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Stability analysis of semi-rigid composite frames

Jing-Feng Wang†

School of Civil Engineering, Tsinghua University, Beijing, 10084, People's Republic of China

Guo-Qiang Li‡

School of Civil Engineering, Tongji University, Shanghai, 200092, People's Republic of China
State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University
Shanghai, 200092, People's Republic of China

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Abstract. Based on stability theory of current rigid steel frames and using the three-column subassemblage model, the governing equations for determining the effective length factor (μ-factor) of the columns in semi-rigid composite frames are derived. The effects of the nonlinear moment-rotation characteristics of beam-to-column connections and composite action of slab are considered. Furthermore, using a two-bay three-storey composite frame with semi-rigid connections as an example, the effects of the non-linear moment-rotation characteristics of connections and load value on the μ-factor are numerically studied and the μ-factors obtained by the proposed method and Baraket-Chen's method are compared with those obtained by the exact finite element method. It was found that the proposed method has good accuracy and can be used in stability analysis of semi-rigid composite frames.

Keywords: effective length factor; semi-rigid connection; composite frame; moment-rotation characteristics.

1. Introduction

The structural behavior of a steel frame is closely related with the characteristics of beam-to-column connections. It is the common engineering practice to assume connections between beams and columns to be either pinned or fully rigid in a frame. However, experiments have shown that the actual behavior of flexible beam-to-column connections, or called semi-rigid connections, lies somewhere between these two idealized conditions. When a moment is applied to a flexible connection, the relation of relative beam column rotation is nonlinear (Kameshki and Saka 2001). In order to perform the design of semi-rigid frames based on the actual behavior of connections, the designer must have a sufficient knowledge of the moment-rotation characteristic of connections and carry out a second-order elastic frame analysis that also takes into account the connection behavior. On the other hand, most steel buildings built in recent years have used concrete floor slabs designed to act compositely with steel beams by means of shear connectors. There are economic and structural benefits to utilize the partially restrained composite connections with some degree of continuity (Liew 2001). Consequently, the effects of semi-rigid connections and composite

†Assistance Researcher, Corresponding Author, E-mail: jfwang@mail.tsinghua.edu.cn
‡Professor, E-mail: gqli@mail.tongji.edu.cn

action of slab should be properly considered in the design of steel frames.

Stability problem has played a very important part in the steel frame design. Presently the effective length method is popularly used for the design of frame columns. Since the effective length of a column is mainly relevant to the end restraints of adjacent members to the column, the simplified practical method for determining the column effective length may be developed on the basis of the subassemblage involving the column. Evaluation of the effective length of semi-rigid steel frames has drawn attention of a number of researchers. Cheong-Siat-Moy (1986) examined the μ-factor paradox for leaning columns. Bridge and Fraser (1987) proposed an iterative procedure for the evaluation of the effective length, which accounts for the presence of axial forces in the restraining members and thus also considers the negative values of rotational stiffness. Aristizabal-Ochoa (1994) proposed two simple formulas to evaluate the effective length of columns with semi-rigid connections, which are simpler to apply than the alignment chart. Hellesland and Bjorhovde (1996) proposed a new restraint demand factor considering the vertical and horizontal interaction in member stability terms. Kishi *et al.* (1997, 1998) proposed an analytical method for determining the effective length of columns in semi-rigid steel frames. Essa (1997) proposed a design method for the evaluation of the effective length of columns in unbraced multi-story frames considering different story drift angles.

To evaluate the effective length factor of columns in semi-rigid frames, the rotational stiffness of the semi-rigid beam-to-column connections at the bucking state of the column should be determined firstly. Many studies used initial connection stiffness for this purpose, such as Liu (1985) etc, but Goto *et al.* (1993) indicated that it could underestimate the μ-factor. As improvement, Brarakat and Chen (1990) recommended the modification of the relative stiffness factor in the alignment for unbraced steel frames and corresponding to a reference plastic rotation for braced frames. Kishi *et al.* (1997, 1998) considered that the alignment chart could be used to determine the column μ-factor in semi-rigid steel frames, with a proper evaluation of the tangent connection stiffness for semi-rigid connections at bucking state.

The objective of this work is to propose a simplified approach for the evaluation of the effective length factor of the column in semi-rigid composite frames, taking into account the behavior of semi-rigid connections. A simplified method for determining rotational stiffness of connections is also evaluated considering the influence of the connection non-linearity. Furthermore, using a two-bay three-storey composite frame with semi-rigid connections as an example, the effects of the non-linear moment-rotation characteristics of connections and load value on the μ-factor are numerically studied and the μ-factors obtained by the proposed method and Barakat-Chen's method are compared with those obtained by the exact FEM.

2. Governing equations for the effective length factor

The three-column subassemblage model is shown in Fig. 1. The model involves the target column ($c2$) as well as the neighbor columns ($c1$ and $c3$) and girders ($b1$, $b2$, $b3$, and $b4$) providing restrains. The beam-to-column connections can be semi-rigid.

The following assumptions are employed for establishing governing equations of μ-factor with the subassemblage model:

(1) All beams and columns are purely elastic;
(2) All members are uniform in cross-section;
(3) For braced frames, rotations at opposite ends of the restraining beams are equal in magnitude but opposite in direction, i.e., beams are bent in single curvature;

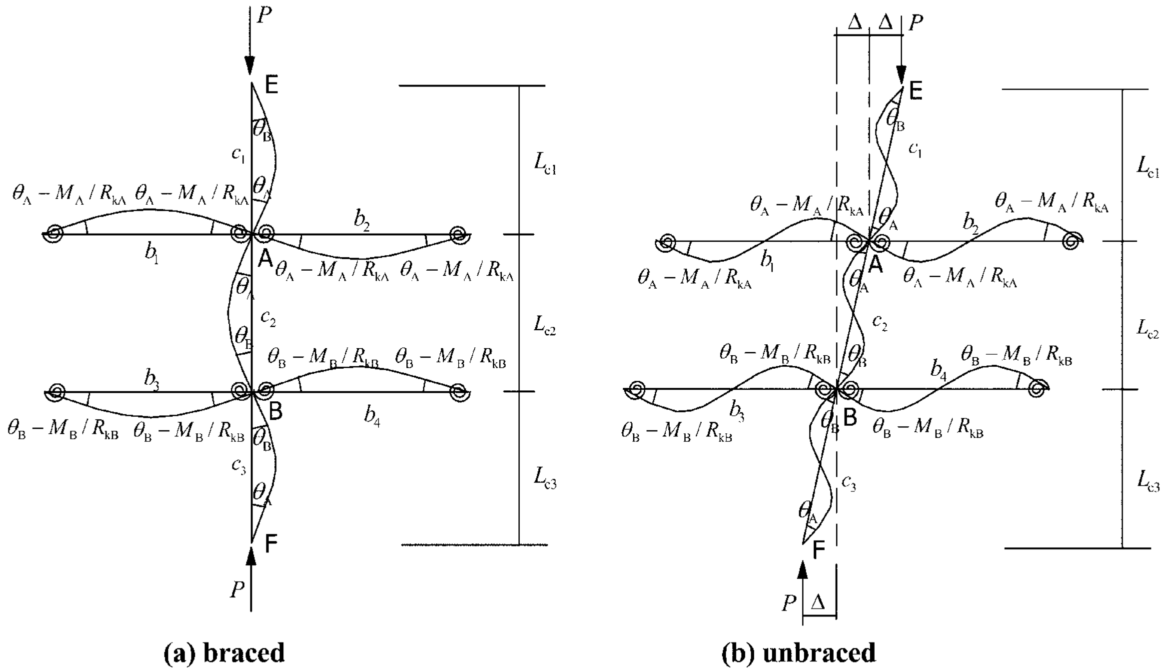


Fig. 1 Three-column subassemblage model

(4) For unbraced frames, rotations at opposite ends of the restraining beams are the same in magnitude and direction, i.e., beams are bent in double curvature;
(5) The stiffness parameters $L_{ci}\sqrt{P/EI_{ci}}$ of all columns are equal;
(6) All columns in a story buckle simultaneously; and
(7) The axial forces in the beams are negligibly small.

Based on the slop-deflection equations of the target column and the equilibrium equations of the joints at the ends of the target column, the governing equation for the column μ-factor in semi-rigid frames can be derived as:

(1) for braced frame

Column $c1$
$$(M_A)_{c1} = \frac{EI_{c1}}{L_{c1}}[S_{ii}\theta_A + S_{ij}\theta_B] \tag{1a}$$

Column $c2$
$$(M_A)_{c2} = \frac{EI_{c2}}{L_{c2}}[S_{ii}\theta_A + S_{ij}\theta_B] \tag{1b}$$

$$(M_B)_{c2} = \frac{EI_{c2}}{L_{c2}}[S_{ii}\theta_A + S_{ij}\theta_B] \tag{1c}$$

Column $c3$
$$(M_B)_{c3} = \frac{EI_{c3}}{L_{c3}}[S_{ii}\theta_A + S_{ij}\theta_B] \tag{1d}$$

Beam $b1$
$$(M_A)_{b1} = 2\alpha_1 \frac{EI_{b1}}{L_{b1}} \theta_A \tag{1e}$$

Beam $b2$
$$(M_A)_{b2} = 2\alpha_1 \frac{EI_{b2}}{L_{b2}} \theta_A \tag{1f}$$

Beam $b3$
$$(M_B)_{b3} = 2\alpha_1 \frac{EI_{b3}}{L_{b3}} \theta_B \tag{1g}$$

Beam $b4$
$$(M_B)_{b4} = 2\alpha_1 \frac{EI_{b4}}{L_{b4}} \theta_B \tag{1h}$$

where S_{kl} (k, l=i or j) are the stability functions:

$$S_{ii} = S_{jj} = \frac{\frac{\pi}{\mu}\sin\frac{\pi}{\mu} - \left(\frac{\pi}{\mu}\right)^2 \cos\frac{\pi}{\mu}}{2 - 2\cos\frac{\pi}{\mu} - \frac{\pi}{\mu}\sin\frac{\pi}{\mu}} \tag{2a}$$

$$S_{ij} = S_{ji} \frac{\left(\frac{\pi}{\mu}\right)^2 - \frac{\pi}{\mu}\sin\frac{\pi}{\mu}}{2 - 2\cos\frac{\pi}{\mu} - \frac{\pi}{\mu}\sin\frac{\pi}{\mu}} \tag{2b}$$

In Eqs. (1a)-(1h), subscripts A and B indicate the values of the columns and girders connecting the ends A and B of the target column, and subscripts b and c denote beam and column, respectively; E is Young modulus of steel material; I_b is the effective moment of inertia of girders; L_b is the corresponding length of girders; I_c is the moment of inertia of columns and L_c is the corresponding length of columns. The flexibility coefficient α_1 representing the connecting conditions can be obtained by

$$\alpha_1 = \left(1 + \frac{6EI_b}{L_b R_{kR}}\right) \Big/ \left[\left(1 + \frac{4EI_b}{L_b R_{kL}}\right)\left(1 + \frac{4EI_b}{L_b R_{kR}}\right) - \left(\frac{EI_b}{L_b}\right)^2 \frac{4}{R_{kL} R_{kR}}\right] \tag{3}$$

in which R_{kL}, R_{kR} are elastic spring constants at left and right ends of semi-rigid beam, respectively.

Formulating joint equilibrium at nodal points A and B, then

$$(M_A)_{c1} + (M_A)_{c2} + (M_A)_{b1} + (M_A)_{b2} = 0 \tag{4a}$$

$$(M_B)_{c2} + (M_B)_{c3} + (M_B)_{b3} + (M_B)_{b4} = 0 \tag{4b}$$

Substituting Eqs. (1a)-(1h) into Eqs. (4a)-(4b), the general governing equation for the μ-factor of column $c2$ in braced frame can be derived as

$$\left[\left(\frac{\pi}{\mu}\right)^2 + 2(K_1' + K_2') - 4K_1'K_2'\right]\frac{\pi}{\mu} \cdot \sin\frac{\pi}{\mu} - 2\left[(K_1' + K_2')\left(\frac{\pi}{\mu}\right)^2 + 4K_1'K_2'\right] \cdot \cos\frac{\pi}{\mu} + 8K_1'K_2' = 0 \tag{5}$$

where K_1', K_2' are the modified relative stiffness factors, given by

$$K_1' = \frac{\sum_A \alpha_1 EI_b / L_b}{\sum_A EI_c / L_c}, \quad K_2' = \frac{\sum_B \alpha_1 EI_b / L_b}{\sum_B EI_c / L_c} \tag{6}$$

(2) for unbraced frame

Column $c1$
$$(M_A)_{c1} = \frac{EI_{c1}}{L_{c1}}\left[S_{ii}\theta_A + S_{ij}\theta_B - (S_{ii}+S_{ij})\frac{\Delta}{L_{c1}}\right] \tag{7a}$$

Column $c2$
$$(M_A)_{c2} = \frac{EI_{c2}}{L_{c2}}\left[S_{ii}\theta_A + S_{ij}\theta_B - (S_{ii}+S_{ij})\frac{\Delta}{L_{c2}}\right] \tag{7b}$$

$$(M_B)_{c2} = \frac{EI_{c2}}{L_{c2}}\left[S_{ij}\theta_A + S_{ii}\theta_B - (S_{ii}+S_{ij})\frac{\Delta}{L_{c2}}\right] \tag{7c}$$

Column $c3$
$$(M_B)_{c3} = \frac{EI_{c3}}{L_{c3}}\left[S_{ij}\theta_A + S_{ii}\theta_B - (S_{ii}+S_{ij})\frac{\Delta}{L_{c3}}\right] \tag{7d}$$

Beam $b1$
$$(M_A)_{b1} = 6\alpha_2\frac{EI_{b1}}{L_{b1}}\theta_A \tag{7e}$$

Beam $b2$
$$(M_A)_{b2} = 6\alpha_2\frac{EI_{b2}}{L_{b2}}\theta_A \tag{7f}$$

Beam $b3$
$$(M_B)_{b3} = 6\alpha_2\frac{EI_{b3}}{L_{b3}}\theta_B \tag{7g}$$

Beam $b4$
$$(M_B)_{b4} = 6\alpha_2\frac{EI_{b4}}{L_{b4}}\theta_B \tag{7h}$$

Formulating joint equilibrium at nodal points A and B, and story sway equilibrium on column $c2$, then

$$(M_A)_{c1} + (M_A)_{c2} + (M_A)_{b1} + (M_A)_{b2} = 0 \tag{8a}$$

$$(M_B)_{c2} + (M_B)_{c3} + (M_B)_{b3} + (M_B)_{b4} = 0 \tag{8b}$$

$$(M_A)_{c2} + (M_B)_{c2} + P\Delta = 0 \tag{8c}$$

where $P = EI_{c2}(\pi/\mu L_c)^2$, $\Delta = \rho L_{c2}$.

Substituting Eqs. (7a)-(7h) into Eqs. (8a)-(8c), the general governing equation for the μ-factor of column $c2$ in unbraced frame can be derived as

$$\left[36K_1'K_2' - \left(\frac{\pi}{\mu}\right)^2\right]\sin\left(\frac{\pi}{\mu}\right) + 6(K_1' + K_2')\frac{\pi}{\mu}\cdot\cos\left(\frac{\pi}{\mu}\right) = 0 \tag{9}$$

where

$$K_1' = \frac{\sum_A \alpha_2 EI_b / L_b}{\sum_A EI_c / L_c}, \quad K_2' = \frac{\sum_B \alpha_2 EI_b / L_b}{\sum_B EI_c / L_c} \tag{10}$$

$$\alpha_2 = \left(1 + \frac{2EI_b}{L_b R_{kR}}\right) \Big/ \left[\left(1 + \frac{4EI_b}{L_b R_{kL}}\right)\left(1 + \frac{4EI_b}{L_b R_{kR}}\right) - \left(\frac{EI_b}{L_b}\right)^2 \frac{4}{R_{kL} R_{kR}}\right] \tag{11}$$

The relation of the rotation angles of beams is associated with the buckling mode of the frame, which influenced by many factors such as the configuration of the frame, stiffness of members and connections, load distribution, and bracing and support conditions (Xu 2002). Therefore, an assumption has to be made on the buckling mode of the frames. The current practice of evaluating the effective length factor for rigid frames is based on the alignment chart method, in which a symmetric buckling mode is applied to braced frames with third assumption, while an asymmetric bucking mode is adopted for unbraced frames with fourth assumption. However, in general, the buckling mode of the frame may be neither symmetric nor asymmetric. Therefore, it is understood that such assumptions may result in inaccuracy in some cases. Xu and Liu (2000) investigated the effect of the fourth assumption on critical buckling loads of unbraced semi-rigid frames and concluded that the maximum error is only 7.92%. So the assumption of the rotation angles of beams used in this paper is feasible and reasonable.

3. Determination of main parameters

3.1 Moment-rotation relation of connections

Modeling of beam-to-column connections requires representation of the non-linear moment-rotation, M-θ_r, curve. The Kishi-Chen's (1990) three-parameter power model of semi-rigid connections can be used to represent the nonlinear M-θ_r relation of a composite connection. The model includes three parameters: the initial connection stiffness, R_{ki}, the ultimate moment capacity of connection, M_u, and the shape parameter n. The dimensional power equations relating moment to relative rotation θ_r is expressed as

$$M = \frac{R_{ki}\theta_r}{[1 + (\theta_r / \theta_0)^n]^{1/n}} \tag{12}$$

$$\theta_r = \frac{M}{R_{ki}[1 - (M / M_u)^n]^{1/n}} \tag{13}$$

where θ_0 is the reference plastic rotation given by $\theta_0 = M_u / R_{ki}$.

This power model is an effective tool for designers to execute the second-order nonlinear structural analysis quickly and accurately. The tangent connection stiffness R_{kt} can be expressed as

$$R_{kt} = \frac{dM}{d\theta_r} = \frac{R_{ki}}{[1 + (\theta_r / \theta_0)^n]^{(n+1)/n}} \tag{14}$$

Based on a larger of number of experiments and the component method, design approaches (Xiao *et al.* 1996, Ahmed and Nethercot 1997, Ahmed *et al.* 1997) to predict the key aspects of connections, such as moment capacity and initial connection stiffness, have been developed. Xiao *et al.* (1996) proposed formulas to predict moment capacity of endplate type connections. Based on a simple force transfer mechanism and consideration of the behavior of individual components, a method has been developed to predict the initial stiffness of composite flush endplate connection proposed by Ahmed and Nethercot (1997).

3.2 Effective stiffness of composite beam

In analysis of composite frames, the composite effect of steel beam and concrete slab on the frame behavior should be considered. However, since the ability of concrete to bear tension is ignorable, the composition of concrete slab with steel beam should not be considered at the location of the beam where the moment makes the concrete slab in tension. Because the moment is various at different locations in the beam of a frame, the effective stiffness of the composite beam may also varies at the locations where the moment makes the concrete slab in compression and in tension. Despite this apparent complexity, the effective second moment of inertia to predict an acceptable behavior of the beam in a composite frame has been proposed by Ammerman and Leon (1990), which is given by

$$I_b = 0.6 I_{pos} + 0.4 I_{neg} \tag{15}$$

where I_b is the effective moment of inertia of the composite beam; I_{pos} and I_{neg} are the moments of inertia of the composite section and pure steel beam section, respectively.

3.3 Connection rotational stiffness

The beam-line equation and moment-rotation curve of connection are applied to determinate tangent connection stiffness, R_{kt}, as shown in Fig. 2. Firstly, through structural analysis, the moment at the end of the beam in the corresponding rigidly connected frames and the rotation at the end of the beam in the corresponding pin-connected frame are obtained. Then the crossing point between the beam-line equation and moment-rotation curve of connection can be determined. Finally, using Eq. (14), the tangent rotational stiffness of the connection with the moment at the crossing point as shown in Fig. 2.

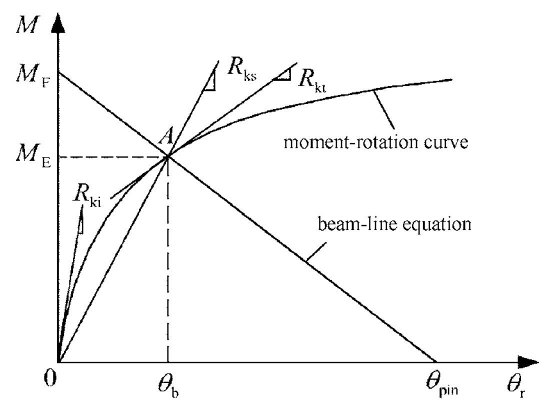


Fig. 2 Rotational stiffness of connection

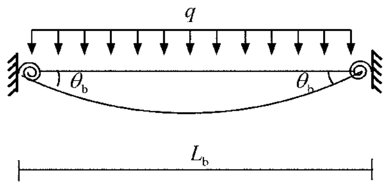


Fig. 3 Beam with semi-rigid connections

For an elastic beam with symmetric and vertical load as shown in Fig. 3, the relation between the moment, M_E, and rotation, θ_b, at the end of the beam may be expressed as

$$M_E = M_F - \frac{2EI}{L_b}\theta_b \tag{16}$$

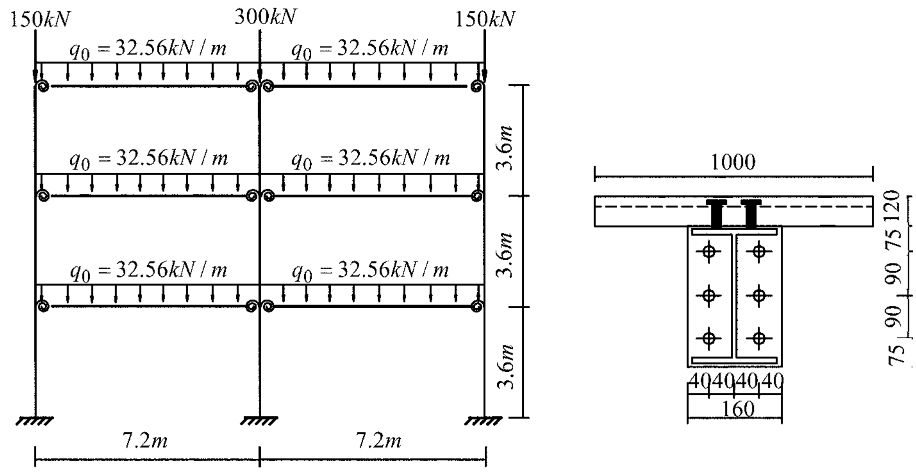
where M_F is the moment of at the end of the corresponding rigidly connected beam.

The Eq. (16) is called the beam-lined equation. According to compatibility of deformation, the beam-line equation and connection curve intersect to a point A as shown in Fig. 2.

4. Numerical studies

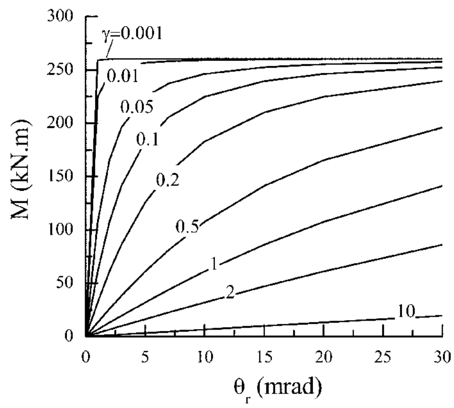
To consider the effects of the non-linear M-θ_r characteristics of beam-to-column connections on the column μ-factor for semi-rigid composite frames, numerical studies are performed using a two-bay three-storey composite frame with semi-rigid connections under uniformly distributed loads, as shown in Fig. 4(a).

Select HN300×150×6.5×9 for steel beams and HW250×250×9×14 for steel columns. A 24 mm diameter bolts are used for this connection. A composite endplate connection is adopted for the semi-rigid beam-to-column connections as shown in Fig. 4(b). The value of the basic distributed load q_0 is specified in Fig. 4(a).

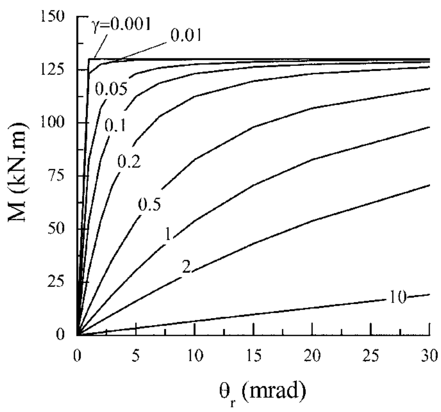


(a) composite frame (b) section of composite beam

Fig. 4 Two-bay three-story semi-rigid composite frame



(a)M_u=260kN.m



(b) M_u=130kN.m

Fig. 5 M-θ_r curves

The distributed load q applied on the frame is varied up to βq_0 using an amplification factor β. Since the column μ-factor of semi-rigid frames depends on the non-linear moment-rotation relations of connections, the initial stiffness R_{ki} and the ultimate moment capacity M_u of connection in the three-parameter power model are taken as parameters for numerical studies.

For universalization, a non-dimensional parameter γ is defined as:

$$\gamma = \frac{EI_b}{L_b R_{ki}} \tag{17}$$

In this study, γ is varied within the range from the nearly rigid state (γ= 0.001) to the nearly pinned state (γ= 10). The ultimate moment capacity of the connection M_u is assumed to be: M_u= M_p (case I) and M_u = 0.5 M_p (case II), in which M_p is the plastic moment capacity of beam and in this study M_p = 260 kN.m. The shape parameter of the connection n is set to be 1.5. Fig. 5 shows the M-θ_r curves of the connections with various values of γ used in the study.

The procedure for calculating the column μ-factor in frames with flexible beam-to-column connections is as follows:

(1) assume γ and determine R_{ki} using Eq. (17);
(2) assume the remaining two parameters of Eq. (12) for a non-linear M-θ_r curve of semi-rigid connections and an applied distributed load q;
(3) calculate relative rotations θ_r by using Eqs. (12) and (16);
(4) estimate the tangent connection stiffness R_{kt} by using Eq. (14);
(5) calculate the flexibility coefficient α_i (i = 1, 2) using Eqs. (3) and (11);
(6) calculate the modified relative stiffness factor K_1', K_2' using Eqs. (6) and (10);
(7) solve for μ-factor of each column from Eq. (5) or Eq. (9).

The comparisons of the μ-factors of column $c1$ among the method proposed here in before, the exact component finite element method (FEM) and Barakat-Chen's method are shown in Figs. 9-12.

4.1 Moment capacity of connection M_u

The comparison of the results for the case of M_u=M_p and M_u=0.5 M_p is shown in Figs. 6(a) and 6(b). The solid and dashed lines in Fig. 6 are for the cases of M_u=M_p and M_u=0.5 M_p, respectively. It shows that

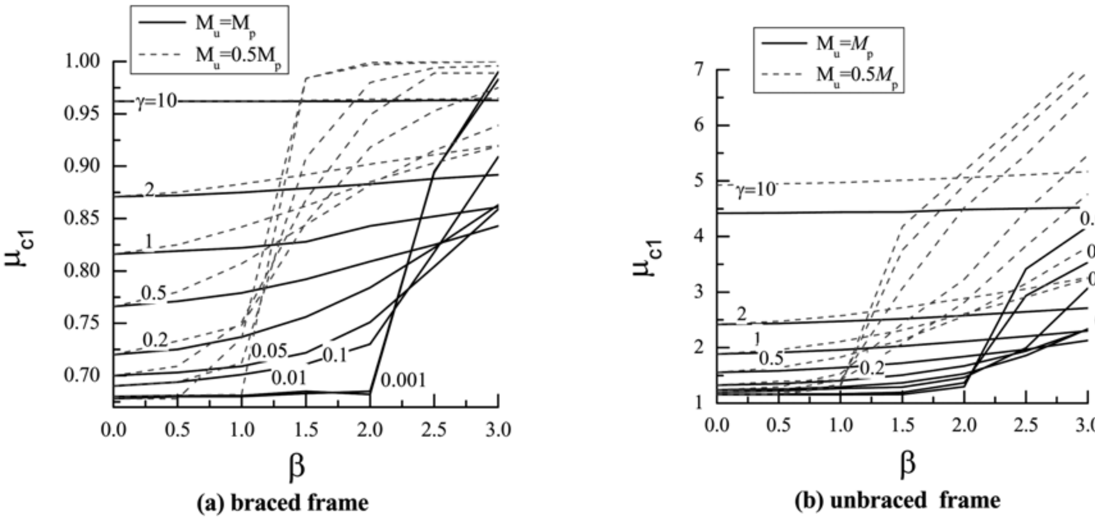


Fig. 6 Effective length factors of column $c1$ for composite frames

when the initial connection stiffness is small, the incremental rate of the column μ-factor with respect to the applied load is large.

From Fig. 6, it is seen that μ-factor of the column $c1$ in the example frame is within the two lines of the results for the braced frames with rigid ($\mu = 0.678$) and pinned ($\mu = 1.0$) connections, or for the unbraced frames with rigid ($\mu = 1.155$) and pinned ($\mu = 6.96$) connections.

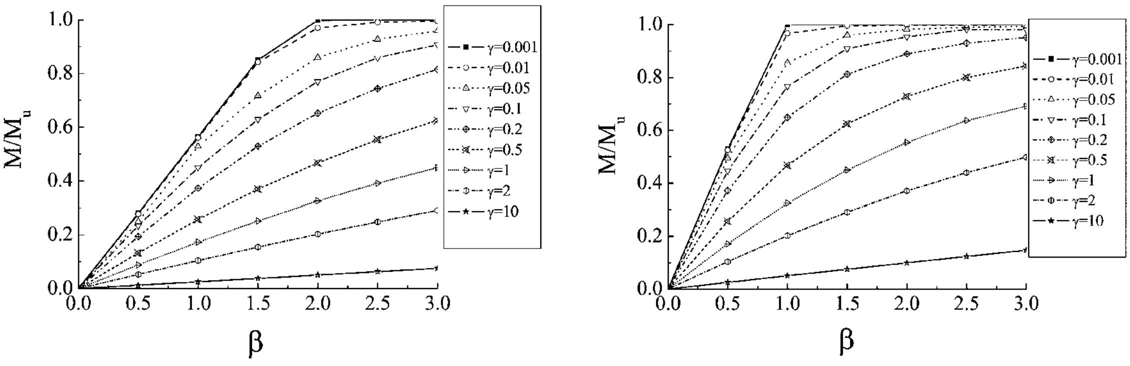
Fig. 6 also shows that the μ-factor becomes large with increasing of load factors β in both cases of $M_u = M_p$ and $M_u = 0.5\ M_p$. The variation of μ-factor for the case of $M_u = 0.5\ M_p$ is larger than that for $M_u = M_p$, because the former tangent connection stiffness are smaller than the later ones. For braced frame, the maximum difference in μ-factor between the two cases is about 0.299; and for unbraced frame, the maximum difference in μ-factor between the two cases is about 3.6.

4.2 M/M_u and β relation

EC3 (2004) suggested that when the moment-rotation characteristic is elastic, the moment in connection is less than 2/3 M_u. In order to study the effects of load value and connection stiffness on μ-factor, M/M_u and α relation with variety of γ are analyzed, as shown in Figs. 7 and 8. The results show that when $M/M_u \leq 2/3$ and $\beta < 2.0$, M/M_u and β relation is approximately linear.

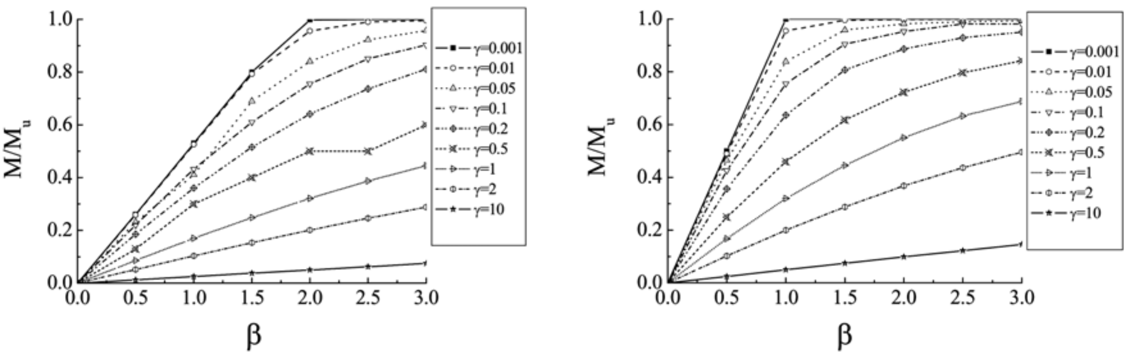
4.3 Braced frame

The results of the μ-factors of column $c1$ in a flexibly jointed and two-bay three-story braced frame in cases of M_u=M_p and M_u=0.5 M_p, obtained by the proposed method and Barakat-Chen's method are compared with those obtained by the exact FEM. The secant connection stiffness R_{ks}, adopted in Barakat-Chen's method corresponds to a reference plastic rotation θ_0 and is less than the initial connection stiffness R_{ki} for the connection model used in this study. In the case of $M_u = M_p$ and $M_u = 0.5\ M_p$, the comparison of the results between the proposed method and the exact FEM are respectively shown in Figs. 9(a) and (b),



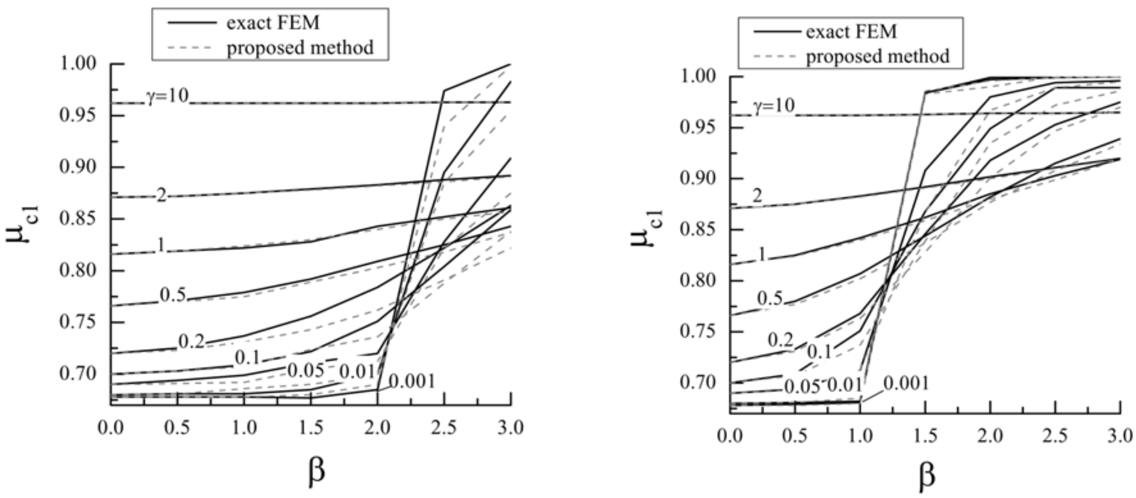
(a) M_u=M_p **(b) M_u=0.5M_p**

Fig. 7 M/M_u and β relation (joint A)



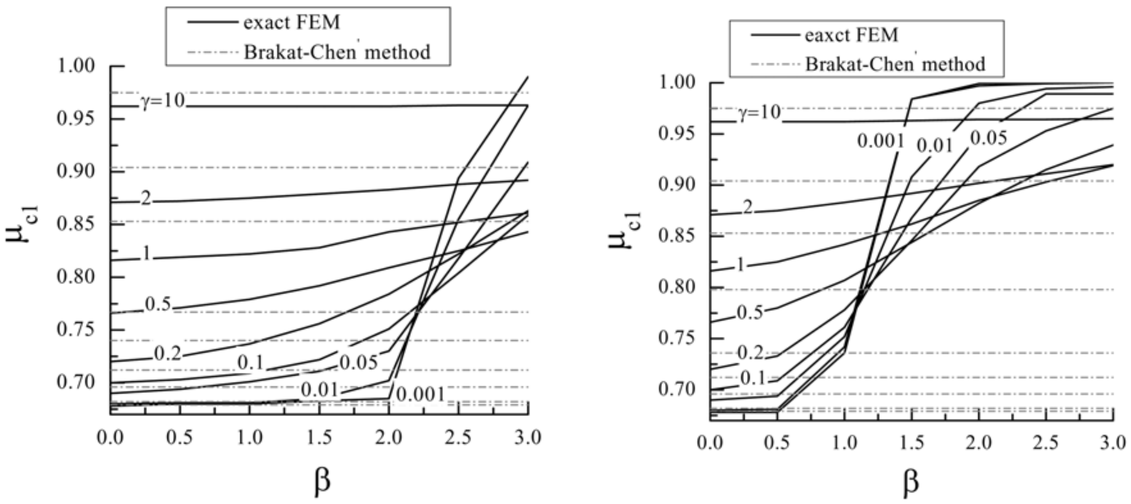
(a) M_u=M_p **(b) M_u=0.5M_p**

Fig. 8 M/M_u and β relation (joint B)



(a) M_u=M_p **(b) M_u=0.5M_p**

Fig. 9 Comparison of μ-factors in braced frame between the proposed method and the exact FEM



(a) M_u=M_p (b) M_u=0.5M_p

Fig. 10 Comparison of μ-factors in braced frame between Barakat-Chen's method and the exact FEM

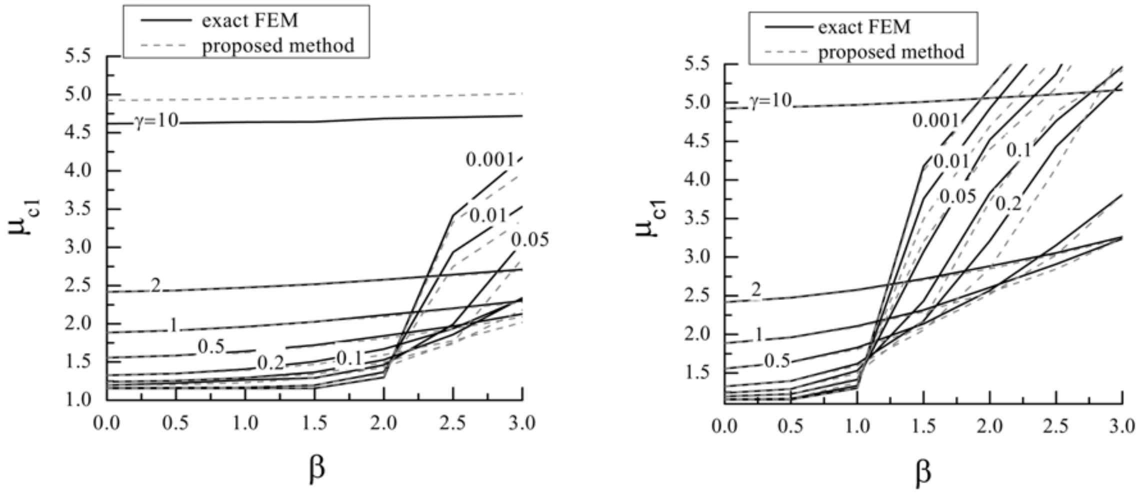
and the comparison of the results between Barakat-Chen's method and the exact FEM are respectively shown in Figs. 10(a) and (b). The results show that the μ-factors obtained by the proposed method are quite close to those obtained by the exact FEM. However, the results obtained by Barakat-Chen's method have much difference with those obtained by the exact FEM in the case of $M_u = M_p$ and $M_u = 0.5\ M_p$.

4.4 Unbraced frame

The results of the μ-factors of column $c1$ in a flexibly jointed and two-bay three-story unbraced frame in cases of $M_u = M_p$ and $M_u = 0.5\ M_p$, obtained by the proposed method and Barakat-Chen's method are compared with those obtained by the exact FEM. The secant connection stiffness R_{ks}, adopted in Barakat-Chen's method corresponds to a reference rotation θ_b ($\theta_b = M_E / R_{ks}$) and is equal to 63% the initial connection stiffness R_{ki} for the connection model used in this study. In the case of $M_u = M_p$ and $M_u = 0.5\ M_p$, the comparison of the results between the proposed method and the exact FEM are respectively shown in Figs. 11(a) and (b), and the comparison of the results between Barakat-Chen's method and the exact FEM are respectively shown in Figs. 12(a) and (b). The results show that the μ-factor obtained by the proposed method are quite close to those obtained by the exact FEM. However, the results obtained by Barakat-Chen's method have much difference with those obtained by the exact FEM in the case of $M_u = 0.5\ M_p$.

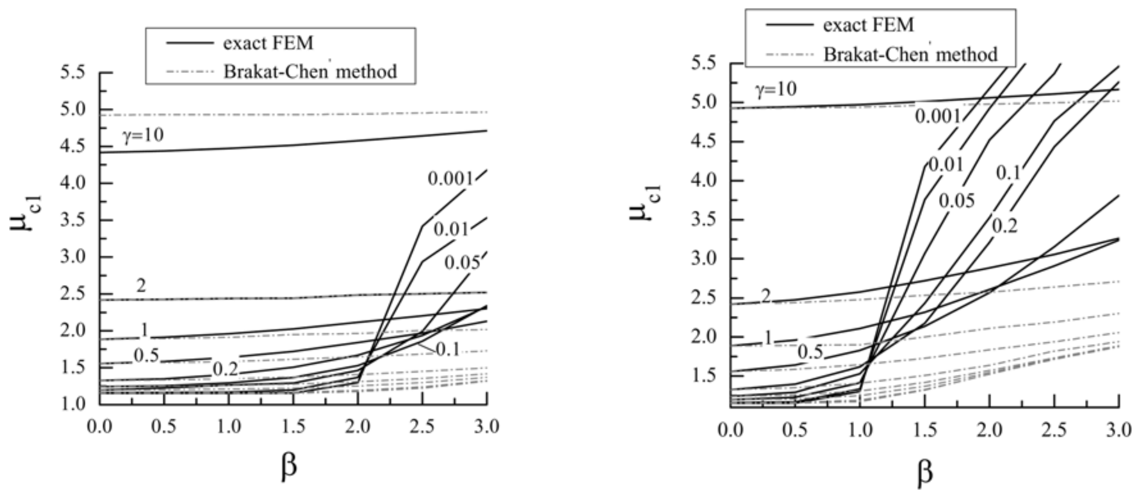
When the moment-rotation characteristic is elastic, i.e., $M/M_u \leq 2/3$ and $\beta < 2.0$, the investigations in before demonstrate that:

(1) The method proposed can predict quite well the column μ-factor in the semi-rigid frames;
(2) The column μ-factor is increased with increment of the initial stiffness of connections for both braced and unbraced frames; and
(3) The column μ-factor is also increased with the increment of the loads applied on frames, except when the ultimate moment capacity of connection M_u, close to the plastic moment of the beam, the increment of the column μ-factor for unbraced frames is negligible.



(a) M_u=M_p (b) M_u=0.5M_p

Fig. 11 Comparison of μ-factors in unbraced frame between the proposed method and the exact FEM



(a) M_u=M_p (b) M_u=0.5M_p

Fig. 12 Comparison of μ-factors in unbraced frame between Barakat-Chen's method and the exact FEM

5. Conclusions

In current specifications for engineering practice, such as in GB50017 (2003), AISC-LRFD (1999), the μ -factor of frame columns must be determined in order to check the structural capacity of steel frames.

In this paper, the governing equations for determining the μ-factor of columns in semi-rigid composite frames are proposed with considering the effects of the nonlinear moment-rotation characteristics of beam-to-column connections and composition of concrete slab with steel beams. The proposed method can predict the μ-factor of columns in semi-rigid composite frames with satisfactory accuracy and be used for stability design of semi-rigid composite frames.

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Nomenclature

EI_b	: Flexural stiffness of composite beam
EI_c	: Flexural stiffness of beam-column
K_1, K_2	: Beam-to-column stiffness factors at A-th and B-th ends
K_1', K_2'	: Modified Beam-to-column stiffness factors at A-th and B-th ends
μ	: effective length factor of column
L_b	: length of beam
L_c	: length of beam-column
P	: Axial force applied on beam-column
M	: Connection moment
M_A, M_B	: End moment at A-th and B-th ends
M_F	: Moment at the end of the corresponding rigidly connected beam
M_p	: Plastic moment capacity of beam
M_u	: Ultimate moment capacity of connection
n	: Shape parameter in the three-parameter power model for beam-to-column connections
θ_r	: Relative rotation of connection
θ_A, θ_B	: Nodal rotation at A-th and B-th ends
θ_0	: Reference plastic rotation, M_u / R_{ki}
R_{kA}, R_{kB}	: Elastic spring constants at A-th and B-th ends of beam
R_{kL}, R_{kR}	: Elastic spring constants at left and right ends of beam
R_{ki}	: Initial connection stiffness
R_{ks}	: Secant connection stiffness
R_{kt}	: Tangent connection stiffness
β	: Load factor
q	: Distributed load applied on the beam, $=\beta q_0$
S_{kl}	: Stability functions (k, l=i or j)
α_1	: Flexibility coefficient representing connecting conditions of beam in braced frame
α_2	: Flexibility coefficient representing connecting conditions of beam in unbraced frame
Δ	: Side-way of column
ρ	: Obliquity of column
I_b	: Effective moments of inertia of the composite beam
I_{pos}, I_{neg}	: Moments of inertia of the composite section and pure steel beam section
γ	: Non-dimensional parameter, $= EI_b/(R_{ki}L_b)$

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