Steel and Composite Structures, Vol. 4, No. 5 (2004) 367-384
DOI: http://dx.doi.org/10.12989/scs.2004.4.5.367

Ultimate section capacity of steel thin-walled I-section beam-columns

Adel Helmy Salem†

Structural Engineering Department, Ain Shams University, Cairo, Egypt

Ezzeldin Yazeed Sayed-Ahmed‡

University of Qatar, Civil Engineering Dept., Doha Qatar, PO Box 2713, Qatar
(on leave from the Structural Engineering Department, Ain Shams University, Cairo, Egypt)

Ahmed Abdelsalam El-Serwi‡ and Mohamed Mostafa Korashy‡†

Structural Engineering Department, Ain Shams University, Cairo, Egypt

(Received December 31, 2003, Accepted October 6, 2004)

Abstract. A numerical model based on the finite element technique is adopted to investigate the behavior and strength of thin-walled I-section beam-columns. The model considers both the material and geometric nonlinearities. The model results were first verified against some of the currently available experimental results. A parametric study was then performed using the numerical model and interaction diagrams for the investigated beam-columns have been presented. The effects of the web depth-to-thickness ratio, flange outstand-to-thickness ratio and bending moment-to-normal force ratio on the ultimate strength of thin-walled I-section beam-columns were scrutinized. The interaction equations adopted for beam columns design by the NAS (North American Specifications for the design of cold formed steel structural members) have been critically reviewed. An equation for the buckling coefficient which considers the interaction between local buckling of the flange and the web of a thin-walled I-section beam-column has been proposed.

Key words: beam column; buckling coefficient; flange outstand; local buckling; interaction diagram; slenderness; thin walled section.

1. Introduction

The behavior of beam-columns has been a major structural engineering research topic in the past several decades. Through enormous experimental and analytical investigations, a clear understanding of the beam-columns behavior having compact and non-compact cross-sections has been achieved.

†Professor Emeritus
‡Associate Professor
‡†MSc Graduate

A beam-column with a slender cross-section distributes the material far from the neutral axis, and hence increases the resistance to overall buckling compared to a compact section of analogous cross-sectional area. As a result, thin-walled members are usually more efficient than compact sections for intermediate to long columns. On the other hand, as the component plates of the section become more slender, local buckling may occur at a load lower than the overall buckling load or the plastic resistance of the column. Consequently, columns having thin-walled sections may not develop their full plastic capacity which is normally developed by columns having compact sections with comparable overall slenderness ratios.

The behaviour of thin-walled beam-columns attracts the interest of many researchers (e.g., Hasham and Rasmussen 1998, El-Serwi 1998a,b, Chick and Rasmussen 1999a,b, Sohal and Sayed 1992, Sputo 1993, Chen and Wang 1999). The effective width concept proved to be efficient in dealing with local and post local buckling behavior of thin-walled members (Von Karaman *et al*. 1932, Dawe and Grondin 1985, March 1988, Yu 2000). However, this concept which is adopted by most codes of practice for design of thin-walled steel members (e.g., NAS-AISI 2001), deals with local buckling of the section components as individual plates and does not account for the interaction between local buckling of the plates comprising the section.

Generally, the structural behavior and the load-carrying capacity of a compressed stiffened element depends on the flat width-to-thickness ratio of the component plate element *w/t* and the boundary conditions along the element longitudinal edges. For small *w/t* ratios, the behavior of the element is usually governed by steel yielding. On the other hand, for large *w/t* ratios, the behavior is governed by local buckling. However, for columns with large *w/t* ratios, failure may not occur for compressed stiffened elements when local buckling is first encountered: an additional load may be sustained in a post-buckling fashion through stress redistribution (Winter 1974). When the critical stress in a compressed stiffened element exceeds the proportional limit of the steel, an interactive failure criterion between local buckling and yielding occurs. The interaction between local buckling of the component plate elements, overall buckling of the column, and yield failure criterion of the steel represents a very important design aspect (Lau and Hancock 1989, Basu and Akhtar 1990, Shen and Zhang 1992, Al-Bermani *et al*. 1994, Ren and Zeng 1997).

The present paper numerically investigates the strength and behavior of thin-walled I-sections subjected to compression and major-axis bending moment considering the web-flange local buckling interaction. A finite element model which considers both the material and geometric nonlinearities was adopted to determine the ultimate capacity of the beam-columns and establish their interaction diagrams. The effects of the flange outstand and the web slenderness on the ultimate strength of the beam-columns were also investigated numerically. A design equation for the buckling coefficients of the component plate elements comprising the cross-section of a thin-walled I-shaped beam-column is proposed. The proposed equation takes into account the interaction between local buckling of the flange and the web of a thin-walled I-section beam-column.

2. The numerical model

A numerical model based on the finite element technique was developed to investigate the behavior of thin-walled I-section beam-columns (Korashy 2002). A parametric study on the ultimate strength of thin-walled I-section beam-columns was performed numerically. Interaction curves representing the relation between the ultimate load and the ultimate moment capacity for these beam-columns were generated. The effects of web depth-to-thickness and flange outstand-to-thickness ratios on the ultimate capacity of beam-columns were investigated. The finite element computer program COSMOS/M, was used for pre-processing of the model data, solution of the finite element equations and post-processing of the results.

The model considered both the material and the geometric nonlinearities. It employed 4-node quadrilateral thick shell elements with six degrees of freedom per node. The element has membrane and bending capabilities and also has isotropic material properties. The uniaxial material behaviour of the steel was conceptually modelled using the bilinear stress-strain curve shown in Fig. 1.

A typical finite element mesh used to model the beam-columns is shown in Fig. 1. The two ends of the beam-column were provided with two rigid end plates which are 20 mm thick. To simulate the simply supported end conditions, a node in the middle of each end plate was restrained from translation

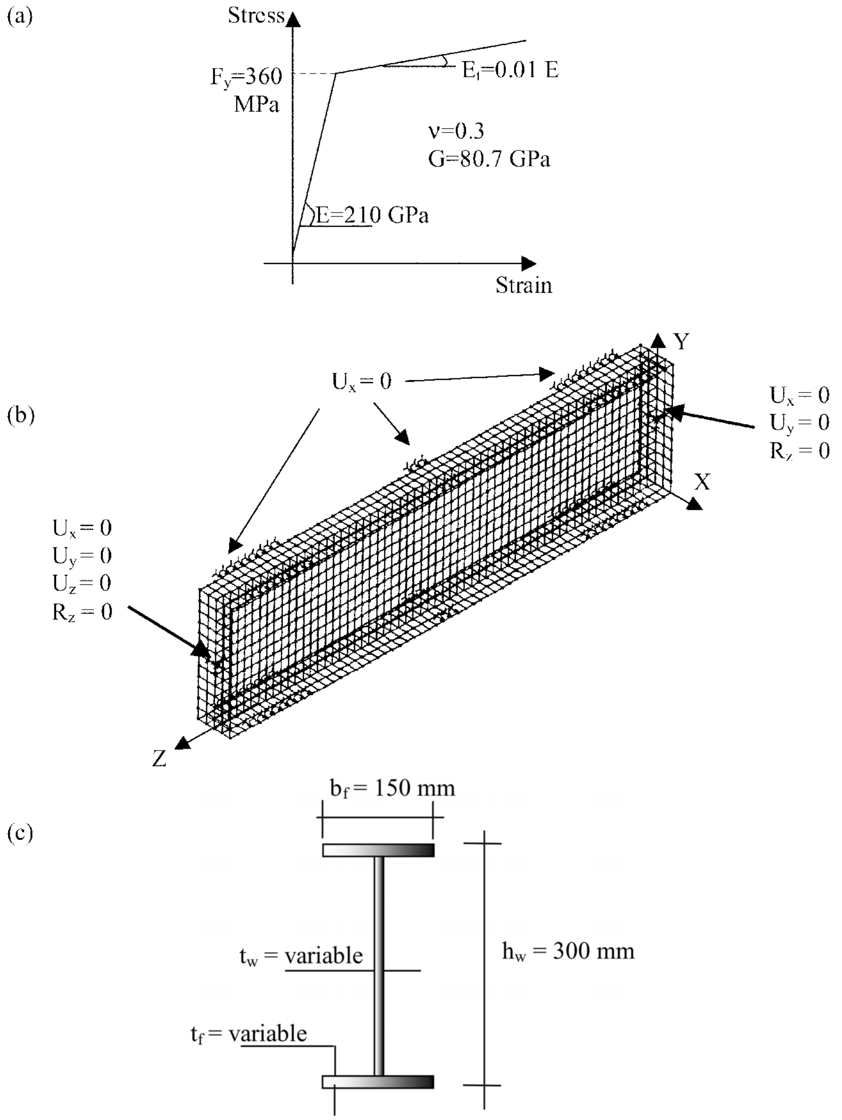


Fig. 1 Material and geometric properties of the analyzed beam-columns: (a) Conceptual uniaxial stress-strain of the steel; (b) Finite element mesh adopted in the analysis; (c) Cross-section of the analyzed beam column

in the X- and Y-directions in addition to the rotation around the beam-column longitudinal axis (Z-axis). The translation of one of these two nodes was also restrained in the Z-direction. To eliminate the effect of lateral torsion-flexural buckling, lateral supports were provided to the compression flange of the beam-column at small intervals. An axial compressive force and a major-axis bending moment (moment acting about the X-axis) were applied at the ends of the beam-columns and were monotonically increased till failure takes place. Ratios of bending moment to normal force M/N of 0, 0.025, 0.05, 0.125, 0.25, 0.5, 1.0 and (meter units) were adopted in the analysis.

The cross-section of the analyzed beam-columns is shown in Fig. 1. The web depth h_w and the flange width b_f were chosen to be 300 mm and 150 mm respectively. Throughout the analysis, the web thickness t_w and the flange thickness t_f were changed to have web depth-to-thickness ratios h_w/t_w varying between 50 and 150 and flange outstand-to-thickness ratios $b_f/2t_f$ varying between 7.5 and 18.75. The length of all the analyzed beam-columns was 1500 mm.

3. Verification of the finite element model

The model was first verified against some of the currently available experimental results (Hasham and Rasmussen 1998). Two series (SI and SII) of welded I-sections fabricated from hot rolled steel plates with a nominal yield stress of 350 MPa were tested. The specimens were subjected to axial compression and a constant major-axis bending moment with the ratios shown in Table 1. Series SI specimens had a section composed of two 105×8 flange plates and a 350×5 web plate while Series SII specimens had a section composed of two 175×5 flange plates and a 250×5 web plate. The two series were chosen such that local buckling of the web would occur prior to reaching the ultimate loads of the specimens. The specimens length and restraint against lateral buckling were chosen such that the full section capacity would be attained with no reduction in load due to lateral instability.

The specimens described above were analyzed from the first application of loads till final failure using the proposed finite element model. The numerical analysis results were explicitly compared to the experimental results (Korashy 2002). A comparison between the numerically-obtained ultimate loads and bending moments and those recorded experimentally (Hasham and Rasmussen 1998) is listed in Table 1. It is evident from this table that the finite element model results are in a good agreement with the experimental results.

Table 1 Verification of the finite element model

Specimen	M/N (m)	Experimental results		F.E.M. results		P_{FE}/P_{exp}
		Ultimate load (kN)	Moment (kN.m)	Ultimate load (kN)	Moment (kN.m)	
SI-1	0	981.7	0	1014.8	0	1.0330
SI -2	0.0668	774.7	51.6	769.7	51.3	0.9935
SI -3	0.3798	327.2	124.3	328.49	124.79	1.0039
SI-4	∞	0	177.0	0	162.6	---
SII-1	0	868.5	0	899.2	0	1.0353
SII-2	0.05427	613.6	33.3	623.62	33.94	1.0163
SII-3	0.3728	229.3	85.5	208.4	77.7	0.9088
SII-4	∞	0	108.7	0	99.5	---

As mentioned earlier, the two series were chosen such that local buckling of the web would occur well before reaching the ultimate loads of the section. Thus, the residual stresses in would have an insignificant effect on the value of the ultimate (post-buckling) strength of the section. This explains why a good agreement between the numerical model results and the experimental investigation prediction was achieved (error between 1% and 10%) despite the fact that the residual stresses were not considered.

4. Results of the numerical investigation

Fig. 2 shows the relationship between the axial load and the bending moment capacities (interaction diagrams) for the analyzed I-section beam-columns with various web depth-to-thickness ratios h_w/t_w (150, 125, 100, 75, and 50) and flange outstand-to-thickness ratios $b_f/2t_f$ (18.75, 12.5, 9.375, and 7.5).

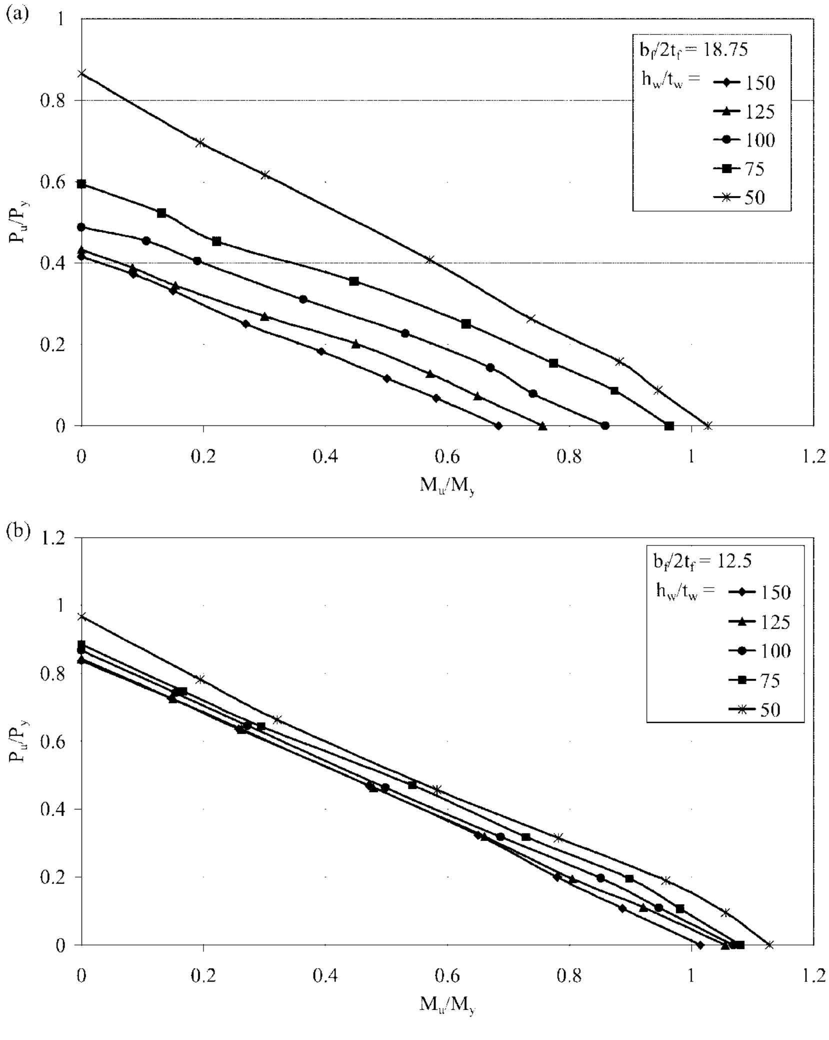


Fig. 2 Interaction diagrams for the numerically analyzed beam-columns

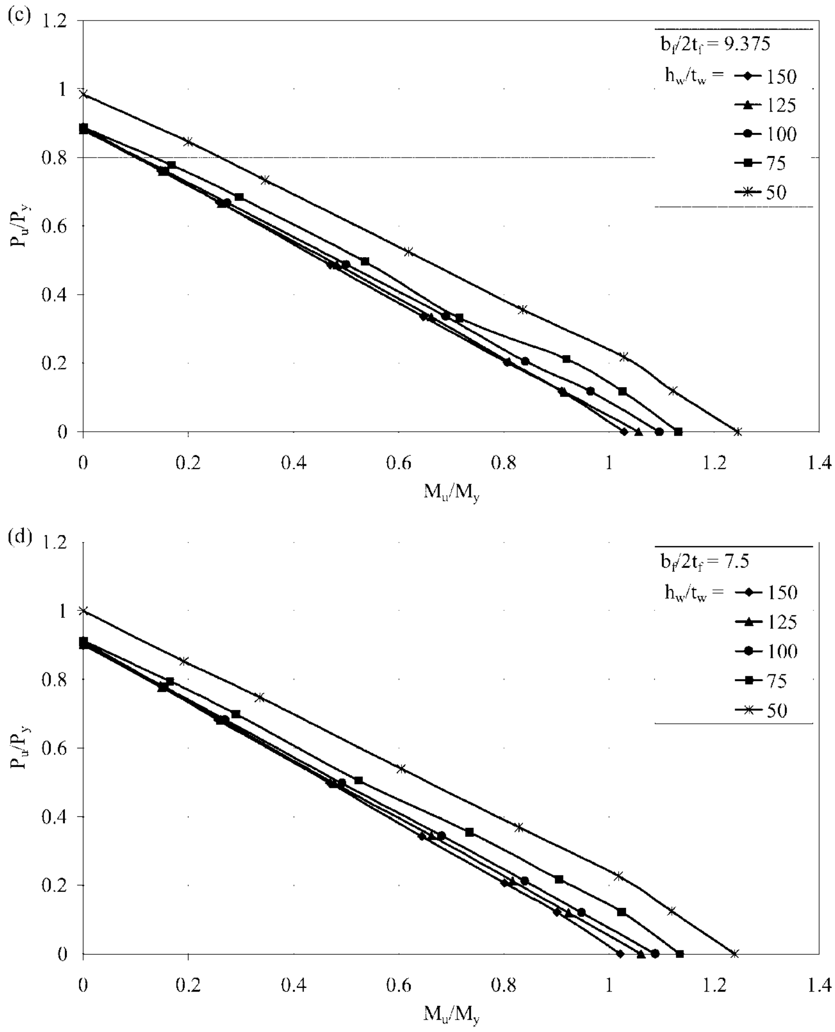


Fig. 2 (continued)

The ultimate axial load P_u was normalized with respect to the yield load of the cross-section P_y ($P_y=A_sF_y$) while the ultimate moment M_u was normalized with respect to the yield moment of the cross-section M_y ($M_y=S_xF_y$). It is evident from Fig. 2 that a linear interaction equation for the strength of thin-walled I-shape beam-column may be used for design purposes.

The numerical analysis revealed that for flange outstand-to-thickness ratios $b_f/2t_f$ of 18.75 and 12.5, failure occurred by local buckling of both the web and the flange (acting interactively) without reaching steel yielding (Korashy 2002). This buckling mode is shown in Fig. 3: contour lines for the displacements in the X-direction and in the Y-direction are plotted on the deformed shape of a beam column having $b_f/2t_f$ of 18.75 and h_w/t_w of 150. The least strength was observed for sections with h_w/t_w of 150, where the axial load capacity was only 45% of the yield load and the ultimate moment was only 68% of the yield moment. Fig. 2 shows that for h_w/t_w equals to 100 and 150, the ultimate strength of the analyzed beam-columns with $b_f/2t_f$ equals to 18.75 is about 50% less than that for beam-columns with

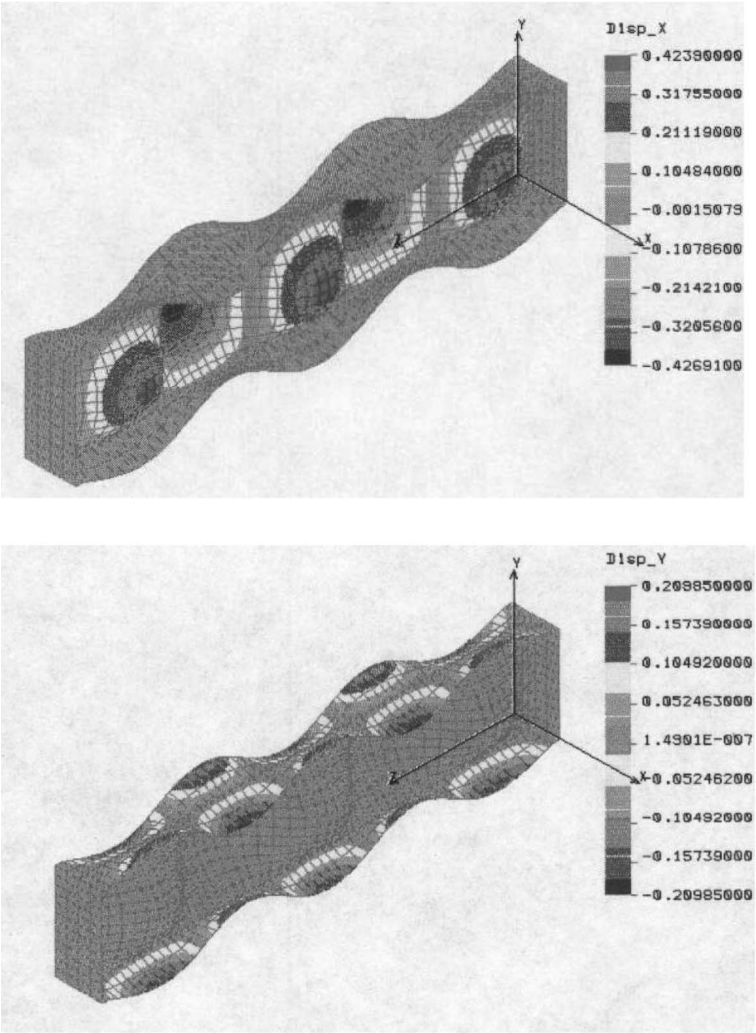


Fig. 3 Interactive local flange and web buckling mode of beam columns with $b_f/2t_f$=18.75 and h_w/t_w=150: contour lines for the displacement in the X-direction (above) and in the Y-direction (below)

$b_f/2t_f$ equals to 12.5. This difference decreases to 38% and 12% for h_w/t_w equals to 75 and 50 respectively. The analysis revealed that for high values of h_w/t_w local buckling occurs at both the flange and the web, while for lower values of h_w/t_w local buckling of the compression flange has the dominant effect on the section strength. On the other hand, for flange outstand-to-thickness ratios $b_f/2t_f$ of 9.375 and 7.5, failure was dominated by local buckling of the web: the flanges were thick enough to prevent local buckling. The local buckling mode of the web is shown in Fig. 4 where contour lines of the displacement in the X-direction are plotted on the deformed shape of a beam column having h_w/t_w of 150 and $b_f/2t_f$ of 7.5. Thus, the effect of flange outstand-to-thickness ratio is not significant for $b_f/2t_f <$ 9.375. Fig. 2 also ahows that for all values of $b_f/2t_f$ with the exception of $b_f/2t_f$ equals to 18.75, h_w/t_w has a significant effect on the ultimate strength of thin-walled I-section beam-columns only when it is less than 75. Changing h_w/t_w between 75 and 150 reduced the ultimate strength by about 3%.

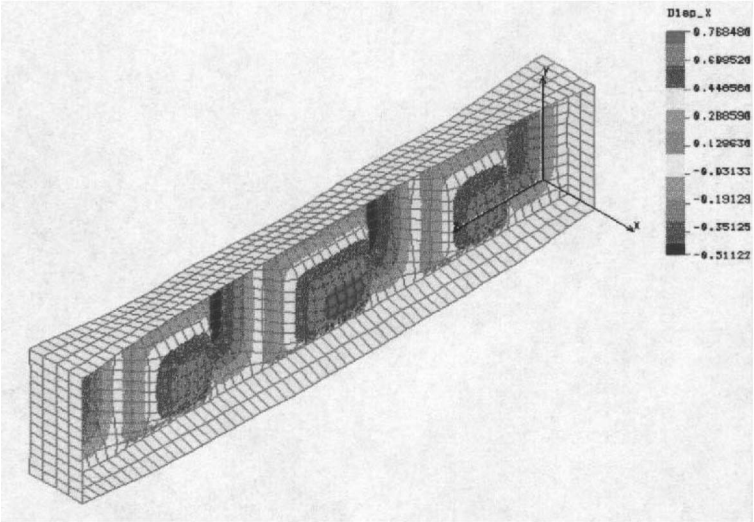


Fig. 4 Local web buckling mode of beam columns with $b_f/2t_f$=7.5 and h_w/t_w=150: contour lines for the displacement in the X-direction

4.1. Effect of web depth-to-thickness ratio

The analysis results were plotted in Fig. 5 to investigate the effect of web depth-to-thickness ratio h_w/t_w on the normalized compressive strength (P_u/P_y) of the analyzed beam-columns. The figure reveals that P_u/P_y is almost constant for $h_w/t_w \geq 100$ for all the analyzed values of $b_f/2t_f$ and M/N. Fig. 5 also reveals that the effect of h_w/t_w is more pronounced for smaller M/N (particularly for $b_f/2t_f = 18.75$) as the increase of the bending moment to the axial force ratio decreases the effect of local buckling due to the tensile stresses produced by the bending moment on one side of the section. Comparing the beam-column strength for the different analyzed ratios of $b_f/2t_f$ reveals that increasing the flange thickness reduces the drop in the normalized compressive strength which is a clear evidence for the fixation effect of the flange on the web.

4.2. Effect of flange outstand-to-thickness ratio

Fig. 6 shows the strength of the analyzed beam-columns plotted versus the flange outstand-to-thickness ratio $b_f/2t_f$. The figure reveals that the beam-column normalized compressive strength decreases slightly with increasing the flange outstand-to-thickness ratio to about 10. This slight difference in the strength is attributed to the fact that the flanges are fully effective, and not subjected to local buckling for $b_f/2t_f < 10$. On the other hand, a significant reduction in the ultimate strength occurs when the flange outstand-to-thickness ratio exceeds this value. For example, when $b_f/2t_f$ increases from 12.5 to 18.75, the strength of an axially loaded column drops by 30% to 50% depending on h_w/t_w. This reduction in the ultimate strength is again attributed in-part to the interaction between local buckling of the web and that of the flanges.

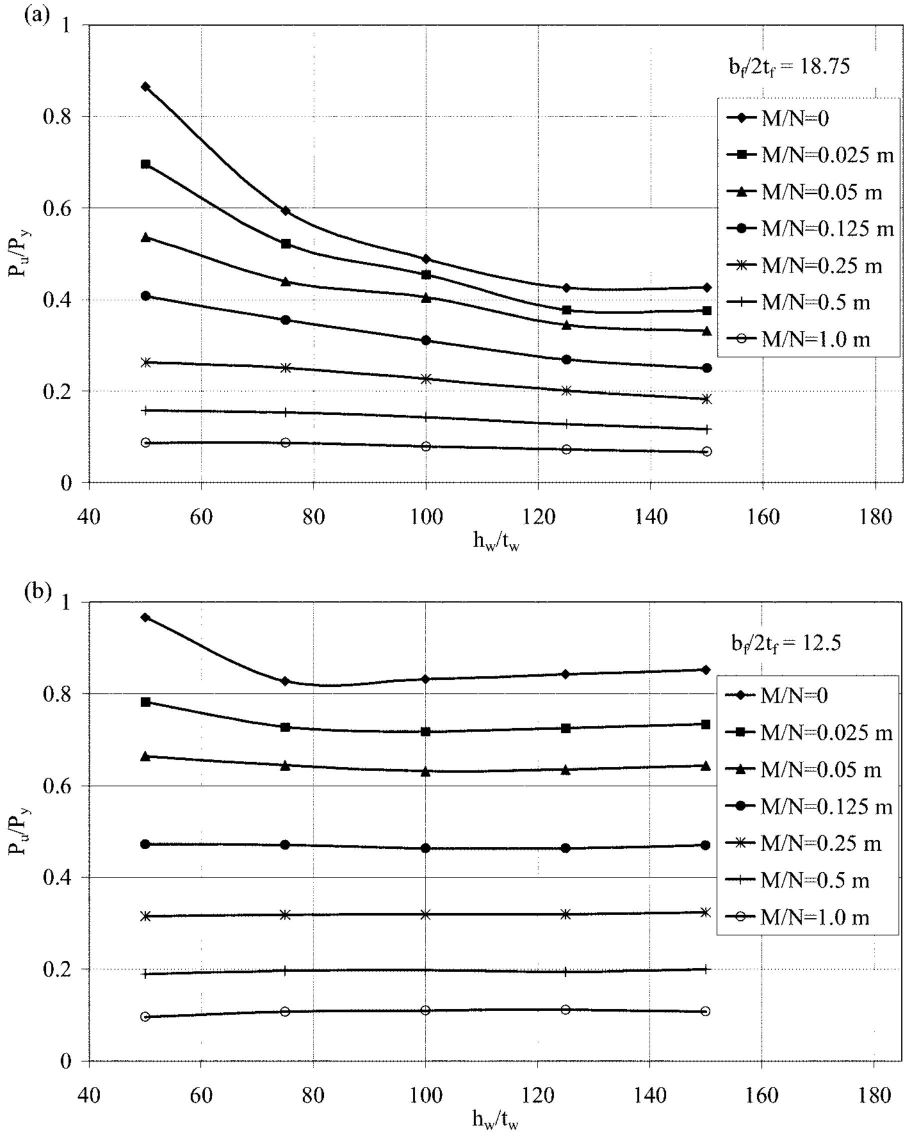


Fig. 5 Effect of h_w/t_w on the strength of the analyzed beam-columns

4.3. Effect of bending moment to normal force ratio

Fig. 7 shows the relationship between $[P_u/P_y + M_u/M_y]$ value and M/N ratio for various web depth-to-thickness ratios. It is evident from this figure that the values given by the interaction equation $[P_u/P_y + M_u/M_y]$ are almost constant for M/N ≥ 0.25 m. The figure also confirms that the effect of local buckling is more pronounced for lower values of M/N.

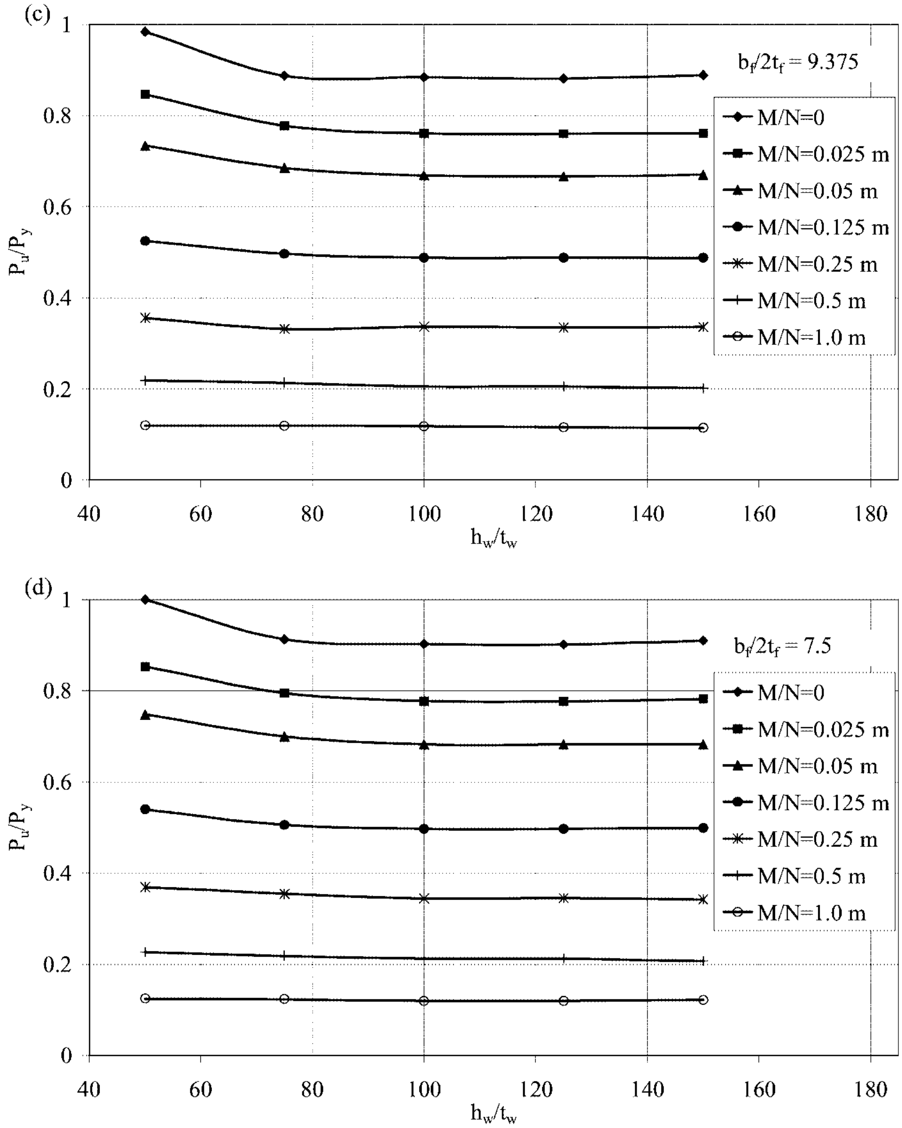


Fig. 5 (continued)

5. Proposed design procedures

A new design equation for the local buckling coefficients of the flange and the web comprising the thin-walled I-section beam-columns was proposed (Korashy 2002). This equation accounts for the interactive local buckling effect between the flange and the web. Based on the proposed equation, the ultimate capacities of the analyzed beam-columns were evaluated by adopting design procedures analogous to those of the NAS (AISI-2001). Then, the results were compared to the prediction of the NAS (AISI 2001) for the strength of thin-walled beam-columns and to the numerical analysis results.

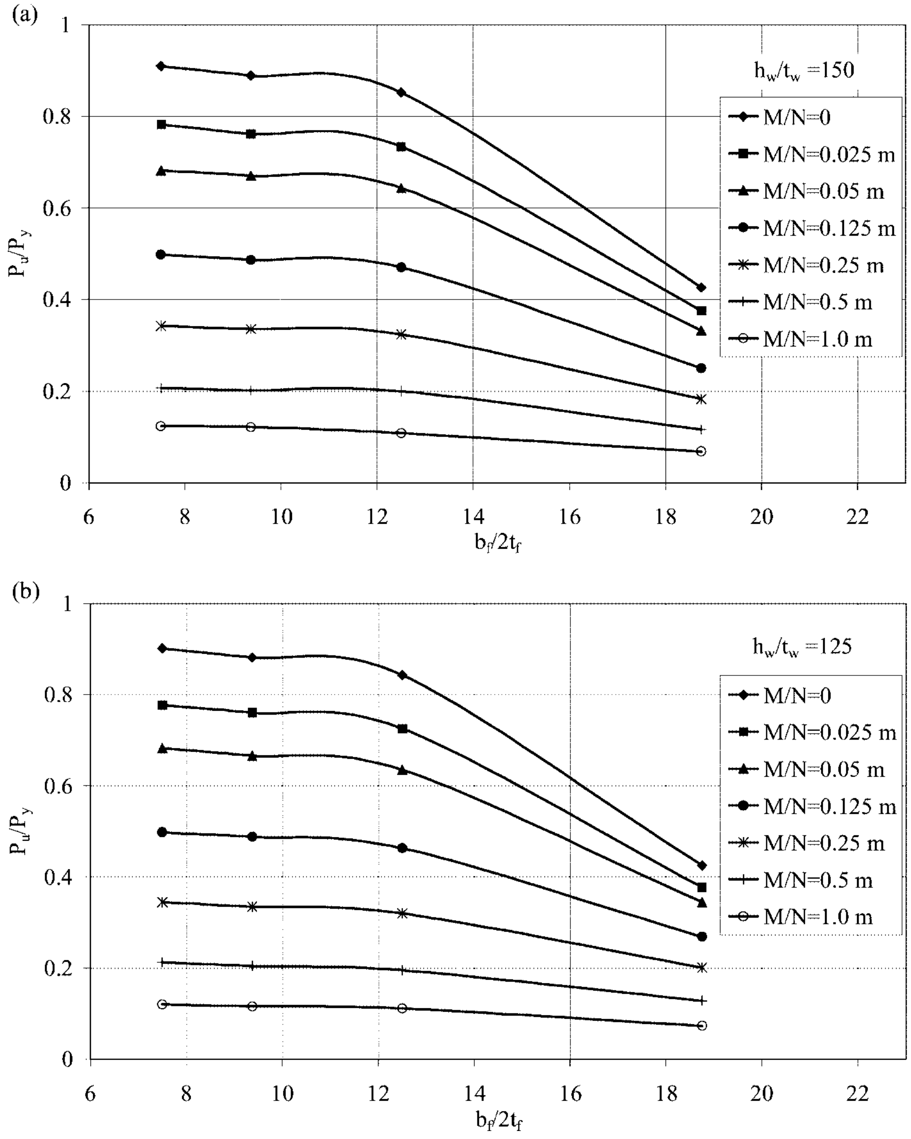


Fig. 6 Effect of $b_f/2t_f$ on the strength of the analyzed beam-columns

The NAS for the design of cold-formed steel structural members (AISI-2001) published recently, supercedes the previous editions of the AISI specification (AISI-1996, 1999) and the S136-94 Standards (CSA-1994). It uses an integrated treatment of the LRFD and the ASD as commonly used in the USA and the LSD commonly used in Canada through including the appropriate resistance factors for use with LRFD and LSD, and the appropriate factors of safety for use with ASD.

The present design equations of the NAS (AISI 2001) include the effect of web and flange local buckling embedded in the calculation procedures for the ultimate axial compression and the nominal moment. However, NAS (AISI 2001) partially consider the web-flange interactive local buckling.

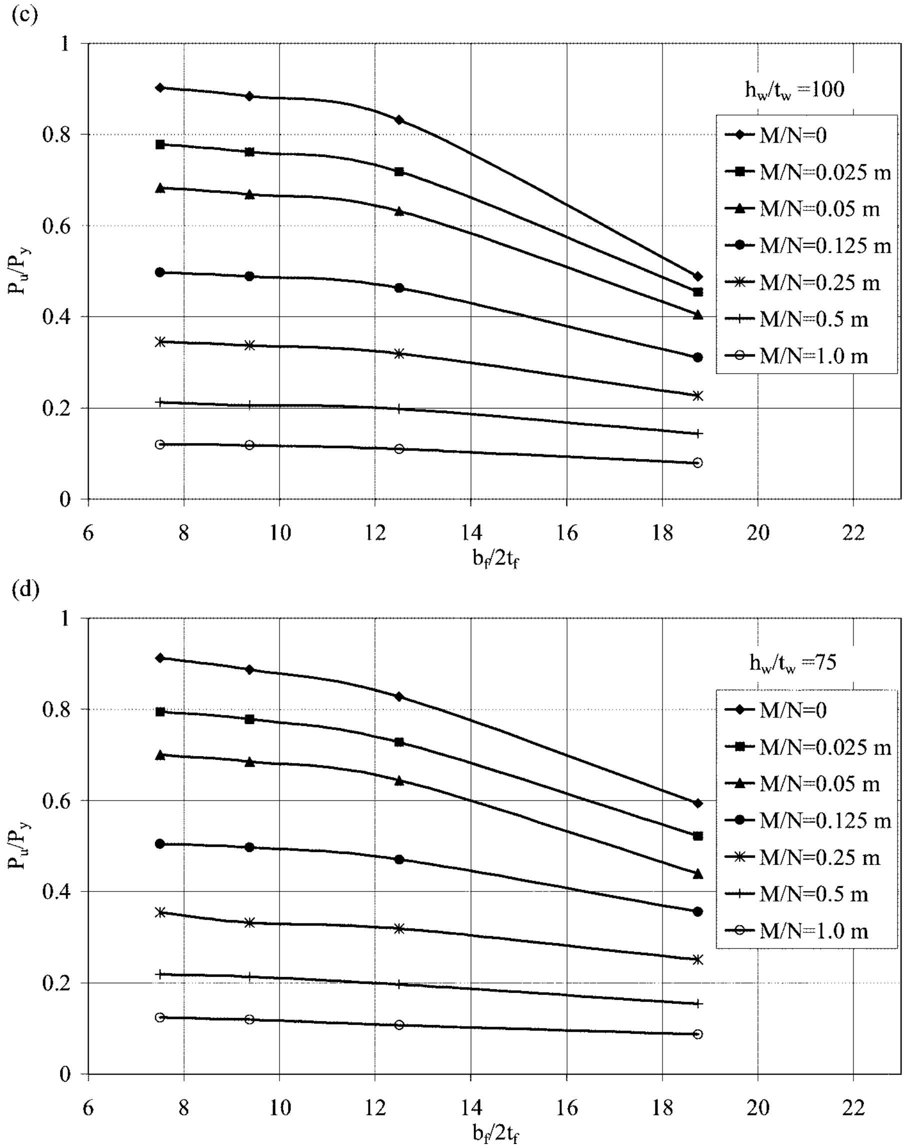


Fig. 6 (continued)

According to NAS, two sets of equations are used for the effective width of the web: as h_w/b increases eventually (at $h_w/b > 4$) a second set of expression is adopted. Nevertheless, the buckling coefficient K is taken equal to 4 in case of stiffened elements and equal to 0.43 in case of unstiffened elements subject to uniform compression, regardless of the relative slenderness of flanges and web. When the section is subjected to a stress gradient, K is calculated using the equations which are only dependent on the stress gradient ($\psi = f_2/f_1$). Thus, the web-flange interaction is only partially considered. Therefore, a new technique for evaluating the buckling coefficient, which accounts for the web-flange interaction, is currently required.

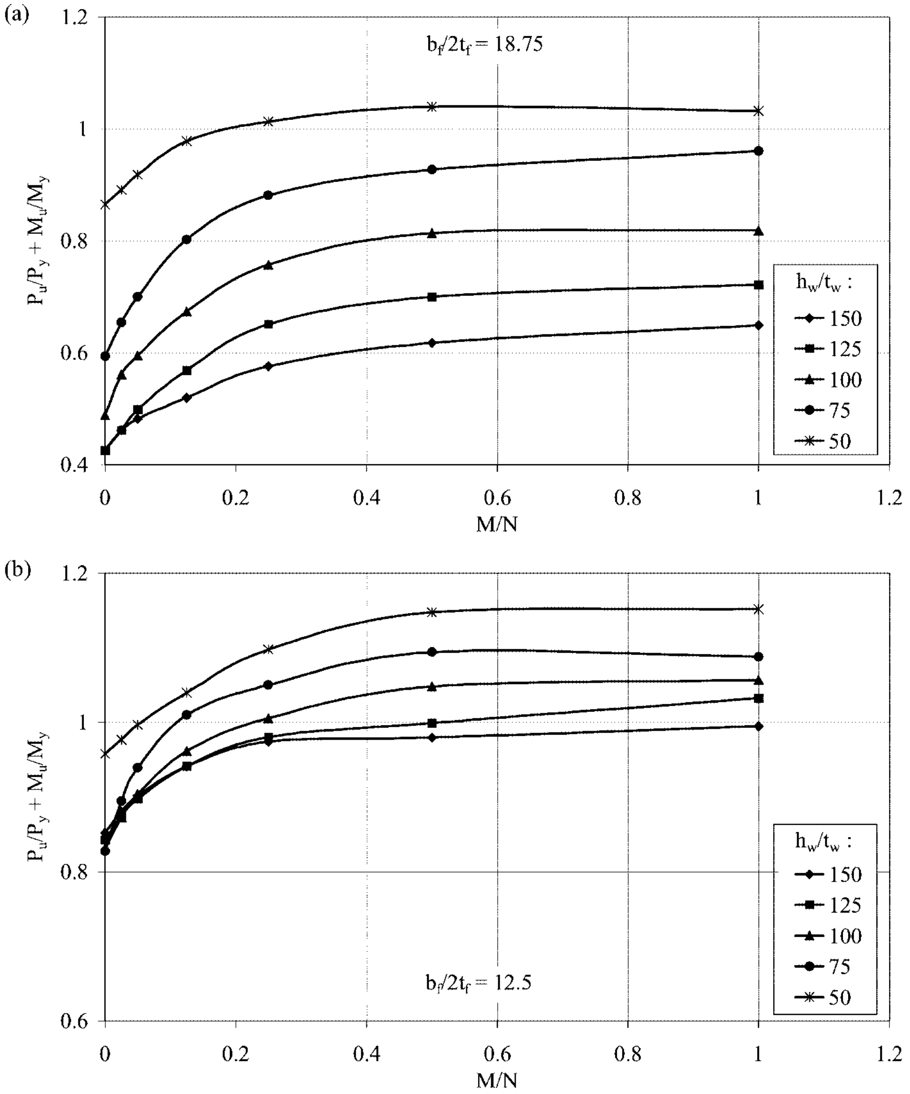
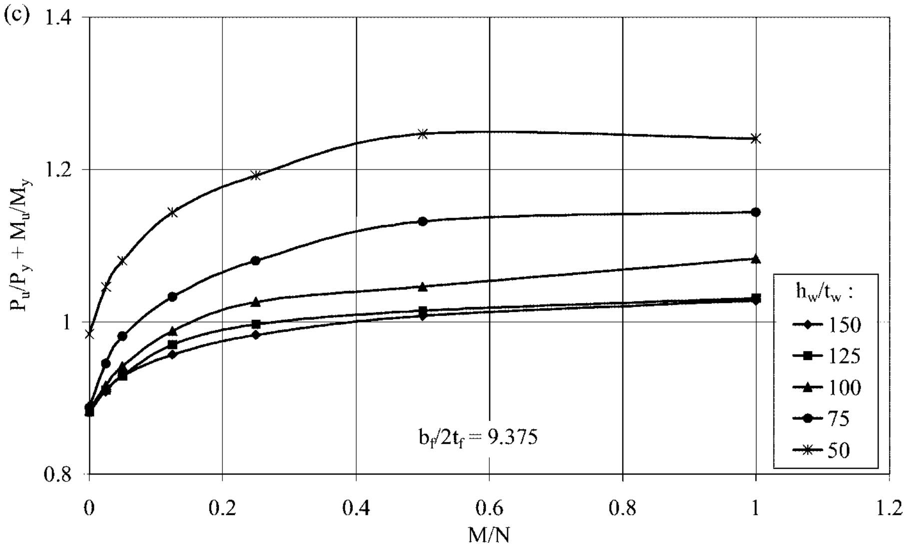


Fig. 7 Effect of M/N on the strength of the analyzed beam-columns

Two parameters α and β are chosen to represent the mutual effect of the slenderness of the flange and the web on each other. Those two parameters may be considered as modified slenderness ratios to account for the web-flange interactive local buckling. They are given by:

$$\alpha = \sqrt[3]{\left(\frac{h_w}{t_w}\right)^2 \times \left(\frac{b_f}{2t_f}\right)} \tag{1}$$

$$\beta = \sqrt[3]{\left(\frac{b_w}{t_w}\right) \times \left(\frac{b_f}{2t_f}\right)^2} \tag{2}$$



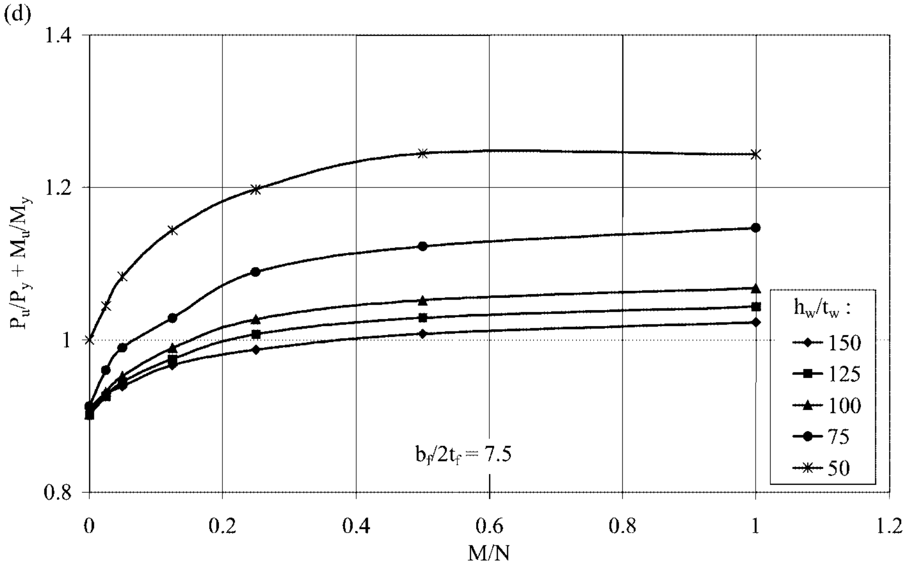


Fig. 7 (continued)

These two parameters may be used to obtain a modified expression for the buckling coefficient K that best fit the finite element results using linear regression analysis (Korashy 2002). Thus, expressions for K_{web} and K_{flange} may be given by:

$$K_{web} = K_o^{web} + 2(1-\psi)^3 + 2(1-\psi) \qquad K_o^{web} = 2 + 0.2 \cdot e^{\left(\frac{80}{\alpha}\right)^3} \leq 6.97 \tag{3}$$

$$K_{flange} = \frac{(4-\psi)}{3} K_o^{flange} \qquad K_o^{flange} = 0.2 + 0.02 \cdot e^{\left(\frac{40}{\beta}\right)^3} \leq 1.27 \tag{4}$$

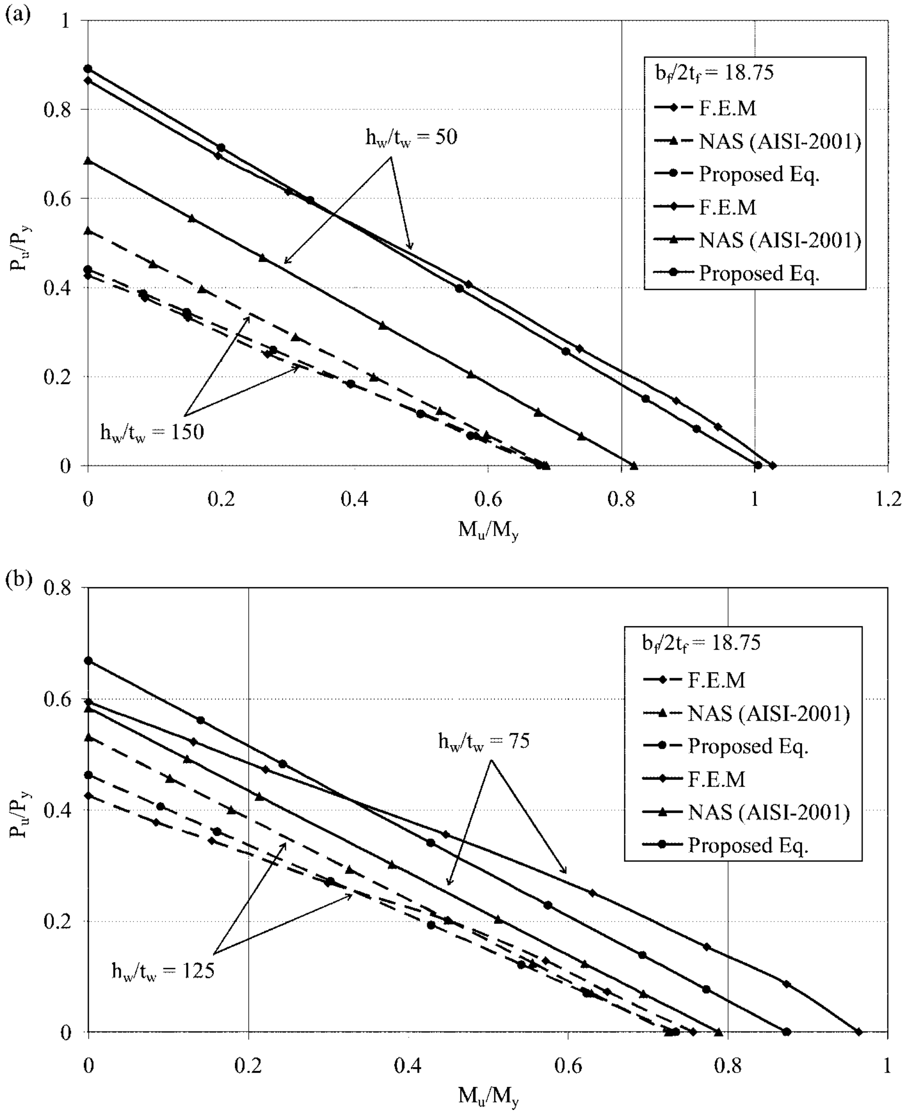


Fig. 8 A typical comparison between beam-column strength predicted based on the NAS (AISI-2001), the numerical analysis and the proposed design method

The limit of K_o^{web} represents the case of a web that is completely clamped on all sides. The provisions of NAS (AISI 2001) may now be adopted for calculating the ultimate strength of beam-columns with the buckling coefficients for the web and the flanges defined by Eqs. (3) and (4).

An explicit comparative study between the interaction diagrams constructed using the numerical analysis results and that obtained using the NAS (AISI 2001) provisions with and without the proposed buckling coefficients equations was carried out (Korashy 2002). It was found that both the AISI currently used provisions deviated from the numerical analysis results by -9% to +23%. On the other hand, using the modified flange and web buckling coefficients produced interaction diagrams which

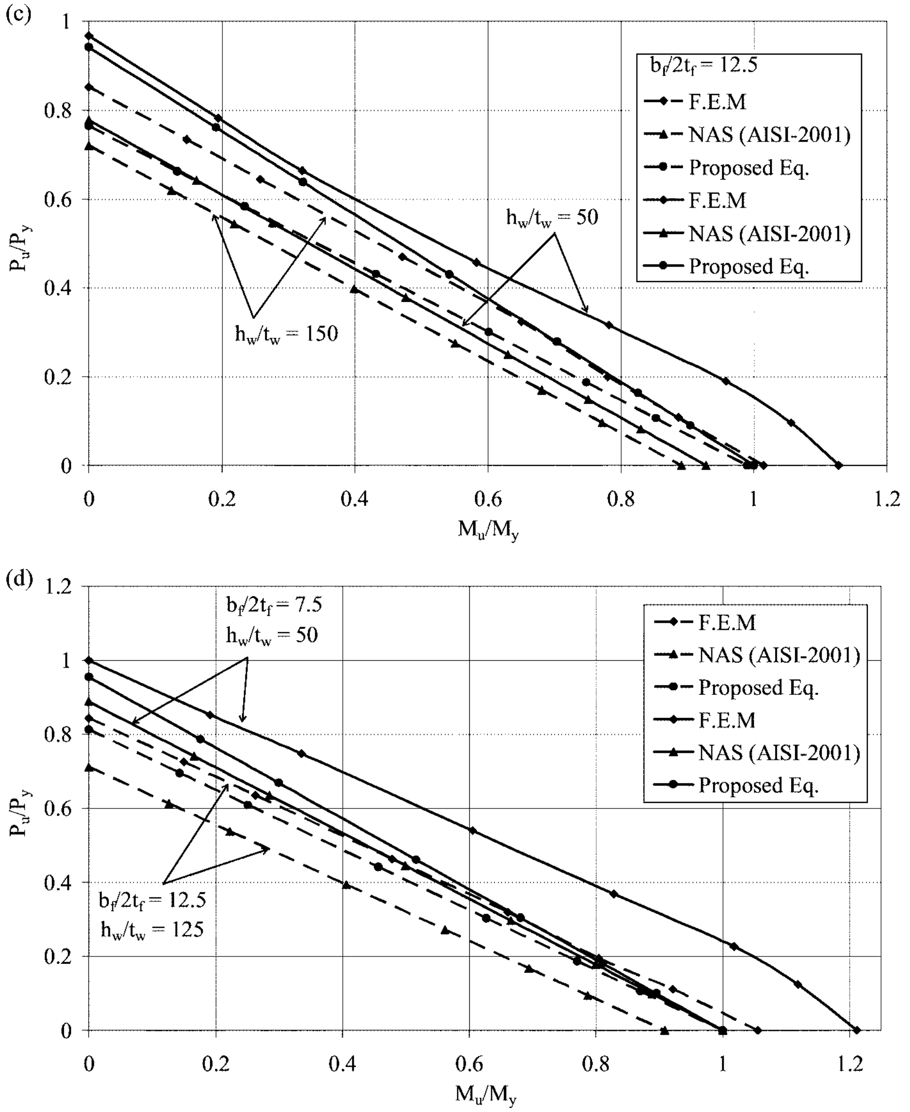


Fig. 8 (continued)

deviated by 3% to 10% from the numerical analysis results. A typical comparison for some of the analyzed data is shown in Fig. 8.

6. Conclusions

A finite element model is proposed to analyze thin-walled I-section beam-columns. The model was verified against some of the experimental results previously published. Using the numerical model, interaction diagrams were constructed for beam-columns subject to axial compression and major axis bending moment which revealed a linear trend in most of the studied cases.

For sections with slender flanges ($b_f/2t_f$=18.75) subjected to pure axial load, the variation of the web depth-to-thickness ratio (h_w/t_w) from 150 to 100 has very slight effect on the ultimate/yield load ratio while changing the web depth-to-thickness ratio (h_w/t_w) from 100 to 50 affects the ultimate/yield load ratio by about 60%. For these sections, full yield strength was not achieved except for h_w/t_w = 50 at M/N greater than 0.25 m.

For sections with stiffer flanges ($b_f/2t_f$=9.375 and 7.5), the variation of the web depth-to-thickness ratio (h_w/t_w) from 150 to 75 has insignificant effect on the ultimate/yield load ratio for all values M/N. For these sections, the flanges were fully effective and not subjected to local buckling. A significant difference in the ultimate/yield load ratio was recorded for beam-columns with $b_f/2t_f$=12.5 compared to those with $b_f/2t_f$=18.75 due to the effect of local flange buckling.

The NAS (AISI 2001) provisions for estimating the strength of thin-walled I-section beam-column were critically reviewed: it produced strengths which deviated from the interaction curves obtained numerically by -9% to +23%. New equations for estimating the buckling coefficients of the flange and the web comprising the thin-walled I-section beam-columns are proposed. The new equations consider the interaction between local buckling of the flange and the web. Considering the proposed buckling coefficients equation in the NAS (AISI 2001) for estimating the beam-column strength, the deviation from the numerically obtained interaction curves was reduced to only 3% to 10%.

References

Al-Bermani, F., Kitipornchai, S. and Chin, S. (1994), "Stability of cold-formed members", *J. Eng. Struct.*, **16**(5), 386-391.

AISI - American Iron and Steel Institute (2001), "North American specifications for the design of cold-formed steel structural members", Washington, USA.

AISI - American Iron and Steel Institute (1996), "Specifications for the design of cold-formed steel structural members", Washington, USA.

AISI - American Iron and Steel Institute (1999), "Specifications for the design of cold-formed steel structural members (Suppl. 1)", Washington, USA.

Basu, P.K. and Akhtar, M.N. (1990), "Interactive and local buckling of thin-walled members", *Thin-Walled Struct.*, **12**, 335-352.

CSA - Canadian Standards Association (1994), "Cold-formed steel structural members. structures (design)", S136-94. Toronto, Ontario, Canada.

Chen, S.J. and Wang, W.C. (1999), "Moment amplification factor for P-δ effect of steel beam-columns", *J. Struct. Eng.*, ASCE, **125**(2), 219-223.

Chick, C.G. and Rasmussen, K.J.R. (1999a), "Thin-walled beam columns, I: Sequential loading and moment gradient tests", *J. Struct. Eng.*, ASCE, **125**(11), 1275-1266.

Chick, C.G. and Rasmussen, K.J.R. (1999b), "Thin-walled beam columns, II: Proportional loading tests", *J. Struct. Eng.*, ASCE, **125**(11), 1267-1276.

Dawe, John L. and Grondin, Gilbert Y. (1985), "Inelastic buckling of steel plates", *J. Struct. Eng.*, ASCE, **111**(1), 95-107.

El-Serwi, A.A. (1998a), "Web buckling of H-shaped cross-sections subjected to axial force and bending moment about major axis", Civil Engineering Research Magazine, Al-Azhar University, Cairo, Egypt, **22**(2).

El-Serwi, A.A. (1998b), "Ultimate capacity of thin-walled H-shaped cross-sections subjected to axial force and major axis bending moment", PSERJ, Faculty of Engineering, Suez Canal University, Port Said, Egypt, **4**(2).

Hasham, A.S. and Rasmussen, K.J.R. (1998), "Section capacity of thin-walled I-section beam-columns", *J. Struct. Eng.*, ASCE, **124**(4), 351-359.

Korashy, M.M. (2002), "Load capacity of thin-walled I-section beam-columns", MSc., Structural Engineering Department, Faculty of Engineering, Ain shams University, Cairo, Egypt.

Lau, S.C.W. and Hancock, G.J. (1989), "Inelastic buckling analyses of beams, columns and plates using the spline finite strip method", *Thin-Walled Struct.*, **7**, 213-238.
March, Cedric (1988), "Design method for buckling failure of plate elements", *J. Struct. Eng.*, ASCE, **124**(7), 850-853.
Ren, W. and Zeng, Q. (1997), "Interactive buckling behavior and ultimate load of I-section steel columns", *J. Struct. Eng.*, ASCE, **123**(9), 1210-1217.
Shen, Z. and Zhang, Q. (1992), "Nonlinear stability analysis of steel members by finite element method", *J. Eng. Mech.*, ASCE, **118**(3), 445-461.
Sohal, I.S. and Syed, N.A. (1992), "Inelastic amplification factor for design of steel beam-columns", *J. Struct. Eng.*, ASCE, **118**(7), 1822-1839.
Sputo, Thomas, (1993), "History of steel beam-column design", *J. Struct. Eng.*, ASCE, **119**(2), 547-557.
Von Karaman, T., Sechler, E.E. and Donnell, L.H., (1932), "The strength of thin plates in compression", Transactions, ASME, **54**.
Winter, G. (1974), "Strength of thin-steel compression flanges", Cornell University, Eng. Exp. Stn., Reprint No. 32.
Yu, W.W. (2000), *Cold-formed Steel Design*, 3rd Edition, John Wiley & Sons Inc., New York, USA.