

# Sway of semi-rigid steel frames - Part 1: Regular frames

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**Abstract.** Lateral sway is most likely to control the design of semi-rigid steel frames where the frame arrangements do not include any form of bracing. This paper investigates the sway behaviour of semi-rigid regular steel frames i.e., frames having the same arrangement of beam and column sections at all levels, and hence proposes some design charts for the prediction of sway that eliminate the need for doing any numerical modelling. Schueller's equation has also been modified to incorporate connection flexibility in addition to its original rigid frame considerations. All the proposed methods have been validated using results obtained from numerical analysis.

**Key words:** ANSYS; connection stiffness; corresponding regular frame; finite element analysis; flexibility factor; multi-storey frames; semi-rigid connection; sway.

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## 1. Introduction

Limiting the lateral sway of building frames is an important design consideration, particularly in those cases for which so-called frame action is relied upon to provide the necessary lateral stiffness. For arrangements that do not include any form of bracing, it is the inherent bending stiffness of the beams and columns themselves that provides the necessary lateral stiffness. In recent years, there has been a growing realisation that a further structural component also contributes some degree of flexibility - both the connections between individual beams and columns and arrangements at the base of the columns where joints are made to the foundation system are now recognised as structural members with their own strength and stiffness. Estimating the lateral sway of frame structures, including the making of realistic allowances for the contributions of the connections, is therefore an important design issue.

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Techniques for analysing frames incorporating semi-rigid joint effects have developed significantly during the last two decades. Much of this work has recently been reviewed (Chan and Chui 2000, Chen *et al.* 1996) and only a few particularly relevant contributions are covered herein. Huang and Morris (1991) conducted a computer analysis on bare steel frames to study the effect of connection properties on lateral sway. The connection moment-rotation relationship was taken from test data. Colson and Bjorhovde (1991) analysed a two storey, two span bare steel frame assuming pin connections for the exterior beam-to-column connections and semi-rigid behaviour for the interior ones using the computer program PEP-Micro. Deierlein (1991) studied semi-rigid bare steel frames using the computer program CU-STAND. All the beam-to-column connections were modelled using zero-length rotational springs to account for their non-linear moment-rotation behaviour. Li *et al.* (1995) analysed a two-span, two-storey semi-rigid bare steel frame using the general-purpose finite element software ABAQUS. They studied the effect of connection length and connection stiffness on the moments at different sections. Ye *et al.* (1996) proposed a finite element model for composite frames using the general purpose software ABAQUS.

It is, of course, possible to utilise a comparatively rigorous approach in which a computer based frame analysis that explicitly allows for the contributions of the joints is undertaken. However, as with all computerised approaches, initial values must be assigned to the key member properties before such a study can be undertaken. It is, of course, helpful if such values can be selected as being realistic in the sense that they are close to those likely to be the result of the final design decisions.

When checking deflections under serviceability conditions, it is important to recognise two features:

- (i) Such calculations are normally undertaken assuming elastic behaviour.
- (ii) The essential design requirement is that the calculated deflection be not greater than the permissible limit i.e., great precision in undertaking the analysis is not necessary unless the calculations produce a result that is very close to the limiting value.

Methods that utilise the principles of elastic analysis in an approximate fashion are therefore likely to be useful when attempting to arrive at an appropriate overall balance between member sizes and joint configuration.

Ammerman and Leon (1990), Ahmed (1996) and Ahsan (1997) proposed some methods for sway prediction of semi-rigid frames. The proposals made by Ammerman and Leon (1990) and Ahmed (1996) were obtained using some specific test data and were applicable only to low-rise frames. Ahsan's (1997) proposed method is applicable only to medium-rise frames ranging from 5 to 8 storeys high. So a more general approach is warranted to predict the sway of semi-rigid frames.

This paper investigates the sway behaviour of semi-rigid steel frames using the Finite Element (FE) package ANSYS V5.4 (1998) to generate results that are then used as the basis for a simplified method for sway prediction. The proposed method is based on design charts that result from a thorough investigation of the key parameters that affect the sway response of semi-rigid frames. Modifications are also made to the approach, originally proposed by Schueller (1977), for estimating the lateral sway of frames with rigid joints, that permits explicit allowance to be made for the influence of the flexibility of the beam-to-column connections. A subsequent paper [Ashraf *et al.* (7)] further extends the approach to deal with the influence of column base effects.

The accuracy of the proposed methods is demonstrated by comparisons against a portfolio of numerical results for a representative set of structures. These have been obtained using the ANSYS package.

## 2. Numerical modelling technique of semi-rigid frame behaviour and its verification

In the present study, a general-purpose finite element program ANSYS V5.4 has been used to model semi-rigid frames. The beams and columns were modelled using 'BEAM3 - 2D Elastic Beam' elements while a rotational spring element 'COMBIN39 - Non-linear spring' was used to model the beam-to-column connections. This spring has the capability to incorporate the non-linear moment-rotation behaviour of connections. The column bases were modelled as fixed.

The developed numerical models have been extensively verified by Ashraf (2001) using all available previous research. The following sections present selected comparisons of the developed models with some previously reported results.

### 2.1. Frame analysed by Lui and Chen (1988)

Lui and Chen (1988) analysed a two storey single bay frame considering both rigid and semi-rigid beam-to-column connections to verify their proposed method for the analysis of sway frames. The moment-rotation behaviour of the extended end plate connection, tested by Johnson and Walpole (1981), was used in the semi-rigid case. The beams and columns were W14×48 and W12×96 sections respectively. Axial compressive loads  $P$  were applied to the top of each of the columns and small lateral forces -  $0.001P$  and  $0.002P$  to the top and bottom storey respectively - were applied to induce sway. This frame was modelled using ANSYS and the results are compared in Figs. 1 and 2.

### 2.2. Sway frames analysed by Ahmed (1996)

Ahmed (1996) analysed a number of semi-rigid sway frames - ranging from one to four storeys and one to two bays to verify his proposed equation. Both the beams and columns were modelled as 203×203 UC 46 a universal column section. The stiffness of the beam-to-column connections was varied from almost 0 (pin connection) to extremely large (rigid connection). A single lateral load was applied at the top of each of the frames. These frames were also modelled using ANSYS as a part of the present study and Fig. 3 shows a comparison of results for two frames described by Ahmed (1996).

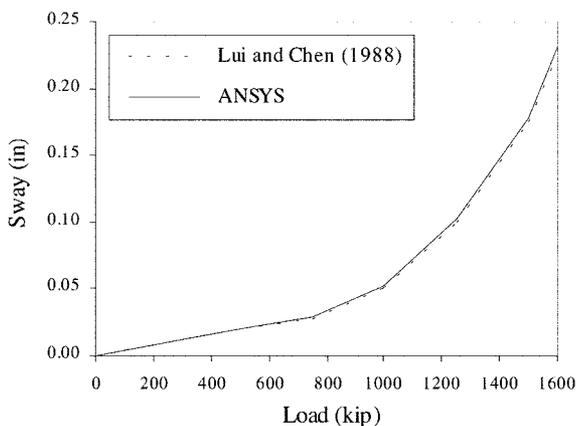


Fig. 1 Comparison for rigid frame analysed by Lui and Chen (1988)

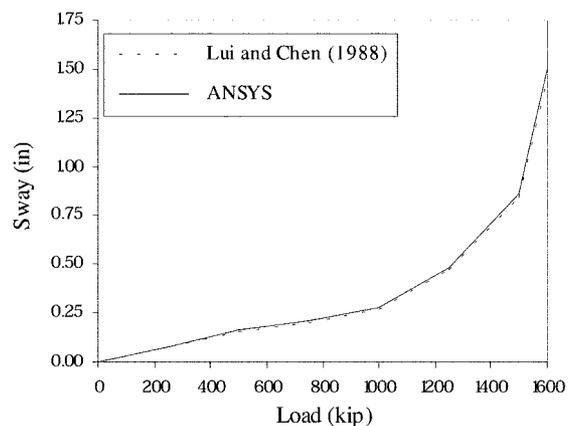


Fig. 2 Comparison for semi-rigid frame analysed by Lui and Chen (1988)

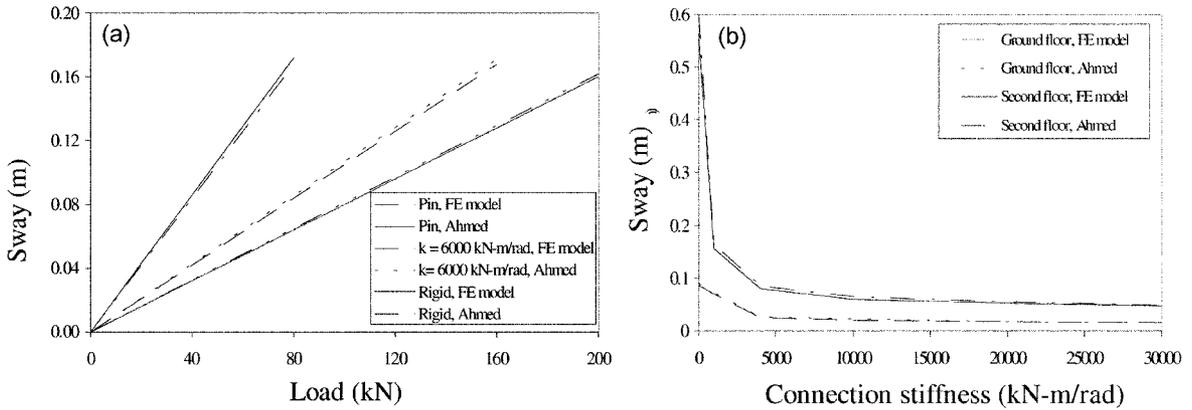


Fig. 3 (a) Comparison for 1 storey 1 bay frame analysed by Ahmed (1996), (b) Comparison for 3 storey 1 bay frame analysed by Ahmed (1996)

Detailed verification of the developed models is available in the M.Sc. thesis by Ashraf (2001). All the analysis and comparison showed that the developed numerical models can accurately represent the response of semi-rigid frames, thereby providing confidence for their use in the parametric studies reported herein.

### 3. Objective and methodology of the present study

Proper understanding of the frame response to various factors is very important in order to make accurate predictions of sway. Frames having a wide range of variation in cross-sectional and geometrical properties were analysed to reveal the key parameters affecting the sway behaviour. The following sections describe the frames considered in the parametric studies reported in this paper.

#### 3.1. Geometric dimensions of the frames

Frames ranging from 5 to 30 storeys, grouped into six categories as shown in Table 1, were considered in the present study. The number of bays was varied from two to five. Unless otherwise specified the bay size and storey height were 6 and 3 metres respectively. In all cases, a representative intermediate frame was taken for analysis from a larger 3D configuration.

Table 1 Beam and column sections used in analyses

Group	No. of storeys	No. of bays	Beam section	Column section	$K_c / K_b$
I	5	2 to 5	254 × 102 UB 25	203 × 203 UC 46	2.68
II	10	2 to 5	254 × 102 UB 25	203 × 203 UC 71	4.50
III	15	2 to 5	305 × 127 UB 37	203 × 203 UC 86	2.64
IV	20	2 to 5	305 × 127 UB 37	254 × 254 UC 132	6.28
V	25	2 to 5	356 × 171 UB 45	254 × 254 UC 167	4.96
VI	30	2 to 5	356 × 171 UB 45	305 × 305 UC 240	10.65

Table 2 Working load considered in frame analyses

Gravity load (kN/m <sup>2</sup> )		Storey no.												
		Wind load (kN/m <sup>2</sup> )												
DL	LL	5	6	7	8	9	10	11	12	13	14	15	16	17
		1.84	1.96	2.06	2.15	2.23	2.31	2.39	2.47	2.52	2.58	2.65	2.72	2.77
3.25	3	18	19	20	21	22	23	24	25	26	27	28	29	30
		2.82	2.87	2.92	3.08	3.08	3.08	3.22	3.22	3.22	3.35	3.35	3.35	3.35

### 3.2. Member properties

Table 1 lists the beam and column sections used (unless otherwise specified) in the parametric studies. The corresponding ratio of column to beam stiffness  $K_c / K_b$  is also listed in the table.

### 3.3. Connection details

Since the main objective of the present study is to examine the influence of the semi-rigid connections on sway, a wide spectrum of connection stiffness was considered. In parametric studies, the beam stiffness  $K_b$  was usually kept constant while connection stiffness  $K_j$  was varied, giving  $K_b / K_j$  ratios varying from 0.125 to 4.0. A linear constant connection stiffness  $K_j$  was used, though the actual  $M-\phi$  relationship is non-linear. A subsequent paper describes the method to predict a constant  $K_j$  from the non-linear  $M-\phi$  behaviour of connections [Ashraf *et al.* (8)].

### 3.4. Loading on frames

The frames were analysed under the working load conditions presented in Table 2. The loading was determined on the basis of the Bangladesh National Building Code, BNBC (1993). While calculating the wind load intensities the type of occupancy of the building was considered as general office, representing a standard occupancy structure for which the structure importance coefficient  $C_I$  is 1.0. The exposure category was assumed to be urban and sub-urban area which is represented as 'Exposure Category A' in BNBC. The basic wind speed was considered as 210 km/h.

## 4. Parametric studies

### 4.1. Introduction

Ahsan (1997) observed that the ratio of column to beam stiffness  $K_c / K_b$  has a significant effect on the sway of steel frames. This section is aimed at identifying the other key parameters that affect the sway response. Most of the parametric studies were carried out for the group of frames given in Table 1 having a constant  $K_c / K_b$  ratio for a specific number of stories.

The basic concept of this study is to establish a relationship between the sway of a semi-rigid frame and that of its corresponding rigid frame. If such a relationship can be devised then the sway of semi-rigid frames could be predicted directly by extending the simplified method already proposed by Ashraf

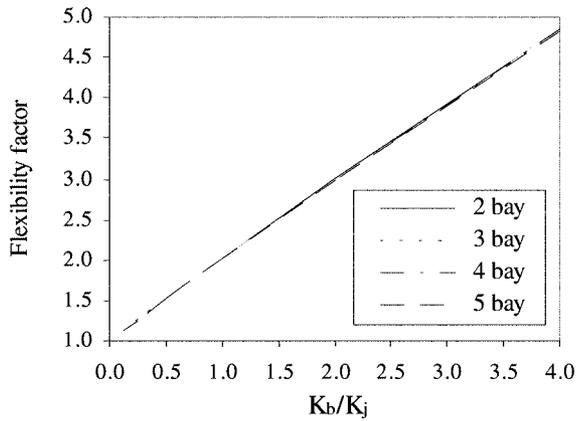


Fig. 4 Sway behaviour of 15 storey frames having different bays

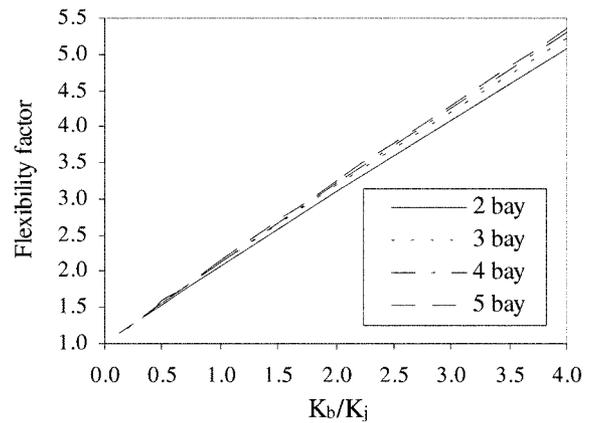


Fig. 5 Sway behaviour of 25 storey frames having different bays

*et al.* [6] to predict the sway of rigid frames. With this point in mind a new term ‘Flexibility Factor (FF)’ is introduced herein to consider the ratio of the maximum lateral sway of the semi-rigid frame to that of the ‘corresponding rigid frame’ i.e.,  $FF = \Delta_{semi-rigid} / \Delta_{rigid}$ . This definition has been used throughout this paper.

#### 4.2. Effect of number of bay on sway

Frames defined by six distinct groups according to the number of stories as specified in Table 1 were analysed. For the first three groups - 5, 10 and 15 storey frames - no significant change was observed in sway behaviour even though the number of bays was varied from 2 to 5. The remaining three groups - 20, 25 and 30 storey frames - exhibited almost identical behaviour except for the 2-bay frames. Figs. 4 and 5 show the behaviour of 15 and 25 storey frames respectively.

The limiting values of FFs for the different groups are listed in Table 3. From this table, it is observed that the number of bay affects only the 2-bay frames with more than 15 storeys. The effect is not so significant though i.e., a maximum of 6.5% in the case of the 30 storey frames.

Table 3 Average values of FFs for different groups

Group	No. of storeys	No. of bay	FF		$K_c / K_b$
			$K_b / K_j = 0.125$	$K_b / K_j = 4.0$	
I	5	2-5	1.12	3.87	2.68
II	10	2-5	1.14	4.80	4.50
III	15	2-5	1.13	4.84	2.64
IV	20	2	1.15	5.24	6.28
		3-5	1.15	5.36	
V	25	2	1.14	5.07	4.96
		3-5	1.15	5.32	
VI	30	2	1.15	5.22	10.65
		3-5	1.16	5.56	

Table 4 Frames considered to study the effect of  $L_b/L_c$  ratio on sway

Number of storey	Number of bay	$K_c/K_b$	$L_b/L_c$	$I_c/I_b$
20	4	5.00	1.00	5.00
			1.50	3.33
			2.00	2.50
25	4	5.00	1.00	5.00
			1.50	3.33
			2.00	2.50
30	4	7.50	1.00	7.50
			1.50	5.00
			2.00	3.75

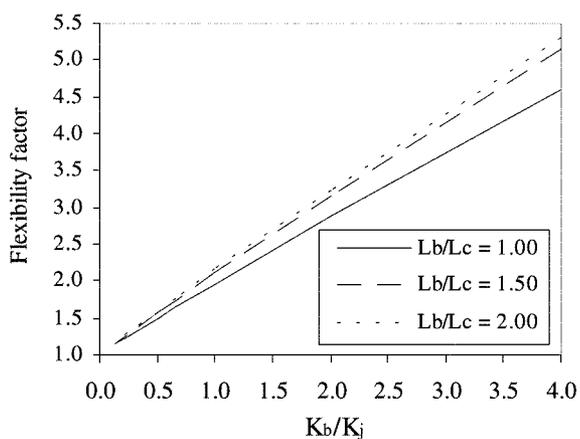


Fig. 6 Sway behaviour of 20 storey 4 bay frames having different  $L_b/L_c$  ratio

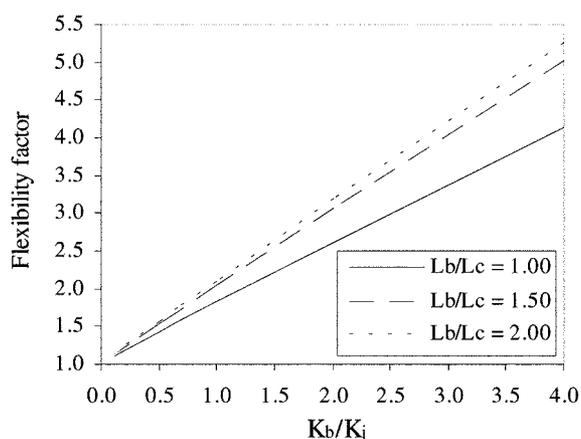


Fig. 7 Sway behaviour of 25 storey 4 bay frames having different  $L_b/L_c$  ratio

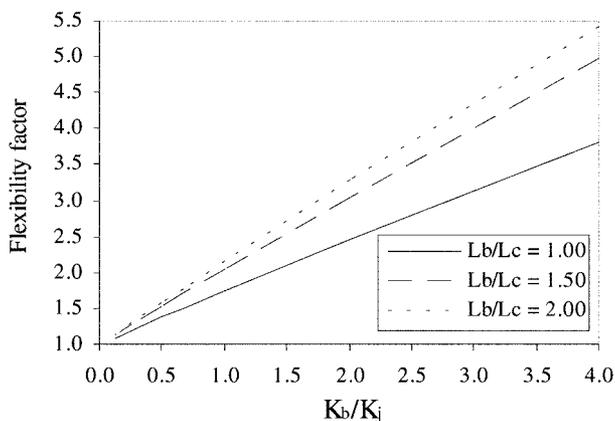


Fig. 8 Sway behaviour of 30 storey 4 bay frames having different  $L_b/L_c$  ratio

#### 4.3. Effect of beam length to column height ratio $L_b/L_c$ on sway

Frames having the same  $K_c/K_b$  ratios can possess different ratios of  $L_b/L_c$  and  $I_b/I_c$ . This section studies the effect of changes in  $L_b/L_c$  ratio (and  $I_b/I_c$  ratio) keeping the  $K_c/K_b$  ratio constant for any specific group. The frames considered for this particular study are given in Table 4. In all these cases, column height  $L_c$  was kept constant (equal to 3 meters) while the beam length  $L_b$  was varied from 3.0 to 6.0 meters.

Figs. 6 to 8 represent the behaviour of the frames described in Table 4. From these figures, it is observed that the  $L_b/L_c$  ratio (as well as the  $I_c/I_b$  ratio) has a pronounced effect on the sway of these frames. Five, ten and fifteen storey frames were also studied with varying  $L_b/L_c$  ratio but the behaviour was almost the same in all the cases. From Figs. 6 to 8, it is observed that the influence of  $L_b/L_c$  decreases significantly as this ratio rises above 1.5. Two different lines can be used to obtain the FFs of frames having different  $L_b/L_c$  ratios:  $L_b/L_c=1.0$  and  $L_b/L_c \geq 1.5$ .

This section shows that the FFs of taller frames are affected by the beam span to column height ratio  $L_b/L_c$  even though the  $K_c/K_b$  ratio remains the same. The following two sections investigate whether the individual values of  $L_b$  and  $L_c$  or  $I_c$  and  $I_b$  affect the FF even if  $L_b/L_c$  and  $I_c/I_b$  remain the same.

#### 4.4. Effect of beam span $L_b$ and column height $L_c$ on sway

Two separate groups of frames were studied in this section. The first group had  $L_b/L_c=2.0$  ( $L_b=5, 6$  and  $7$  m with  $L_c=2.5, 3.0$  and  $3.5$  m respectively) and the second group had  $L_b/L_c=1.5$  ( $L_b=3.75, 4.5$  and  $5.25$  m with  $L_c=2.5, 3.0$  and  $3.5$  m respectively). The observed behaviour is given in Figs. 9 and 10. The characteristic straight lines for different frames almost overlap each other in these figures. Thus, it can be concluded that FFs for a particular semi-rigid frame are not affected by the individual values of  $L_b$  and  $L_c$  but by their ratio  $L_b/L_c$ .

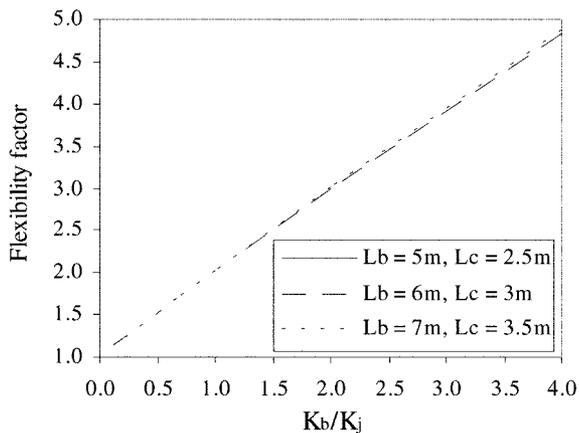


Fig. 9 Sway behaviour of 15 storey 4 bay frames with different  $L_b$  and  $L_c$

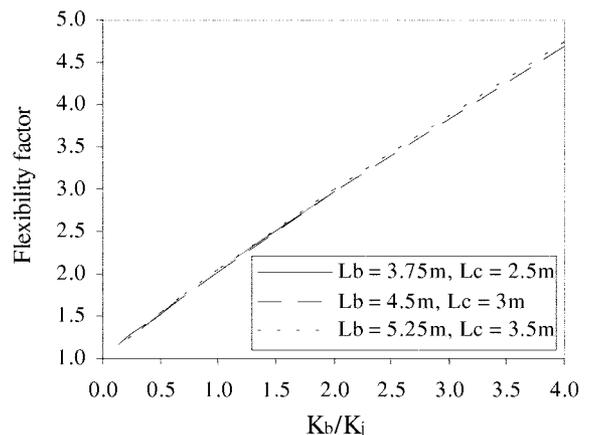


Fig. 10 Sway behaviour of 10 storey 3 bay frames with different  $L_b$  and  $L_c$

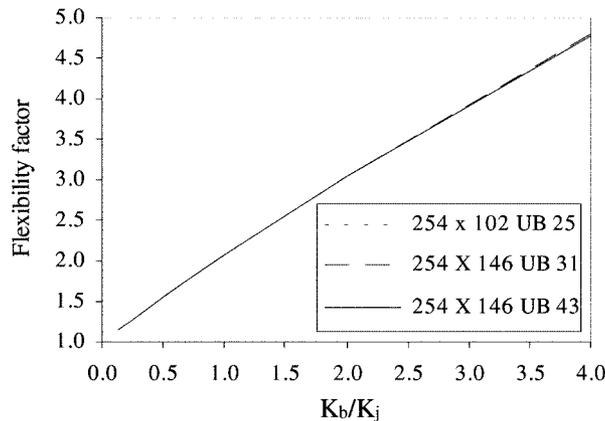


Fig. 11 Sway behaviour of 10 storey 4 bay frames with different beam sections

4.5. Effect of individual beam and column sections on sway

Three different cases were chosen for a 10 storey 4 bay frame by changing the beam and column sections but keeping the  $I_c / I_b$  ratio constant. It is worth mentioning that the  $L_b / L_c$  ratio and hence the  $K_c / K_b$  ratio were the same for all three cases. The obtained behaviour is presented in Fig. 11.

This clearly shows that FFs for a particular frame are not affected by the individual beam and column sections as long as the ratio of  $I_c / I_b$  remains constant.

5. Design charts for calculating flexibility factors

Sway behaviour of steel frames is largely affected by the  $K_c / K_b$  ratio (as established by Ahsan 1997). The parametric studies carried out in the previous section revealed some other parameters - number of bays, number of storeys and the ratio of beam length to column height  $L_b / L_c$  - that affect the sway behaviour of comparatively taller frames i.e., frames higher than 15 storeys. This leads to the following simplified design proposal.

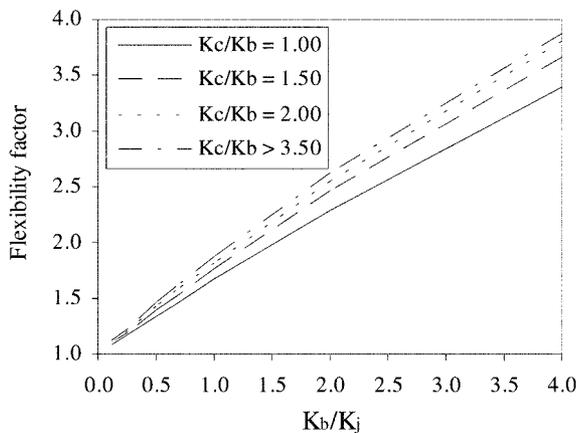


Fig. 12 Design chart for 5 storey frames

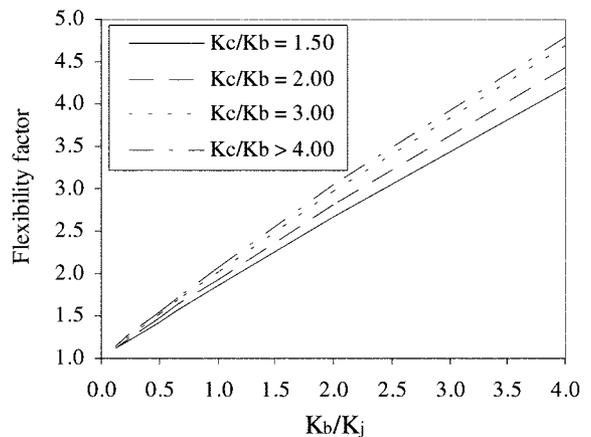


Fig. 13 Design chart for 10 storey frames

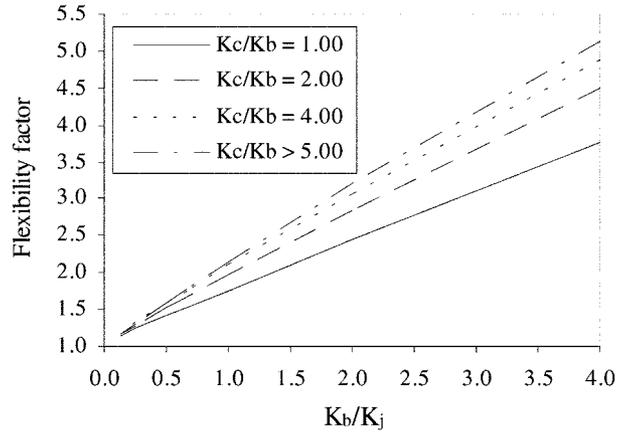
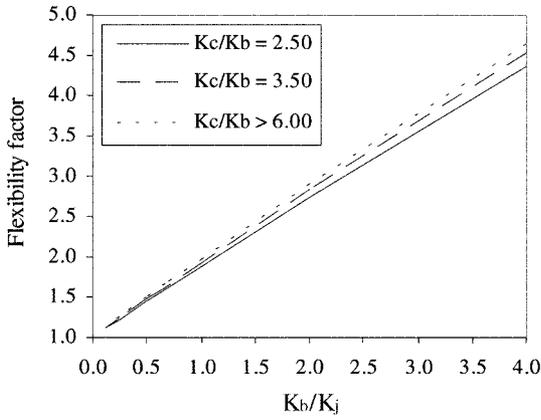
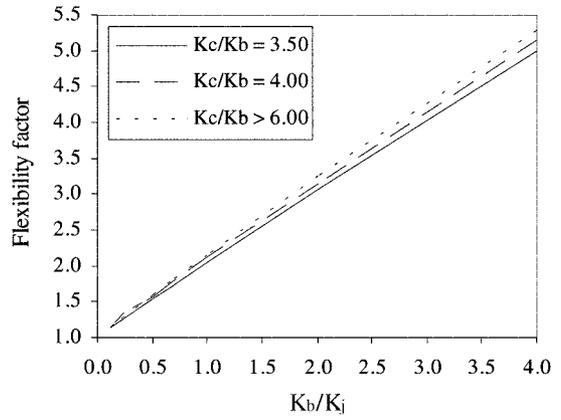


Fig. 14 Design chart for 15 storey frames

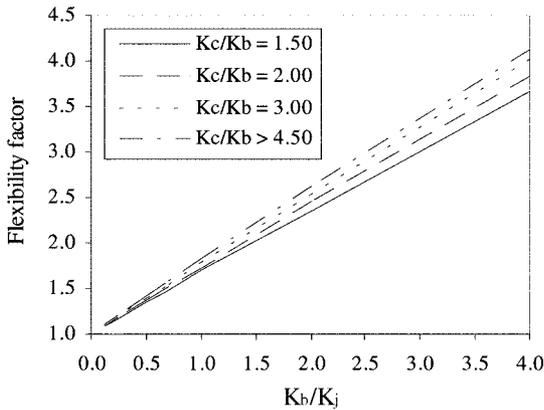


(a) Frames with  $L_b/L_c = 1.0$

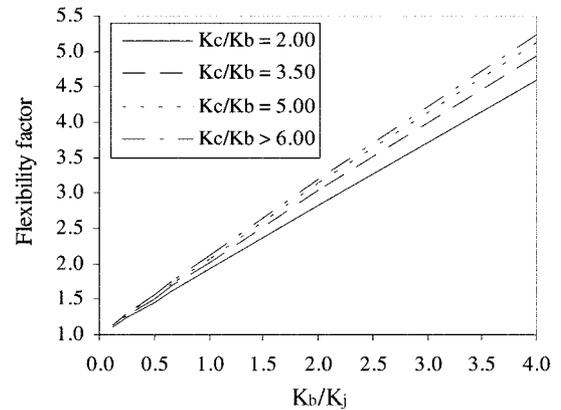


(b) Frames with  $L_b/L_c \geq 1.5$

Fig. 15 Design charts for 20 storey frames



(a) Frames with  $L_b/L_c = 1.0$



(b) Frames with  $L_b/L_c \geq 1.5$

Fig. 16 Design charts for 25 storey frames

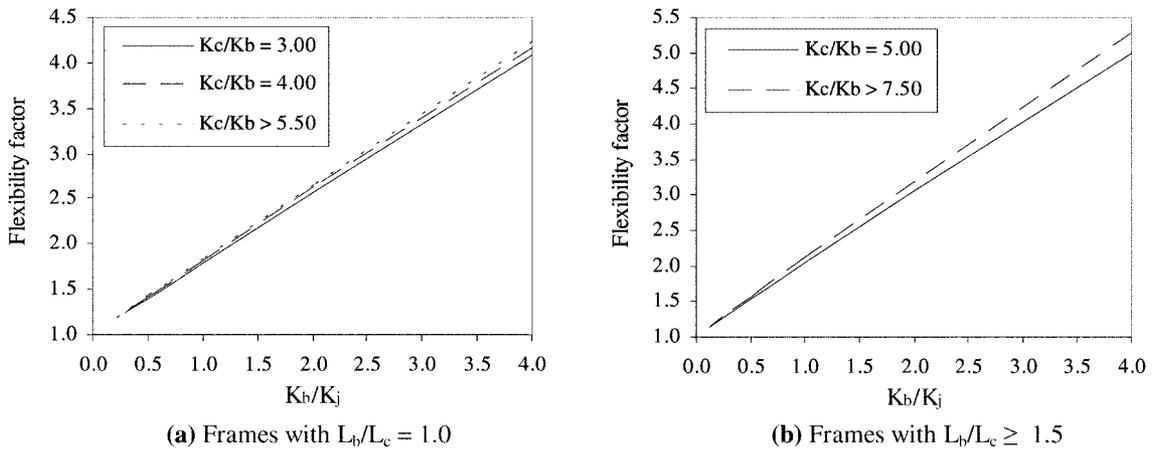


Fig. 17 Design charts for 30 storey frames

Frames grouped into six categories according to the number of storeys, Table 1, were analysed to reveal the sway pattern for all possible combinations. Figs. 12 to 17 show the sway behaviour of semi-rigid frames ranging from 5 to 30 storeys high. These figures give the FF for a particular semi-rigid frame and hence the sway can be determined by knowing the sway of its corresponding rigid frame. The design steps are described in the following section.

## 6. Design steps for the sway prediction of a semi-rigid frame

### 6.1 Step 1: Determination of FF

Figs. 12 to 17 give the FF for a particular semi-rigid frame. Their use is described below in a step by step manner.

- (i) The number of storeys and number of bays for the given frame are obtained from the frame geometry.
- (ii) By knowing the values of  $L_b/L_c$  and the number of storeys, the appropriate design chart(s) should be selected.
- (iii) The  $K_c/K_b$  value for the given frame is determined using the following equation:

$$\frac{K_c}{K_b} = \frac{I_c L_b}{I_b L_c} \quad (1)$$

- (iv) Knowing the value of  $K_b/K_j$ , the FF(s) can be determined using the design chart(s).
- (v) When determining FFs from the design chart(s) linear interpolation(s), if necessary, should be made in the following order:
  - a.  $K_c/K_b$  ratio
  - b. Number of storeys
  - c.  $L_b/L_c$  ratio

## 6.2. Step 2: Determination of $\Delta_{rigid}$

The FF establishes a relationship between a semi-rigid frame and its corresponding rigid frame. To obtain the sway of a semi-rigid frame  $\Delta_{semi-rigid}$  the sway of its corresponding rigid frame  $\Delta_{rigid}$  needs to be determined first.

$\Delta_{rigid}$  can be obtained by using readily available FE packages which can analyse rigid frames. Otherwise the following equation, proposed by Schueller (1977), can be used.

$$\Delta_{rigid} = \frac{HV_c h^2}{12EI_c} + \frac{HV_g L^2}{12EI_g} + \frac{2N_c H^2}{3EA_c B} \quad (2)$$

When using this equation the level of column shear  $V_c$  should be obtained using Fig. 18. This modification to this equation was proposed by Ashraf *et al.* (2004) and was found to give predictions very close to those obtained from FE analysis.

The symbols used in Eq. (2) have the following meanings:

$N_c$ =Axial force in exterior column at the base due to wind

$V_c$ =Shear force in exterior column due to wind at the level specified by Fig. 18

$I_c$ =Moment of inertia of column at the same level as  $V_c$  about axis of bending

$A_c$ =Area of the exterior column at the base

$V_g$ =Shear force in girders due to wind at the same level as  $V_c$

$I_g$ =Moment of inertia of girder about x-axis at the same level as  $V_c$

$H$ =Total height of the frame

$B$ =Total base width of the frame

$h$ =Typical story height

$L$ =Girder span

$E$ =Modulus of elasticity

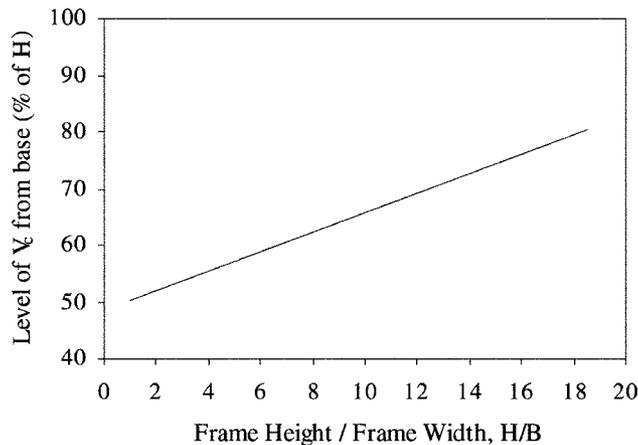


Fig. 18 Level of column shear  $V_c$  to be considered in Schueller's equation

### 6.3. Step 3: Determination of $\Delta_{semi-rigid}$

Finally,  $\Delta_{semi-rigid}$  can be obtained by multiplying the FF as obtained in Step 1 by  $\Delta_{rigid}$  obtained from Step 2.

## 7. Sway prediction using modified Schueller's equation

Schueller's equation was originally proposed for the sway prediction of rigid frames considering the bending of beams and columns and axial deformation of columns. In the case of semi-rigid frames, an additional term is required to take into account the rotation of the connections ( $\phi$ ). If Schueller's equation is modified to incorporate connection rotations the general form of the equation will be:

$$\Delta_{semi-rigid} = \left( \frac{HV_c h^2}{12EI_c} + \frac{HV_g L^2}{12EI_g} + \frac{2N_c H^2}{3EA_c B} \right) + \frac{MH}{K_j} \quad (3)$$

But under the working load the moments and rotations induced in the connections vary according to their location within the frame. A representative location is required in a similar way to the situation for column shear  $V_c$  as described by Fig. 18. In the present study, it was observed that if the rotation is considered at the same level as  $V_c$ , the predictions remain within the 10% of FE analysis. Moments at this level can be taken as  $V_g L/2$  and the corresponding rotation  $\phi$  will be  $V_g L/(2K_j)$ . Finally, sway due to this rotation can be considered as  $V_g LH/(2K_j)$ . So Eq. (3) can be replaced by Eq. (4) which can be used to predict sway of semi-rigid frames.

$$\Delta_{semi-rigid} = \left( \frac{HV_c h^2}{12EI_c} + \frac{HV_g L^2}{12EI_g} + \frac{2N_c H^2}{3EA_c B} \right) + \frac{V_g LH}{2K_j} \quad (4)$$

It is worth mentioning that Schueller (1977) proposed the terms enclosed within the bracket for rigid frames, while the fourth term is proposed from the present study as a way of incorporating the effect of connection flexibility.

## 8. Illustrations and comparisons

The design steps presented in Section 6 are illustrated here using worked examples. Two completely different frames were considered to cover all the relevant features of the methods, especially interpolation when using more than one design chart for sway prediction.

**Example 1:** A 12 storey, 3 bay regular frame with typical storey height of 3 m and bay size of 6 m is subjected to a gravity load of 37.5 kN/m. Wind pressure acting on the frame has an intensity 1.5 times higher than the typical BNBC load as mentioned in Table 2. If the beam and column sections are 305×102 UB 28 and 203×203 UC 86 respectively and the connection stiffness  $K_j$  is 5600 kN-m/rad, calculate the sway of this frame.

#### Determination of key parameters

$$L_b/L_c = 6.0/3.0 = 2.00$$

$$K_c/K_b = I_c/I_b \times L_b/L_c = 9462/5415 \times 6.0/3.0 = 3.50$$

$$K_b/K_j = 4 \times (2.07 \times 10^8) \times (5415 \times 10^{-8}) / (6.0 \times 5600) = 1.33$$

#### Selection of design charts

Since the frame is 12-storey, interpolations must be made between 10 and 15 storey frames. So the design charts of Figs. 13 and 14 are used:

#### Determination of FFs

From Fig. 13, for 10 storey frames with  $K_b/K_j = 1.33$ , FFs are 2.35 and 2.41 for  $K_c/K_b = 3.0$  and 4.0 respectively. So for  $K_c/K_b = 3.50$ , FF = 2.38.

Similarly using Fig. 14, for 15 storey frames with  $K_b/K_j = 1.33$ , FFs are 2.27 and 2.43 for  $K_c/K_b = 3.0$  and 4.0 respectively. So for  $K_c/K_b = 3.50$ , FF = 2.39.

So, for a 12 storey frame with  $K_c/K_b = 3.50$ , FF = 2.384.

#### Determination of $\Delta_{rigid}$

For Schueller's equation the magnitudes of the various parameters are:

$$h = 3.0 \text{ m}; H = 12 \times 3.0 = 36 \text{ m}$$

$$L = 6.0 \text{ m}; B = 6.0 \times 3 = 18.0 \text{ m}$$

$$I_c = 9462 \text{ cm}^4, I_g = 5415 \text{ cm}^4, A_c = 110.1 \text{ cm}^2$$

For this frame,  $H/B = 36.0/18.0 = 2.00$

So, from Fig. 18, it is observed that  $V_c$  should be considered at 0.5 2H from the base. Considering this fact, the following values were obtained from the portal method,

$$N_c = 700.39 \text{ kN}, V_c = 61.28 \text{ kN}, V_g = 56.63 \text{ kN}$$

So, finally  $\Delta_{rigid}$  was found to be 64.48 cm.

#### Determination of $\Delta_{semi-rigid}$

For the given frame,  $\Delta_{rigid} = 64.48 \text{ cm}$  and FF = 2.384

So,  $\Delta_{semi-rigid} = 64.48 \times 2.384 = 153.72 \text{ cm}$

**Example 2:** An 18 storey, 4 bay regular frame with typical storey height of 3 m and bay size of 5 m is subjected to a gravity load of 37.5 kN/m. Wind pressure acting on the frame is as given in Table 2. If the beam and column sections are 356×127 UB 39 and 305×305 UC 97 respectively and the connection stiffness  $K_j$  is 14980 kN-m/rad, calculate the sway of this frame.

#### Determination of key parameters

$$L_b/L_c = 5.0/3.0 = 1.67$$

$$K_c/K_b = I_c/I_b \times L_b/L_c = 22202/10054 \times 5.0/3.0 = 3.68$$

$$K_b/K_j = 4 \times (2.07 \times 10^8) \times (10054 \times 10^{-8}) / (5.0 \times 14980) = 1.11$$

#### Selection of design charts

Since the frame has 18 storeys and its  $L_b/L_c > 1.50$ , interpolations should be made between 15 and 20 storey frames having  $L_b/L_c > 1.50$ . So design charts shown in Figs. 14 and 15(b) should be used.

Table 5 Comparison among the predictions obtained from proposed methods

Description of the frame	FE Results (cm)	Design charts		Eq. (5)	
		Sway (cm)	Variation with FE analysis	Sway (cm)	Variation with FE analysis
12 storey 3 bay	153.83	153.72	-0.07%	173.69	+12.91%
18 storey 4 bay	87.11	86.31	+0.92%	93.64	+7.50%

#### Determination of FFs

From Fig. 14, for 15 storey frames with  $K_c/K_b = 2.0$  and  $4.0$ , FFs are 1.95 and 2.11 respectively. So for  $K_c/K_b = 3.68$ , FF = 2.084.

From Fig. 15(b), for a 20 storey frame with  $K_c/K_b = 3.5$  and  $4.0$ , FFs are 2.18 and 2.25 respectively. So for  $K_c/K_b = 3.68$ , FF = 2.205.

So, for an 18 storey frame with  $K_c/K_b = 3.68$ , FF = 2.16.

#### Determination of $\Delta_{rigid}$

For Schueller's equation the magnitudes of the various parameters are:

$$h = 3.0 \text{ m}; H = 18 \times 3.0 = 54 \text{ m}$$

$$L = 5.0 \text{ m}; B = 5.0 \times 4 = 20.0 \text{ m}$$

$$I_c = 22202 \text{ cm}^4, I_g = 10054 \text{ cm}^4, A_c = 123.3 \text{ cm}^2$$

For this frame,  $H/B = 54.0/20.0 = 2.70$

So, from Fig. 18, it is observed that  $V_c$  should be considered at  $0.5 \text{ } 4H$  from the base. Considering this fact, the following values were obtained from the portal method,

$$N_c = 1073.12 \text{ kN}, V_c = 41.79 \text{ kN}, V_g = 59.56 \text{ kN}$$

So, finally  $\Delta_{rigid}$  was found to be 39.96 cm.

#### Determination of $\Delta_{semi-rigid}$

For the given frame,  $\Delta_{rigid} = 39.96 \text{ cm}$  and FF = 2.16.

So,  $\Delta_{semi-rigid} = 39.96 \times 2.16 = 86.31 \text{ cm}$ .

These frames were also analysed by using Eq. (4). Predictions obtained from this equation and those obtained using design charts are compared with the FE analysis in Table 5. Comparisons show that the design chart predictions are very accurate, while the proposed equation gives slightly conservative results. Further study is required to find out the exact location of the representative rotation to make the predictions from the equation closer to FE analysis.

## 9. Conclusions

The key parameters affecting the sway behaviour of semi-rigid regular steel frames are identified and hence design charts are proposed to predict the sway of such frames. Schueller's equation, originally proposed for rigid frames, has also been modified to incorporate the effect of connection stiffness and proposals are made to add a new term to the equation. All the proposed methods are explained using

worked examples, with results being compared against the FE results. Comparisons show that the design charts accurately predict the sway of a semi-rigid frame without using any FE packages. The proposed equation overestimates the sway by about 10% which can be considered as reasonable and safe at the preliminary design stage. The proposed design charts permit prediction of the overall sway of a multi-storey semi-rigid regular frame without the need to resort to numerical modelling.

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