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Combined effect of the horizontal components of earthquakes for moment resisting steel frames

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Abstract. The commonly used seismic design procedures to evaluate the maximum effect of both horizontal components of earthquakes, namely, the Square Root of the Sum of the Squares (SRSS) and the 30-percent (30%) combination rules, are re-evaluated. The maximum seismic responses of four three-dimensional moment resisting steel frames, in terms of the total base shear and the axial loads at interior, lateral and corner columns, are estimated as realistically as possible by simultaneously applying both horizontal components. Then, the abovementioned combination rules and others are evaluated. The numerical study indicates that both, the SRSS rule and the 30% combination method, may underestimate the combined effect. It is observed that the underestimation is more for the SRSS than for the 30% rule. In addition, the underestimation is more for inelastic analysis than for elastic analysis. The underestimation cannot be correlated with the height of the frames or the predominant period of the earthquakes. A basic probabilistic study is performed in order to estimate the accuracy of the 30% rule in the evaluation of the combined effect. Based on the results obtained in this study, it is concluded that the design requirements for the combined effect of the horizontal components, as outlined in some code-specified seismic design procedures, need to be modified. New combination ways are suggested.

Key words: seismic inelastic response; three-dimensional moment resisting steel frames; horizontal components; time history analysis; multi degree of freedom systems.

1. Introduction

For numerical evaluation of the seismic response of buildings, earthquake motions are generally represented by three orthogonal components: two horizontal and one vertical. The effect of the vertical component is usually smaller than those of the horizontal components and is consequently neglected. Additional bases to neglect the vertical component effect are that building designs allow for gravity loads which provides for a high factor of safety in the vertical direction, and that the vertical component is significantly out of phase with the horizontal components. For these reasons, the horizontal motions are usually considered as the earthquake load. Most building codes with earthquake provisions require that simplified equivalent lateral loads be considered because of the horizontal motions. In such

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approaches, the effect of each component is evaluated independently and combined according to some empirical rules. Some codes (UBC 1997, Eurocode 1998), however, require considering the three seismic components.

Among the first investigations conducted on the topic are those of Newmark (1975) and Rosenblueth and Contreras (1977). They proposed a rule known as the *Percentage Rule* which approximates the combined response as the sum of the 100% of the response resulting from one component and some percentage (λ) of the responses resulting from the other two components. Newmark (1975) suggested to use λ = 40%, arguing that the combined response would be conservative with respect to that given by the Square Root of the Sum of the Squares (SRSS). Rosenblueth and Contreras (1977) proposed to use λ = 30% in order to minimize the errors introduced by this linear approximation.

There have been many other studies regarding the evaluation of the responses of structures considering the three (or the two horizontal) seismic components. Wilson and Button (1982) presented a simple method to determine the critical angle of structures without considering any correlation between the horizontal ground motion components. López and Torres (1997) proposed a method to estimate the critical angle of incidence of structures. The method was illustrated with one-story and nine-story buildings. Results of this investigation also showed that the method proposed by Wilson and Button (1982) was inaccurate. Fernández-Dávila *et al*. (2000) studied the seismic elastic response of concrete buildings considering three degrees of freedom per floor. Correnza and Hutchinson (1994) analyzed one-story models with and without transverse elements subjected to one and to bi-directional earthquakes. De Stefano and Faella (1996) studied the biaxial inelastic response of simplified single mass two-degree-of-freedom concrete structures. A formulation known as the *Complete Quadratic Combination* (CQC), which is based on the theory of random vibrations, was suggested by Der Kiureghian (1981) and by Wilson *et al*. (1981). This rule was used to combine the modal responses due to a single seismic component. Smeby and Der Kiureghian (1985) proposed an extension of the CQC rule. The rule, known as the CQC3 rule, can estimate the modal responses due the three seismic components. Smeby and Der Kiureghian (1985) and Lopez and Torres (1996) focused the application of the CQC3 rule on building-type structures with rectangular geometry. Menun and Der Kiureghian (1998) extended the two previous investigations by using more complex three-dimensional structures subjected to the two horizontal components. They considered a curved bridge structure. They compared the results of the CQC3 rule with those of the SRSS, the 30% (λ = 0.3) and the 40% (λ = 0.4) rules and examined the shortcomings of the last three rules. López *et al*. (2001) evaluated the accuracy of the above-mentioned combination rules (SRSS, 30% and 40%) by comparing the resulting estimates of response with the critical value of response determined in accordance with the CQC3 rule. In such evaluation the two horizontal components were considered. It was presented first in a generic way, which is valid for any structure, then for a range of one-storey systems with symmetrical and unsymmetrical plan, and finally for two multistorey buildings. Recently, Hernández and López (2003) extended the work done by López *et al.* (2001) by considering the effect of the vertical component. The critical response was calculated for two cases: a) assuming that a principal seismic component is along the vertical direction (CQC3 rule) and b) when a component departs from the vertical direction (GCQC3 rule). They showed that the inclination of a principal component with respect to the vertical direction could significantly reduce the value of the ratio of the approximated response to the critical response.

In spite of the contributions of the previous investigations, the general limitations on most of these studies are that elastic analysis and/or concrete frames were used and that the models were too simplified (a few stories and plane frames connected by rigid diaphragms). Consequently, the inelastic behavior of all the structural elements (including beams and columns), energy dissipation, and contribution of higher modes were not properly considered. In addition, steel frames modeled as multi-degree of freedom systems have not been studied. This type of structures is considered in this study.

Among the different structural configurations used for steel frames in regions of high seismicity, moment resisting steel frames (MRSF) have been the most popular for several reasons. Architects and owners prefer to use this system because of lack of bracing, which provides maximum flexibility for space utilization. Even though member sizes of MRSF are normally increased to meet the building codes drift requirements, it is customarily considered a good exchange for bracing elements. In addition, MRSF are popular because they are considered as highly ductile systems. Code-specified seismic procedures typically assign the largest force reduction factor (R) to MRSF, implying the largest ductility and the lowest lateral design forces. The force reduction factor mainly depends on the ductility (μ) of the frame and the structural overstrength (Ω) (Uang 1991).

Some seismic design provisions permit to use two types of MRSF: strong-column weak-beam (SCWB) and weak-column strong-beam (WCSB) steel frames. In the first type, the beams are the weak elements and are expected to develop plastic hinges, while in the second type the columns are expected to develop them. It has been shown (Reyes-Salazar *et al*. 2000) that the force reduction factors depend on the amount of dissipated energy, which in turn is larger for SCWB steel frames. It implies greater ductility. For this reason, SCWB steel frames have been preferred over WCSB steel frames.

If a SCWB steel frame is modeled as a frame with rigid diaphragms, one of the most important sources of energy dissipation, i.e. dissipation of energy at plastic hinges, won't be considered and the structural behavior will be modified. Numerical studies (Wang and Wen 2000) showed that, for response evaluation of SCWB steel buildings, conventional shear-beam assuming rigid diaphragms might significantly underestimate the structural response. Consequently, SCWB steel frames should be analyzed as multi-degree of freedom (MDOF) systems. As stated below, in this paper MRSF SCWB-type are used and are modeled as MDOF systems.

In this study, the Square Root of the Sum of the Squares (SRSS), the 30-percent (30%) and other combination rules, commonly used in code-specified seismic design procedures to evaluate the maximum effect of both horizontal components of earthquakes, are re-evaluated. Using a time domain nonlinear finite element program developed by the authors and their associates, the maximum inelastic seismic responses of frames are estimated as realistically as possible by simultaneously applying both horizontal components. The response is expressed in terms of the maximum total base shear and the axial loads at interior, lateral and corner columns, at the base of the frames. Then, the above-mentioned combinations rules are evaluated. Four three-dimensional moment resisting steel frame structures, representing different dynamic characteristics, are used in the study. The frames are modeled as complex MDOF systems. Consequently, energy dissipation and higher modes response are explicitly considered. Six degrees of freedom per node are considered. The frames are excited by several recorded time histories, which were selected to represent the different characteristics of strong motions. The effect of the vertical component of the used earthquakes is neglected and the horizontal components are assumed to be uncorrelated (Yamamura and Tanaka 1990).

2. Combinations rules

Buildings respond in both horizontal directions at the same time to the action of a strong motion. Therefore, it is necessary to consider the effect on both directions of the horizontal components of earthquakes. Usually, the effect of each horizontal component is estimated individually and combined according to several ways. The common ways of combination are the 30% and SRSS rules (Newmark and Hall 1982). These two procedures are briefly explained below.

Let us define as R_X the maximum absolute value of the effect (moment, shear, etc.) at a particular point in a particular member of a given structure arising from the horizontal component in X direction of a given earthquake. Let us also define as R_Y the peak absolute value of the same effect arising from the horizontal component of the earthquake in Y direction. Then, the combined effect can be calculated as the most unfavorable of:

$$R_{C1} = R_X + \lambda R_Y \text{ or, } R_{C1} = \lambda R_Y + R_Y \tag{1}$$

If $\lambda = 0.3$ in Eq. (1), it represents the 30% combination rule. According to the SRSS rule the combined response is given by

$$R_{C2} = \sqrt{R_X^2 + R_Y^2} \tag{2}$$

The accuracy of the above procedures is evaluated by comparing the R_{C1} (for λ=30%) and R_{C2} values with the exact response. The accuracy of using other combination rules is also evaluated. This is elaborated further in Sections 5 and 6 of the paper. The exact solution is obtained from the simultaneous application of both horizontal components.

At this state, it is important to establish a relationship between the R_{C1} and R_{C2} parameters. Let us assume that R_X is the larger of the two peak effects and consequently R_Y will be the smaller one. If Q denotes the ratio of the smaller to the larger effect, that is $Q = R_Y / R_X$, then R_{C2} can be expressed as:

$$R_{C2} = \sqrt{R_X^2 + (QR_X)^2} = R_X\sqrt{1 + Q^2} \tag{3}$$

$$\frac{R_{C2}}{R_X} = \sqrt{1 + Q^2} \tag{4}$$

In the same manner R_{C1} can be expressed as:

$$R_{C1} = R_X + \lambda Q R_X \tag{5}$$

$$\frac{R_{C1}}{R_X} = 1 + Q\lambda \tag{6}$$

Therefore, the ratio of the two combined responses, defined as R, is given by

$$R = \frac{R_{C2}}{R_{C1}} = \frac{\sqrt{1 + Q^2}}{1 + Q\lambda} \tag{7}$$

The values of R are plotted in Fig. 1 for several values of Q and λ. It is observed from this figure that for λ= 0.3, R_{C2} and R_{C1} are close each other. If the SRSS combination rule were the exact solution, the minimum introduced error by using $\lambda = 0.3$ would be zero for $Q = 0.67$. The maximum overestimation error would be about 4% for $Q = 0.3$ and the maximum underestimation

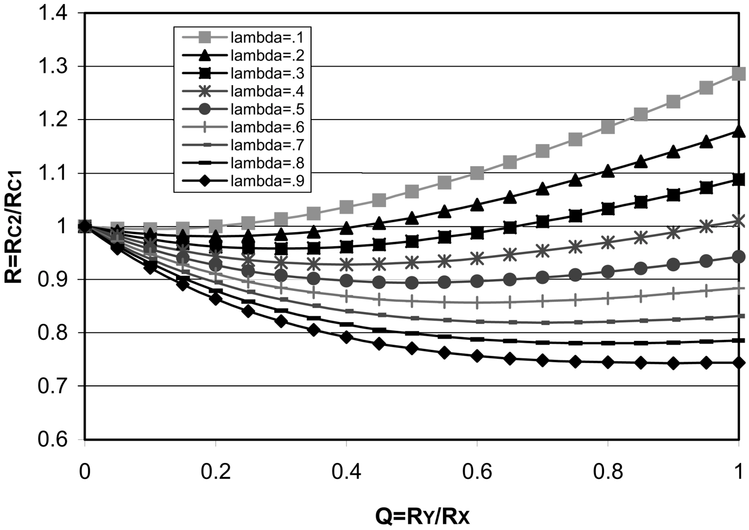


Fig. 1 Variation of the ratio of the SRSS and λ combinations

error would be about 8% for Q = 1.0. These results make one thinks that the 30% combination rule is based on the assumption that the SRSS rule gives the exact solution. However, as shown in Sections 5 and 6, the combined response according to the SRSS rule can be quite different from that of the exact solution. Thus, if λ = 0.3 is used in Eq. (1), significant errors can be introduced according to this rule.

3. Mathematical formulation

An efficient finite element-based time-domain nonlinear analysis algorithm already developed for the authors and their associates (Gao and Haldar 1995, Reyes-Salazar 1997) is used to estimate the effect of both horizontal components on the overall structural response. The procedure estimates nonlinear seismic responses of steel frames considering all major sources of energy dissipation. Material nonlinearity and geometric nonlinearity are considered. Considering its efficiency, particularly for steel frame structures, the assumed stress-based finite element method (Kondo and Atluri 1987) is used. Using this approach, an explicit form of the tangent stiffness matrix is derived without any numerical integration. Fewer elements can be used in describing a large deformation configuration without sacrificing any accuracy. Furthermore, information on material nonlinearity can be incorporated in the algorithm without losing its basic simplicity. It gives very accurate results and is very efficient compared to the displacement-based approach. The procedure has been studied and verified with existing theoretical and experimental results. Only the basic concepts are given here due to lack of space.

The geometric and material nonlinearities are considered in the tangent stiffness matrix. The mathematical details of the derivation are not shown here, but can be found in the literature (Kondo and Atluri 1987). The material is considered to be linear elastic except at plastic hinges. Concentrated plasticity behavior is assumed at plastic hinge locations. In the past, several analytical procedures were proposed to predict the deformation of elasto-plastic frames under increasing seismic and static loads. However, most of these formulations were based on small deformation theory. In this study, each elasto-plastic beam-column element can experience arbitrary large rigid deformations and small relative deformations.

Thus, in addition to the elastic stress-strain relationships, the plastic stress-strain relationships need to be incorporated into the constitutive equations if a given yield condition is satisfied. Several yield criteria have been proposed in the literature in terms of stress components or nodal forces. Since the nodal forces can be obtained directly from the proposed method, the yield criteria used here is expressed in terms of nodal forces. When the combined action of the resultant stresses satisfy a prescribed yield function at a given end of an element, a plastic hinge is assumed to occur instantaneously at that location. Plastic hinges are considered to form at the ends of the beam-columns elements. The yield function (or interaction equation) depends on both, the type of section and loading acting on the beam-column element (Mahadevan and Haldar 1991). The yield function for three-dimensional beam-column elements has the following general form:

$$f(P, M_X, M_Y, M_Z, \sigma_y) = 0 \text{ at } X = l_p \tag{8}$$

where P is the axial force, M_X and M_Y are the bending moments with respect to the mayor and minor axis, respectively, M_Z is the torsional moment, σ_y is the yield stress, and l_p is the location of the plastic hinge. For the W-type sections used in this study, this equation has the following particular form:

$$\left(\frac{P}{P_n}\right)^2 + \left(\frac{M_X}{M_{nX}}\right)^2 + \left(\frac{M_Y}{M_{nY}}\right)^2 + \left(\frac{M_z}{M_{nZ}}\right)^2 - 1 = 0 \tag{9}$$

where P_n is the axial strength, M_{nX} and M_{nY} are the flexural strength with respect to the major and minor axis, respectively and M_{nZ} is the torsional strength.

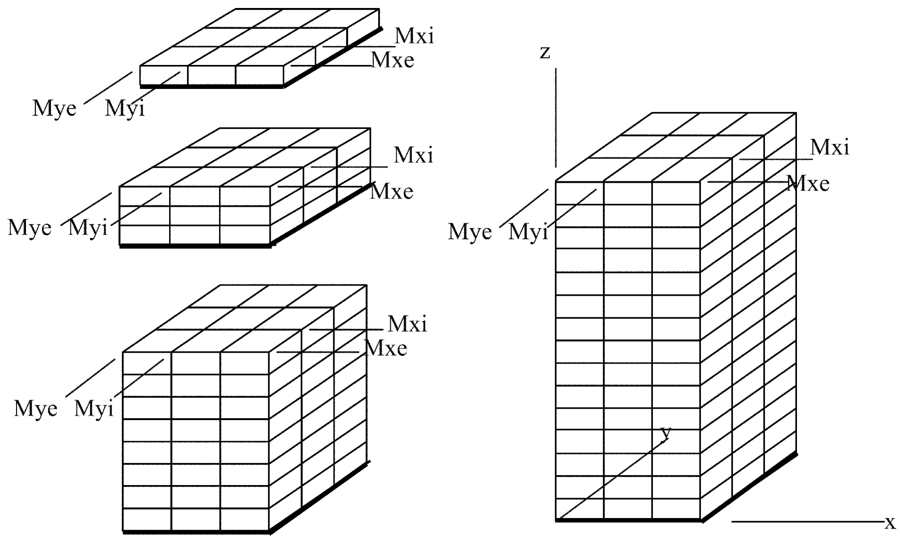
The additional axial deformations and relative rotations produced by the presence of plastic hinges are taken into account in the stiffness matrix and the internal force vector of the plastic stage. Explicit expressions for the elasto-plastic tangent stiffness matrix and the elasto-plastic internal force vector are also developed. The mathematical derivations can be found in the literature (Kondo and Atluri 1987). Depending on the level of earthquake excitation, in a typical structure, all the elements may remain elastic, or some of the elements will remain elastic and the rest will yield. The structural stiffness matrix and the internal force vector can be explicitly developed from the individual elements and the particular state (elastic or plastic) they are in.

Based on an extensive literature review, it is observed that viscous Rayleigh-type damping is commonly used in the profession and is used in this study too (Clough and Penzien 1993). The consideration of both the tangent stiffness and the mass matrices is a rational approach to estimate the energy dissipated by viscous damping in a nonlinear seismic analysis. The mass matrix is assumed to be concentrated-type.

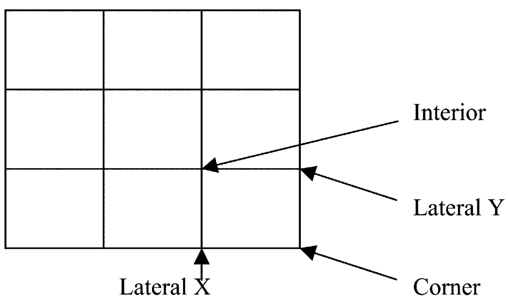
The step-by-step direct integration numerical analysis procedure and the Newmark β method (Bathe 1982) are used to solve the nonlinear seismic governing equation of the problem. A computer program has been developed to implement the solution procedure. The program was extensively verified using information available in the literature. The structural response behavior in terms of members' forces (axial load, shear force and bending moment), total base shear and interstory displacement, can be estimated using this computer program.

4. Structural models and earthquakes

Four three-dimensional moment resisting steel frame structures are used in the study. The geometry of the frames is shown in Fig. 2(a), the location of the columns in Fig. 2(b) and their member sizes in Table 1. They will be denoted hereafter as Models 1, 2, 3 and 4. For each model, four plane frames are indicated in Fig. 2(a): two interior (M_{xi} and M_{yi}) and two exterior (M_{xe} and M_{ye}). The story height for all the models is a constant of 3.66 m and their bay width is 7.32 m in both directions. The plane frames in both directions were designed according to the UBC standards and then modified following the strong column-weak beam (SCWB) concept. The seismic design load was computed for seismic zone 4. The dead and live loads were 5.8 and 2.9 kN/m^2 (≈120 and 60 psf), respectively. With the exception of exterior joints and the joints located on the top floor, for both sets of plane frames, the ratio of the sum of the plastic moments of the beams framing into a given beam-column joint to the sum of the plastic moments of the columns framing into the same joint, ranges from 0.68 to 0.91. The fundamental



a). Structural models



b) Location of columns

Fig. 2 Structural models and location of columns periods of the models in the major direction (*X* direction) are 0.21, 0.67, 1.15 and 1.32. respectively. The corresponding values for the minor direction are 0.34, 1.04, 1.61 and 1.93 sec., respectively. In all these frames, the columns are assumed to be made of Grade-50 steel and the girders of A36 steel. In the seismic analysis of these frames, equivalent nodal forces were calculated, as required for the assumed stress-based finite element formulation used in this study. One node was placed at the mid-span of each of the girders. Each node is considered to have six degrees of freedom. These four models with

Table 1 Member sizes

Model	Frame	Story	Ext. Col.	Int. Col.	Girders
1	M_{xe}	1	W14x74	W14x109	W21x44
	M_{xi}	1	W14x99	W14x145	W21x57
	M_{ye}	1	W14x74	W14x99	W14x26
	M_{yi}	1	W14x109	W14x145	W14x38
2	M_{xe}	1-2	W14x82	W14x99	W21x73
		3	W14x82	W14x99	W18x40
	M_{xi}	1-2	W14x99	W14x159	W18x71
		3	W14x99	W14x159	W18x71
	M_{ye}	1-2	W14x82	W14x99	W18x40
		3	W14x82	W14x99	W16x26
	M_{yi}	1-2	W14x99	W14x159	W18x71
		3	W14x99	W14x159	W16x40
3	M_{xe}	1-2	W14x120	W14x159	W24x94
		3	W14x109	W14x159	W24x94
		4-5	W14x109	W14x145	W24x84
		6-7	W14x82	W14x132	W24x84
		8	W14x82	W14x132	W21x50
	M_{xi}	1-2	W14x159	W14x211	W24x131
		3	W14x145	W14x211	W24x131
		4-5	W14x145	W14x193	W24x117
		6	W14x132	W14x176	W24x104
		7	W14x132	W14x176	W24x104
		8	W14x132	W14x176	W21x68
	M_{ye}	1-2	W14x120	W24x117	W24x55
		3	W14x109	W14x159	W24x55
		4-5	W14x109	W14x145	W21x57
		6-7	W14x82	W14x132	W18x46
		8	W14x82	W14x132	W16x31
	M_{yi}	1-2-3	W14x159	W14x211	W24x68
		4-5	W14x145	W14x193	W21x73
		6-7	W14x132	W14x176	W18x71
		8	W14x132	W14x176	W16x40
4	M_{xe}	1-2-3	W14x283	W14x370	W27x217
		4-5	W14x257	W14x342	W27x194
		6-7	W14x233	W14x311	W27x178
		8-9	W14x211	W14x283	W27x161
		10-11	W14x193	W14x257	W27x146
		12-13	W14x176	W14x233	W27x114
		14-15	W14x145	W14x176	W21x68
	M_{xi}	1-2-3	W14x398	W14x500	W27x258
		4-5	W14x370	W14x455	W27x258
		6-7	W14x342	W14x398	W27x235
		8-9	W14x311	W14x370	W27x217
		10-11	W14x283	W14x342	W27x194
		12-13	W14x257	W14x311	W27x161
		14-15	W14x211	W14x233	W21x93

Table 1 continued

Model	Frame	Story	Ext. Col.	Int. Col.	Girders
4	M_{ye}	1-2-3	W14x283	W14x398	W27x129
		4-5	W14x257	W14x370	W27x114
		6-7	W14x233	W14x342	W27x102
		8-9	W14x211	W14x311	W27x94
		10-11	W14x193	W14x283	W27x84
		12-13	W14x176	W14x257	W27x84
		14	W14x145	W14x211	W24x76
		15	W14x145	W14x211	W21x44
	M_{yi}	1-2-3	W14x370	W14x500	W27x146
		4	W14x342	W14x455	W27x146
		5	W14x342	W14x455	W27x129
		6	W14x311	W14x398	W27x129
		7	W14x311	W14x398	W27x114
		8-9	W14x283	W14x370	W27x114
		10	W14x257	W14x342	W27x114
		11	W14x257	W14x342	W27x102
		12	W14x233	W14x311	W27x102
		13	W14x233	W14x311	W24x84
		14	W14x176	W14x233	W24x84
		15	W14x176	W14x233	W21x50

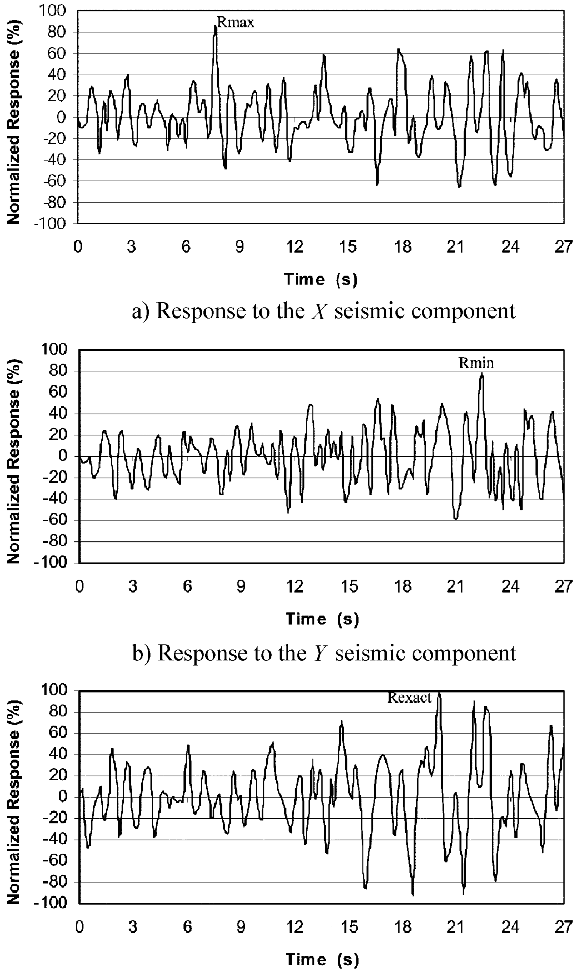
Table 2 Earthquake models

Earthquake Number	Earthquake Name	Station	Predominant Period (SEC)
1	EL Salvador 2001	Relaciones Exteriores	0.11
2	EL Centro	ELC7	0.19
3	Northridge	Los Angeles, Wadsworth V.A. Hospital	0.25
4	México 1985	Cayaco, Michoacán, México	0.29
5	Northridge	Topanga Fire Station	0.31
6	Northridge	Irvine, 2603 Main	0.38
7	EL Centro	ELC8	0.39
8	Northridge	Los Angeles, Brentwood V.A. Hospital	0.49
9	Northridge	Los Angeles, Griffith Observatory	0.51
10	Northridge	Los Angeles, Wadsworth V.A. Hospital	0.55
11	México 1985	Villita, Michoacán, México	0.55
12	México 1985	Atoyac, Michoacán, México	0.58
13	Northridge	Hawthorne Faa Bldg.	0.60
14	EL Centro	ELC0	0.68
15	México 1985	Apatzingan, Michoacán, México	0.91
16	EL Salvador 2001	Ahuachapan	1.03
17	México 1985	Chilpancingo, Guerrero, México	1.05
18	EL Centro	ELC1	1.29
19	EL Centro	ELC5	2.10
20	EL Centro	ELC2	2.20

different dynamic characteristics are subjected to twenty strong motions which are scaled down or up in such a way that all the frames approximately develop a maximum interstory displacement of 1.5%. The earthquakes are given in Table 2. They are presented in increasing order for their predominant period. The predominant period of each earthquake is the period value corresponding to the largest peak in the response spectra. These earthquakes are denoted hereafter as Earthquakes 1 through 20.

5. Evaluation of the 30% combination rule

For each one of the time history records, the horizontal component with the largest peak ground acceleration is selected to be applied to the strong direction of the models and the other horizontal component is applied to the weak direction. In order to analytically evaluate the accuracy of the 30% combination method, the following analyses are performed:



a) Response to the *X* seismic component

b) Response to the *Y* seismic component

c) Response to the *X* and *Y* seismic components

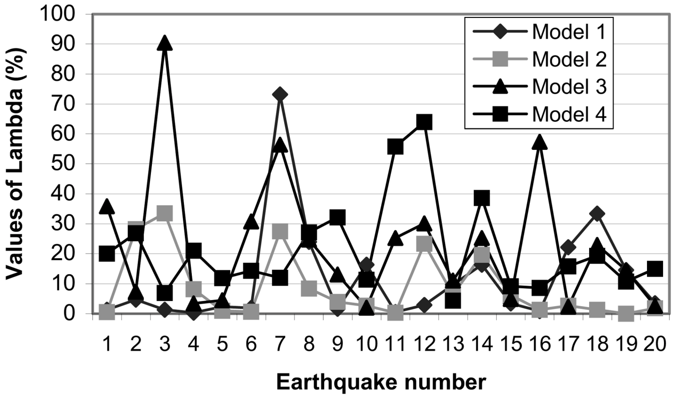
Fig. 3 Definitions of $R_{\max}$, $R_{\min}$ and R_{exact}

a) the frames are excited for each component separately. The peak values for a given response parameter are denoted as R_X and $R_{Y,}$ as previously discussed in Section 2, and the greater one is identified. For clarification purposes, R_X and R_Y are redefined: $R_{\max}$ and $R_{\min}$ represent the larger and the smaller peak values of the individual responses, respectively. Fig. 3 shows these values for a hypothetical case.

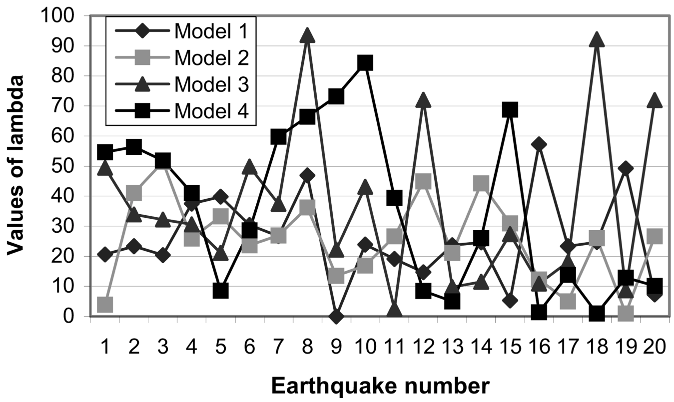
b) the frames are excited for both horizontal components acting simultaneously. The results obtained from these analyses are assumed to be the peak exact solution (R_{exact}). Then the λ parameter is estimated as:

$$\lambda = \frac{R_{exact} - R_{\max}}{R_{\min}} \tag{10}$$

The four steel models are excited for the twenty earthquakes given in Table 2 and the 30% combination method is evaluated. Elastic and inelastic analyses are considered. The λ parameter given in Eq. (1) is estimated for the maximum axial loads at ground level columns (Fig. 2b) and for the maximum total base shear. Results according to elastic analysis, for interior columns are presented in



a) Elastic analysis



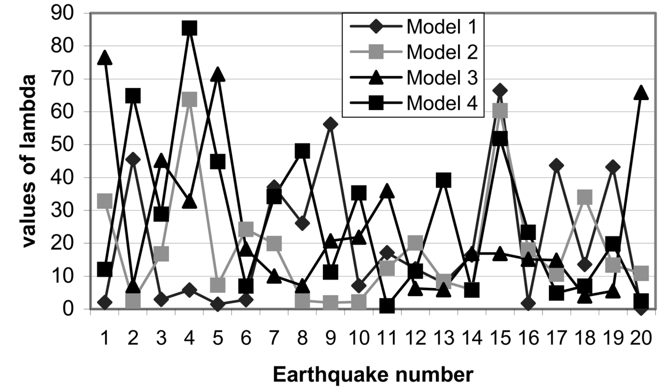
b) Inelastic analyisis

Fig. 4 Values of λ for interior columns

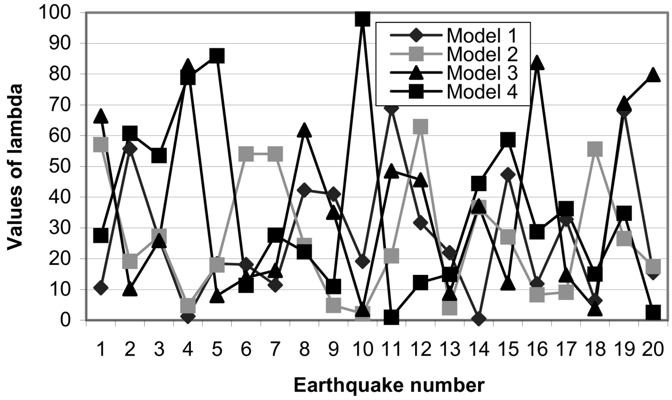
Fig. 4(a), for all the models. It is observed that, for a given model, the values of λ significantly vary from one earthquake to another, without shown any correlation. In addition, for a given earthquake, no correlation is observed between λ and the fundamental period of the models. The most important observation that can be made is that the 30% value is exceeded in many cases. If the values of λ are significantly larger than 30%, this rule could underestimate the combined structural response.

The results for inelastic analysis for interior columns of all the models are presented in Fig. 4(b). The major observations made for elastic analysis also apply to inelastic analysis. The only additional observation that can be made is that the number of cases in which the 30% value is exceeded is much larger for inelastic than for elastic analysis. For example for Model 1, this number is two for elastic analysis, but the corresponding number is five for inelastic analysis.

Values of λ for lateral columns on X direction are shown in Figs. 5(a) and 5(b), for elastic and inelastic analysis, respectively. By comparing Figs. 4 and 5, it is observed that the underestimation is more for lateral than for interior columns. Results for corner columns are also estimated, but they are not shown because of lack of space. However, results indicate that the number of cases of underestimation is larger for corner than for lateral columns, which in turn is larger than for interior columns. The values of λ for lateral columns on Y direction are similar to those of X direction.



a) Elastic analysis



b) Inelastic analyisis

Fig. 5 Values of λ for lateral columns

The results for the maximum total base shear for both X and Y directions are similarly estimated but are not shown. As for the case of axial load, the values of λ are quite different from 30% for many of the cases. It is also observed that the values of the λ parameter significantly vary from one earthquake to another and from one frame to another. No correlation is observed between the λ parameter and the predominant periods of the earthquakes or between the λ values and the fundamental periods of the models. The underestimation is more for inelastic than for elastic analysis.

It is important to note that the accuracy of evaluating the combined response by using $\lambda = 30\%$, will significantly depend on the values of the Q parameter, as observed from Eq. (5). If Q tends to zero, for any value of λ the combined response R_{C1} will tend to R_X, which in turn will tend to the exact solution. Thus, if the "real" λ is larger or smaller than 0.3, the combined response will be correctly estimated. On the other hand, if Q tends to one, the combined response will tend to 1.3 R_X. Consequently, if the "real" λ is larger than 0.3, the combined response will be underestimated.

From the above discussion, it can be concluded that the values of λ vary with the fundamental period of the frames, the predominant period of the earthquakes, the selected response parameter and its location, and the type of analysis. The most important observation so far is that the values of λ can be significantly larger than 30%.

6. SRSS and 30% combination rules vs. exact solution

The combined effect, according to the SRSS and the 30% rules, is estimated and compared to that given by the exact solution. In addition, the combined effect is calculated by using $\lambda = 40\%$ instead 30%, and by increasing the maximum of the two peak responses 20% (it is, the combined response is 1.2 times $R_{\max}$). These two additional combination rules will be referred hereafter as the 40% and the 1.2 $R_{\max}$ rules, respectively. For comparison purposes, the following error terms are defined:

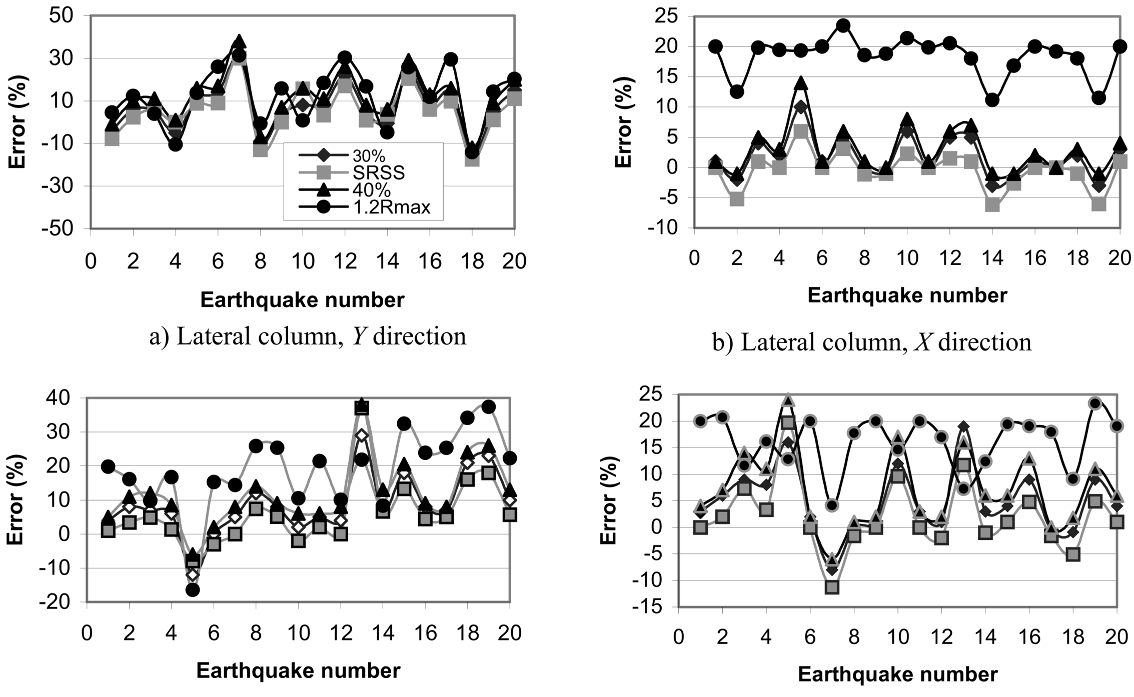
$$E_{30\%} = \frac{30\% \text{ value} - \text{exact value}}{\text{exact value}} \tag{11}$$

$$E_{SRSS} = \frac{\text{SRSS value} - \text{exact value}}{\text{exact value}} \tag{12}$$

$$E_{40\%} = \frac{40\% \text{ value} - \text{exact value}}{\text{exact value}} \tag{13}$$

$$E_{1.2R\max} = \frac{1.2R_{\max} \text{ value} - \text{exact value}}{\text{exact value}} \tag{14}$$

where the terms exact value, 30% value, SRSS value, 40% value and $1.2R_{\max}$, represent the combined effect according to the exact solution, $\lambda = 30\%$, SRSS rule, $\lambda = 40\%$ and the maximum peak value increased by 20%, respectively. A negative error in the above equations implies that the combination rule under consideration underestimates the combined effect of both components; in other words, the results are unconservative. A positive value of the errors indicates that the combination rules overestimate the combined effect, and thus are conservative. The errors are calculated for the axial loads at ground level columns and for the total base shear for all the models. Both, elastic and inelastic analyses are considered.



a) Lateral column, Y direction

b) Lateral column, X direction

c) Corner column

d) Interior column

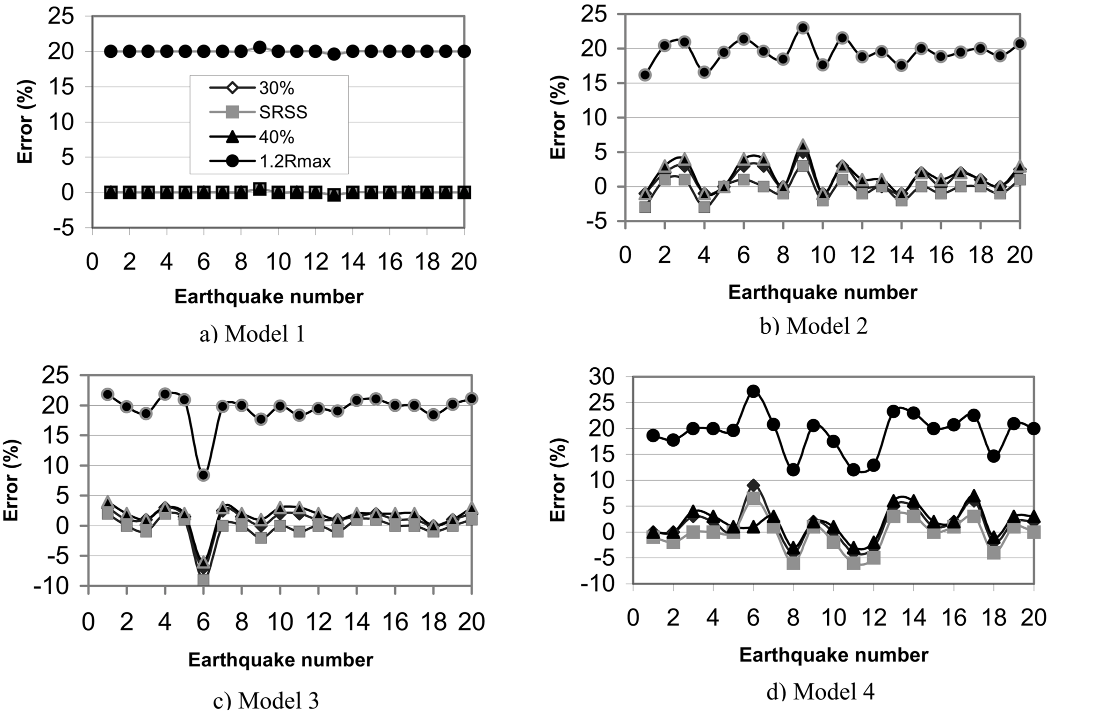
Fig. 6 Errors for axial load on base columns of Model 1, elastic analysis

6.1 Elastic analysis

Results for elastic analysis are discussed first. The errors for the axial load on columns of Model 1 are shown in Figs. 6(a) through 6(d). It is observed that both the 30% and the SRSS rules, commonly used in seismic design procedures, underestimate the combined response in some of the cases. In general, the curve for the 30% rule is over the corresponding curve for the SRSS rule. In other words, the 30% rule is more conservative than the SRSS rule. The implication of this is that the values of the Q ratio, presented in Fig. 1, are smaller than 0.67 in all the cases. No correlation is observed between the magnitude of the errors and the predominant period of the earthquakes.

Results indicate that increasing λ from 30 to 40%, does not necessarily reduce the error with respect to that of 30%. This is supported by the results shown in Fig. 1 and Eq. (5) where it is observed that the combined response depends on the value of the Q parameter. If Q is small, increasing λ from 30% to even 90% may result in negligible increments of R_{C1} and consequently the error remains practically the same. Fig. 6 also indicates that the $1.2R_{\max}$ combination rule reasonably overestimate the combined response for most of the cases. As for the other rules, no correlation is observed between the magnitude of the errors and the predominant period of the earthquakes.

The errors for the axial load on columns of Models 2, 3 and 4, are similarly estimated but are not shown because of lack of space. The major observations made for Model 1, also apply to these models. From an analysis of the results for all columns and models, it is observed that there is no correlation between the introduced errors and the fundamental period of the frames. It is also shown that the $1.2R_{\max}$ rule is more conservative for lateral than for corner and interior columns.



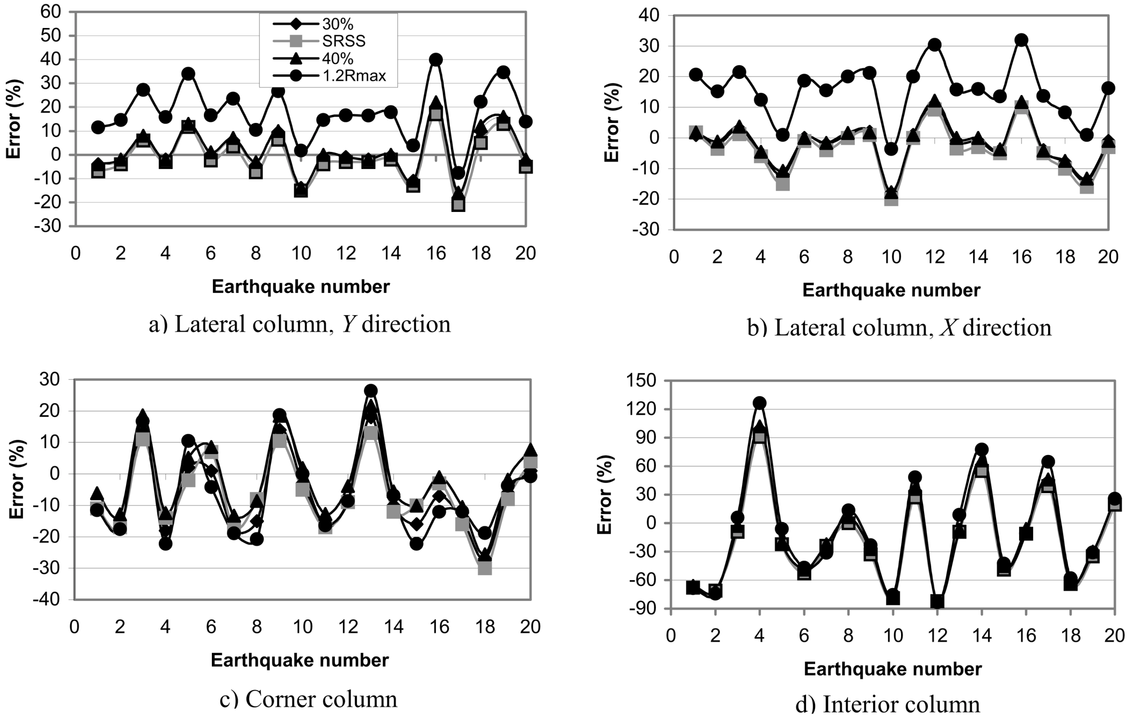
a) Model 1 b) Model 2 c) Model 3 d) Model 4

Fig. 7 Errors for total base shear in *X* direction, elastic analysis

The errors in terms of the total base shear in *X* direction are shown in Fig. 7 for all the models and combination rules. Unlike the error for the axial load in columns, both the 30% and the SRSS combination rules accurately estimate the combined effect for the total base shear. It is observed that for these two rules and the 40% rule the error ranges between 5 and −5% in most of the cases. This is expected. In the case of total base shear, one of the peak responses (R_X or R_Y) used in Eqs. (1) and (2), will be very large in comparison with the other one. Then, R_{C1} and R_{C2} will be very close each other and at the same time will be close to the exact solution. Consequently, the introduced errors according to Eqs. (11), (12) and (13), will be very small. It is observed that the scatter in the error values tends to increase with the fundamental period of the frames. As for the case of axial loads, the underestimation is more for the SRSS rule than for the 30% rule. It is also observed that the $1.2R_{\max}$ rule, overestimate the response in all of the cases. In addition, since R_{C1}, R_{C2} and the exact solution are close each other and because of Eq. (14), the introduced error according to this rule is close to 20% for most of the cases. The errors in terms of the total base shear in *Y* direction are similarly estimated. The results are similar to those of the *X* direction.

6.2 Inelastic analysis

The error for the axial load on columns of the base of Model 1 is given in Figs. 8(a) through 8(d). As for the case of elastic analysis, for a given rule, no correlation is observed between the magnitude of the errors and the predominant period of the earthquakes. It is also observed that both the 30% and SRSS rules underestimate the combined response in some cases. However, the number of cases and the



a) Lateral column, *Y* direction

b) Lateral column, *X* direction

c) Corner column

d) Interior column

Fig. 8 Errors for axial load on base columns of Model 1, inelastic analysis

magnitude of the underestimation are larger for the inelastic case. For example, for elastic analysis, the maximum negative error according to the SRSS rule for lateral columns on *X* direction was about 5% and occurred in three cases (Fig. 6b). For the inelastic case, the corresponding error was larger than –5% in about 8 cases (Fig. 8b). The errors were even larger than –15% in at least three cases. In addition, for these two rules the underestimation is in general larger for corner and interior than for lateral columns.

It is also observed from Fig. 8 that, for lateral columns (Figs. 8a and 8b), the $1.2R_{max}$ rule reasonably overestimates the combined response. For corner and interior columns, however, unlike for elastic analysis, the $1.2R_{max}$ rule may significantly underestimate the combined response. For example, for corner columns this rule underestimates the combined effect only in one case (Fig. 6c) for elastic analysis. For inelastic analysis however, the combined response is underestimated in fifteen cases (Fig. 8c). This indicates that the results for elastic analysis may be quite different from those of inelastic analysis. The implication of this is that the results obtained from elastic analysis of steel frame structures subjected to strong motions may be a very crude approximation. The errors in terms of axial load on columns of Models 2, 3 and 4 are also estimated. The results are similar to those of Model 1.

The errors for the total base shear in *X* direction are shown in Figs 9(a) through 9(d). It is observed that, as for the case of elastic base shear, the 30% and SRSS rules, in general estimate the combined effect very well. Excepting for Model 1 and Earthquake10 (Fig 9a), whenever the combined effect is underestimated (negative errors), the corresponding error is relatively small. The underestimation is always more for the SRSS than for the 30% rule. Results also indicate that the $1.2R_{max}$ rule, excepting for Model 1 and Earthquake 10, reasonably overestimate the combined effect. The errors for the total base shear in *Y* direction are also estimated but are not shown. The results are similar to those of *X* direction.

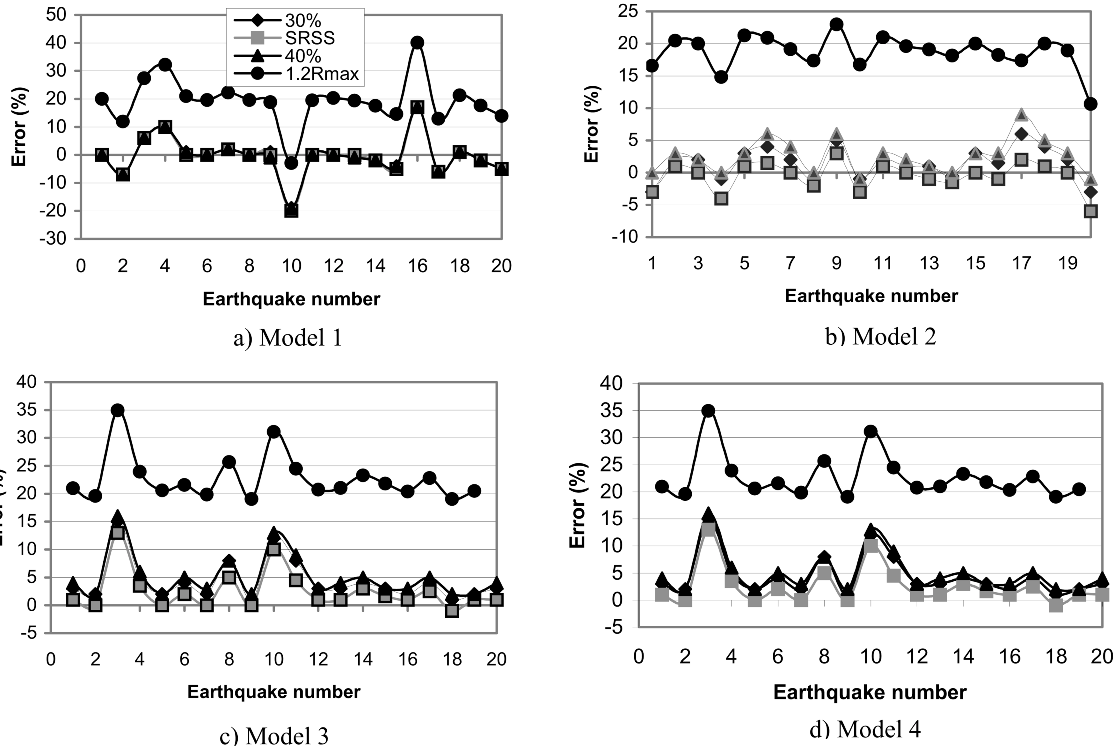


Fig. 9 Errors for total base shear in *X* direction, inelastic analysis

In summary, whether elastic or inelastic analysis is used the combined total base shear is reasonably overestimated by all the rules. The 30% and the SRSS rules, however, can underestimate the combined response in terms of axial loads. The $1.2R_{\max}$ rule reasonably estimates the combined axial load for elastic analysis but it can be significantly underestimated for inelastic analysis, particularly for corner and interior columns.

7. Probabilistic analysis for λ

The results of this study show that the combined response, according to the different rules, for both, axial and total base shear, significantly vary from one earthquake to another and from one model to another even though the maximum relative deformation (intersory displacement) produced on the models is approximately the same for all time histories (≈1.5%). It indicates that the seismic response of frames is highly sensitive to the characteristics of the time histories used. This is particularly true for inelastic analysis. In addition, the combined response is different from each rule, which in turn may be quite different from the exact solution. This shows the necessity of a probabilistic analysis of the problem.

In the design of an engineered system, it is usually stated that the capacity should at least satisfy the demand. Different terminology is used to describe these concepts depending upon the specific problem under consideration. In structural engineering, for example, the capacity is usually expressed in terms of resistance and the demands in terms of applied loads or their effect. However, the parameters related to capacity and demand are, in general, random variables. Therefore, the associated uncertainty needs to be quantified. The primary task in the design of a system is to ensure satisfactory performance. Satisfactory performance cannot be absolutely guaranteed. Instead, assurance can only be given in terms of the probability of success (p_s) according to some performance criterion. In engineering, this probabilistic assurance of performance is referred as reliability. Another way of looking the problem is to consider the probability of failure (p_f), which is also commonly defined as risk. Risk and reliability are complementary terms.

The problem of estimating an appropriate value for λ in Eq. (1) can be circumscribed in the abovementioned context. Even though, strictly speaking, it is not exactly a "demand and capacity problem" this formulation can give probabilistic bases to estimate the accuracy of the 30% rule in evaluating the combined response. Any mathematical model satisfying the properties of Probability Density Function (PDF) and Cumulative Distribution Function (CDF) can be used to evaluate the uncertainties of the problem (Haldar and Mahadevan 2000). However, if a random variable cannot have negative values, as in the case of the λ parameter, the lognormal variation is appropriate, since it automatically eliminates the possibility of negative values.

If λ is assumed to be a lognormal variable [that is, $LN(\eta_\lambda\ \varsigma\lambda)$], then another random variable can be defined as (Haldar and Mahadevan 2000):

$$Y = \ln \lambda \tag{15}$$

Since λ is lognormal, Y will be a normal variable [$Y \sim N(\mu_Y, \sigma_Y)$]. It must be remembered that the relationship between the parameters of a normal variable (mean μ, and standard deviation σ) and the parameters of a lognormal variable (η and ς) is given by

$$\begin{aligned} \eta &= \ln \mu - \frac{1}{2}\zeta^2 \\ \zeta^2 &= \ln(1+\delta^2) \end{aligned} \tag{16}$$

where $\delta = \sigma/\mu$ is the coefficient of variation (COV).

According to the framework of the earthquake design criteria presented in standard seismic codes, very low probabilities of exceedence are assigned to design seismic motions. Since the value of λ somehow defines the actions acting on a frame, it is a load-related random variable. Therefore, its *design value* should be selected in such a way that its probability of exceedence is also very low. In this paper, it is assumed that this value corresponds to the 90th percentile value (that is, the probability of being exceeded or "probability of failure p_f" is 10%). The 95th percentile value is also calculated for comparison purposes only. If λ is a lognormal variable, it can be shown (Haldar and Mahadevan 2000) that the probability of $\lambda = a$ being exceeded (a = *design value*) can be calculated as

$$p_f = p(\lambda > a) = 1 - \Phi\left(\frac{\ln a - \eta_\lambda}{\zeta_\lambda}\right) \tag{17}$$

where Φ is the CDF of the standard normal variable.

The values of a for axial loads, corresponding to the 90th and 95th percentile values, are calculated by using Eq. (17) and presented in Table 3. The results are presented for two cases: for individual models

Table 3 Design values of λ, in percent, for axial load and base shear corresponding to 90th and 95th percentile

Response parameter		Percentile	Elastic					Inelastic				
			M1	M2	M3	M4	ALL	M1	M2	M3	M4	ALL
Axial load	Lateral X	90th	31	28	37	40	34	40	39	54	53	47
		95th	32	29	39	41	35	41	40	55	55	48
	Corner	90th	35	26	53	38	39	52	54	51	65	56
		95th	36	28	54	39	40	54	56	52	67	57
	Interior	90th	17	13	35	31	24	36	36	54	52	45
		95th	18	14	36	32	26	37	37	56	54	46
	Lateral Y	90th	34	25	45	38	36	53	50	31	56	48
		95th	36	26	47	40	37	54	52	33	58	50
Base shear		90th	2	51	41	46	37	25	39	41	45	38
		95th	2	52	42	47	39	26	40	42	46	39

(M1,…., M4) and for all the models (ALL). In both cases, the results are separated according to the column location. It is observed from the table that the values of *a* for axial load significantly vary with the type of analysis, the location of the column and the model considered. The values of *a* are larger for inelastic than for elastic analysis. In addition, the values are also in general larger for taller frames (Models 3 and 4) than for low-rise frames (Models 1 and 2). Results also indicate that the values of the *a* parameter are larger for corner columns. The probabilistic results for *a* for all the models and all the columns (not shown in the table) are as follow. The 90th percentile values are 33 and 51 for elastic and inelastic analyses, respectively. The corresponding 95th percentile values are 35 and 53. Based on the results of this study and on the assumption of the 90th percentile-design value, a value of 50% is proposed for λ to estimate the combined effect for axial loads.

The values of *a* for the total base shear are also shown in Table 3. The major conclusions made for axial loads also apply to this parameter. The only additional observations that can be made is that for a given percentile value, the values of *a* for all the models are similar for elastic and inelastic analyses. From these results, a value of 40% is proposed for λ for the combined response in terms of the total base shear.

8. Conclusions

The Square Root of the Sum of the Squares (SRSS) rule and the 30-percent (30%) combination method, commonly used in code-specified seismic design procedures to evaluate the maximum effect of both horizontal components of earthquakes, are re-evaluated. Four three-dimensional moment-resisting steel frame structures modeled as complex multi-degree of freedom representing different dynamic characteristics are considered in the study. Using a time domain nonlinear finite element program developed by the authors, the maximum inelastic seismic responses of the models are evaluated by simultaneously applying both components. Then, the abovementioned combination rules and others are evaluated. The response is expressed in terms of the total base shear, and the axial loads at interior, lateral and corner ground level columns. The numerical study indicates that both the SRSS rule and the 30% combination method may underestimate the combined effect in terms of axial loads. It is observed that the underestimation is more for the SRSS than for the 30% rule. In addition, the underestimation is more for inelastic analysis than for elastic analysis. This indicates that the results for elastic analysis may be quite different from those of inelastic analysis. The implication of this is that the results obtained from elastic analysis of steel frames structures subjected to strong motions may be a very crude approximation. The underestimation cannot be correlated with the height of the frames or the predominant period of the earthquakes. Based on the results obtained in this study, it is concluded that the design requirements for the combined effect of the horizontal components, as outlined in some code-specified seismic design procedure, need to be modified. Values of λ of 50% and 40% are proposed in this study for axial load and total base shear, respectively. It is also concluded that the $1.2R_{\max}$ rule is more conservative than the 30% and the SRSS rules and it is also suggested in this study.

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