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Free vibration analysis of composite conical shells using the discrete singular convolution algorithm

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Abstract. The discrete singular convolution (DSC) algorithm for determining the frequencies of the free vibration of single isotropic and orthotropic laminated conical shells is developed by using a numerical solution of the governing differential equations of motion based on Love's first approximation thin shell theory. By applying the discrete singular convolution method, the free vibration equations of motion of the composite laminated conical shell are transformed to a set of algebraic equations. Convergence and comparison studies are carried out to check the validity and accuracy of the DSC method. The obtained results are in excellent agreement with those in the literature.

Keywords: composite conical shells; free vibration; discrete singular convolution.

1. Introduction

Because of the practical importance of the free vibration analysis of the conical shell in structural, aerospace, nuclear, petrochemical, submarine hulls, and mechanical applications, investigators have made efforts to deal with free vibration analysis of this type of structures (Bert and Francis 1974, Reddy 1996, Bacon and Bert 1967, Siu and Bert 1970, Irie *et al*. 1984, 1982, Sivadas and Ganesan 1992, Yang 1974, Tong 1993a,b, Tong and Wang 1992). Layered composites are increasingly used in these structures because of their possible higher specific stiffness and better damping absorbing properties over the isotropic ones. More recently, Shu (1996a,b, 1997), Hua (2000a,b), Hua and Lam (2000) and Lam and Hua (1997) presented the differential quadrature method to study the free vibration of orthotropic and laminated rotating conical shells. Liew *et al*. (1995) also studied the effects of initial twist and thickness variation on the vibration behavior of shallow conical shells. Some selected works in this research topic includes those of Liew *et al*. (2005, 1994), Lim *et al*. (1998, 1995), Wu and Wu (2000), Leissa (1973), Soedel (1996), Civalek (1998, 2004), Lim and Kitipornchai (1999) and Markus (1988). More detailed information can be found in related references (Chang 1981, Kapania 1989, Wu *et al*. 2005, Hu *et al*. 2002, Lee *et al*. 2002). The focus in this work is on the application of the DSC method to the differential equation, which governs the free vibration analysis of laminated, orthotropic, and single isotropic conical shells. To the best knowledge of author, it is the first time the discrete singular convolution algorithm has been successfully applied to composite laminated, orthotropic, and isotropic conical shell problem for free vibration analysis.

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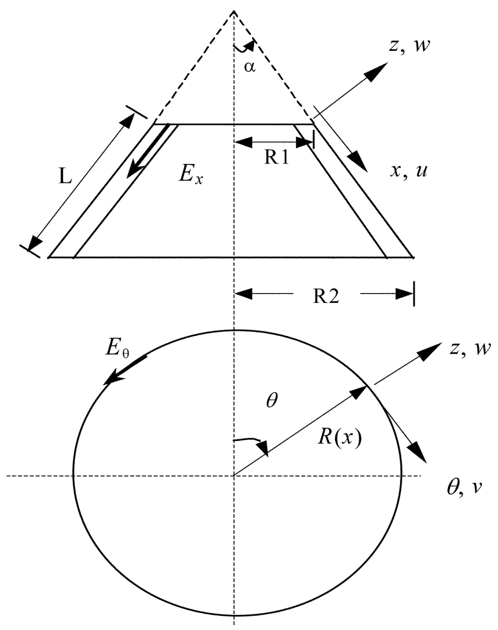

Fig. 1 Geometry and notation of laminated conical shell

2. Formulation

A typical laminated conical shell is given as shown in Fig. 1. The cone semivertex angle, thickness of the shell, and cone length are denoted by α, h and L, respectively. R1 and R2 are the radii of the cone at its small and large edges. The conical shell is referred to a coordinate system (x, θ, z) as shown in Fig. 1. The components of the deformation of the conical shell with references to this given coordinate system are denoted by u, v, w in the x, θ and z directions, respectively. The equilibrium equation of motion in terms of the force and moment resultants can be written as Tong (1993b)

$$\frac{\partial N_x}{\partial x} + \frac{1}{R(x)}\frac{\partial N_{x\theta}}{\partial \theta} + \frac{\sin\alpha}{R(x)}(N_x - N_\theta) = \rho h \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R(x)}\frac{\partial N_\theta}{\partial \theta} + \frac{\cos\alpha}{R(x)}\frac{\partial M_{x\theta}}{\partial x} + \frac{\cos^2\alpha}{R^2(x)}\frac{\partial M_\theta}{\partial \theta} + 2\frac{\sin\alpha}{R(x)}N_{x\theta} = \rho h \frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R(x)}\frac{\partial^2 M_{x\theta}}{\partial \theta \partial x} + \frac{1}{R^2(x)}\frac{\partial^2 M_\theta}{\partial \theta^2} + \frac{2\sin\alpha}{R(x)}\frac{\partial M_x}{\partial x} - \frac{1}{R(x)}\left(\sin\alpha\frac{\partial M_\theta}{\partial x} + \cos\alpha N_\theta\right) = \rho h \frac{\partial^2 w}{\partial t^2} \tag{3}$$

where

$$R(x) = R1 + x\sin\alpha \tag{4}$$

$$\rho_a(x, \theta) = \frac{1}{h}\int_{-h/2}^{h/2} \rho(x, \theta, z)dz \tag{5}$$

Where ρ and ρ_a are, respectively, the density and density per unit length.

Based on the Love's first approximation theory the strain components are defined as linear functions of the normal (thickness) coordinate z, namely

$$\varepsilon_x = \varepsilon_1 + zk_1, \quad \varepsilon_\theta = \varepsilon_2 + zk_2, \quad \varepsilon_{x\theta} = \gamma + 2z\tau \tag{6}$$

where $\{\varepsilon\}^T = \{\varepsilon_1, \varepsilon_2, \gamma\}$ and $\{\kappa\}^T = \{\kappa_1, \kappa_2, 2\tau\}$ are respectively the strain and curvature vectors of the reference surface. They are defined by

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \quad \varepsilon_2 = \frac{1}{R(x)}\frac{\partial v}{\partial \theta} + \frac{u\sin\alpha}{R(x)} + \frac{w\cos\alpha}{R(x)}, \quad \gamma = \frac{1}{R(x)}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} - \frac{v\sin\alpha}{R(x)}$$

$$\kappa_1 = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_2 = -\frac{1}{R^2(x)}\frac{\partial^2 w}{\partial \theta^2} + \frac{\cos\alpha}{R^2(x)}\frac{\partial v}{\partial \theta} - \frac{\sin\alpha}{R(x)}\frac{\partial w}{\partial x}$$

$$\tau = \left[-\frac{1}{R(x)}\frac{\partial^2 w}{\partial \theta \partial x} + \frac{\sin\alpha}{R^2(x)}\frac{\partial w}{\partial \theta} + \frac{\cos\alpha}{R(x)}\frac{\partial v}{\partial x} - \frac{v\sin\alpha\cos\alpha}{R^2(x)}\right] \tag{7}$$

For a thin and generally orthotropic layer, the stresses are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11}^* & Q_{12}^* & Q_{16}^* \\ Q_{12}^* & Q_{22}^* & Q_{26}^* \\ Q_{16}^* & Q_{26}^* & Q_{66}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{\theta x} \end{Bmatrix} \tag{8}$$

where $\{\varepsilon_k^*\}^T = \{\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}\}$ is the strain vector. The transformed reduced stiffness matrix of the kth layer is defined by

$$[Q_k^*] = [K][Q_k][K]^{-1} \tag{9}$$

Where

$$[K] = \begin{bmatrix} \cos^2\varphi & \sin^2\varphi & 2\sin\varphi\cos\varphi \\ \sin^2\varphi & \cos^2\varphi & -2\sin\varphi\cos\varphi \\ -\sin\varphi\cos\varphi & \sin\varphi\cos\varphi & \cos^2\varphi - \sin^2\varphi \end{bmatrix} \tag{10}$$

in which $[K]$ is the transformation matrix between the material principal coordinate of the kth layer and the geometric coordinate of the laminated composite conical shell; φ is the angle between these two coordinate directions. The force and moment resultants are given in terms of displacements u, v and w by

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \\ c_{61} & c_{62} & c_{63} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{11}$$

where

$$c_{i1} = A_{i1}\frac{\partial}{\partial x} + A_{i2}\frac{\sin\alpha}{R(x)}, \quad c_{i2} = \frac{A_{i2}}{R(x)}\frac{\partial}{\partial\theta}$$

$$c_{i3} = -A_{i2}\frac{\cos\alpha}{R(x)} - B_{1i}\frac{\partial^2}{\partial x^2} - B_{i2}\frac{\sin\alpha}{R(x)}\frac{\partial}{\partial x} - \frac{B_{i2}}{R^2(x)}\frac{\partial^2}{\partial\theta^2}$$

$$c_{31} = \frac{A_{66}}{R(x)}\frac{\partial}{\partial\theta}, \quad c_{32} = A_{66}\left(\frac{\partial}{\partial x} - \frac{\sin\alpha}{R(x)}\right), \quad c_{33} = -B_{66}\frac{\partial}{\partial x}\left(\frac{1}{R(x)}\frac{\partial}{\partial\theta}\right)$$

$$c_{ji} = B_{1i}\frac{\partial}{\partial x} + \frac{B_{i2}\sin\alpha}{R(x)}, \quad c_{j2} = \frac{B_{i2}}{R(x)}\frac{\partial}{\partial\theta}$$

$$c_{j3} = -D_{1i}\frac{\partial^2}{\partial x^2} - D_{i2}\frac{\sin\alpha}{R(x)}\frac{\partial}{\partial x} - \frac{D_{i2}}{R^2(x)}\frac{\partial^2}{\partial\theta^2} - B_{i2}\frac{\cos\alpha}{R(x)}$$

$$c_{61} = \frac{B_{66}}{R(x)}\frac{\partial}{\partial\theta}, \quad c_{62} = B_{66}\left(\frac{\partial}{\partial x} - \frac{\sin\alpha}{R(x)}\right), \quad c_{63} = -2D_{66}\frac{\partial}{\partial x}\left[\frac{1}{R(x)}\frac{\partial}{\partial\theta}\right] \tag{12}$$

where i = 1,2 and j = 3+i. A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending stiffnesses and calculated from the following equations:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q^*_{ij}(1, z, z^2)dz \tag{13}$$

For an arbitrarily laminated composite shell, these stiffnesses can be given as

$$(A_{ij}) = \sum_{k=1}^{N_L} Q_{ij}^{(k)}(h_k - h_{k-1}), \quad (B_{ij}) = \frac{1}{3}\sum_{k=1}^{N_L} Q_{ij}^{(k)}(h_k^2 - h_{k-1}^2), \quad (D_{ij}) = \frac{1}{3}\sum_{k=1}^{N_L} Q_{ij}^{(k)}(h_k^3 - h_{k-1}^3) \tag{14}$$

Where N_L is the number of total layers of the laminated conical shell, $Q_{ij}^{(k)}$, the element of the transformed reduced stiffness matrix for the kth layer, and h_k and $h_{k\text{-}1}$ denote distances from the shell reference surface to the outer and inner surfaces of the kth layer. By substituting Eq. (11) into Eqs. (1)-(3), governing equations for the linear free vibration analysis of composite laminated conical shells are obtained;

$$L_{11}u + L_{12}v + L_{13}w = \rho h\frac{\partial^2 u}{\partial t^2} = 0 \tag{15a}$$

$$L_{21}u + L_{22}v + L_{23}w = \rho h\frac{\partial^2 v}{\partial t^2} = 0 \tag{15b}$$

$$L_{31}u + L_{32}v + L_{33}w = \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{15c}$$

where L_{ij} is the differential operators. These can be found in related literatures (Shu 1996a). The displacement terms are taken as

$$u = U(x) \cdot \cos(n\theta) \cdot \cos(\omega t) \tag{16a}$$

$$u = V(x) \cdot \sin(n\theta) \cdot \cos(\omega t) \tag{16b}$$

$$w = W(x) \cdot \cos(n\theta) \cdot \cos(\omega t) \tag{16c}$$

In this study, the following three type main boundary conditions and four subclasses boundary conditions for clamped edge are considered. These are defined as:

Simply supported edge (S)

$$V = 0, \quad W = 0, \quad N_x = 0, \quad M_x = 0 \tag{17}$$

Clamped edge (C)

$$U = 0, \quad V = 0, \quad W = 0 \quad \text{and} \quad W_x = 0 \tag{18}$$

Free edge (F)

$$\overline{Q}_x = 0, \quad M_x = 0, \quad N_x = 0, \quad S_{x\theta} = 0 \tag{19}$$

Type-1 Clamped boundary (C-C1)

$$W = 0, \quad N_x = 0, \quad N_{x\theta} = 0, \quad W_x = 0 \tag{20a}$$

Type-2 Clamped boundary (C-C2)

$$W = 0, \quad U = 0, \quad N_{x\theta} = 0, \quad W_x = 0 \tag{20b}$$

3. Discrete singular convolution (DSC)

Discrete singular convolutions (DSC) algorithm introduced by Wei (1999). As stated by Wei (2001a,b) singular convolutions (SC) are a special class of mathematical transformations, which appear in many science and engineering problems, such as the Hilbert, Abel and Radon transforms. Wei and his co-workers first applied the DSC algorithm to solve solid and fluid mechanics problem (Wei *et al.* 2002a,b). Zhao *et al.* (2002a,b) analyzed the high frequency vibration of plates and plate vibration under irregular internal support using DSC algorithm. Zhou and Wei (2002) adopted the DSC in the vibration analysis of rectangular plates with non-uniform boundary conditions. Numerical solution of unsteady incompressible flows using DSC is given by Wan *et al.* (2002). A good comparative accuracy of DSC and generalized differential quadrature methods for vibration analysis of rectangular plates is presented by Ng *et al.* (2004). More recently, Hou *et al.* (2005) and Lim *et al.* (2005) presented the DSC-Ritz method for the free vibration analysis of Mindlin plates and thick shallow shells. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by

$$F(t) = (T*\eta)(t) = \int_{-\infty}^{\infty} T(t-x)\,\eta(x)dx \tag{21}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type

$$T(x) = \delta^{(n)}(x); \quad (n = 0,1,2,...,) \tag{22}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n > 1$ are essential for numerically solving differential equations. More recently, the use of some new kernels and regularizer such as delta regularizer (Wei 2001a) was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{23}$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (23) is practically and has an essentially compact support for numerical interpolation. Eq. (23) can also be used to provide discrete approximations to the singular convolution kernels of the delta type

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\alpha,\sigma}^{(n)}(x-x_k)f(x_k); \quad (n = 0,1,2,...,) \tag{24}$$

When the regularized Shannon's delta kernel (RSDK) is used, the detailed expressions for $\delta_{\Delta,\sigma}(x)$, $\delta_{\Delta,\sigma}^{(1)}(x)$, $\delta_{\Delta,\sigma}^{(2)}(x)$, $\delta_{\Delta,\sigma}^{(3)}(x)$ and $\delta_{\Delta,\sigma}^{(4)}(x)$ can be easily obtained. Detailed formulations on these coefficients are found in Wei (1999, 2001a). First order derivative, for example, are given as:

$$\delta_{\pi/\Delta,\sigma}^{(1)}(x_m-x_k) = \frac{\cos(\pi/\Delta)(x-x_k)}{(x-x_k)}\exp[-(x-x_k)^2/2\sigma^2] - \frac{\sin(\pi/\Delta)(x-x_k)}{\pi(x-x_k)^2/\Delta}\exp[-(x-x_k)^2/2\sigma^2]$$
$$-\frac{\sin(\pi/\Delta)(x-x_k)}{(\pi\sigma^2/\Delta)}\exp[-(x-x_k)^2/2\sigma^2] \tag{25}$$

Substituting Eqs. (16) into Eqs. (15), the governing equations can be written in DSC forms as

$$G_{111}U_{i,j} + G_{112}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)U_{i+k,j} + G_{113}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)U_{i+k,j} + G_{121}V_{i,j}$$
$$+ G_{122}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)U_{i,j+k} + G_{131}W_{i,j} + G_{132}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)W_{i,j+k} = -\rho h\omega^2 U_{i,j} \tag{26a}$$

$$G_{211}U_{i,j} + G_{212}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)U_{i+k,j} + G_{221}V_{i,j} + G_{122}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)V_{i,j+k} + G_{223}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)V_{i+k,j}$$

$$+G_{231}W_{i,j}+G_{232}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)W_{i,j+k}+G_{233}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)W_{i+k,j}=-\rho h\omega^2 V_{i,j} \quad (26b)$$

$$G_{311}U_{i,j}+G_{312}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)U_{i+k,j}+G_{321}V_{i,j}+G_{322}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)V_{i,j+k}+G_{323}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)V_{i+k,j}$$

$$+G_{331}W_{i,j}+G_{332}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta x)W_{i,j+k}+G_{333}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)W_{i+k,j}+G_{334}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(3)}(k\Delta x)W_{i+k,j}$$

$$+G_{335}\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(4)}(k\Delta x)W_{i+k,j}=-\rho h\omega^2 W_{i,j} \quad (26c)$$

Thus, the governing equations are spatialy discretized by using the DSC algorithm. From the above procedures, one can derive the general form of eignvalue equation as follows

$$GU=\Omega BU \quad (27)$$

where U is the displacement vector defined as follows:

$$U=[U_{ijk} \quad V_{ijk} \quad W_{ijk}]^T \quad (28)$$

In Eq. (27), G and B are the matrices derived from the governing equations described by (26) and the boundary conditions considered in Eqs. (17-20). In the above eigenvalue equation, Ω is the nondimensional frequency parameter.

4. Numerical applications

4.1. Numerical results for single isotropic conical shells

To check whether the purposed formulation and programming are correct, an isotropic conical shell is analysed first. The numerical results are given by the dimensionless frequency parameter Ω, defined by

$$\Omega = R_2\sqrt{\frac{\rho h}{A_{11}}}\omega$$

where ω is referred to as the frequency parameter. During the numerical applications, the unit of cone angle(α) is taken as degree(^{o}), for simplicty. For example, in Table 1, $\alpha=15$ means, the cone angle is 15^o.

Table 1 shows the convergence of computed frequency parameters Ω for an isotropic conical shell with $L\sin\theta/R_2$=0.25 and circumferential wave number, n=0. The results given by Irie *et al.* (1984), Tong (1993a), and Shu (1996b) are also given in this Table 1. In order to examine the influence of bandwidth on the accuracy, five values of N = 8,11,16,21, and 32, with corresponding regularization parameters being σ/Δ = 1,73, 2,15, 2.46,2,8 and 3.2, and we choose $M=N$ with r being optimally selected. From the Table 1, it is shown that the convergence of DSC results is very good. By comparing with the results of Irie et *al.* (1984), the DSC results using 16 uniform grid points are very accurate. When the number

Table 1 Frequency parameters of C-S conical shells; $\nu = 0.3$, $h/R2 = 0.01$, $n = 0$

	L sin$a$ / $R2$ = 0.25				
	$\alpha = 15$	$\alpha = 30$	$\alpha = 45$	$\alpha = 60$	$\alpha = 75$
N=8	0.8983	0.9058	0.8204	0.7704	0.6629
N=11	0.8355	0.8901	0.8151	0.7556	0.6447
N=16	0.7851	0.8935	0.8043	0.7357	0.6228
N=21	0.7856	0.8941	0.8047	0.7361	0.6234
N=32	0.7856	0.8941	0.8047	0.7361	0.6234
Irie *et al.* (1984)	-	0.8938	0.8041	0.7353	-
Tong (1993a)	-	0.8938	0.8041	0.7353	-
Shu (1996b)	-	-	-	0.7366	-

Table 2 Frequency parameters of S-S conical shells; $\nu = 0.3$; $h/R2 = 0.01$; $\alpha = 45$

	L sinα / $R2$ = 0.5					
	DSC					
Cicumferential wave number (n)	N=16	N=21	FEM Civalek (1998)	HDQ Civalek (2004)	DQ Wang *et al.* (1999)	Irie *et al.* (1984)
2	0.6313	0.6312	0.6435	0.6313	0.6319	0.6310
3	0.5066	0.5064	0.5102	0.5064	0.5063	0.5065
4	0.3947	0.3947	0.4016	0.3946	0.3947	0.3947
5	0.3348	0.3348	0.3428	0.3348	0.3348	0.3348

of grid points is larger than 16, the DSC results are independent of grid. Table 2 shows the convergence of computed frequency parameters Ω for an isotropic conical shell with $L\sin\theta/R_2 = 0.5$. The results given by Irie *et al.* (1984), Wang *et al.* (1999), and Civalek (2004) are also given in this Table. By comparing with the results of Irie et *al.* (1984), the DSC results using 16 uniform grid points are very accurate. When the number of grid points is larger than 16, the DSC results are independent of grid. The results given by Civalek (1998) are obtained using the finite element method (FEM) and the result given by Wang *et al.* (1999) are obtained by differential quadrature (DQ) method. From the Table 2, it is shown that the convergence of DSC results is very good for only 16 grid points. The DSC results are generally in agreement with the results produced from the analytical (Irie *et al.* 1984) and harmonic differential quadrature (HDQ) results (Civalek 2004). It is also seen, the DSC results compare very well with the FEM solutions from reference (Civalek 1998) for only 16 grid points. The present numerical solutions are in close agreement with the HDQ (Civalek 2004), FEM (Civalek 1998), DQ (Wang *et al.* 1999), and analytical solutions (Irie *et al.* 1984) available in the literature. But it is impossible to state that the DSC method is superior to finite elements or DQ in all cases or problems by only depending on this study. Each method has its own advantages and application areas.

4.2. Numerical results for orthotropic conical shells

The frequency parameters of simply supported orthotropic conical shells for $L\sin\alpha / R2 = 0.25$, $h/R2 = 0.01$, $\mu_{x\phi} = 0.3$, $s = E_x/E_\theta$, $E_x = 2.1 \times 10^6$ and $G_{x\theta} = 807692$ are presented in Figs. 2. These figures show the effects of the ratio s on the values of Ω. It is observed that the values of Ω decrease when the ratio s increases. The variation is only marginal for larger value of s, irrespective of cone

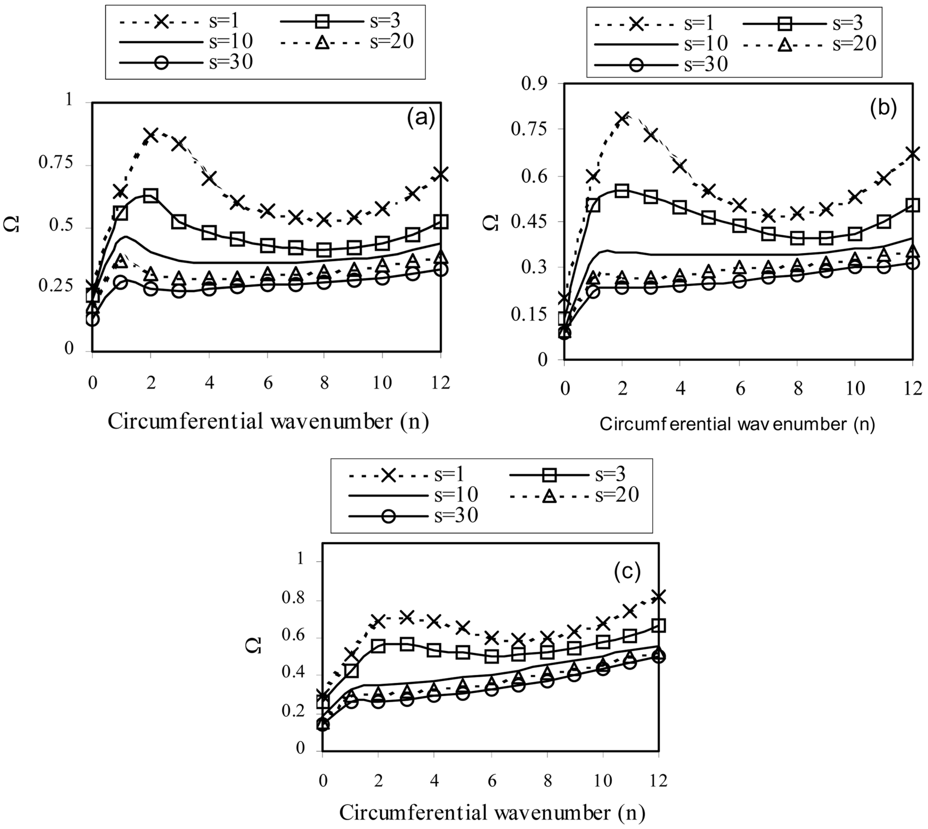

Fig. 2 Effect of $s = E_x/E_\theta$ ratio on frequencies with the S-S boundary condition (a) $\alpha = 15$ (b) $\alpha = 30$ (c) $\alpha = 45$

angles. In general, it is seen that the frequencies increase considerably with circumferential wave number for larger value of s (i.e., $s > 10$). Variations of frequency are shown in Figs. 3 for various cone angles of S-S boundary condition. Two different orthotropic parameters, $s = 3$, $s = 10$ are considered. For the cases under consideration, axisymmetric frequencies ($n = 0$) are not the lowest frequencies. The lowest frequencies occur for a higher value of n.

4.3. Numerical results for orthotropic laminated conical shells

It is noted that the extension bending coupling terms reach their maximum values with two plies ($N_L = 2$) and become zero with an infinite number of plies ($N_L = \infty$). Thus the case of $N_L = 2$ is referred to as "maximum coupling" and the case of $N_L = \infty$ is referred to as "without coupling". The effect of extension-bending coupling on frequency response with circumferential wave number is shown in Figs. 4-5 for antisymmetric cross-ply laminated conical shell. Two different semivertex angles such as 30 and 45 and four different boundary conditions are taken into consideration. In order to discuss the influences of $h/R2$ on the frequency characteristics for S-S boundary conditions, Fig. 6 is obtained for two different cone angles, i.e., $\alpha = 30$ and $\alpha = 45$. With the increase of ratio $h/R2$, frequency parameter Ω increases rapidly. Generally, the decreasing magnitude of cone angle decreases the frequency parameter Ω. Fig. 7 show the effect of $h/R2$ on the frequency. It is concluded that, the frequency

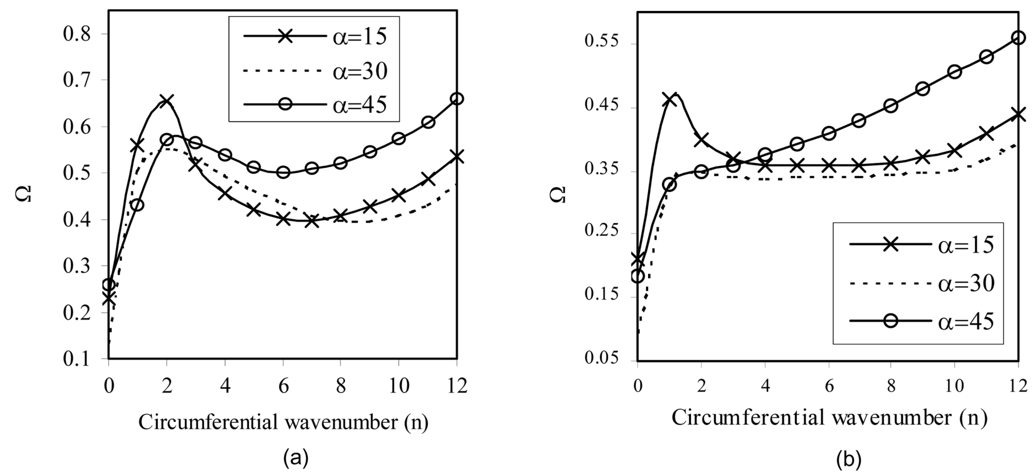

Fig. 3 Variation of frequency for various cone angles for S-S boundary condition ($L\sin\alpha/R2$=0.25, $h/R2$=0.01) (a) $s = E_x/E_\theta = 3$ (b) $s = E_x/E_\theta = 10$

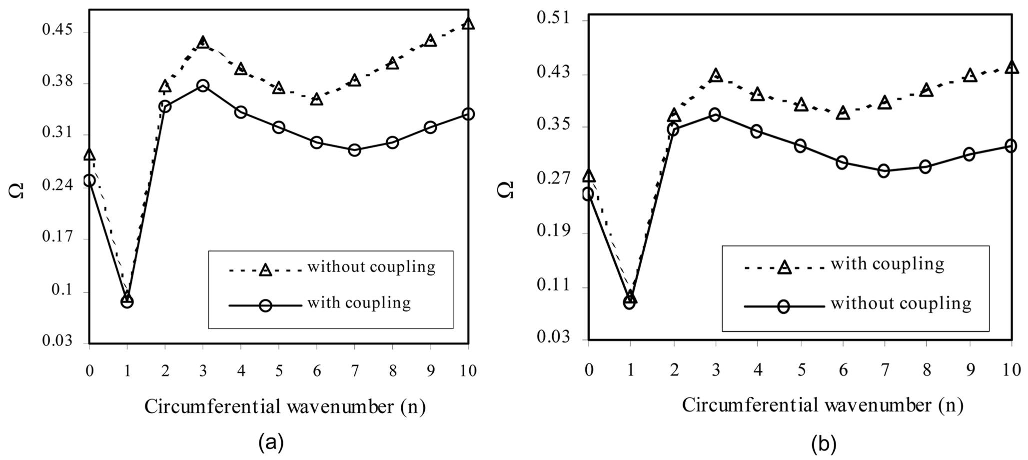

Fig. 4 Effect of extension-bending coupling on frequency parameter for C-C1 conical shells (a) $\alpha = 30$ (b) 45

parameter is uniformly increase with increasing the ratio $h/R2$.

For ratio $L/R2 = 0.5$, the influence of geometric ratio $h/R2$ on the relation between frequency parameter Ω is shown in Fig. 8 for four different cone angle, α. It can be concluded that the influence of $h/R2$ ratio on the relation between frequency parameter Ω and the cone angle is significant. It is shown that the increasing value of α always increases the frequency parameter Ω. It is also observed that the influence of boundary condition on the frequency parameter Ω with $h/R2$ is significant. Frequency parameters of (0/90/0) laminated conical shells with S-S boundary conditions for the ratio $L/R1 = 5$ is given in Fig. 9. The layer material properties are $\nu_{12} = 0.25$, $\nu_{22} = 0.25$, $E_{11}/E_{22} = 25$, $G_{12}/E_{22} = 0.5$; $G_{22}/E_{22} = 0.2$ These figures show the effects of the ratio $h/R1$ on the values of Ω. The variation is only marginal for larger value of n, irrespective of cone angles. For the cases under consideration, axisymmetric frequencies ($n = 0$) are not the lowest frequencies. The lowest frequencies occur for a higher value of n.

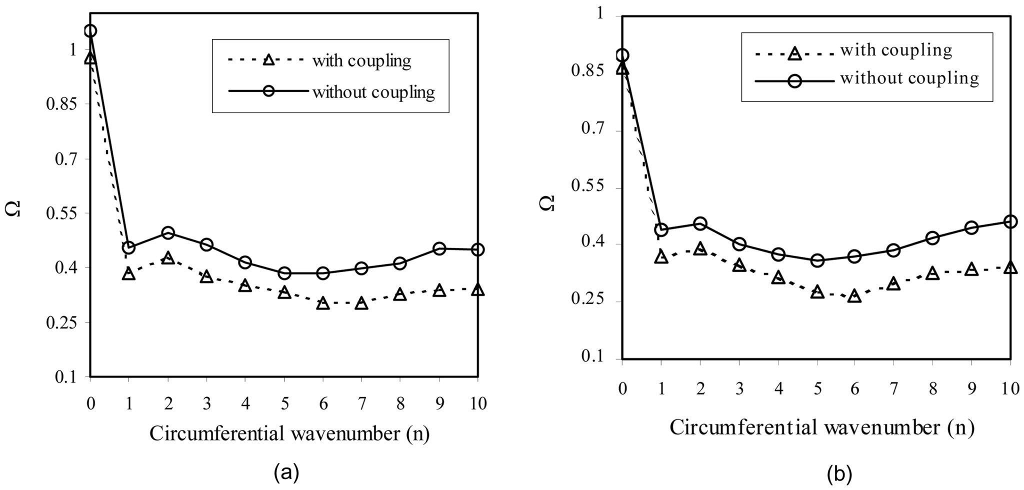

Fig. 5 Effect of extension-bending coupling on frequency for C-C2 conical shells (a) $\alpha = 30$ (b) 45

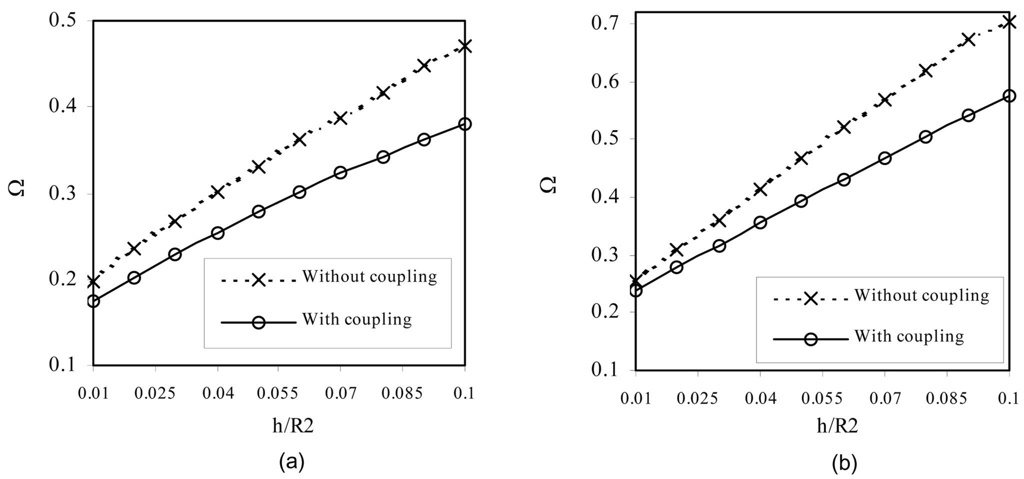

Fig. 6 Variation of frequency versus geometric ratio $h/R2$ for S-S conical shell (a) $\alpha = 30$ (b) $\alpha = 45$

5. Conclusions

The numerical solution of partial differential equations plays a considerable role in the areas of engineering. In many cases all that is desired is a moderately accurate solution at a few points which can be calculated rapidly. Therefore, an effective numerical technique for the solution of partial equations is very desirable. In seeking a more efficient numerical method that requires fewer grid points yet achieves acceptable accuracy, the method of DSC was introduced by Wei (1999).

The present paper focusses on the application of DSC method. Hence, free vibration of orthotropic conical shell problem is chosen. In conjunction with the DSC method, the free vibration of orthotropic laminated conical shells is presented. Typical numerical results are presented illustrating the effect of various geometric and material parameters. Convergence tests are performed to validate the proposed approach for handling various combinations of two types of boundary conditions. The cone angle α,

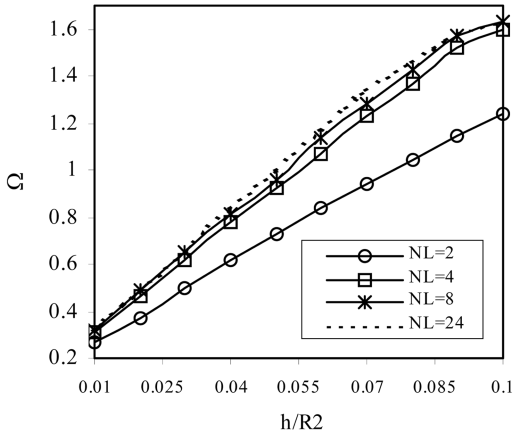

Fig. 7 Variation of frequency versus geometric ratio $h/R2$ for C-S conical shell ($\alpha = 30$, $L\sin\alpha / R2 = 0.25$)

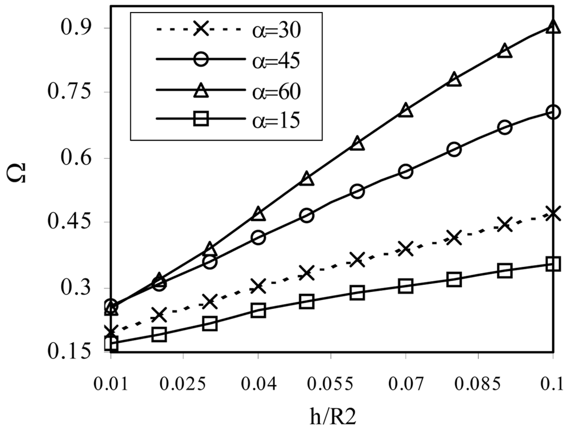

Fig. 8 Variation of frequency versus geometric ratio $h/R2$ for various cone angles of S-S conical shell ($L/R2 = 0.5$; $h/R2 = 0.01$; $\nu = 0.3$)

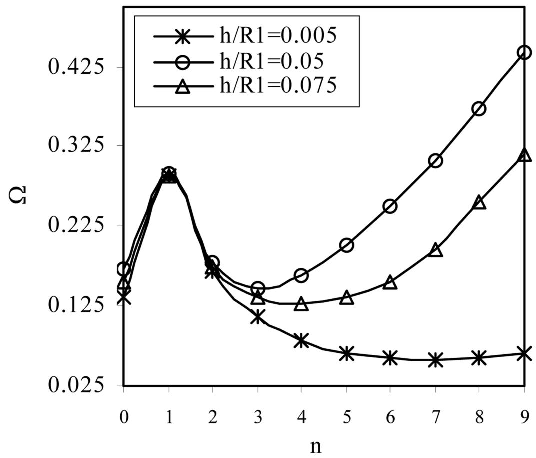

Fig. 9 Frequency parameters of (0/90/0) laminated conical shells with S-S boundary conditions ($L/R1$=5; $\alpha = 30$)

$L\sin\alpha/R_2$ ratio and orthotropic parameter s has been found to have significant influence on the frequency parameters of the conical shell. It is also shown from numerical examples that the boundary condition has a great effect on the frequency characteristics of orthotropic conical shell and that such effect is significant for the case of small circumferential wave number or low rotating geometric parameter or orthotropic parameter.

Several test examples have been selected to demonstrate the convergence properties, accuracy and simplicity in numerical implementation of DSC procedures. This has verified the accuracy and applicability of the DSC method to the class of problem considered in this study. The discretizing and programming procedures are straightforward and easy. Furthermore, the known boundary conditions are easily incorporated in the DSC. Numerical results indicate that the DSC is a simple and reliable method for free vibration analysis of isotropic and orthotropic conical shells.

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