost of steel this could mean important cost savings. Furthermore, using the appropriate semi-rigid connection design, the stability of the frames against lateral loads may entirely rely on the joint stiffness, thus avoiding bracing systems. Taking these considerations into account, the semi-rigid concept becomes highly competitive when compared to the pinned and rigid alternatives. The execution cost of semi-rigid connections with respect to the pinned ones is very low: it only requires the continuity of the slab reinforcement through the column. When compared to the rigid ones, the use of stiffeners in the columns, prestressing of the bolts and preparation of the surfaces are avoided.

New design rules appear in Eurocode 3 (EC3) and Eurocode 4 (EC4) for steel and composite connections, respectively. The font size in this paragraph is smaller than that of the rest of the text. The rules in EC4 are an extension of those in EC3 and include the contribution of additional components such as concrete and reinforcement. There are other very useful publications (SCI (1998), Lawson and Gibbons (1995), Nethercot and Li (1995)) that provide design guides for composite semi-rigid connections, and which clarify and complement the component method proposed in EC4. Not all the design methods are based on EC4. Leon *et al.* (1991) have supplied moment-rotation ($M-\theta$) curves for semi-rigid composite connections derived from tests and FE parametric studies.

Meanwhile, the way of determining more accurately the resistant characteristics of semi-rigid connections is being investigated, especially in steel. In composite structures the research work is not so abundant. It is worth pointing out the work of Ahmed and Nethercot (1997) who carry out a revision of the methods proposed for determining the rotation capacity and stiffness of connections, and propose new methods that are validated by experimental results. The finite element method has also been used (Kattner and Crisinel 2000) to determine the connection characteristics. However, as reported in SSEDTA 2001, this method presents some difficulties when modelling local phenomena such as the sliding taking place in the interface concrete-steel, the crushing of the concrete against the column or the cracking of the concrete.

With the aim of knowing the real behaviour of semi-rigid composite joints, a great number of experimental studies have been developed under monotonically applied loads: Brown and Anderson (2001), Simoes da Silva *et al.* (2001), Simoes da Silva *et al.* (2001) (in which the influence of concrete encasement in columns and the behaviour of the perimeter joints are studied), Liew *et al.* (2000), Amadio and Fragiaco (2003). The work for reversal loads carried out by Liew *et al.* (2004), and under dynamic loads by Rassati *et al.* (2000) and Calado *et al.* (2000) are also very interesting. Most of these results have helped validating the analytical methods proposed in the different codes.

All this research work and the EC4 present thorough methods of analysis but they do not provide enough insight from the design point of view. In order to analyse each connection, and find its strength and stiffness, the connection must be previously defined (designed). As a consequence, the connections have to be fully detailed prior to carry out a global analysis of the structure, which requires the stiffness and resistance of the connections. This leads to a highly iterative procedure, since no connection can be designed without knowing the moment and shear forces acting on them, and which result from a global analysis of the structure.

The purpose of this paper is to introduce simplified and more design-oriented methods of analysis that will facilitate the work of a structural analyst starting from the global analysis of the composite frame and ending with the final connection design. The starting point is the definition of the rotational stiffness that leads to an optimal moment distribution in the composite beams and, at the same time provides enough stability against lateral loads. Although any technique can be used in this process, the simplified methods presented below for joint design are mostly based on the component method defined in EC3 and EC4.

2. Optimum semi-rigid design

The proposed method starts with the definition of the initial rotational stiffness and strength of the connections that will lead to the best possible moment distribution in the composite beams. Composite beams have different behaviour depending on whether the beam is under hogging or sagging bending moments. Considering the resistant characteristics of the composite beams, it is not economical to design them under simply supported conditions, since their size is determined by the mid-span section while the rest of the beam becomes over dimensioned. Similarly, there is a waste of material in clamped beams since the maximum moments occur at the supports, where the strength of the composite beam is smaller. This makes the mid span zone be over dimensioned, since it carries a smaller moment and has a greater resistance.

In this respect, semi-rigid composite connections can lead to an optimal use of the material. In the case of steel beams the stiffness of the connection may be chosen so that the maximum moments at the supports and mid span are the same and equal to $qL^2/16$. However, in the case of composite beams the moments at the supports must be smaller than that in the middle of the beam, since the resistant moment of the composite beam varies along its length, being smaller under hogging than under sagging conditions.

2.1 Design assumptions

In order to find the most appropriate bending moment distribution, the following design assumptions are adopted:

Full shear connection is assumed for both, hogging and sagging moment regions.

Following Lawson and Chung (1994), the beam span/height for uniformly distributed loads will be kept between 18 and 20.

According to Lawson and Gibbons (1995), the amount of reinforcement will be within the following interval:

- 1% minimum to guarantee an appropriate rotation capacity (Dissanayake *et al.* 1999).
- 1.5% maximum to avoid column web stiffeners with semi-rigid behaviour. A larger amount of reinforcement is unnecessary due to the fact that the strength does not increase, and the failure of the connection will take place at the column web under compression.

The moment of inertia in the hogging moment region is always less than that in the sagging moment region.

The sagging plastic resistant moment will always be larger than the hogging counterpart.

Following Leon *et al.* (1991), a ratio among the plastic resistant moment of the composite section in sagging bending moment and the one of the steel section of approximately 1.8 will be maintained.

According to these design assumptions, the hogging plastic resistant moment will vary from 0.7 to 0.8 times the plastic resistant moment under sagging. Similarly, the moment of inertia in the hogging region will be approximately 0.6 times the moment of inertia in the sagging region. Accordingly, the following assumptions are made:

$$M_{pl,Rd}(-) = 0.75M_{pl,Rd}(+)$$

$$I(-) = 0.6I(+)$$

Once the relationship between the section characteristics in hogging and sagging moment regions have been established, the optimal moment distribution will be the one showed in Fig. 1. The value of

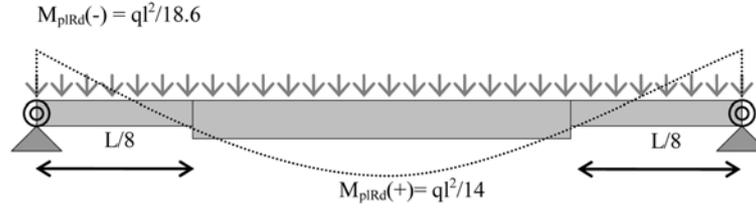


Fig. 1 Proposed moment distribution in a composite beam with semi-rigid composite connections

the moment at the ends of the beam is $qL^2/18.6$ and the value in the centre is $qL^2/14$. The length of the beam under hogging is between $L/8$ and $L/7$ at each side.

2.2 Optimal rotational stiffness

EC3 defines the connection stiffness as a function of a parameter k and the adjacent beam stiffness (see Eq. 1). The parameter k may vary from zero to infinity. Another way to measure the stiffness is the one proposed by Chen (2000) which is characterised by Eq. (2). The end fixity factor r , which oscillates between 0 and 1, gives a more precise measure of the connection stiffness than the parameter k , since it becomes 0 for pinned connections and 1 for rigid ones.

$$S_{j,ini} = kEI_b / L_b \quad (1)$$

$$r = \frac{1}{1 + \frac{3EI_{eq}}{S_j L}} \quad (2)$$

In composite beams, the stiffness for which the optimal moment distribution is obtained varies depending on the beam span. Taking into account the considerations of Section Design assumptions, the values of the parameters k and r that lead to the optimal moment distributions in composite beams of lengths varying from 6 to 12 meters, are shown in Table 1.

The joint stiffness has been defined based on the equivalent beam inertia I_{eq} , for which several expressions have been proposed. Leon *et al.* (1991) propose the following one, which is widely used:

$$I_{eq} = 0.6I_{pos} + 0.4I_{neg} \quad (3)$$

In Eq. (3), the coefficients that affect the inertia depend on the percentage of beam length that is under sagging or hogging bending. Another expression has been suggested by Hensman and Nethercot (2001):

Table 1 Optimal stiffness coefficient and end fixity factor for composite beams

Span (m)	Stiffness parameter k	End fixity factor r
6	4.0	0.57
8	3.2	0.51
10	2.5	0.45
12	1.7	0.36

$$I_{eq} = \frac{7.5I_{pos}I_{neg}^2}{9I_{neg}^2 + 2I_{pos}I_{neg}} \quad (4)$$

In this paper, the following equation is proposed to determine the required joint stiffness.

$$I_{eq} = 0.7I_{pos} + 0.3I_{neg} \quad (5)$$

The difference between Eq. (5) and Eq. (3) lies on the fact that in Eq. (5) the beam lengths under hogging and sagging are equal to those corresponding to a composite beam with an optimal moment distribution (see Fig. 1).

3. Methods of analysis

Until recently the usual procedure for frame design consisted in making some decisions with regard to the structural morphology and choosing a joint typology that was mostly restricted to pinned or rigid. The characterization of these joints for global analysis is straight-forward, since they are assigned a zero or infinite stiffness, respectively. After the global analysis, the internal forces could be used to design the sections and the connections.

If the methods proposed by modern codes (such as EC3 and EC4) which include semi-rigid designs are used, the connections have to be fully detailed prior to carrying out the global analysis of the structure, because the latter requires the stiffness and resistance of all the connections. However, no connection can be designed without knowing the moment and shear forces acting on them, which in turn are obtained from a global analysis. Thus, a trial and error procedure has to be implemented which may lead to a tedious and highly iterative process.

In what follows and with the aim of either avoiding or substantially reducing the number of iterations, a practical method of analysis is proposed which will facilitate the work of a structural analyst starting from the global analysis of the composite frame and ending on the final connection design.

3.1 Design assumptions

For the global structural analysis the following assumptions are made:

When performing plastic analysis, the plastic hinges are to be formed in the connections or beams, not in the columns.

The behaviour of the joint will be ductile after the yielding, that is, it will have enough rotation capacity so that a plastic redistribution of moments can take place. This happens if the weak part of the joint is either one of the following: the column web in shear, the end plate in bending, the column flange in bending or the reinforcement in tension.

The starting point is the stiffness that leads to an optimal moment distribution, given by the values shown in Table 1.

3.2 Modelling of the elements

Prior to carry out the global analysis, it is necessary to introduce the behaviour of the composite beams in the structural model, as well as that of the joints by means of their $M-\phi$ curves. However,

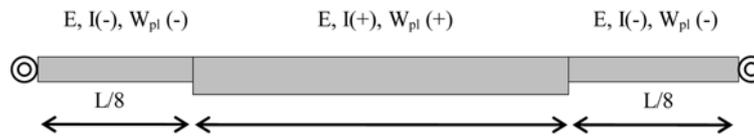


Fig. 2 Modelling a composite beam with semi-rigid connections

most of current available software does not have the possibility to identify and to assign different resistant characteristics to an element, whether it is under hogging or sagging bending moments.

In order to introduce the characteristics of the composite beams for the structural global analysis, the beam is divided into three parts of different lengths, and they are assigned the resistant characteristics shown in Fig. 2. Liew *et al.* (2001) observed that the use of the cracked length equal to 15% of the span length in elastic analysis yields end moments which are at most 2% different from those obtained using the exact cracked length. In this paper the lengths of each part are the ones corresponding to a beam with semi-rigid composite connections and the moment distribution of Fig. 1.

If the available software allows it, a rotational spring with the joint characteristics (strength and stiffness) is added to the end of the beam (see Fig. 2). If this is not possible, the joint can be modelled as an equivalent beam element of reduced dimensions and equivalent characteristics to the rotational spring as described in SSEDTA (2001). This procedure has the added advantage of considering the joint size.

3.3 Structural global analysis of frames

3.3.1 Elastic analysis

The flowchart on the left of Fig. 3 shows the procedure for an elastic global analysis of a composite frame with semi-rigid composite joints. The size of the sections must be determined in the first step. The beams may be pre-sized according to the moment distribution shown in Fig. 1. In order to optimise the structure, the stiffness coefficient that yields an optimal moment distribution is chosen from Table 1, according to the length and the equivalent inertia of the beam (see Section Optimal rotational stiffness). The required stiffness ($S_{j,req}$) of each joint is obtained using Eq. (1), and then it is introduced in the structural model. A second order elastic analysis is carried out, verifying that the conditions for ultimate and serviceability limit states are satisfied.

Once the beams and columns are correctly sized, the moment applied on each joint ($M_{j,Ed}$) corresponds to the required moment resistance ($M_{j,req}$). These applied moments allow us to determine the transformation parameter β for each joint. This parameter takes into account the influence of the moments, coming from the beams at each side of the joint, on the shear in the column web panel. The parameter β is defined in EC3 part 1-8, clause 5.3.

Using the theoretical required characteristics ($S_{j,req}$ and $M_{j,req}$) and the value of β , the joint can be pre-designed so that the real stiffness (S_j) and the moment resistance ($M_{j,Rd}$) may be obtained. In the following section a practical method is described to pre-design the connection without having to perform a complete and exhaustive design of each connection which would have required a full detailing. The global elastic analysis is then carried out to check the U.L.S. and the S.L.S. under all the possible load combinations.

The iteration process illustrated in Fig. 3 is due to the dispersions generated by the required and the obtained joint moments and stiffness. When this variation is less than 5%, and provided that the requirements for ULS and SLS are satisfied, the iteration process may be stopped, the joint design is considered valid, and the final connection design and detailing is performed. If not, the transformation

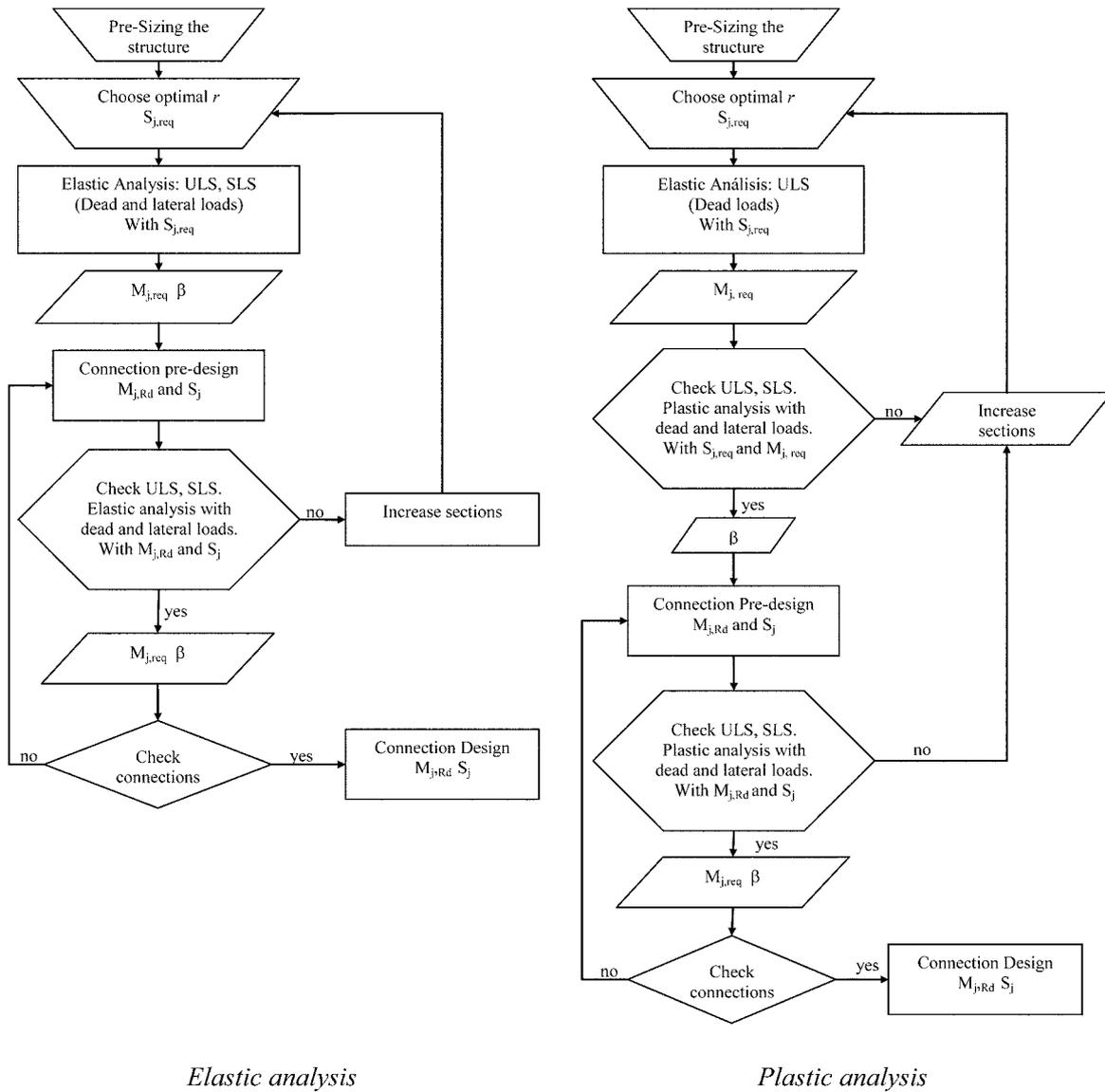


Fig. 3 Flowcharts for elastic and plastic analysis

parameter β must be re-calculated, new required values for the moments must be determined and, the joints pre-designed again.

3.3.2 Plastic analysis

The method for plastic analysis is shown in the second flowchart of Fig. 3. The two first steps are the same as those in elastic analysis. In the following step, only the dead loads are introduced in the structural model, the second order elastic analysis is carry out and the Ultimate Limit State is checked.

The applied design moment on each joint ($M_{j,Ed}$) corresponds to the required moment resistance ($M_{j,req}$). The joint required moments are introduced in the model in order to carry out the second order

plastic analysis, including the lateral loads. The U.L.S. and S.L.S. are then checked. Once the beams and columns are correctly sized, the transformation parameter β may be calculated and the practical method proposed in Section 4 may be used to pre-design the connection and obtain $M_{j,Rd}$ and S_j .

The U.L.S. and S.L.S. are checked performing a global analysis for the different load combinations. Again an iteration process is established that will take into account the difference between the required and the obtained joint moments and stiffness. When the variation is less than 5%, and provided that the requirements for ULS and SLS are satisfied, the iteration process may be stopped, the joint design is considered valid, and the final connection design and detailing is performed. If not, the parameter β must be re-calculated, new required values for the moments must be determined and, the joints pre-designed again.

Two examples are presented in Section 5 to illustrate the application of these methods of analysis to a composite frame.

4. A practical joint pre-design method

Semi-rigid design involves a number of iterations. Considering both types of analyses elastic and plastic, once the structural analysis has been made, the resistant characteristics of the joint must be obtained by means of the available methods (components method, codes, etc), which demand that the joint be fully detailed. In order to avoid the complete and exhaustive design of each joint in every iteration, a pre-design method is proposed, which is based on the component method. This approach not only provides the resistance and stiffness of the connection but also allows us to assess if it is possible for the joint to reach the required nominal values, thus leaving the complete design and detailing only for the very last step.

4.1 Components method

The behaviour of the joint is defined by the non-linear moment rotation curve shown in Fig. 4. In order to define the $M-\phi$ curve, the EC4 and EC3 propose the component method. This method has been considered the most suitable for the analytical treatment of the connection properties, and it has been preferred over the FEM due to its better ability to treat and model local phenomena.

Each component is a specific part of the joint, which contributes in a defined way towards its global behaviour. Each one of the components is characterised as a translational spring with strength

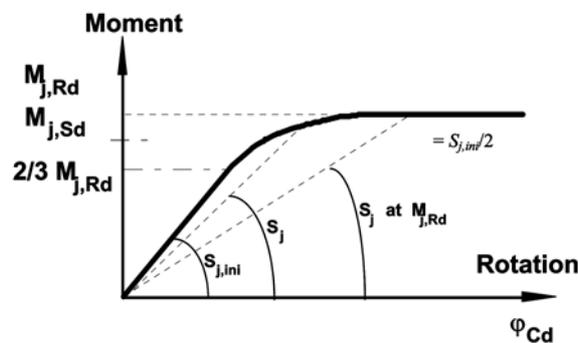


Fig. 4 $M-\phi$ curve for semi rigid connections

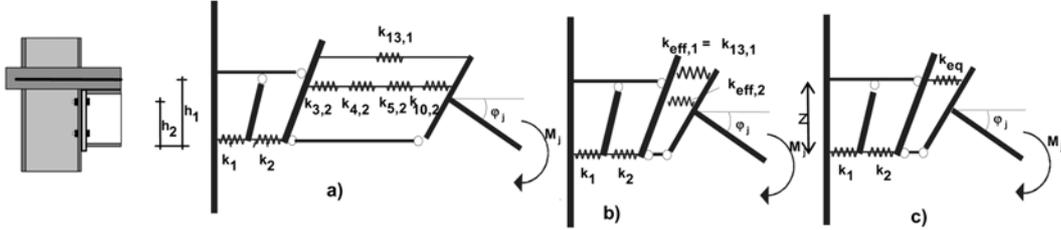


Fig. 5 Component assembly in a composite connection with flush end plate

and stiffness, which is assembled with the rest of the components in order to obtain the behavioural curve. Regarding the composite joints, it is necessary to take into account other components in addition to those considered in steel joints, namely, the reinforcement in tension, the longitudinal shear studs and the concrete.

The component method considers the real dimensions and the behaviour and follows the procedure of identification, characterisation and assembly of the different components (SSEDTA 2001). The configuration, interaction and final assembly procedure of the components of a composite semi-rigid connection, with flush-end plate, is illustrated in Fig. 5. This type of connection is the one that will be dealt with in this paper, due to the fact that it is one of the most widely used. The best results are obtained when the resistances of the components in the compression and tension zones are well balanced.

The notation used in Fig. 5 is defined as follows: k_1 represents the column web panel in shear; k_2 the column web panel in compression; k_3 the column web in tension; k_4 the column flange in bending; k_5 the end plate in bending; k_{10} the bolts in tension; k_{13} the reinforcement; k_{eff} the effective stiffness of one row, after assembling k_3 , k_4 , k_5 and k_{10} ; and k_{eq} the equivalent stiffness of the components in tension.

On each row, the components that act in series are assembled, and an effective stiffness k_{eff} is obtained. Then, assuming the centre of rotation at the beam bottom flange, a k_{eq} is formed. The axial stiffness of the components acting in the compression zone depends mainly on the characteristics of the column that configures the joint. The end result is the initial rotational stiffness:

$$S_{j,int} = \frac{E_a z^2}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{eq}}} \quad (6)$$

where, z is the equivalent lever arm (see Fig. 5).

The resistant moment can be calculated by means of the following equation:

$$M_{j,Rd} = \sum F_{L,i,Rd} h_i \quad (7)$$

where, the summation is extended to every bolt row and the layer of reinforcement placed in the tension zone. In practice, it may occur that the compression strength or shear resistance of the joint is smaller than that of the components in tension. Regarding equilibrium, the total force of the components in tension must exceed neither the strength of the compression group $F_{c,Rd}$, nor the strength of the web panel in shear $V_{S,Rd} \beta$.

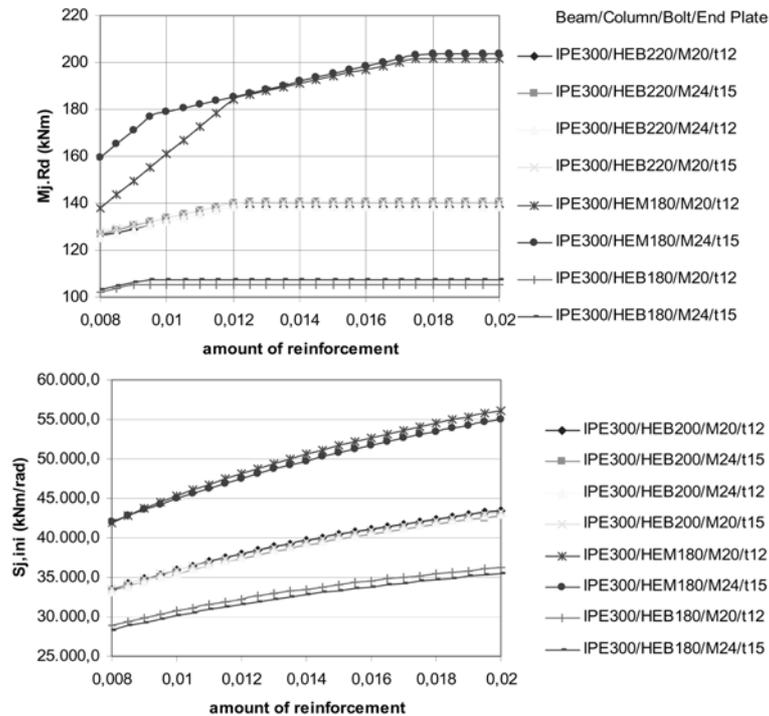


Fig. 6 Connection stiffness and strength versus the amount of reinforcement

4.2 Parametric study

The strength and stiffness of a connection depend on different variables, namely, the amount of reinforcement, number and type of bolts, type, dimensions and shear force acting on the column web, type and thickness of end plates, etc. The purpose of the parametric study described in this section is to assess the influence of these variables on both the resistance and stiffness of the connection. The study is based on the assessment of each of the different components of the connection as defined in the EC4, along with its influence in the global behaviour of the connection. The aim is to come up with a simple expression that will lead to the required connection characteristics.

4.2.1 Amount of reinforcement

The charts of Fig. 6 illustrate the strength and the stiffness as a function of the amount of reinforcement, while the rest of the parameters are kept constant (location of the bolts, end plate thickness, covering of reinforcement, etc.).

Regarding the strength, there is a first interval where the slope of the curve is steeper. This corresponds to the joint failure due to the components in tension (bolts and reinforcement). The next interval, with a lesser slope, corresponds to the yielding of the reinforcement or the column web under tension. The final stage with zero slope supposes the failure of one of the components in compression (column web or beam bottom flange) or the column web panel in shear. However, it may be noticed that there is no slope discontinuity in the stiffness; also, the slope is smaller when the amount of reinforcement grows, and it becomes nearly horizontal when the amount of reinforcement is very high.

The variation of the bolts diameter and end plate thickness, for a given amount of reinforcement, does not imply an important variation of the stiffness and strength of the connection. This finding is also shared by Amadio and Fragiaco (2003).

It is worth pointing out that there is some disagreement between different authors about the calculation of the reinforcement stiffness k_r (determined following Eq. 8), and in particular to what refers to the effective length used (l_r).

$$k_r = \frac{A_{s,r}}{l_r} \quad (8)$$

where $A_{s,r}$ is the reinforcement area within the effective width of the slab.

The EC4 provides the Eqs. (9) to (13) below, in which the effective length, l_r , depends on the transformation parameter β and, consequently, on the beam moments acting at each side of the joint, as follows:

For an external connection $l_r = 3.6 h_c$, where h_c is the height of the column (9)

For an internal connection the following expressions for l_r are given depending on the values of the moments at each side of the joint:

$$l_r = 0.5 h_c \quad \text{If } M_{Ed,1} = M_{Ed,2} \quad (10)$$

$$l_r = h_c \left(\frac{1 + \beta}{2} \right) k_\beta \quad \text{If } M_{Ed,1} > M_{Ed,2}, \text{ for the connection with } M_{Ed,1} \quad (11)$$

$$k_\beta = \beta(4.3\beta^2 - 8.9\beta + 7.2) \quad (12)$$

$$l_r = h_c \left(\frac{1 - \beta}{2} \right) \quad \text{if, } M_{Ed,1} > M_{Ed,2} \text{ for the connection with } M_{Ed,2} \quad (13)$$

Eqs. (11) and (13) are very sensitive to the values of the moments $M_{Ed,1}$ and $M_{Ed,2}$ and therefore to the parameter β . Large variations on l_r are obtained for small changes of $M_{Ed,1}$ and $M_{Ed,2}$.

As an alternative, Anderson and Najafi (1994) also use Eq. (10) for both, external and internal joints. Thus, l_r does not depend on the moments acting in the joint. Furthermore, they include the flexibility of the shear studs by increasing l_r up to the location of the first row of studs. Brown and Anderson (2001) also use Eq. (10) with no modification for the shear studs flexibility.

Ren and Crisinel (1995) consider the following effective length for all types of joints:

$$l_r = 2\eta(60 + 1.3ks) \quad (14)$$

where $\eta = 0.35$, s is the spacing of reinforcement and $k = 1$ for pure tension and $k = 0.5$ for simple bending.

Ahmed and Nethercot (1997) propose the following equation which is widely used in practice:

$$l_r = h_c/2 + p_1 + p_2 \quad (15)$$

where, p_1 is the distance between the column flange to the first shear stud and p_2 is the pitch of the shear studs.

Liew *et al.* (2000) proposed the following effective length which is also used by Fang *et al.* (1999):

$$l_r = h_c/2 + p_1 \quad (16)$$

Finally Rassati *et al.* (2004) use the following expression:

$$l_r = h_c(1 + 2,8 - 0,5K_{trans}) \tag{17}$$

where $K_{trans} = A_{s,r} / (A_{s,r,b} + 0,64t_{fb}b_b)$, $A_{s,r,b}$ is the longitudinal reinforcement area in the beam's section adjacent to the joint.

After reviewing the profuse literature it has been observed that several authors consider that the use of Eq. (10) is unsafe, except for internal joints with equal moments at each side of the column. Nevertheless, as already mentioned, the last proposal of EC4, reduces in some cases the reinforcement stiffness too much. As a consequence, the equation used in this paper is Eq. (15), which depends on the column height, the distance between the external face of the column flange and the first shear stud and the pitch of the shear studs.

4.2.2 Transformation parameter β

The transformation parameter β depends on the shear in the column web and, consequently, on the beam moments that act at each side of the column. Since these are not known at the beginning of the analysis, an iteration process is needed that will start with some guessed initial values. The EC4 recommends a value of $\beta = 1$ for external connections; $\beta = 0$ for internal ones with equal moments at each side of the column, and $\beta = 1$ for different moments. The value $\beta = 2$ is considered when there are reversal moments at each side of the column. As it will be seen later on in the discussion, in most cases the assumption of $\beta = 1$ for internal joints is conservative, unless there is a significant difference between the moments at each side.

It may be observed in Fig. 7, that the stiffness is quite sensitive to β , and it drops heavily for low values of β to approach a horizontal slope when β tends to 2. The strength also varies significantly

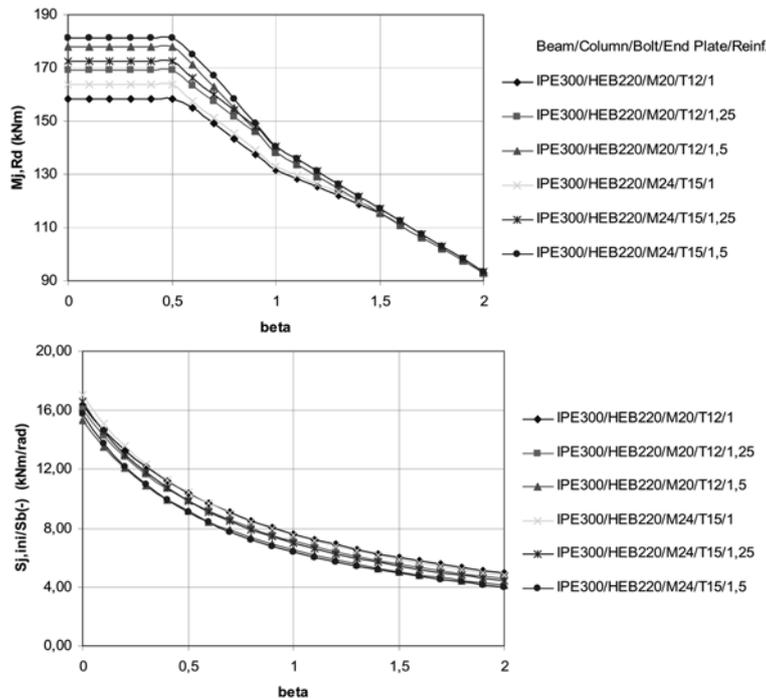


Fig. 7 Connection stiffness and strength versus β

as a function of this parameter but in a different manner. The strength remains constant from 0 to a certain intermediate value of β between 0 and 1. From that value, the strength drops until $\beta = 1$, and with a lesser slope until $\beta = 2$. It may be seen, therefore, that $\beta = 1$ is conservative enough in most of internal joints.

A method that eliminates the use of the parameter β for global analysis and design has been proposed by Bayo *et al.* (2006).

4.2.3 Vertical distance between the bolts under tension and the beam flange

The plots depicted in Fig. 8 clearly show that the vertical distance between the bolts under tension and the beam flange do not affect significantly either the stiffness or the strength of the connection. The lesser the distance is, the larger the stiffness and the strength.

4.2.4 Horizontal distance between bolts (g)

Similar to the results of the previous section, the variation of the stiffness and strength in terms of the horizontal distance between bolts, g , (see Fig. 9) is very low and almost negligible when compared to the amount of reinforcement or the β parameter. The value of g only affects the axial stiffness of two components: the column flange in bending and the end plate in bending. This influence diminishes after assembling the axial stiffness of these two components with those corresponding to the bolts in tension and the column web in tension to find the effective stiffness of the bolt row. Finally, after assembling it with the rest of the stiffness components (reinforcement, compression and shear components) its influence is minimal. The plot corresponding to the strength is not shown because it remains flat, without any influence of the variation in horizontal distance between bolts.

4.3 A practical predesign method

As mentioned above, the complete component method is not a practical method for the design of composite connection, the reason being that a complete and detailed definition of all the parts and dimensions of the connection are needed prior to the analysis. This leads to a tedious and work intensive trial and error procedure for the global analysis of the structure. A more design-oriented approach is needed, by which the global analysis (Section 3) can be performed and the overall configuration of the connection established prior to a detailed definition.

Thus, a new simplified component method is proposed in this section in which a required stiffness and strength act as starting points and inputs for the global analysis. Then, the configuration of the connection (amount of reinforcement, flush end plate thickness, bolt diameters, etc.) may be obtained in a simple and useful way. It is foreseen that a substantial reduction of the work and number of iterations may be accomplished.

4.3.1 Stiffness

The parametric study of Section Parametric study showed that the size of the bolts and the thickness of the end plate, for a determined amount of reinforcement, have little influence in the connection stiffness.

As a consequence, the following simplified expression for the stiffness is proposed, which only depends on the amount of reinforcement and the column web in shear and compression:

$$S_{j,int} = \frac{E_a z_r^2}{\frac{1}{k_r} + \frac{1}{k_{cc}} + \frac{1}{k_{vc}}} \quad (18)$$

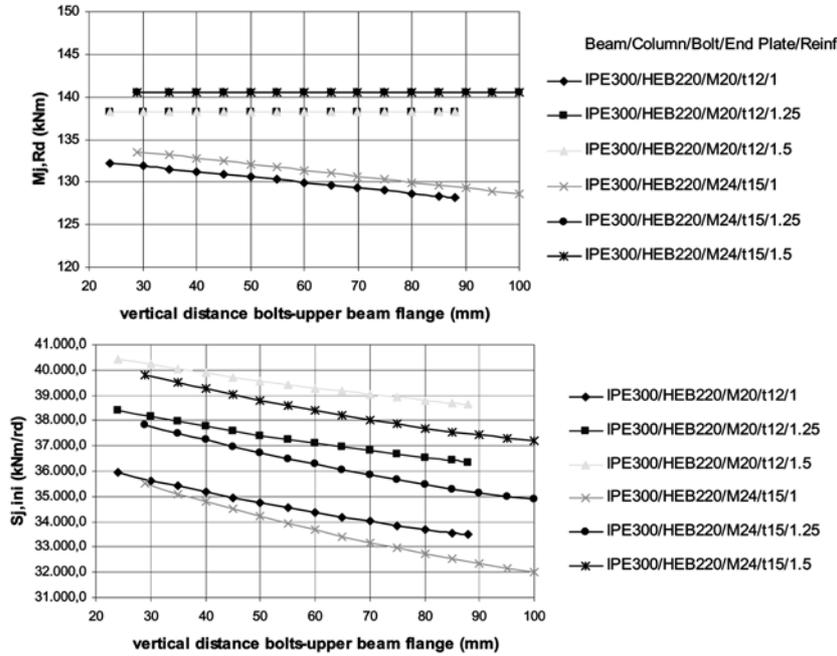


Fig. 8 Connection stiffness and strength versus the distance from the bolts to the beam flange

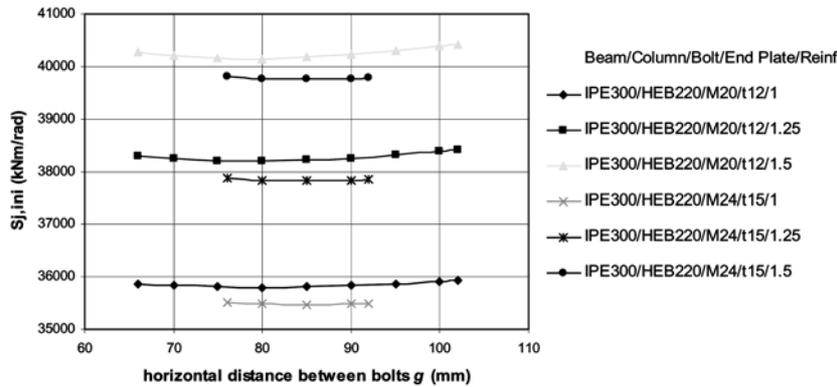


Fig. 9 Relation between the stiffness and the horizontal distance between the bolts

where, k_r is the reinforcement stiffness $k_r = A_{s,r} / l_r$; k_{cc} is the stiffness of column web in compression $k_{cc} = 0.7 b_{eff,c,wc} t_{wc} / d_c$; $b_{eff,c,wc}$ is the effective width; k_{vc} is the stiffness of column web in shear $k_{vc} = (0.38 A_{VC}) / (\beta z)$; z is the lever arm; A_{vc} is the shear area and z_r is the distance from beam bottom flange to the reinforcement.

Starting from a required stiffness $S_{j,ini}$ and knowing the column section, with the values of k_{vc} and k_{cc} , the application of Eq. (18) will yield the value of k_r and the necessary amount of reinforcement. If the value of $S_{j,ini}$ can not be reached with a reasonable amount of reinforcement, a larger column will be necessary. The main point is that the analyst will know if the desired stiffness can be reached without having to detail the entire connection, as current practice may require.

Fig. 10 shows the results obtained by the complete model and the proposed simplified model, which

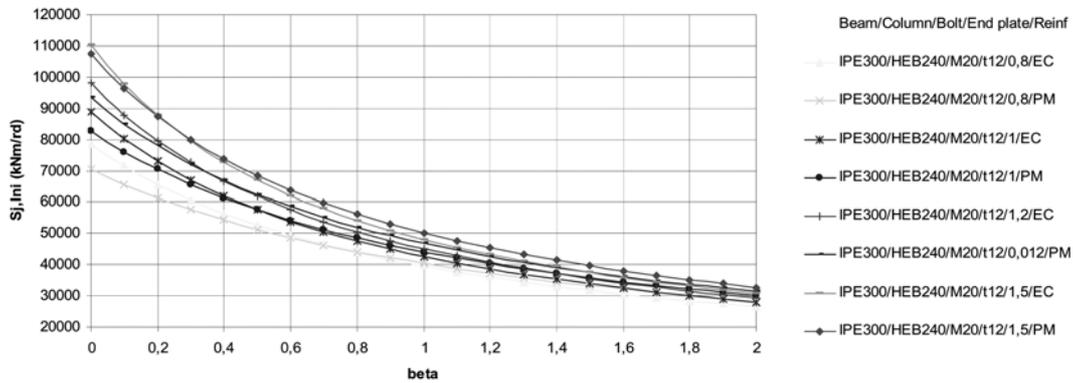


Fig. 10 Comparison of stiffness obtained by means of components method (EC) and proposed one (PM)

includes only the stiffness of the reinforcement and the column web components (column web in shear and column web in compression). The complete model includes those ones plus the end plate in bending, the column flange in bending, the column web in tension and the bolts in tension. It may be seen that the stiffness obtained by both models are practically the same, except when, for an amount of reinforcement less than 1%, the value of β drops below 0,5. However, as mentioned above, amounts of reinforcements below 1% are not recommended due to the necessary rotation capacity.

4.3.2 Strength

The following design assumptions, taken from the limitations and recommendations suggested by the Eurocodes and the publications mentioned below, are considered in order to find a simplified expression for the connection strength:

The relation between the end plate thickness and bolt diameters (EC3) in order to ensure suitable rotation capacity is:

$$t \leq 0.36 d \sqrt{\frac{f_{ub}}{f_y}} \quad \text{For 10.9 bolts and steel S275: } t_{ep} \leq 0.6 d$$

The welding of flanges is $a_f = 0.6 t_{fb}$ and the welding of webs is $a_w = 0.4 t_{wb}$ (Quintero and Cudós 1996).

The arrangement of the bolts according to EC3 is:

Distances		Minimum	Maximum
To upper edge	e_1	$1.2 d$	$4 \min(t_{ep}, t_{fc}) + 40 \text{ mm}$
To lateral edge	e_2	$1.2 d$	$4 \min(t_{ep}, t_{fc}) + 40 \text{ mm}$
Horizontal distance between bolts	p_1	$2.2 d$	$\min(14 \cdot \min(t_{ep}, t_{fc}), 200)$
Vertical distance between bolts	p_2	$2.4 d$	$\min(14 \cdot \min(t_{ep}, t_{fc}), 200)$

The arrangement of the bolts according to Murray and Shoemaker (2002) is:

Distancias		Minimum	Maximum
Vertical bolt- beam flange	m_{ep2}	$d + 13 \text{ mm}$	-

The parametric study showed that the stiffness is slightly higher when m_{ep2} is smaller, and that the

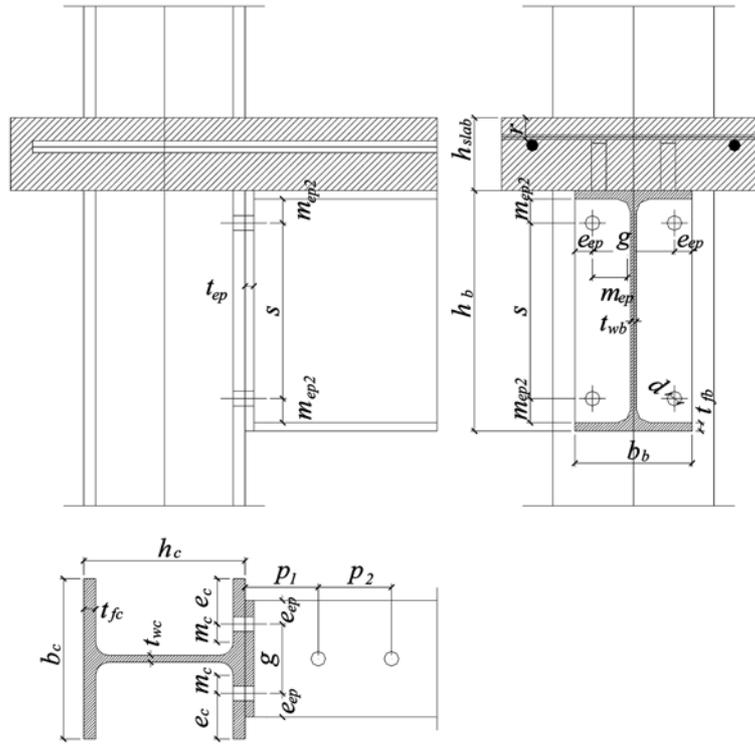


Fig. 11 Joint configuration

horizontal distance between bolts, g , hardly has an influence. None of these values affects the strength significantly. This is the reason why, instead of guessing initial values and fitting them later to obtain the required strength, the following constant distances are considered:

$$e_{ep} = e_2 = 1.2d, \quad m_{ep} = b_b/2 - e_{ep} - t_{fb}, \quad m_{ep2} = d + 13\text{mm}, \quad t_{fb} > t_{ep} \cong 0.6d$$

All the parameters are defined in Fig. 11.

4.3.2.1 Strength of the bolt row

By keeping these dispositions, the most common failure mode is the one that involves the first row of bolts and the end plate. The corresponding resistance of the bolt row may be obtained by the following expression:

$$F_b = \frac{2M_p + 2e_{ep}P'_t}{e_{ep} + m_{ep}} \quad \text{where } M_p = \frac{l_{eff}^2 t_{ep}^2 f_y}{4}$$

The effective length of the flush end plate will be $l_{eff} = \min(\alpha m_{ep}, 2\pi m_{ep})$ and, as the maximum value of α is 2π , the effective length will always be αm_{ep} . Therefore, in order that the previous expression should only depend on the beam, the column and the bolt diameter, the following simplified expression is proposed:

Table 2 Moment resistance for a composite flush end plate

1	2	3
$\min(F_c, V_{wp,Rd}) > F_r + F_b$	$F_r < \min(F_c, V_{wp,Rd}) < F_r + F_b$	$\min(F_c, V_{wp,Rd}) \leq F_r$
$M_{pl,Rd} = F_r z_r + F_b z_b$	$M_{pl,Rd} = F_r z_r + (\min(F_c, V_{wp,Rd}) - F_r) z_b$	$M_{pl,Rd} = \min(F_c, V_{wp,Rd}) z_r$

$$F_b = \frac{1}{2} \alpha \left(\frac{0.5b_b - t_{wb} - 1.2d}{0.5b_b - t_{wb}} \right) t_{ep}^2 f_y + 2 \left(\frac{1.2d}{0.5b_b - t_{wb}} \right) P'_i \quad (19)$$

where, $P'_i = 2(0, 9f_{ub}A_s / 1, 25)$; A_s is the resistant area of the bolt and f_{ub} is the bolt strength.

An intermediate value of $\alpha = 5$ may be considered for predesign connections.

4.3.2.2 Joint strength M_{plRd}

Regarding the strength, Fig. 6 (Section 4.2.1) shows dropping slopes in three intervals. In the first interval, as already said before, the failure of the joint is produced by the yielding of the reinforcement and the row of bolts, and one may observe the strength difference produced by several bolt diameters. In the second interval, with a lesser slope, the failure is produced by either the yielding of the reinforcement or the yielding of the column web. The third interval with zero slope supposes the failure of one of the components in compression or the column web panel in shear. Since it does not lead to an increase in the strength due to the fact that the joint failure is produced in the column web, any increase in reinforcement is redundant. Depending on the relation of the strength of the components, different equations must be used for each of the stages. This may be observed in Table 2.

F_r is the resistance of the reinforcement, F_c is the resistance of the compression zone: minimum of $F_{c,wc,Rd}$ (unstiffened column web subject to transverse compression) and $F_{c,fb,Rd}$ (design compression resistance of the combined beam flange and web), and $V_{wp,Rd}$ is the strength of the column web panel in shear, according to EC 3. The simplification is applied by replacing the effective width of the column web in compression $b_{eff,c,wc}$ from the Eurocode by the following equation, which only depends on the dimensions of the beam and the column.

$$b_{eff,c,wc} = 3t_{fb} + 12t_{fc} \quad (20)$$

The amount of reinforcement that is necessary to reach the required stiffness is determined, as explained before, using Eq. (18). The suitable bolt diameter and end plate thickness in order to achieve the required strength can be obtained by means of Table 2 knowing the amount of reinforcement and the strength of the components in the shear and compression zone. If it is not possible to reach the required values, then it may be necessary to increase the section of the column.

If $\min(F_c, V_{wp,Rd}) \leq F_r$ then $M_{pl,Rd} = \min(F_c, V_{wp,Rd}) z_r$ and the bolts will be chosen to bear the shear. This situation is not recommended, since the column web becomes the weakest part.

If $\min(F_c, V_{wp,Rd}) > F_r$ then

$$F_b = \min(F_c, V_{wp,Rd}) - F_r = \frac{1}{2} \alpha \left(\frac{0.5b_b - t_{wb} - 1.2d}{0.5b_b - t_{wb}} \right) t_{ep}^2 f_y + 2 \left(\frac{1.2d}{0.5b_b - t_{wb}} \right) P'_i$$

The use of this expression will result in the bolt diameter and the end plate thickness. This also yields an optimal situation because the tension and compression stresses of the joint are balanced, and the

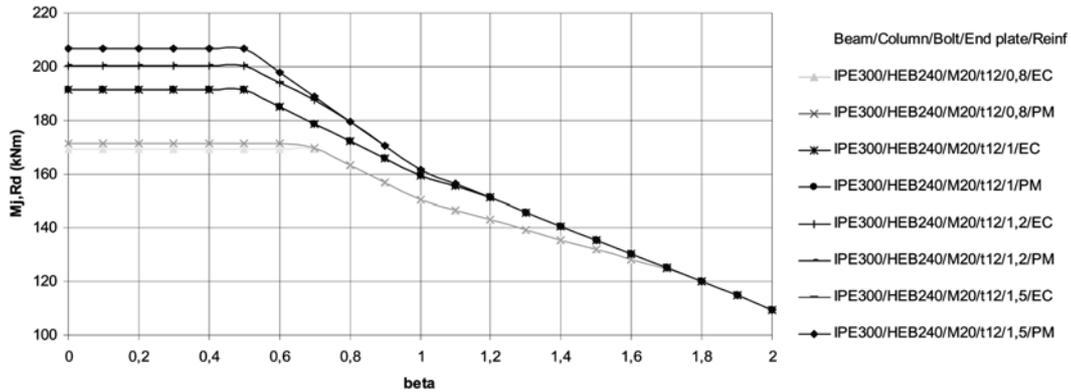


Fig. 12 Comparison of strength obtained by means of component method (EC) and simplified one (PM)

materials are used to the maximum of their possibilities.

Fig. 12 shows the difference between the results of components method and the proposed simplified one. If the amount of reinforcement is less than 1%, there is a minimum dispersion. If the amount of reinforcement is higher, there is no longer any dispersion, since the joint failure is produced by column web yielding, and it is not necessary to simplify the equations in order to calculate the resistant moment.

5. Examples

5.1 Example 1

The two bay and two story sway frame shown in Fig. 13 is chosen as a design example. The plastic global analysis method and the simplified method for joint design are applied. The material properties used in the example are: structural steel S275, concrete C25/30, bolts steel 10.9 and reinforcement B500S. The limitations considered are $H/150$ for lateral deflections, $L/400$ for floor beam deflections and $L/250$ for roof beam deflections. The columns are continuous in the whole length.

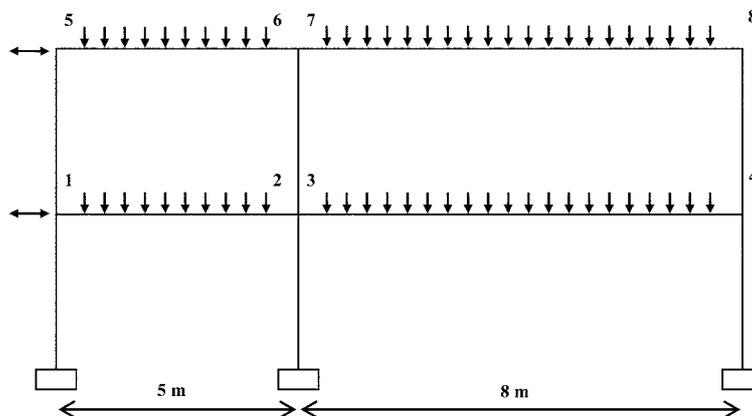


Fig. 13 Configuration of the frame

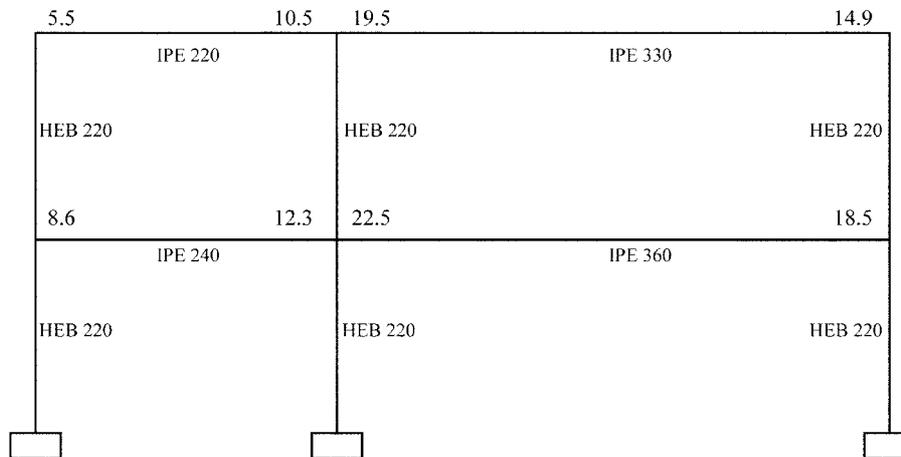


Fig. 14 Sections and required moments for the frame

The loads considered are:

Floors

Self weight (hollow core sections +compression slab, 10 cm)	5,0 kN/m ²
Pavement	0,8 kN/m ²
Imposed load due to partitions	0,5 kN/m ²
Imposed load due to occupancy	3,0 kN/m ²

Roofing

Pass Roofing to the next page right above the corresponding text. Or place the Fig. 13 before the definition of the loads.

Self weight (hollow core sections+compression slab, 8 cm)	5,0 kN/m ²
Deck	0,8 kN/m ²
Imposed load due to maintenance	1,0 kN/m ²
<u>Wind loads</u>	1,2 kN/m ²

The steps described in Fig. 3 are carried out as follows:

i) Optimal stiffness, elastic analysis (dead loads) and $M_{j,req}$

The stiffness coefficients that lead to an optimal moment distribution are chosen: for 8 m beams $k = 4.5$ and for 5 m beams $k = 2.5$. These values, just as the necessary sections and the dead loads, are introduced in the structural model. The elastic analysis is carried out and the Ultimate Limit State is checked. Fig. 14 shows the beam and column sections and the moments acting at the joints in kNm. From the preliminary design, this moments are taken as the required ones ($M_{j,req}$), and they are introduced in the model.

ii) Plastic Analysis U.L.S. (dead and lateral loads and β):

Lateral loads are added, the plastic analysis is carried out and the U.L.S. is checked. The moments acting at the joints are used for the calculation of the parameter β . The collapse load factors for both wind directions are 1.089 and 1.086, respectively.

The SLS is also checked for both wind directions.

iii) Joint design

The joints are pre-designed by means of the practical proposed method described above, and the most demanding solution is considered. The connection configuration is shown in Table 3.

Table 3 Joint configuration

		% Reinf.	Reinforcement	Bolt / End plate thickness
1	external	1.40%	8 Ø12	M16/t10
2	internal	1.30%	8 Ø 12	M16/t10
3	internal	1.20%	12 Ø 12	M24/t15
4	external	1.50%	14 Ø 12	M24/t15
5	external	1.20%	7 Ø 12	M16/t10
6	internal	1.40%	8 Ø 12	M16/t10
7	internal	1.10%	10 Ø 12	M24/t15
8	external	1.30%	12 Ø 12	M24/t15

Table 4 Variation of acting moments and β

	Wind direction 1: Recalc. β				Wind direction 2: Recalc. β				
	M (kNm) 1 st iteration	β 1 st it.	M (kNm) 2 nd iteration	β' 2 nd it.	M (kNm) 1 st iteration	β 1 st it.	M (kNm) 2 nd iteration	β' 2 nd it.	
1	5.45	1	4.25	1	1	8.63	1	10.61	1
2	12.29	0.36	12.70	0.40	2	9.23	1.44	7.72	1.72
3	16.76	0.27	17.83	0.29	3	22.48	0.59	21.02	0.63
4	18.50	1	16.61	1	4	12.71	1	13.55	1
5	3.99	1	3.21	1	5	5.53	1	6.32	1
6	10.47	0.28	10.39	0.33	6	8.39	1.13	8.20	1.14
7	13.39	0.22	13.86	0.25	7	17.87	0.53	17.54	0.53
8	13.82	1	14.03	1	8	8.78	1	9.64	1

iv) The stiffness and the strength values are updated by the obtained ones and a new iteration is carried out. The collapse load factor for both wind directions now become 1.069 and 1.07, respectively. New values of moments are obtained, from which the parameter β is recalculated. The moments and the parameter β from the first to the second iteration are shown in Table 4.

Joints are rechecked due to the fact that the variation of the moments in the 1st and 2nd analysis is higher than 5%. In this case, the variation of the parameter β only implies a slight variation in the resulting values of stiffness and strength but not in the configuration of the connections. Therefore, the same joint configuration of Table 3 still prevails. A second order plastic analysis is performed with the new values.

A third iteration is not necessary since the collapse load factor, the parameter β and the moments are nearly the same and within 5% of error. Therefore, the joints with the characteristics shown in Table 3 are the ones to be fully detailed by means of the complete component method or other desired one. It is worth mentioning again that this full detailing is only done at the very end of the design process, as done in the case of rigid and/or pin connections.

5.2 Example 2: Comparison of results

A three bay and three story sway frame is also analysed following the elastic and plastic global analysis methods, and the semi-rigid results are compared with those obtained with pinned and rigid joints.

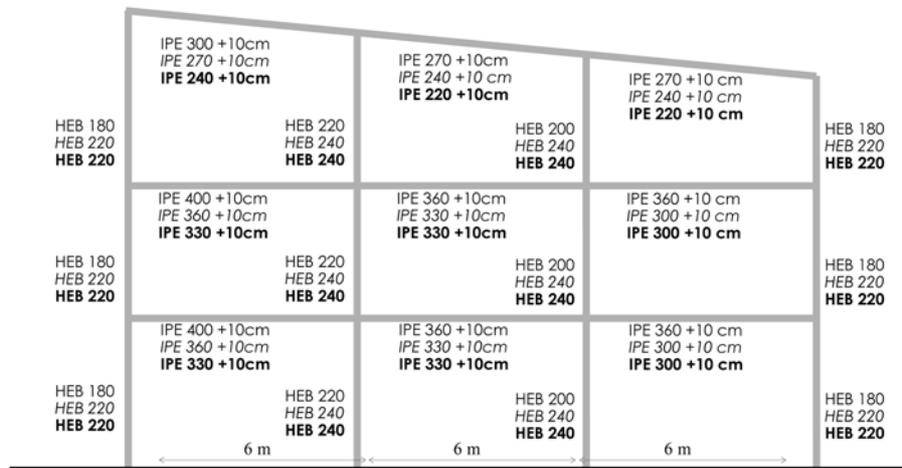


Fig. 15 Frame design and elastic analysis

In this example the loads considered are:

Floors

Self weight (hollow core sections +compression slab, 10 cm)	5,5 kN/m ²
Pavement	0,6 kN/m ²
Imposed load due to partitions	0,5 kN/m ²
Imposed load due to occupancy	4,0 kN/m ²

Roofing

Self weight (hollow core sections+compression slab, 8 cm)	5,0 kN/m ²
Deck	0,2 kN/m ²
Imposed load due to snow/maintenance	1,0 kN/m ²
<u>Wind loads</u>	1,2 kN/m ²

5.2.1 Elastic analysis

The composite sections obtained under elastic analysis with pinned, rigid and semi-rigid connections are shown in Fig. 15. The top value corresponds to the sections obtained with pinned connections; the middle one with rigid connections, and the bottom one with semi-rigid connections.

The results obtained with pinned joints are mere guidelines because it is not appropriate to make a direct comparison, due to the fact that the pinned frame is a non-sway frame and the others are sway frames. It may be observed that although the difference between the sections is not large, the beams with semi-rigid connections are smaller than those with pinned or rigid connections. Therefore savings may be obtained when using semi-rigid connections, not only because of steel savings but also because of the lesser cost of the connections themselves. A semi-rigid connection is much cheaper than the rigid one because it does not need stiffeners, preparation of surfaces and high strength friction grip bolting.

5.2.2 Plastic analysis

The plastic analysis is applied to the frame with rigid and semi-rigid joints. For the rigid joint the collapse load factor of the structure is 1.21. In the case of semi-rigid design the collapse load factor is 1.01.

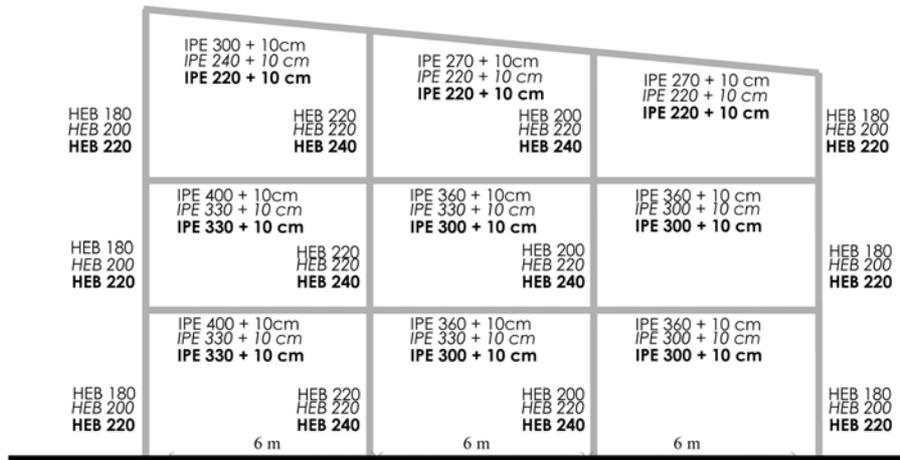


Fig. 16 Frame design and plastic analysis

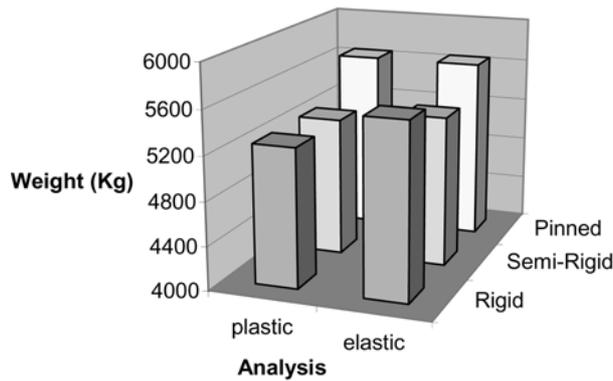


Fig. 17 Frame weight comparison according to the type of analysis and connection

The composite sections obtained for plastic analysis with rigid and semi-rigid connections are shown in Fig. 16. Again, the top value corresponds to the sections obtained with pinned connections; the middle one with rigid connections, and the bottom one with semi-rigid connections. The column sizes have been increased in the semi-rigid structure in order to reach the required values of stiffness and strength. It may be observed a slightly more favourable behaviour of the semi-rigid design than the rigid as far as the sizes of the beams are concerned.

A comparison of the structural weight is established for every analysis and type of connection and may be seen in Fig. 17. Semi-rigid design in composite structures for both, elastic and plastic analysis, leads to structural weight savings. The heaviest structure is the one with pinned joints, although the one with rigid joints could be more expensive due to the price of connection fabrication.

Table 5 Frame cost estimation

FRAME	Connections	Steel	Total	
Rigid	3114 €	8944 €	12058 €	122.7%
Semi-Rigid	878 €	8949 €	9828 €	100.0%
Pinned	374 €	9615 €	9989 €	101.6%

Plastic analysis is more favourable than the elastic because it allows optimizing the material in all cases. Semi-rigid composite design is more suitable than the one with pinned or rigid connections due to the optimized moment distribution. This holds even when bracing systems are used.

Finally, with the aim of obtaining a more accurate comparison, a cost estimation for the frame with the three types of joints (pinned, rigid and semi-rigid) has been done. Typical joints for each type were defined and Spanish Steel Contractors estimated their cost. Obviously, this cost may vary depending on the country, time or other circumstances. In order to see the contribution of each part of the structure: connections and steel, a cost breakdown is shown in Table 5. It may be observed that the cost of the rigid connections is more than three times the cost of the semi-rigid connections, and eight times the cost of the pinned ones. However, as the weight of steel in the pinned frame is higher, the total cost of the primary frame for pinned and semi-rigid connections is very similar, and the rigid one is 20% more expensive than the other two. In addition, the cost of bracing must be added to the cost of the pinned frame, which makes a greater difference when compared to the semi-rigid one.

It may be concluded that the semi-rigid design is in this case the most competitive one.

6. Conclusions

The component method presented in EC4 for semi-rigid composite joints provides a very good analysis tool, however it is not very useful from the design point of view. The proposed procedure simplifies the structural and connection analyses and designs, and leads to a reduced number of iterations. In addition, the proposed design approach is accurate and makes it possible to obtain the joint configuration with no need for intermediate connection analysis and detailing. It is hoped that this method will facilitate the use of composite semi-rigid design in everyday practice, and also bring about its benefits.

The fact of getting the best moment distribution for composite beams by means of the optimal required stiffness allows not only to simplify the procedure but also to economise the materials. A cost estimation and comparison between semi-rigid, rigid and pinned connections has proved that the semi-rigid design in composite structures leads to more economical and competitive solutions than the traditional designs based on rigid or pinned connections.

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