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# Out-of-plane buckling and bracing requirement in double-angle trusses

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**Abstract.** Truss members built-up with double angles back-to-back have monosymmetric cross-section and twisting always accompanies flexion upon the onset of buckling about the axis of symmetry. Approximate formulae for calculating the buckling capacity are presented in this paper for routine design purpose. For a member susceptible only to flexural buckling, its optimal cross-section should consist of slender plate elements so as to get larger radius of gyration. But, occurrence of twisting changes the situation owing to the weakness of thin plates in resisting torsion. Criteria for limiting the leg slenderness are discussed herein. Truss web members in compression are usually considered as hinged at both ends for out-of-plane buckling. In case one (or both) end of member is not supported laterally by bracing member, its adjoining members have to provide an elastic support of adequate stiffness in order not to underdesign the member. The stiffness provided by either compression or tension chords in different cases is analyzed, and the effect of initial crookedness of compression chord is taken into account. Formulae are presented to compute the required stiffness of chord member and to determine the effective length factor for inadequately constrained compressive diagonals.

**Key words:** truss; double-angle section; flexural-torsional buckling; width-thickness ratio; elastic support; lateral stiffness; bracing; effective length.

## 1. Introduction

Trusses with members built-up with twin angles back-to-back are very popular in mill building construction. These members with monosymmetric section buckle in a flexural-torsional pattern about their axis of symmetry. But, for a long period, this buckling problem has often been treated simply as flexural buckling, leading to underdesign of these members.

Marsh (1997) pointed out the necessity of considering flexural-torsional buckling for single and double angle struts and presented simplified formulae in this regard. But the simplification effort focused mainly on two equal-leg angles in close contact with scare interconnection. This study presents simplified formulae ready-to-use for routine design work that covers both equal- and unequal-leg angles, with stitch plates in-between their backs.

When a member is susceptible only to flexural buckling, its constituent plate elements should be slender so as to obtain larger radius of gyration. But, in case twisting interacts with flexion, sections

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with slender plate elements may no more be economic owing to their weakness in resisting torsion. This is especially true when twisting is predominant in the buckling pattern. Advisable width-thickness ratios for double-angle compression members are given in this paper.

It is a common practice to take the out-of-plane effective length of a truss compressive diagonal to be equal to its geometric length. In so doing, it is taken for granted that both ends of this diagonal are supported laterally. In the actual layout of the truss-bracing system, only a part of panel points is provided with lateral bracing, and the chord members concurrent at the unsupported points have to act as elastic support to the diagonal.

Fisher (1983) stressed for the first time on the importance of tension chord bracing to stabilize the compression diagonal and analyzed the restraining effect of lower chord to web member. But the benefit of the tensile force to stiffen the chord was overlooked in his study, thus leading to too stringent requirement of bracing system.

Comprehensive analysis is carried out in the present work on the restraint provided by both compressive and tensional chords, taking account of their axial force. The effect of geometric imperfection on chord stiffness as well as that of twisting is discussed. Formulae are presented for checking the adequacy of chord stiffness and for determining the effective length of web member in the event of inadequate stiffness.

## 2. Practical formulae for flexural-torsional buckling

### 2.1. Equivalent slenderness ratio

The common approach to deal with flexural-torsional buckling is to find out an equivalent slenderness ratio thus transforming the problem into a flexural buckling one. This equivalent slenderness as determined by elastic stability theory is given by the following expressions

$$\lambda_{yz}^2 = \lambda_y^2\left[\frac{1}{2}\left(1+\frac{\lambda_z^2}{\lambda_y^2}\right)+\frac{1}{2}\sqrt{\left(1+\frac{\lambda_z^2}{\lambda_y^2}\right)^2-4k\frac{\lambda_z^2}{\lambda_y^2}}\right] \tag{1a}$$

and

$$\lambda_{yz}^2 = \lambda_z^2\left[\frac{1}{2}\left(1+\frac{\lambda_y^2}{\lambda_z^2}\right)+\frac{1}{2}\sqrt{\left(1+\frac{\lambda_y^2}{\lambda_z^2}\right)^2-4k\frac{\lambda_y^2}{\lambda_z^2}}\right] \tag{1b}$$

where $\lambda_y$ – slenderness ratio about the axis of symmetry $y$, $\lambda_y = l_{oy} / i_y$ (Fig. 1)

$\lambda_z$ – slenderness ratio for torsional buckling, $\lambda_z^2 = \pi^2 EA \Big/ \left[\frac{1}{i_0^2}\left(GI_t + \frac{\pi^2 EI_\omega}{l_{oy}^2}\right)\right]$

$k$ – factor, $k = 1 - e_0^2/i_0^2$

$e_0$ – ordinate of shear center

$i_0$ – radius of gyration about the shear center, $i_0^2 = e_0^2 + i_x^2 + i_y^2$

$GI_t$ – St. Venant torsional rigidity

$EI_\omega$ – warping torsional rigidity

The above formulae (1a) and (1b) although different in format, are totally equivalent in themselves.

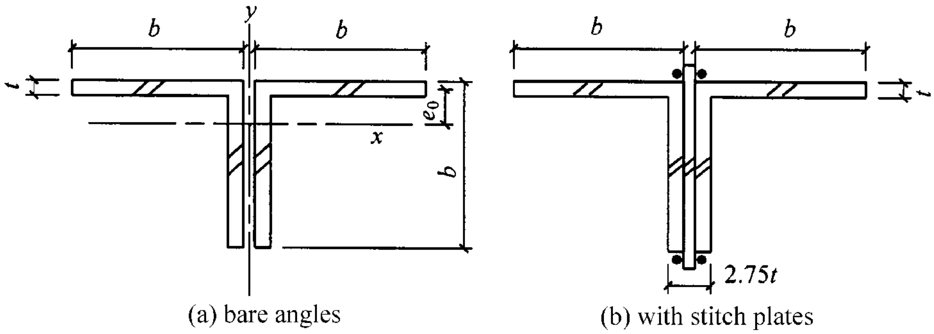


Fig. 1 Equal-leg double-angle sections

They are preferable for use in cases $\lambda_y > \lambda_z$ and $\lambda_y < \lambda_z$ respectively. The first criterion means that flexural buckling is weaker than torsional one so that $\lambda_{yz}$ should be calculated on the bases of $\lambda_y$. On the other hand, when twisting governs at the onset of buckling, Eq. (1b) has to be used.

### *2.2. Simplified formulae for design use*

Formulae (1a) and (1b) are rather complicate for routine design work and can be simplified by adopting approximate geometric properties of the section. For sections built-up with two equal-leg angles, we can write (Fig. 1)

$$i_x = 0.305b, \quad e_o = 0.236b$$

and assuming a gap width of 0.75$t$ between angle backs

$$i_y = 0.441b$$

The St. Venant torsion constant of the two bare angles is equal to

$$I_{t1} = At^2/3$$

$A$ being the total sectional area, whereas that of the section with stitch plate is equal to

$$I_{t2} = \frac{2(b-t)t^3}{3} + \frac{b(2.75t)^3}{3}$$

or approximately $I_{t2} = 1.98At^2$.

For the compound member, assuming that 15% of its length is filled with gusset and stitch plate, the St. Venant torsion constant will be the weighted mean

$$I_t = 0.85I_{t1} + 0.15I_{t2} \approx 0.58At^2$$

Sections built-up with unequal-leg angles (Fig. 2) are treated similarly and the leg width ratio adopted is $b_1 : b_2 = 1.55$.

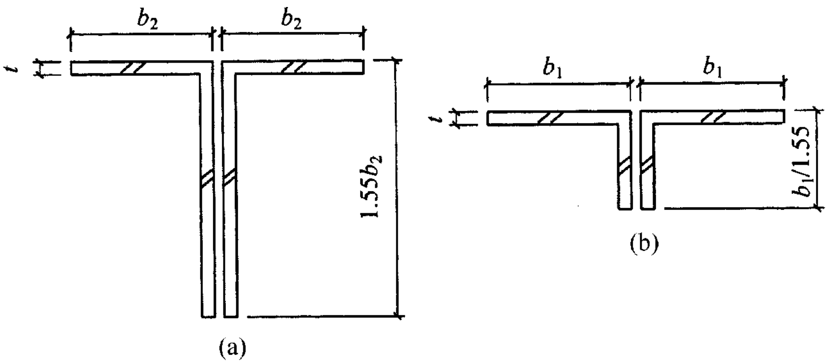


Fig. 2 Unequal-leg double-angle sections

The warping constant $I_\omega$ is neglected so that $\lambda_z$ reduces to

$$\lambda_z = 5.07 i_0 \sqrt{A/I_t}$$

The above approximation leads to the following simplified expressions of equivalent slenderness ratios. For double angles of equal legs (Fig. 1a)

$$\lambda_{yz} = \lambda_y \left(1 + \frac{0.475b^4}{l_{oy}^2 t^2}\right) \quad \text{when } b/t \le 0.58 l_{oy}/b \tag{2a}$$

$$\lambda_{yz} = 3.9\frac{b}{t}\left(1 + \frac{l_{oy}^2 t^2}{18.6b^4}\right) \quad \text{when } b/t > 0.58 l_{oy}/b \tag{2b}$$

For double angles of unequal legs with short leg outstanding (Fig. 2a)

$$\lambda_{yz} = \lambda_y \left(1 + \frac{1.09b_2^4}{l_{oy}^2 t^2}\right) \quad \text{when } b_2/t \le 0.48 l_{oy}/b_2 \tag{3a}$$

$$\lambda_{yz} = 5.1\frac{b_2}{t}\left(1 + \frac{l_{oy}^2 t^2}{17.4b_2^4}\right) \quad \text{when } b_2/t > 0.48 l_{oy}/b_2 \tag{3b}$$

For double angles of unequal legs with long leg outstanding (Fig. 2b)

$$\lambda_{yz} = \lambda_y \left(1 + \frac{0.19b_1^4}{l_{oy}^2 t^2}\right) \quad \text{when } b_1/t \le 0.56 l_{oy}/b_1 \tag{4a}$$

$$\lambda_{yz} = 3.7\frac{b_1}{t}\left(1 + \frac{l_{oy}^2 t^2}{52.7b_1^4}\right) \quad \text{when } b_1/t > 0.56 l_{oy}/b_1 \tag{4b}$$

### 2.3. Experimental verification

Kennedy and Murty (1972) reported experimental results of twelve sets of equal-leg double-angle specimens subject to concentric compression. Seven of them failed by flexural-torsional buckling. Except the set having unusual leg width-thickness ratio of 18.7, the remaining six sets, each comprising three identical specimens, are compared with the bearing capacity calculated with equivalent slenderness ratio of Eqs. (2a) and (2b). Table 1 reveals that test results agree well with calculated ones. Comparison is carried out between strength reduction factors $\varphi$. The experimental $\varphi_t$ is the ratio of the buckling stress to the yield strength of specimen, namely

$$\varphi_t = \frac{\sigma_{c.yz}}{f_y}$$

The calculated strength reduction factor $\varphi_c$ is obtained right away with equivalent slenderness ratio $\lambda_{yz}$ from the specific table of the Chinese Code for Design of Steel Structures (GBJ 17-88). All the ratios $\varphi_t/\varphi_c$ are larger than unity, with mean value 1.075 and standard deviation 0.059.

Table 1 Correlation between suggested formulae and test data–equal-leg angles

| Specimen set | Section $b\times t$ (mm) | $b/t$ | $f_y$ (MPa) | $\lambda_y$ | $\lambda_{yz}$ | $\varphi_t$ | $\varphi_c$ | $\varphi_t/\varphi_c$ |
|---|---|---|---|---|---|---|---|---|
| DH2 | 51×3.2 | 15.6 | 315.8 | 53.2 | 62.2 | 0.751 | 0.738 | 1.02 |
| DH4 | 63.5×4.8 | 13.2 | 349.6 | 43.3 | 52.7 | 0.821 | 0.781 | 1.05 |
| DH5 | 76.2×6.4 | 11.5 | 374.4 | 36.2 | 46.6 | 0.849 | 0.814 | 1.05 |
| DF1 | 44.5×3.2 | 13.0 | 333.7 | 29.6 | 53.2 | 0.890 | 0.789 | 1.13 |
| DF3 | 51×4.8 | 10.5 | 320.6 | 25.8 | 42.6 | 0.882 | 0.857 | 1.03 |
| DF5 | 76.2×6.4 | 11.3 | 393.7 | 17.8 | 48.8 | 0.932 | 0.798 | 1.18 |

Cao (1983) conducted a test program comprising five specimens of 75×50×6 double-angle struts with short leg outstanding. The gap between angle backs was 6 mm and the yield strength of the material 326.3 MPa. Table 2 shows the comparison of tested and calculated strength reduction factors $\varphi_t$ and $\varphi_c$. The ratio $\varphi_t/\varphi_c$ ranges from 1.13 to 1.25 with mean value 1.188 and standard deviation 0.039. The higher bearing capacity of test specimens may be attributed to flat platens 25 mm thick welded to their ends which enhanced the twisting resistance as explained by Marsh (1997).

Table 2 Correlation between suggested formulae and test data – unequal-leg angles

| Specimen | $l_{oy}$ (cm) | $\lambda_y$ | $\lambda_{yz}$ | $\varphi_t$ | $\varphi_c$ | $\varphi_t/\varphi_c$ |
|---|---|---|---|---|---|---|
| SJ1-1 | 255 | 123 | 126.6 | 0.374 | 0.311 | 1.20 |
| SJ2-1 | 205 | 99 | 103.5 | 0.475 | 0.422 | 1.13 |
| SJ2-2 | 205 | 99 | 103.5 | 0.492 | 0.422 | 1.17 |
| SJ2-3 | 205 | 99 | 103.5 | 0.503 | 0.422 | 1.19 |
| SJ3-1 | 155 | 75 | 80.9 | 0.734 | 0.585 | 1.25 |

## 3. Selection of width-thickness ratio

The behavior of flexural-torsional buckling may be classified in two categories where either flexion or twisting is the predominant mode of deformation. Eqs. (2) through (4) are thus divided into two expressions, the first of which applies to the case where flexion prevails over twisting, while the second applies to the case vice versa.

It can be seen from Eqs. (2) through (4) that the equivalent slenderness ratio $\lambda_{yz}$ is very sensitive to the increase of the leg slenderness $b/t$ in the range Eqs. (2b)~(4b) apply. Fig. 3 depicts the strength reduction factor $\varphi$ versus member length $l_{oy}$ of two sections of about the same cross-sectional area but with quite different $b/t$ ratios. The section built-up with two angles 160×10 has higher bearing capacity for larger value of unsupported length $l_{oy}$. This capacity keeps on increasing substantially with decreasing $l_{oy}$ up to 4.4 m. On the other hand, the section composed of two angles 100×16 having smaller radius of gyration, lags behind in capacity for large $l_{oy}$, but exceeds the wider and thinner section at $l_{oy} = 3$ m. The turning point at $l_{oy} = 4.4$ mm for angles 160×10 is the boundary between Eqs. (2a) and (2b), and can be obtained from $b/t = 0.58\, l_{oy}/b$. This latter expression means actually the equality $\lambda_y = \lambda_z$ for twin equal-leg section. In the cause of material saving, it is always advisable to choose angle size that fulfills the criterion

$$\lambda_y \leq \lambda_z \tag{5}$$

The leg width-thickness ratios satisfying this expression are given in Table 3, for the three categories of double-angle section.

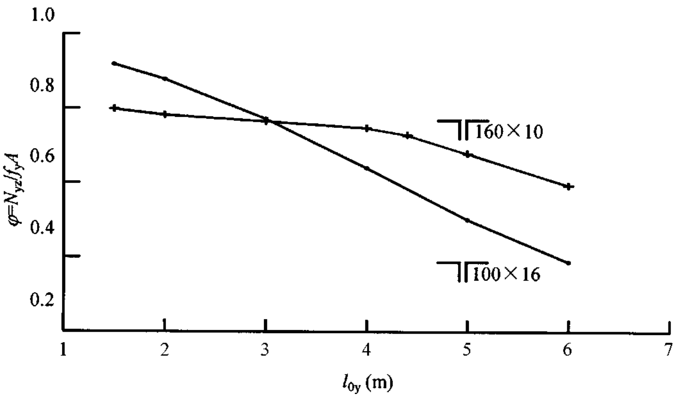


Fig. 3 Comparison of sections with wide and narrow leg

Table 3 Advisable width-thickness ratios of double angles section

| $\lambda_y$ | Equal-leg angle $b/t$ | Unequal-leg angle | |
|---|---|---|---|
| | | Short leg outstanding $b_2/t$ | Long leg outstanding $b_1/t$ |
| ≥ 70 | 16 | 10 | 16 |
| 60 | 15 | 10 | 16 |
| 50 | 12.5 | 10 | 13.5 |
| 40 | 10 | 8 | 10.8 |
| 30 | 7.5 | 6 | 8.1 |

## 4. Chord stiffness requirement for supporting web members

### *4.1. Elastically supported struts and effective length*

Struts elastically supported laterally at both ends may be dealt with as hinged-hinged only when the stiffness of the elastic supports satisfy the following condition (Simitses 1976)

$$S_1 S_2/(S_1+S_2) \geq N_{Ed}/l_d \tag{6}$$

where $S_1$ and $S_2$ – stiffness of elastic support at upper and lower ends respectively
$N_{Ed}$ – Euler critical load of the diagonal, $N_{Ed} = \pi^2 EI_{yd}/l_d^2$
$I_{yd}$ – moment of inertia of the diagonal for out-of-plane bending
$l_d$ – length of the diagonal

In case one of the ends is supported by lateral bracing, Eq. (6) reduces to

$$S \geq N_{Ed} / l_d \tag{7}$$

If criterion (6) or (7) is not satisfied, the diagonal must have an effective length factor larger than 1.0, namely

$$\mu = \frac{\pi}{l_d}\sqrt{\frac{EI_{yd}}{l_d} \cdot \frac{S_1+S_2}{S_1 S_2}} \tag{8}$$

or

$$\mu = \frac{\pi}{l_d}\sqrt{\frac{EI_{yd}}{l_d S}} \tag{8a}$$

### *4.2. Support stiffness provided by chord members*

Consider the compressive diagonals *AE* and *BG* of the truss shown in Fig. 4, which has lateral bracing at joints *A*, *C* and *D*, *F*, *H*. The end diagonal *AE* has hinged support at its lower end and an elastic lateral support at its upper end while the diagonal *BG* has elastic lateral supports at both ends.

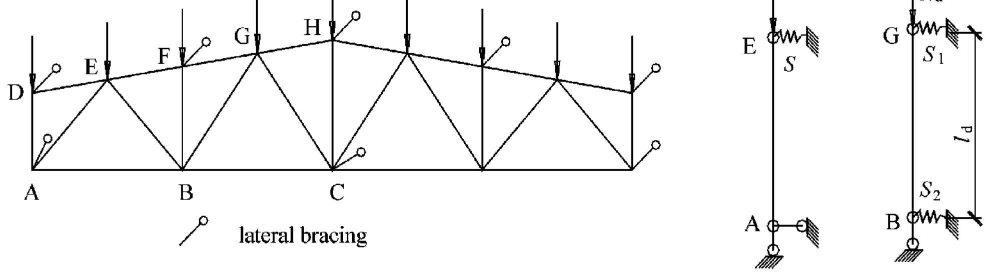


Fig. 4 Web members in compression

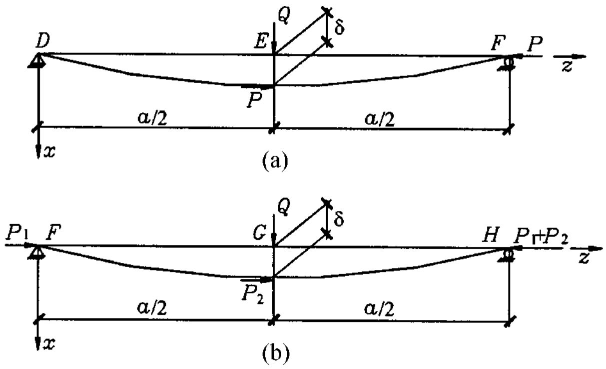


Fig. 5 Upper chord lateral stiffness

The stiffness of the lateral elastic support at *E* and *G* provided by the upper chord can be calculated according to the schemes of Fig. 5. Subject to a concentrated lateral load *Q* at points *E* and *G*, the chords *DF* and *FH* deflect out of the truss plane by $\delta$ and the relevant stiffness is given by

$$S_0 = Q/\delta$$

The analytical solution of the more general case (Fig. 5b) can be easily obtained as

$$S_{0c} = \frac{Q}{\delta} = \frac{8EI_{yc}}{a^3}\left[\frac{\frac{u_1}{\mathrm{tg}u_1}\left(2+\frac{u_2^2}{u_1^2}\right)+\frac{u_3}{\mathrm{tg}u_3}\left(2-\frac{u_2^2}{u_3^2}\right)-\frac{u_2^2}{u_1^2}+\frac{u_2^2}{u_3^2}}{\frac{1}{u_3^2}\left(1-\frac{u_3}{\mathrm{tg}u_3}\right)+\frac{1}{u_1^2}\left(1-\frac{u_1}{\mathrm{tg}u_1}\right)}\right] \tag{9}$$

where

$$u_1 = \frac{a}{2}\sqrt{\frac{P_1}{EI_{yc}}}, \qquad u_2 = \frac{a}{2}\sqrt{\frac{P_2}{EI_{yc}}}, \qquad u_3 = \frac{a}{2}\sqrt{\frac{P_1+P_2}{EI_{yc}}}$$

A simpler solution by Rayleigh-Ritz method, adopting a deflection function

$$x = \delta_1 \sin\frac{\pi z}{a} + \delta_2 \sin\frac{2\pi z}{a}$$

is

$$S_{0c} = \frac{Q}{\delta} = \frac{\pi^4 EI_{yc}}{2a^3}\left[1-\frac{2}{\pi^2}(2u_1^2+u_2^2)-\frac{32u_2^4}{9\pi^4(\pi^2-2u_1^2-u_2^2)}\right] \tag{10}$$

The error of this expression is within 2% and can be further simplified by neglecting the last term of minor importance. Thus

$$S_{0c} = \frac{\pi^4 EI_{yc}}{2a^3}\left[1-\frac{2}{\pi^2}(2u_1^2+u_2^2)\right] \tag{11}$$

This expression, corresponding to the case where equal and opposite forces $P_1 + P_2/2$ act at the two ends, can be used for routine design purpose.

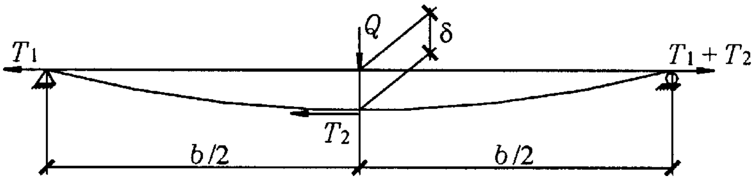


Fig. 6 Lower chord lateral stiffness

For the end panel *DF*, $P_1 = 0$ and $P_2 = P$ (Fig. 5a), Eq. (11) reduces to

$$S_{0c} = \frac{\pi^4 EI_{yc}}{2a^3}\left[1 - \frac{2u^2}{\pi^2}\right] \tag{12}$$

Similarly, the stiffness of the lateral elastic support at *B* provided by the lower chord *AC* is calculated according to the scheme of Fig. 6 and the solutions obtained are:
by analytical approach

$$S_{0t} = \frac{8EI_{yt}}{b^3}\left[\frac{\frac{u'_1}{\text{th}u'_1}\left(2+\frac{u'^2_2}{u'^2_1}\right)+\frac{u'_3}{\text{th}u'_3}\left(2-\frac{u'^2_2}{u'^2_3}\right)-\frac{u'^2_2}{u'^2_1}+\frac{u'^2_2}{u'^2_3}}{-\frac{1}{u'^2_3}\left(1-\frac{u'_3}{\text{th}u'_3}\right)-\frac{1}{u'^2_1}\left(1-\frac{u'_1}{\text{th}u'_1}\right)}\right] \tag{13}$$

and by Rayleigh-Ritz approach with further simplification

$$S_{0t} = \frac{\pi^4 EI_{yt}}{2b^3}\left[1 + \frac{2}{\pi^2}(2u'^2_1 + u'^2_2)\right] \tag{14}$$

where

$$u'_1 = \frac{b}{2}\sqrt{\frac{T_1}{EI_{yt}}}, \qquad u'_2 = \frac{b}{2}\sqrt{\frac{T_2}{EI_{yt}}}, \qquad u_3{}' = \frac{b}{2}\sqrt{\frac{T_1+T_2}{EI_{yt}}}$$

*b* being the distance between laterally braced points.

Trusses with longer span may have lower chord unsupported by a length of three panels as shown in Fig. 7. To solve the lateral stiffness at point *B* and *C* analytically will be too involved. The problem is first solved by replacing chord tension $T_1$, $T_2$ and $T_3$ by the average value $T = (T_1 + T_2 + T_3)/3$ in all the three panels (Fig. 7b). The analytical solution in this case of symmetry is

$$S_{0t} = \frac{EI_{yt}}{b^3}\left[\frac{(3u')^3\text{sh}3u'}{u'\text{sh}3u' - \text{sh}u'(\text{sh}2u' + \text{sh}u')}\right] \tag{15}$$

where

$$u' = \frac{b}{3}\sqrt{\frac{T}{EI_{yt}}}$$

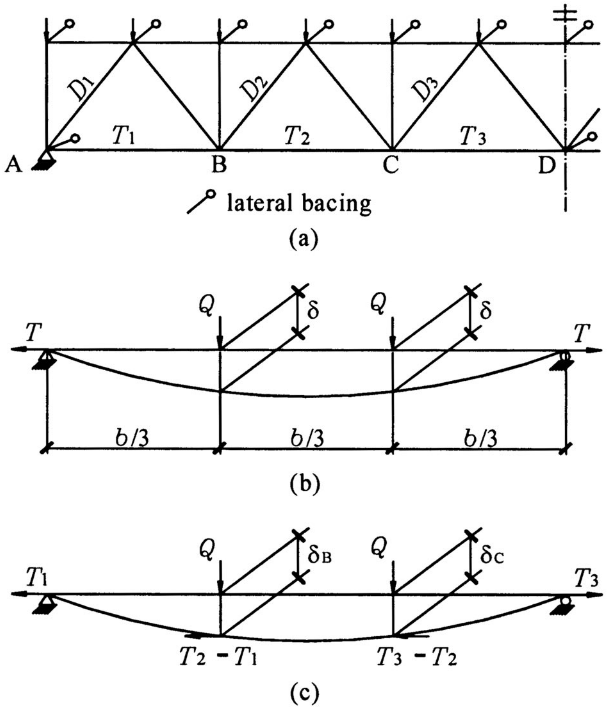


Fig. 7 Lower chord of three panels

The Rayleigh-Ritz solution using a single half-wave of sine curve as deflection function is

$$S_{0t} = \frac{\pi^4 EI_{yt}}{3b^3}[1 + 0.101(3u')^2] \tag{16}$$

and proves itself to be accurate enough by comparison to Eq. (15).

The above approximate solution is afterwards verified by the finite element method, with chord tension increasing from $T_1$ to $T_3$. It is found that the difference between deflections at point $B$ and point $C$ cannot be overlooked. Through parametric calculation and regression of results, it can be shown that the lateral stiffness at point $B$ and $C$ may be obtained from same Eq. (16), but with different $u'$:
for point $B$

$$u_B' = \frac{b}{3}\sqrt{\frac{0.65T_1 + 0.35T_3}{EI_{yt}}} \tag{17a}$$

for point $C$

$$u_C' = \frac{b}{3}\sqrt{\frac{0.5(T_1 + T_3)}{EI_{yt}}} \tag{17b}$$

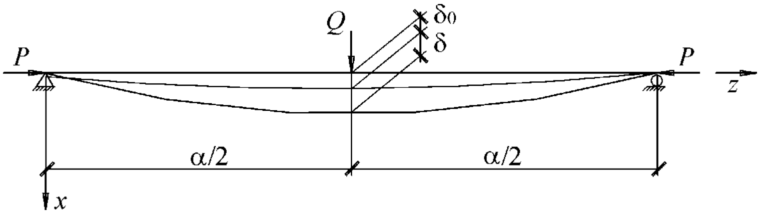


Fig. 8 Compression chord with initial crookedness

### *4.3. Influence of imperfection*

The foregoing analysis is based on perfectly straight members. As compression chord are sensitive to initial crookedness, due attention should be paid to its effect. Assuming an initial deflection in the form of sine curve

$$x_0 = \delta_0 \sin\frac{\pi z}{a}$$

the deflection at mid-span of the member caused by simultaneous action of lateral load $Q$ and axial compression $P$ can be obtained by superposition (Timoshenko and Gere 1961) (Fig. 8).

$$\delta = \delta_0\left(\frac{1}{1-P/P_E}-1\right)+\frac{Qa^3}{48EI_{yc}}\cdot\frac{3(\mathrm{tg}u-u)}{u^3}$$

or

$$\delta = \frac{(2u/\pi)^2}{1-(2u/\pi)^2}\delta_0+\frac{Qa^3}{48EI_{yc}}\cdot\frac{3(\mathrm{tg}u-u)}{u^3} \tag{18}$$

where

$$u = \frac{a}{2}\sqrt{\frac{P}{EI_{yc}}}$$

The maximum allowable initial deflection $\delta_0$ is $a$ / 1000, whereas the deflection [$Qa^3$ / 48$EI_{yc}$] due to a concentrated load $Q$ is usually of the order $a$ / 500, i.e., two times the initial $\delta_0$. Taking account of the deleterious effect of residual stresses on the member stiffness, $\delta_0$ is enlarge to

$$\delta_0 = \frac{Qa^3}{48EI_{yc}}$$

and Eq. (18) becomes

$$\delta = \frac{Qa^3}{48EI_{yc}}\left[\left(\frac{3(\mathrm{tg}u-u)}{u^3}\right)+\frac{(2u/\pi)^2}{1-(2u/\pi)^2}\right] \tag{19}$$

For a member devoid of imperfection, the second term at the right side of Eq. (19) vanishes. Therefore, the initial imperfection magnifies the deflection by the factor

$$\eta = 1 + \frac{(2u/\pi)^2}{1-(2u/\pi)^2} \cdot \frac{u^3}{3(\mathrm{tg}u-u)} \tag{20}$$

Double-angle trusses often keep constant cross-section in their compression (or tension) chord so that the latter is not fully stressed except at mid-span. $P = 2P_E/3 = \frac{2}{3}\frac{\pi^2 EI_{yc}}{a^2}$ is deemed large enough to be introduced into Eq. (20). The factor $\eta$ thus obtained is 1.67 and the imperfection reduces the stiffness of member to 1/1.67 = 0.6 times that of a perfect one. $S_{0c}$ in Eqs. (11) and (12) should be multiplied by 0.6 to obtain the upper support stiffness $S_1$.

Tension in chord members reduces their initial bow in lieu of enlarging it and its effect on stiffness can be neglected. But, residual stresses still exist and a stiffness reduction factor of 0.85 is recommended for $S_{0t}$ of Eqs. (14) and (16) to obtain the lower support stiffness $S_2$.

In case a diagonal is laterally supported by rigid bracing at its top but by tension chord at the bottom, Eq. (7) is still valid. The limiting value of chord unsupported length satisfying this criterion is given by

$$b \le l_d \sqrt[3]{0.85\gamma_t I_{yt}/I_{yd}} \tag{21}$$

where

$$\gamma_t = \frac{\pi^2}{2}\left[1 + \frac{2(2u_1'^2 + u_2'^2)}{\pi^2}\right] \qquad \text{for 2-panel pattern (Fig. 6)}$$

and

$$\gamma_t = \frac{\pi^2}{3}(1 + 0.91u'^2) \qquad \text{for 3-panel pattern (Fig. 7)}$$

### 4.4. Example of calculation

The member forces and cross-section of a truss of 24 m span depicted in Fig. 9 are given in Table 4. The laterally braced points are shown in Fig. 9.

End diagonal $D_1$ is laterally supported at its lower end but elastically supported at the top. The stiffness provided by the upper chord is calculated by Eq. (12) which is to be multiplied by 0.6. We

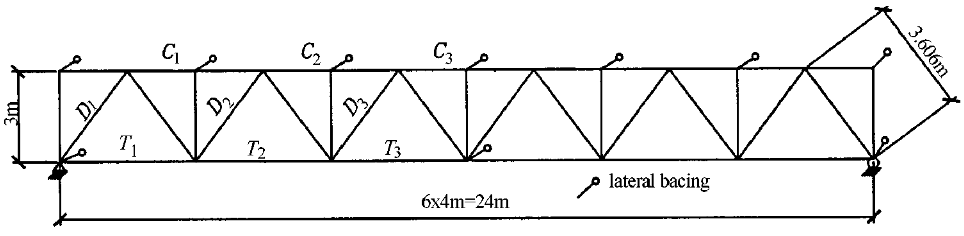


Fig. 9 Example of calculation

Table 4 Member forces and cross-section properties

| | Upper chord | | | Lower chord | | | Compression diagonal | |
|---|---|---|---|---|---|---|---|---|
| | $C_1$ | $C_2$ | $C_3$ | $T_1$ | $T_2$ | $T_3$ | $D_1$ | $D_2$ |
| Forces (kN) | -366.7 | -533.3 | -600.0 | +183.3 | +450.0 | +583.3 | -330.6 | -210.4 |
| | | | | | Mean: +405.5 | | | |
| Sections | | ┐┌ 110×8 | | | ┘└ 100×7 | | ┐┌125×80×8 | ┐┌ 80×7 |
| $I_y$ (cm$^4$) | | 822.6 | | | 549.0 | | 342.2 | 293.6 |

have successively

$$u = \frac{a}{2}\sqrt{\frac{C}{EI_{yc}}} = 200\sqrt{\frac{366.7}{20600 \times 822.6}} = 0.93$$

and

$$S_{0c} = \frac{\pi^4 EI_{yc}}{2a^3}\left[1 - \frac{2u^2}{\pi^2}\right] = 10.6 \text{ kN/cm}$$

The stiffness required is

$$\frac{N_{Ed}}{l_d} = \frac{\pi^2 EI_{yd}}{l_d^3} = \frac{\pi^2 \times 20600 \times 342.2}{360.6^3} = 1.48 \text{ kN/cm} < 0.6 \times 10.6 = 6.36 \text{ kN / cm}$$

This end diagonal may be calculated with effective length factor $\mu = 1$.

Diagonal $D_2$ The stiffness provided by the upper chord is calculated by Eq. (11) where $u_1 = 0.93$ as obtained above, and

$$u_2 = 200\sqrt{\frac{533.3 - 366.7}{20600 \times 822.6}} = 0.627$$

substituting these parameters into Eq. (11), one obtains

$$S_{0c} = 7.35 \text{ kN/cm}, \qquad \text{and } S_1 = 0.6\, S_{0c} = 4.41 \text{ kN/cm}$$

The stiffness provided by the lower chord is calculated by Eqs. (16) and (17a) as follows

$$u' = 400\sqrt{\frac{0.65 \times 183.3 + 0.35 \times 583.3}{20600 \times 549}} = 2.14$$

$$S_{0t} = \frac{\pi^4 \times 20600 \times 549}{3 \times 1200^3}[1 + 0.101(3 \times 2.14)^2] = 1.10 \text{ kN/cm}$$

$$S_2 = 0.85 S_{0t} = 0.935 \text{ kN/cm}$$

The left side of Eq. (6) is

$$\frac{S_1 S_2}{S_1+S_2}=\frac{4.41\times 0.935}{4.41+0.935}=0.77\ \text{kN/cm}$$

While the right side is

$$\frac{N_{Ed}}{l_d}=\frac{\pi^2 EI_{yd}}{l_d^3}=\frac{\pi^2\times 20600\times 293.6}{360.6^3}=1.27\ \text{kN/cm}>0.77\ \text{kN/cm}$$

The support stiffness provided by the lower chord is inadequate to ensure the stability of diagonal $D_2$ if its effective length factor is taken as 1.0. Even though a lateral bracing is provided at diagonal top, the lower chord is still too flexible because $S = 0.935$ cannot satisfy Eq. (7). Notice that the slenderness ratio of the lower chord $b/i_{yt} = 1200/4.46 = 269$ does not violate the allowable or recommended value of 350 (Chinese Code GBJ 17-88) or 300 (American Code AISC LRFD-99). The minimum laterally unsupported length of the lower chord required for this last case of $D_2$ can be obtained form Eq. (21) with $\gamma_t = \pi^2/3(1+0.91\times 2.14^2)=17$ :

$$b=360.6\times\sqrt[3]{0.85\times\frac{549}{293}\times 17}=1083\ \text{cm}$$

unless the unbraced length of the lower chord is reduced to 8 m, the diagonal should be given an effective length factor

$$\mu=\frac{\pi}{l_d}\sqrt{\frac{EI_{yd}}{l_d S_2}}=\frac{\pi}{360.6}\sqrt{\frac{20600\times 293.6}{360.6\times 0.77}}=1.29$$

### *4.5. The influence of twisting on chord stiffness*

In double-angle trusses, members intersect at a joint with their centroidal axes (or working lines) away from chord shear center $S$. The chord, subject to lateral load $Q$, will twist about its shear center axis simultaneously with bending out of the truss plane. The deflection at the centroid $C$ will be increased thereby. Denoting the distance $\overline{SC}$ by $e$ and the average compression $P_1 + P_2/2$ by $P_m$, the additional deflection at $C$ for compression chord is (Fig. 10)

$$\delta_1=\frac{Qe^2 a}{4(GI_t-P_m i_0^2)}$$

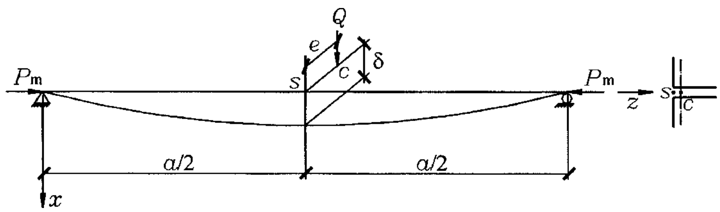


Fig. 10 Influence of twisting

Calculation reveals that the magnitude of this additional deflection is within 10% of the flexural deflection. For tension chord this addition is still smaller because tension enhances the torsional rigidity.

Considering that the tension diagonal at the joint has some stiffening effect, the additional deflection due to twisting may well be neglected.

## 5. Conclusions

Although the double angle truss is quite popular in use for a long period, not all its features are yet well understood or carefully treated, especially regarding the out-of-plane buckling of component members. Twisting at the onset of buckling about the axis of symmetry not only reduces member capacity but also affects the selection of cross-section. Demand on the stiffness of chord members for providing adequate restraint to compressive web members should be given due consideration in design work as well. Bracing system of lower chord laid-out in accordance with the recommended slenderness ratio of tension members may be inadequate.

Making use of the simplified formulae of equivalent slenderness ratio presented in section 2.2 of this paper, the problem of flexural-torsional buckling of double-angle struts can be easily solved in combination with any national design code and Table 3 gives guidance to avoid too wide angle size for short compressive members.

Whether a compression diagonal can be treated as hinged-hinged strut has to be verified with criteria Eq. (6) or (7). If the relevant criterion is not satisfied, Eq. (8) or (8a) is to be used to determine the effective length factor for the diagonal. Compression diagonals laterally braced only at the upper joint, need restraint from the lower chord whose minimum unsupported length can be obtained directly from Eq. (21).

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