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Assessment of non-polynomial shear deformation theories for thermo-mechanical analysis of laminated composite plates

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Abstract. In the present work, the recently developed non-polynomial shear deformation theories are assessed for thermo-mechanical response characteristics of laminated composite plates. The applicability and accuracy of these theories for static, buckling and free vibration responses were ascertained in the recent past by several authors. However, the assessment of these theories for thermo-mechanical analysis of the laminated composite structures is still to be ascertained. The response characteristics are investigated in linear and non-linear thermal gradient and also in the presence and absence of mechanical transverse loads. The laminated composite plates are modelled using recently developed six shear deformation theories involving different shear strain functions. The principle of virtual work is used to develop the governing system of equations. The Navier type closed form solution is adopted to yield the exact solution of the developed equation for simply supported cross ply laminated plates. The thermo-mechanical response characteristics due to these six different theories are obtained and compared with the existing results.

Keywords: thermo-mechanical analysis; shear deformation theory; Navier solution; laminated plate

1. Introduction

In modern times, the composite materials are extensively preferred in automobile, civil, aerospace, and marine applications due to their higher specific stiffness and strength, thermal and chemical resistivity, flexibility in designing, and impact resistance. Due to the usage of these advanced materials in variable thermal environment, it has become a field of keen interest for the researchers to model and analyse these layered structures for their optimal design to an edge.

In order to analyse these composite structures, the researchers have used various approaches in the past. Reissner (1945) and Mindlin (1951) analysed the plates taking into account of transverse shear in the deformation leading to the development of first order shear deformation theory (FSDT). However, the results predicted by FSDT are dependent on the choice of shear correction factor whose value is dependent on the parameters such as boundary conditions, lamination sequence, etc. (Pai 1995). The researchers have focused on various modelling approaches for laminated composite plates taking into consideration of the complicating effects of such structures. In the axiomatic approach based on the displacement field, the theories are categorized in equivalent single layer (ESL) theories, zig-zag (ZZ) theories and layer wise (LW) theories. The extensive review on the devolvement of the theories for modelling of laminated composites are found in the literature (Reddy 1990, Reddy and Robbins 1994, Ghugal and Shimpi 2002, Carrera 2003a, Zhang and Yang 2009, Khandan *et al*. 2012, Singh and Grover 2013). The complexity in FSDT due to the requirement of shear correction factor is eliminated by the development of higher order shear deformation theories (HSDTs). The HSDTs can be further grouped into polynomial shear deformation theories (PSDTs) and non-polynomial shear deformation theories (NPSDTs). The displacement field of PSDT's are the polynomial expansion of the transverse coordinate. (Reddy 1984, Maiti and Sinha 1994, Kant and Khare 1997, Kant and Swaminathan 2002, Shimpi and Patel 2006, Talha and Singh 2010). However, in NPSDTs, a non-polynomial function of thickness coordinate such as trigonometric (Touratier 1991, Shimpi *et al*. 2003, Ferreira *et al*. 2005, Mantari *et al*. 2012, Grover *et al*. 2013a, Thai and Vo 2013, Tounsi *et al*. 2013), inverse trigonometric (Grover *et al*. 2013a, Thai *et al*. 2015), exponential (Karama *et al*. 2003, Aydogdu 2009), hyperbolic (Akavci 2010, Meiche *et al*. 2011, Daouadji *et al*. 2013, Zenkour 2013) and inverse hyperbolic (Grover *et al*. 2013b, Joshan *et al*. 2017) is used in the displacement field. Mantari and Ore (2015) and Merdaci *et al*. (2016) developed ESL theories with four degrees of freedom to analyze the composite plates. Carrera (1998) developed unified formulation (CUF) expanding the displacement field to N-order using Taylor's expansion. Furthermore, Carrera (2001, 2003b) implemented LW approach to analyze the laminated structures. Sahoo and Singh (2013, 2014) used non-polynomial zig-zag models to evaluate structural response of laminated composite plates.

The laminated plates subjected to thermo-mechanical loading have been investigated by the researchers using the

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developed displacement models. Wu and Tauchert (1980) analyzed the response of anti-symmetric cross ply and angle-ply laminates by implementing Kirchhoff's hypothesis using classical laminated plate theory (CLPT) subjected to constant and linearly varying thermal loading. Reddy and Hsu (1980) used FSDT for the analysis of the cross ply laminated plates under thermal bending using Navier type closed form solution. Khdeir and Reddy (1991) used Levy type closed form solution to analyze thermo static behavior of cross plylaminated plates using CLPT, FSDT and HSDT under various boundary constraints. Bhaskar *et al*. (1996) investigated thermo-elastic response of laminates and presented results for angle-ply strips under linearly varying thermal field. Fares and Zenkour (1999) investigated thermal bending behavior of composite plates implementing mixed variational formulation. Khare *et al*. (2003) analyzed the laminated cross plycylindrical shells using HSDT under thermal gradient. Zenkour (2004) developed analytical solution for thermo-mechanical analysis of laminated plates employing sinusoidal plate theory (SPT) developed by Touratier (1991). Mechab *et al*. (2012) examined thick orthotropic plates subjected to thermal bending using HSDT and SPT. Cetkovic (2015) used LW approach for studying the thermo-mechanical behaviour of laminated and sandwich plates. Chattibi *et al*. (2015) used four variable sinusoidal theory for thermo-mechanical analysis of laminated plates. Bouchafa *et al*. (2015) implemented refined hyperbolic shear deformation theory for thermal stress analysis of functionally graded sandwich plates. Ramos *et al*. (2016) analyzed the effect of thermal load on laminated plates based on CUF. Panduro and Mantari (2017) analyzed laminated plates under hygro-thermo-mechanical loading using CUF.

It is observed from the existing literature that a number of ESL theories are developed and used for the analysis of the composite structures due to their simplicity and computational economical behavior. However, the recently developed theories are generally implemented for the structural behavior of composite plates due to mechanical loads. Due to the development of a large number of theories, there is a need to incorporate the recently developed theories for investigating the thermo-mechanical response characteristics of laminated composite plates in order to accurately predict the results. Moreover, the considered shear deformation theories have proven their credibility to investigate the response of composite plates for free vibration, buckling and bending analysis. The objective of the present work is to analytically investigate the thermo-mechanical response characteristics of composite plates using the recently developed shear deformation theories. The assessment of these theories for the prediction of thermo-mechanical response of laminated composite plates is addressed and a detailed comparison of the quantitative results is presented.

2. Mathematical formulation

In the present work, a multi-layered laminated plate having n layers of equal thickness and total thickness of h and having dimensions ($a \times b$) along x and y co-ordinates is considered. The mid plane of laminated plate is considered at the $z = 0$ plane in the Cartesian coordinates as shown in Fig. 1.

2.1 Displacement field

In the present formulation, the displacement field at any point (x, y, z) of the laminated plate is defined in terms of displacements (u, v, w) along x, y and z directions respectively. The displacement field in the framework of a non-polynomial shear deformation theory in terms of a function of a transverse co-ordinate is expressed as follows

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

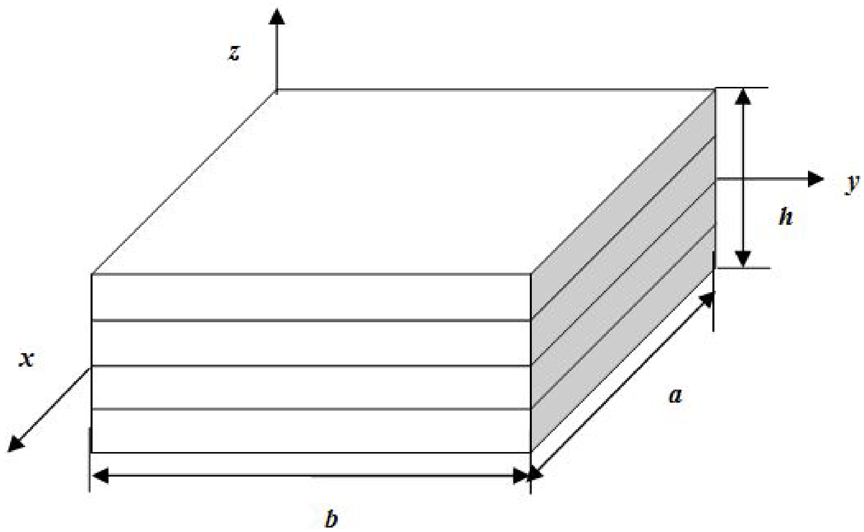

Fig. 1 Coordinate system of the laminated plate

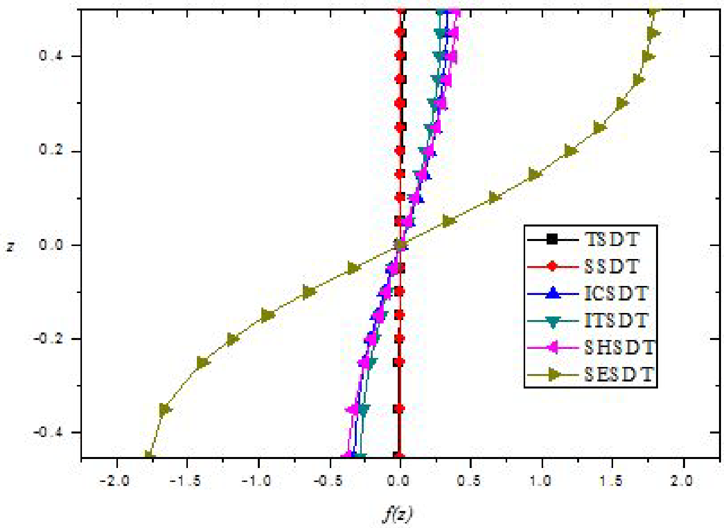

Fig. 2 Variation of shear deformation functions along the thickness of the plate

Table 1 Shear deformation functions of various theories

Theory	Notation	$g(z)$	Ω
Akavci (2010)	SHSDT	$z \sec h(\pi z^2/h^2)$	$z \sec h(\pi/4)(\pi/2(\tanh(\pi/4)) - 1)$
Mantari *et al*. (2011)	SESDT	$\sin(\pi z/h)e^{1/2(\mathrm{coz}(\pi z/h))}$	$\pi/2h$
Mantari *et al*. (2012)	TSDT	$\tan(mz)$	$-m(\sec^2 mh/2)$; $m = 0.2$
Grover *et al*. (2013a)	SSDT	$z\sec(rz/h)$	$-\sec(r/2)(1+(r/2)(\tan(r/2)))$; $r = 0.1$
Grover *et al*. (2013a)	ICSDT	$\cot^{-1}(rh/z)$	$-4r/h(4r^2+1)$; $r = 0.42$
Thai *et al*. (2015)	ITSDT	$h\tan^{-1}(2z/h) - z$	0

where the parameters u_0, v_0 are the in plane mid surface displacements along x and y directions respectively and w_0 is the transverse displacement of the mid surface of the plate. The quantities θ_x and θ_y are the shear rotations around x and y directions respectively. The function $f(z)$ is chosen such that (Soldatos and Timarci 1993)

$$\int_{-h/2}^{h/2} f(z)dz = 0$$

and

$$f'(\pm h/2) = 0$$

where (‘) denotes the derivative of the function and both $f(z)$ and $f'(z)$ are the continuous functions in the domain ($-h/2$) to ($h/2$). Further the function $f(z)$ is chosen in such a way that

$$f(z) = g(z) + \Omega z \tag{2}$$

where $g(z)$ is a non-polynomial function of transverse co-ordinate and Ω is constant introduced in the equation in order to equate the value of shear stains to zero at the top and bottom surface disregarding the need of shear correction factor. The various functions recently used in the literature in order to predict the behavior of laminated plates are illustrated in Table 1. The accuracy of the predicted results depends significantly on choice of the function $f(z)$. The variation of the functions considered in Table 1 along the thickness of the plate is plotted in Fig. 2.

2.2 Temperature field

The laminated plate is subjected to a temperature field (T) across the thickness in accordance with non-polynomial displacement field as

$$T(x, y, z) = T_1(x, y) + \frac{z}{h}T_2(x, y) + \frac{f(z)}{h}T_3(x, y) \tag{3}$$

where the parameters T_1, T_2 and T_3 are constants and respectively represent the coefficients of constant temperature field, linearly varying temperature field across the thickness and non-linearly varying temperature field across the thickness. The non-linear term ($f(z)\ T_3/h$) in the temperature field depends on the choice of function ($f(z)$) and hence is different for each theory as mentioned in Table 1. However the constant (T_1) and linear (zT_2/h) are not affected by the choice of the function ($f(z)$) and hence the temperature field is same for each theory in this case.

2.3 Stress strain relations

The constitutive relations for each layer of composite plate characterize the material behavior of the composites. For a general k^{th} layer, the stress components $\{\sigma\} = [\sigma_{xx}\ \sigma_{yy}\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}]^T$ are related to strain components $\{\varepsilon\}$ and temperature induced strains $\{\varepsilon^{Th}\}$ as follows

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} & 0 & 0 \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} & 0 & 0 \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q_{44}} & \overline{Q_{45}} \\ 0 & 0 & 0 & \overline{Q_{45}} & \overline{Q_{Q_{26}}} \end{bmatrix}^k \times \begin{Bmatrix} \varepsilon_{xx} - \alpha_{xx}\Delta T \\ \varepsilon_{yy} - \alpha_{yy}\Delta T \\ \gamma_{xy} - \alpha_{xy}\Delta T \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \tag{4}$$

or

$$\{\sigma\}^{(k)} = \left[\overline{Q}_{ij}\right]^{(k)} \left(\{\varepsilon\}^{(k)} - \{\varepsilon^{Th}\}^{(k)}\right)$$

where the matrix $[\bar{Q}_{ij}]^{(k)}$ depicts the transformed stiffness matrix for the k^{th} layer which is a function of reduced stiffness matrix $[Q_{ij}]$ and the angle of fiber orientation (Θ) of the layer (Reddy 2004). The parameters α_{xx}, α_{yy} and α_{xy} are the coefficients of thermal expansion in the respective co-ordinates.

2.4 Strain displacement relationship

The linear strain-displacement relationship is implemented in the considered structural problem since the laminated plate is considered to undergo linear deformation. These relations are as follows

$$\begin{aligned}
\varepsilon_{\text{xx}} &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \theta_x}{\partial x} \\
\varepsilon_{\text{xx}} &= \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \theta_x}{\partial x} \\
\gamma_{\text{xy}} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - z\frac{2\partial^2 w_0}{\partial x \partial y} \\
&\quad + f(z)\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) \\
\gamma_{\text{xz}} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = f'(z)\theta_x \\
\gamma_{\text{yz}} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = f'(z)\theta_y
\end{aligned} \tag{5}$$

The parameters ε_{xx} and ε_{yy} are the normal strains in x and y directions respectively, γ_{xy} is the in-plane shear strain, γ_{xz} and γ_{yz} are the shear strains along the transverse directions.

2.5 Governing equations

The principle of virtual work is employed in order to derive the governing equations in terms of stress and moment resultants.

$$\int_0^T (\delta U + \delta W)dt = 0 \tag{6}$$

where the quantity δU represents the virtual strain energy and δW denotes the virtual work done by the applied load on the laminated plate. These are redefined as follows

$$\delta U = \int_{\Omega_0} \left\{ \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} \\ + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} \\ + \tau_{xz}\delta\gamma_{xz} \end{bmatrix} dz \right\} dxdy \tag{7}$$

$$\delta W = \int_{\Omega_0} q\delta w_0 dxdy \tag{8}$$

where q denotes the applied mechanical load onto the plate. The values of virtual strain energy and virtual work are substituted in Eq. (6) and the strains are introduced in terms of displacements using strain displacement relations defined in Eq.(5). The product law of differential calculus is used and corresponding coefficients of five primary variables (u_0, v_0, w_0, θ_x, θ_y) are equated to zero. The governing differential equations in the form of stress and moment resultants are obtained in the process and are as follows

$$\begin{aligned}
&\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \frac{\partial N_{xx}^T}{\partial x} + \frac{\partial N_{xy}^T}{\partial y} \\
&\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = \frac{\partial N_{yy}^T}{\partial y} + \frac{\partial N_{xy}^T}{\partial x} \\
&\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + q = \\
&\quad \frac{\partial^2 M_{xx}^T}{\partial x^2} + \frac{\partial^2 M_{yy}^T}{\partial y^2} + 2\frac{\partial^2 M_{xy}^T}{\partial x \partial y} \\
&\Omega\frac{\partial M_{xx}}{\partial x} + \frac{\partial P_{xx}}{\partial x} + \Omega\frac{\partial M_{xy}}{\partial y} + \frac{\partial P_{xy}}{\partial y} - \Omega S_1 - K_1 \\
&\quad = \Omega\frac{\partial M_{xx}^T}{\partial x} + \frac{\partial P_{xx}^T}{\partial x} + \Omega\frac{\partial M_{xy}^T}{\partial y} + \frac{\partial P_{xy}^T}{\partial y} \\
&\Omega\frac{\partial M_{yy}}{\partial y} + \frac{\partial P_{yy}}{\partial y} + \Omega\frac{\partial M_{xy}}{\partial x} + \frac{\partial P_{xy}}{\partial x} - \Omega S_2 - K_2 \\
&\quad = \Omega\frac{\partial M_{yy}^T}{\partial y} + \frac{\partial P_{yy}^T}{\partial y} + \Omega\frac{\partial M_{xy}^T}{\partial x} + \frac{\partial P_{xy}^T}{\partial x}
\end{aligned} \tag{9}$$

where the parameters N, M and P are resultants of in-plane stresses and moments due to applied mechanical load. The quantities S and K are resultants due to transverse shear stresses. The parameters N^T, M^T and P^T are in-plane stresses and moments due to thermal load. These parameters are redefined in terms of integrals of in-plane and transverse stresses (Zenkour 2004).

$$\begin{bmatrix} N_{xx} & M_{xx} & P_{xx} \\ N_{yy} & M_{yy} & P_{yy} \\ N_{xy} & M_{xy} & P_{xy} \end{bmatrix} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} [1 \;\; z \;\; g(z)] dz \tag{10}$$

$$\begin{bmatrix} N^T_{xx} & M^T_{xx} & P^T_{xx} \\ N^T_{yy} & M^T_{yy} & P^T_{yy} \\ N^T_{xy} & M^T_{xy} & P^T_{xy} \end{bmatrix} = \int_{\frac{-h}{2}}^{\frac{h}{2}} [\overline{Q}]^k_{5\times5} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \\ 0 \\ 0 \end{Bmatrix} \times [1 \;\; z \;\; g(z)] T dz \tag{11}$$

$$\begin{bmatrix} S_2 & K_2 \\ S_1 & K_1 \end{bmatrix} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} [1 \;\; g'(z)] dz \tag{12}$$

In order to express the governing equations of the plate in terms of primary variables, the stress strain relations defined in Eq. (4) are implemented. In the formulation, the following integrals are used in order to the express the governing equations in partial differential equations of mid-plane displacements and rotations (Zenkour 2004)

$$\lfloor A_{ij} \;\; B_{ij} \;\; D_{ij} \;\; E_{ij} \;\; F_{ij} \;\; H_{ij} \rfloor = \int_{\frac{-h}{2}}^{\frac{h}{2}} \left[\overline{Q}_{ij}\right]^{(k)} \times \left[1 \;\; z \;\; z^2 \;\; g(z) \;\; z.g(z) \;\; (g(z))^2\right] dz \tag{13}$$

$$\left[A^T_i \;\; B^T_i \;\; D^T_i \;\; B^{AT}_i \;\; D^{AT}_i \;\; F_i^{AT}\right] \int_{\frac{-h}{2}}^{\frac{h}{2}} \left[\overline{Q}_{ij}\right]^{(k)} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \\ 0 \\ 0 \end{Bmatrix} \times \left\{1 \;\; z \;\; z^2 \;\; f(z) \;\; z.f(z) \;\; (f(z))^2\right\} T dz \tag{14}$$

where *i*, *j* are 1, 2, 4, 5, 6

$$\left[K_{ij} \quad L_{ij}\right] = \int_{\frac{-h}{2}}^{\frac{h}{2}} \left[\overline{Q}_{ij}\right]^{(k)} \left[g'(z) \quad (g'(z))^2\right] d \tag{15}$$

where *i*, *j* are 4, 5.

The generalized partial differential equations obtained can be represented in the form

$$[R]\{\Delta\} = \{F\} \tag{16}$$

where $[R]$ is a differential operator matrix, $\{\Delta\}$ is displacement vector of primary displacements and rotations and $\{F\}$ is force vector constituting both thermal and mechanical load terms. For cross ply laminates, the stiffness characteristics are defined in Eq. (17).

$$\begin{aligned} &A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0 \\ &E_{16} = E_{26} = F_{16} = F_{26} = H_{16} = H_{26} = 0 \\ &A_{45} = B_{45} = D_{45} = E_{45} = F_{45} = H_{45} = K_{45} = l_{45} = 0 \\ &A^T_6 = B^T_6 = D^T_6 = B^{AT}_6 = D^{AT}_6 = F^{AT}_6 = 0 \end{aligned} \tag{17}$$

These stiffness characteristics are implemented in Eq. (16) to obtain the explicit partial differential equations for cross ply laminates as follows

$$\begin{aligned} &A_{11}\left(\frac{\partial^2 u_0}{\partial x^2}\right) + B_{11}\left(\Omega\frac{\partial^2\theta_x}{\partial x^2} - \frac{\partial^3 w_0}{\partial x^3}\right) \\ &+ E_{11}\left(\frac{\partial^2\theta_x}{\partial x^2}\right) + A_{12}\left(\frac{\partial^2 v_0}{\partial x\partial y}\right) \\ &+ B_{12}\left(\Omega\frac{\partial^2\theta_y}{\partial x\partial y} - \frac{\partial^3 w_0}{\partial x\partial y^2}\right) + E_{12}\left(\frac{\partial^2\theta_y}{\partial x\partial y}\right) \\ &+ A_{66}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x\partial y}\right) \\ &+ B_{66}\left(\Omega\left(\frac{\partial^2\theta_x}{\partial x^2} + \frac{\partial^2\theta_y}{\partial x\partial y}\right) - 2\left(\frac{\partial^3 w_0}{\partial x\partial y^2}\right)\right) \\ &+ E_{66}\left(\frac{\partial^2\theta_x}{\partial x^2} + \frac{\partial^2\theta_y}{\partial x\partial y}\right) = A^T_1\left(\frac{\partial T_1}{\partial x}\right) + B^T_1\left(\frac{\partial T_2}{\partial x}\right) \\ &+ B^{AT}_1\left(\frac{\partial T_3}{\partial x}\right) + \Omega B^T_1\left(\frac{\partial T_3}{\partial x}\right) + A^T_6\left(\frac{\partial T_1}{\partial y}\right) \\ &+ B^T_6\left(\frac{\partial T_2}{\partial y}\right) + B^{AT}_6\left(\frac{\partial T_3}{\partial y}\right) + \Omega B^T_6\left(\frac{\partial T_3}{\partial y}\right) \end{aligned} \tag{18a}$$

$$\begin{aligned} &A_{22}\left(\frac{\partial^2 v_0}{\partial x^2}\right) + B_{22}\left(\Omega\frac{\partial^2\theta_y}{\partial y^2} - \frac{\partial^3 w_0}{\partial y^3}\right) \\ &+ E_{22}\left(\frac{\partial^2\theta_y}{\partial y^2}\right) + A_{12}\left(\frac{\partial^2 u_0}{\partial x\partial y}\right) \\ &+ B_{12}\left(\Omega\frac{\partial^2\theta_x}{\partial x\partial y} - \frac{\partial^3 w_0}{\partial x^2\partial y}\right) + E_{12}\left(\frac{\partial^2\theta_x}{\partial x\partial y}\right) \\ &+ A_{66}\left(\frac{\partial^2 u_0}{\partial x\partial y} + \frac{\partial^2 v_0}{\partial x^2}\right) + \\ &B_{66}\left(\Omega\left(\frac{\partial^2\theta_x}{\partial x\partial y} + \frac{\partial^2\theta_y}{\partial x^2}\right) - 2\left(\frac{\partial^3 w_0}{\partial x^2\partial y}\right)\right) \\ &+ E_{66}\left(\frac{\partial^2\theta_x}{\partial x\partial y} + \frac{\partial^2\theta_y}{\partial x^2}\right) = A^T_2\left(\frac{\partial T_1}{\partial y}\right) + B^T_2\left(\frac{\partial T_2}{\partial y}\right) \\ &+ B^{AT}_2\left(\frac{\partial T_3}{\partial y}\right) + \Omega B^T_2\left(\frac{\partial T_3}{\partial y}\right) + A^T_6\left(\frac{\partial T_1}{\partial x}\right) \\ &+ B^T_6\left(\frac{\partial T_2}{\partial x}\right) + B^{AT}_6\left(\frac{\partial T_3}{\partial x}\right) + \Omega B^T_6\left(\frac{\partial T_3}{\partial x}\right) \end{aligned} \tag{18b}$$

$$B_{11}\left(\frac{\partial^3 u_0}{\partial x^3}\right) + D_{11}\left(\Omega\frac{\partial^3\theta_x}{\partial x^3} - \frac{\partial^4 w_0}{\partial x^4}\right) \tag{18c}$$

$$
\begin{aligned}
&+F_{11}\left(\frac{\partial^3\theta_x}{\partial x^3}\right)+B_{12}\left(\frac{\partial^3 u_0}{\partial x\partial y^2}+\frac{\partial^3 v_0}{\partial x^2\partial y}\right)\\
&+D_{12}\left(\Omega\left(\frac{\partial^3\theta_y}{\partial x^2\partial y}+\frac{\partial^3\theta_x}{\partial x\partial y^2}\right)-2\frac{\partial^4 w_0}{\partial x^2\partial y^2}\right)\\
&+F_{12}\left(\frac{\partial^3\theta_y}{\partial x^2\partial y}+\frac{\partial^3\theta_x}{\partial x\partial y^2}\right)+B_{22}\left(\frac{\partial^3 v_0}{\partial y^3}\right)\\
&D_{22}\left(\Omega\frac{\partial^3\theta_y}{\partial y^3}-\frac{\partial^4 w_0}{\partial y^4}\right)+F_{22}\left(\frac{\partial^3\theta_y}{\partial y^3}\right)\\
&+2B_{66}\left(\frac{\partial^3 u_0}{\partial x\partial y^2}+\frac{\partial^3 v_0}{\partial x^2\partial y}\right)\\
&+2D_{66}\left(\Omega\left(\frac{\partial^3\theta_y}{\partial x^2\partial y}+\frac{\partial^3\theta_x}{\partial x\partial y^2}\right)-2\frac{\partial^4 w_0}{\partial x^2\partial y^2}\right)\\
&+2F_{66}\left(\frac{\partial^3\theta_y}{\partial x^2\partial y}+\frac{\partial^3\theta_x}{\partial x\partial y^2}\right)+q=B_1^T\left(\frac{\partial^2 T_1}{\partial x^2}\right)\\
&+D_1^T\left(\frac{\partial^2 T_2}{\partial x^2}\right)+D_1^{AT}\left(\frac{\partial^2 T_3}{\partial^2 x}\right)+\Omega D_1^T\left(\frac{\partial^2 T_3}{\partial^2 x}\right)\\
&+\left(2\left(\begin{aligned}&B_6^T\left(\frac{\partial^2 T_1}{\partial x\partial y}\right)+D_6^T\left(\frac{\partial^2 T_2}{\partial x\partial y}\right)\\&+D_6^{AT}\left(\frac{\partial^2 T_3}{\partial x\partial y}\right)+\Omega D_6^T\left(\frac{\partial^2 T_3}{\partial x\partial y}\right)\end{aligned}\right)\right)\\
&+B_2^T\left(\frac{\partial^2 T_1}{\partial y^2}\right)+D_2^T\left(\frac{\partial^2 T_2}{\partial y^2}\right)+D_2^{AT}\left(\frac{\partial^2 T_3}{\partial y^2}\right)\\
&+\Omega D_2^T\left(\frac{\partial^2 T_3}{\partial y^2}\right)
\end{aligned}
\tag{18c}
$$

$$
\begin{aligned}
&\left(\Omega B_{11}+E_{11}\right)\left(\frac{\partial^2 u_0}{\partial x^2}\right)+\left(\Omega D_{11}+F_{11}\right)\\
&\times\left(\Omega\frac{\partial^2\theta_x}{\partial x^2}-\frac{\partial^3 w_0}{\partial x^3}\right)+\left(\Omega F_{11}+H_{11}\right)\left(\frac{\partial^2\theta_x}{\partial x^2}\right)\\
&+\left(\Omega B_{12}+E_{12}\right)\left(\frac{\partial^2 v_0}{\partial x\partial y}\right)\\
&+\left(\Omega D_{12}+F_{12}\right)\\
&\times\left(\Omega\frac{\partial^2\theta_y}{\partial x\partial y}-\frac{\partial^3 w_0}{\partial x\partial y^2}\right)+\left(\Omega F_{12}+H_{12}\right)\left(\frac{\partial^2\theta_y}{\partial x\partial y}\right)\\
&+\left(\Omega B_{66}+E_{66}\right)\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}\right)\\
&+\left(\Omega D_{66}+F_{66}\right)\left(\Omega\left(\frac{\partial^2\theta_x}{\partial x^2}+\frac{\partial^2\theta_y}{\partial x\partial y}\right)-2\left(\frac{\partial^3 w_0}{\partial x\partial y^2}\right)\right)\\
&+\left(\Omega F_{66}+H_{66}\right)\left(\frac{\partial^2\theta_x}{\partial x^2}+\frac{\partial^2\theta_y}{\partial x\partial y}\right)\\
&-\left(\Omega^2 A_{55}+2\Omega K_{55}+L_{55}\right)\theta_x=
\end{aligned}
\tag{18d}
$$

$$
\begin{aligned}
&\left(\begin{aligned}&\left(\Omega B_1^T+B_1^{AT}\right)\left(\frac{\partial T_1}{\partial x}\right)+\left(\Omega D_1^T+D_1^{AT}\right)\left(\frac{\partial T_2}{\partial x}\right)\\&+\left(\Omega D_1^{AT}+F_1^{AT}\right)\left(\frac{\partial T_3}{\partial x}\right)+\Omega\left(\Omega D_1^T+D_1^{AT}\right)\left(\frac{\partial T_3}{\partial x}\right)\end{aligned}\right)\\
&\left(\Omega B_6^T+B_6^{AT}\right)\left(\frac{\partial T_1}{\partial y}\right)+\left(\Omega D_6^T+D_6^{AT}\right)\left(\frac{\partial T_2}{\partial y}\right)\\
&+\left(\Omega D_6^{AT}+F_6^{AT}\right)\left(\frac{\partial T_3}{\partial y}\right)+\Omega\left(\Omega D_6^T+D_6^{AT}\right)\left(\frac{\partial T_3}{\partial y}\right)
\end{aligned}
\tag{18d}
$$

$$
\begin{aligned}
&\left(\Omega B_{22}+E_{22}\right)\frac{\partial^2 v_0}{\partial x^2}+\left(\Omega D_{22}+F_{22}\right)\\
&\times\left(\Omega\frac{\partial^2\theta_y}{\partial y^2}-\frac{\partial^3 w_0}{\partial y^3}\right)+\left(\Omega F_{22}+H_{22}\right)\frac{\partial^2\theta_y}{\partial y^2}\\
&+\left(\Omega B_{12}+E_{12}\right)\frac{\partial^2 u_0}{\partial x\partial y}+\left(\Omega D_{12}+F_{12}\right)\\
&\times\left(\Omega\frac{\partial^2\theta_x}{\partial x\partial y}-\frac{\partial^3 w_0}{\partial x^2\partial y}\right)+\left(\Omega F_{12}+H_{12}\right)\frac{\partial^2\theta_x}{\partial x\partial y}\\
&+\left(\Omega B_{66}+E_{66}\right)\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial^2 v_0}{\partial x^2}\right)+\\
&\left(\Omega D_{66}+F_{66}\right)\left(\Omega\left(\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{\partial^2\theta_y}{\partial x^2}\right)-2\left(\frac{\partial^3 w_0}{\partial x^2\partial y}\right)\right)\\
&+\left(\Omega F_{66}+H_{66}\right)\left(\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{\partial^2\theta_y}{\partial x^2}\right)\\
&-\left(\Omega^2 A_{44}+2\Omega K_{44}+L_{44}\right)\theta_y=\left(\Omega B_2^T+B_2^{AT}\right)\left(\frac{\partial T_1}{\partial y}\right)\\
&+\left(\Omega D_2^T+D_2^{AT}\right)\left(\frac{\partial T_2}{\partial y}\right)+\left(\Omega D_2^{AT}+F_2^{AT}\right)\left(\frac{\partial T_3}{\partial y}\right)\\
&+\Omega\left(\Omega D_2^T+D_2^{AT}\right)\left(\frac{\partial T_3}{\partial y}\right)\\
&\left(\Omega B_6^T+B_6^{AT}\right)\left(\frac{\partial T_1}{\partial x}\right)+\left(\Omega D_6^T+D_6^{AT}\right)\left(\frac{\partial T_2}{\partial x}\right)\\
&+\left(\Omega D_6^{AT}+F_6^{AT}\right)\left(\frac{\partial T_3}{\partial x}\right)+\Omega\left(\Omega D_6^T+D_6^{AT}\right)\left(\frac{\partial T_3}{\partial x}\right)
\end{aligned}
\tag{18e}
$$

2.6 Solution methodology

The partial differential equations defined in Eq. (18) are solved using the Navier type closed form solution for simply supported cross ply laminates. The boundary conditions for simply supported plate are as follows

$$
\begin{aligned}
u_0 = w_0 = \theta_x = N_{yy} = M_{yy} = 0 \quad &\text{at} \quad y = 0, b\\
v_0 = w_0 = \theta_y = N_{xx} = M_{xx} = 0 \quad &\text{at} \quad x = 0, a
\end{aligned}
\tag{19}
$$

The Navier solution for the cross ply laminated plates for investigating thermo-mechanical behaviour is as follows (Reddy 2004)

$$u_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\cos(\alpha x)\sin(\beta y)$$
$$v_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\sin(\alpha x)\cos(\beta y)$$
$$w_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin(\alpha x)\sin(\beta y) \tag{20}$$
$$\theta_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}\cos(\alpha x)\sin(\beta y)$$
$$\theta_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}\sin(\alpha x)\cos(\beta y)$$
$$\alpha = \frac{m\pi}{a}, \beta = \frac{n\pi}{b}$$

where the quantities U_{mn}, V_{mn}, W_{mn}, X_{mn} and Y_{mn} are arbitrarily parameters to be determined by substituting the mid-plane displacements in Eq. (18). The mechanical load and the thermal load applied on the plate are taken as

$$q = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(\alpha x)\sin(\beta y)$$
$$T_1 = \overline{T_1}\sin(\alpha x)\sin(\beta y)$$
$$T_2 = \overline{T_2}\sin(\alpha x)\sin(\beta y) \tag{21}$$
$$T_3 = \overline{T_3}\sin(\alpha x)\sin(\beta y)$$

where $q_{mn} = q_0$ for sinusoidal loading and $q_{mn} = 16q_0\pi^2/\text{mn}$ for uniformly distributed loading. The solution and loading conditions are substituted into differential equations defined in Eq. (18) and algebraic form of the equations is obtained. The solution of the quantities(U_{mn}, V_{mn}, W_{mn}, X_{mn} and Y_{mn}) are retrieved in the form

$$\{U_{mn} \quad V_{mn} \quad W_{mn} \quad X_{mn} \quad Y_{mn}\} = [\overline{R^C}]^{-1}\{\overline{F^C}\} \tag{22}$$

The mid plane displacements (u_0, v_0, w_0) and rotations (θ_x, θ_y) can be obtained following the solution methodology. The coefficients of the resultant matrix $\left[R^C\right]$ are given in Appendix A.

3. Numerical results and discussion

In this section, a study of thermo-mechanical behavior of the laminated plates implementing various non-polynomial shear deformation theories is presented. Using the developed mathematical formulation and solution methodology, a MATLAB code for each of the theory is developed in order to solve the mathematical problem. A number of results are presented in order to validate the present formulation and the effects of various parameters such as side to thickness ratio, aspect ratio, lamination sequence, loading conditions and material anisotropy on thermo-mechanical response of cross ply laminates are discussed and few new results are also presented. Each plate

Table 2 Effect of thickness on dimensionless deflection of orthotropic square plate subjected to sinusoidal temperature field (α_{yy}/α_{xx} = 3)

a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
6.25	1.0602	1.0597	10595	1.0597	1.0583	1.0597	1.0597	1.0588	1.0590
10	1.0440	1.0439	1.0438	1.0438	1.0434	1.0439	1.0439	1.0436	1.0436
12.5	1.0396	1.0396	1.0396	1.0395	1.0393	1.0396	1.0396	1.0394	1.0394
20	1.0346	1.0346	1.0346	1.0346	1.0345	1.0346	1.0346	1.0345	1.0345
25	1.0334	1.0334	1.0334	1.0334	1.0333	1.0334	1.0334	1.0333	1.0333
50	1.0317	1.0317	1.0317	1.0317	1.0317	1.0317	1.0317	1.0317	1.0317
100	1.0313	1.0313	1.0313	1.0313	1.0313	1.0313	1.0313	1.0313	1.0313

Table 3 Effect of thickness on dimensionless deflection of simply supported anti symmetric cross ply [$0^0/90^0$] square plate under sinusoidal temperature field (α_{yy}/α_{xx} = 3)

a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
6.25	1.6765	1.6848	1.6858	1.6821	1.6916	1.6883	1.6883	1.6883	1.6879
10	1.6765	1.6798	1.6802	1.6787	1.6825	1.6812	1.6812	1.6812	1.6810
12.5	1.6765	1.6786	1.6789	1.6779	1.6804	1.6795	1.6795	1.6795	1.6794
20	1.6765	1.6773	1.6774	1.6771	1.678	1.6777	1.6777	1.6777	1.6777
25	1.6765	1.6770	1.6771	1.6769	1.6775	1.6773	1.6773	1.6773	1.6772
50	1.6765	1.6767	1.6767	1.6766	1.6768	1.6767	1.6767	1.6767	1.6767
100	1.6765	1.6766	1.6766	1.6765	1.6766	1.6766	1.6766	1.6766	1.6766

Table 4 Effect of thickness on dimensionless deflection of symmetric cross ply [$0^0/90^0/0^0$] square plate subjected to sinusoidal temperature field (α_{yy}/α_{xx} = 3)

a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
6.25	1.1870	1.2057	1.2077	1.1995	1.1958	1.2057	1.2057	1.2118	1.2113
10	1.1365	1.1463	1.1475	1.1428	1.1438	1.1463	1.1463	1.1502	1.1498
12.5	1.1224	1.1292	1.1300	1.1267	1.1278	1.1291	1.1291	1.1319	1.1316
20	1.0158	1.1087	1.1090	1.1076	1.1083	1.1087	1.1087	1.1099	1.1098
25	1.1018	1.1036	1.1039	1.1029	1.1034	1.1036	1.1036	1.1044	1.1044
50	1.0963	1.0967	1.0968	1.0966	1.0967	1.0967	1.0967	1.0969	1.0969
100	1.0949	1.0950	1.0950	1.0949	1.095	1.0950	1.0950	1.0950	1.0950

Table 5 Effect of aspect ratio and span to thickness ratio on dimensionless deflection for simply supported three layered symmetric cross ply[$0^0/90^0/0^0$] rectangular plate subjected to sinusoidal temperature field (α_{yy}/α_{xx} = 3)

a/b	a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
0.5	10	1.0959	1.1008	1.1014	1.0813	1.0993	1.0922	1.0923	1.0981	1.1026
	20	1.0795	1.0808	1.0810	1.0759	1.0806	1.0786	1.0786	1.0801	1.0813
	100	1.0741	1.0741	1.0741	1.0739	1.0741	1.074	1.074	1.0741	1.0742
2.0	10	0.7508	0.7455	0.7449	0.7376	0.7449	0.7525	0.7525	0.7477	0.7437
	20	0.7601	0.7583	0.7581	0.7563	0.7581	0.7608	0.7608	0.7591	0.7577
	100	0.7643	0.7642	0.7642	0.7642	0.7642	0.7644	0.7644	0.7643	0.7642

is considered of equal thickness layers and the material properties of each layer of the laminated plate are assumed as

$$E_1/E_2 = 25, G_{12}/E_2 = 0.5, G_{13}/E_2 = 0.5,$$
$$G_{23}/E_2 = 0.2, \nu_{12} = 0.25, \alpha_{xx}/\alpha_{yy} = 1/3, \alpha_{xy} = 0$$

3.1 Thermo-mechanical analysis of cross ply laminates under linearly varying temperature field

The thermo-mechanical response characteristics of simply supported laminated plates are investigated for an orthotropic plate [0^0], two layered anti-symmetric [$0^0/90^0$] and three layered symmetric [$0^0/90^0/0^0$] plates subjected to linearly varying sinusoidal thermal field (T_2 = 100, T_1 = T_3 = 0, q = 0). The response characteristics of the considered plates are expressed in terms of maximum non-dimensional transverse deflection ($\overline{w}$) defined as

$$\overline{w} = (10wh)/(\alpha_{xx}\overline{T}_2 a^2) \tag{23}$$

The results obtained for the considered plates using the respective theories are compared with the existing results due to FSDT (Reddy and Hsu 1980), HSDT (Zenkour 2004) and SPT (Zenkour 2004).

The non-dimensional deflection for orthotropic plate [0^0], two layered anti-symmetric [$0^0/90^0$] and three layered symmetric [$0^0/90^0/0^0$] plate is obtained for a variety of span to thickness ratio so as to ensure the applicability for thick and thin plates. These results are enlisted in Tables 2-4. All the considered higher order theories (HSDT, SPT, SHSDT, SESDT, SSDT, TSDT, ICSDT, and ITSDT) have a good agreement of the results with FSDT for thin cross ply plates. However, for the thick cross ply plates, due to the higher order contributions of SHSDT, SSDT and TSDT, these non-polynomial theories predict the results more accurately relative to FSDT. The SESDT, ICSDT and ITSDT predict the results more accurately as comparable to SPT, SHSDT, SSDT and TSDT.

Further, a three layered symmetric [$0^0/90^0/0^0$] rectangular cross ply laminated plate is considered. A linearly varying sinusoidal temperature load is applied on the laminated plate. The effect of aspect ratio on non-dimensional transverse deflection is illustrated in Table 5 and the results given by the higher order non-polynomial theories are compared with FSDT (Reddy and Hsu, 1980), HSDT (Zenkour 2004) and SPT (Zenkour 2004). It is observed that for a thin plate (a/h = 100), the response predicted due to higher order theories (HSDT, SPT, SHSDT, SESDT, SSDT, TSDT, ICSDT, ITSDT) are identical as predicted by FSDT. However, for thick plates (a/h = 10), the SHSDT, SESDT, ICSDT and ITSDT yield accurate results relative to other shear deformation theories.

The variation of non-dimensional deflection with change in span to thickness ratio for four layered symmetric cross ply [$0^0/90^0/90^0/0^0$] and anti- symmetric cross ply [$0^0/90^0/0^0/90^0$] square plate under linearly varying

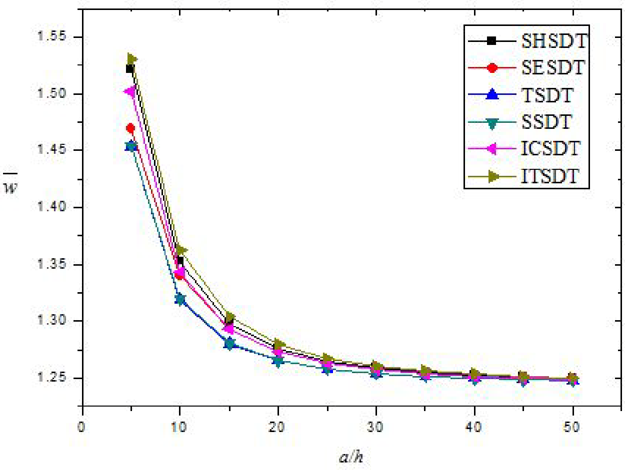

Fig. 3 Variation of maximum dimensionless deflection with side to thickness ratio for symmetric cross ply $[0^0/90^0/90^0/0^0]$ plate subjected to sinusoidal temperature field

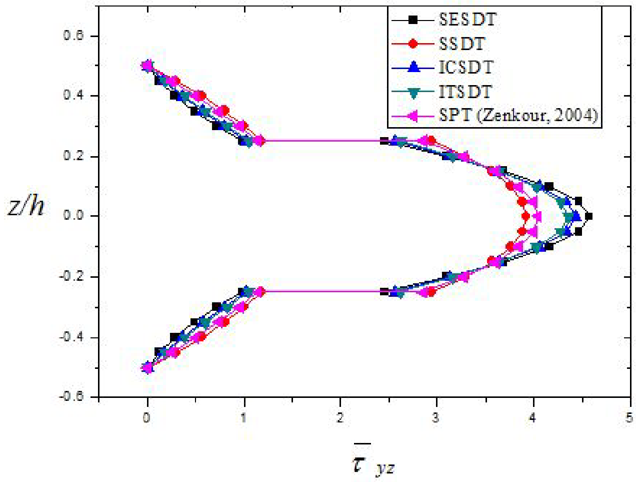

Fig. 5 Variation of transverse shear stress across the thickness for symmetric cross ply $[0^0/90^0/90^0/0^0]$ plate subjected to sinusoidal temperature field

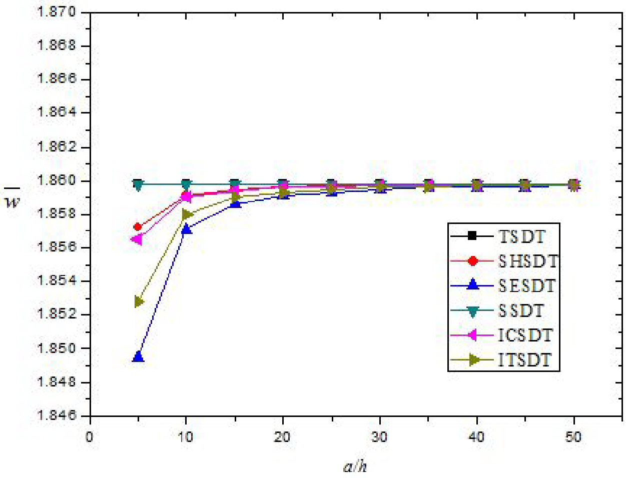

Fig. 4 Variation of dimensionless deflection at the centre of the plate with side to thickness ratio for anti-symmetric cross ply$[0^0/90^0/0^0/90^0]$ plate subjected to sinusoidal temperature field

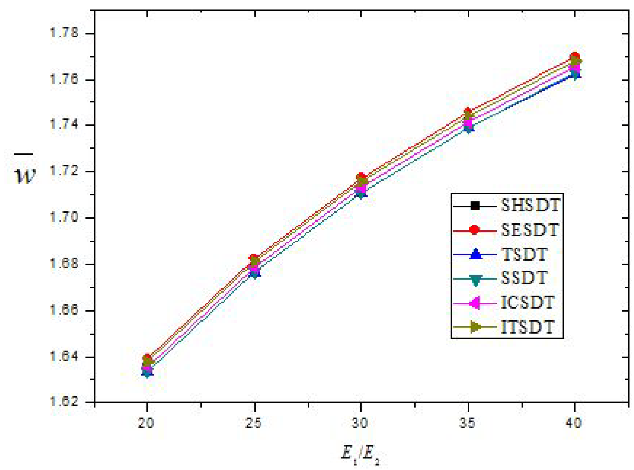

Fig. 6 Variation of dimensionless deflection with material anisotropy for anti-symmetric cross ply plate $[0^0/90^0]$ ($T_3 = 0$)

sinusoidal temperature field ($T_2 = 100$, $T_1 = T_3 = 0$, $q = 0$) is illustrated in Figs. 3 and 4. It is observed that for thin plates i.e., $a/h = 50$, all the considered theories (SHSDT, SESDT, SSDT, TSDT, ICSDT, and ITSDT) yield the same result. However, for thick symmetric cross ply plate, SHSDT, ITSDT and ICSDT yield accurate results and for anti-symmetric cross ply plate, SESDT ITSDT and ICSDT produce accurate results. Further, the variation of transverse shear stress $(\bar{\tau}_{yz})$ across the thickness of the considered four layered symmetric cross ply $[0^0/90^0/90^0/0^0]$ laminated plate is presented in Fig. 5. The non-dimensional relation used for transverse shear stress $(\bar{\tau}_{yz})$ is given as follows

$$\bar{\tau}_{yz} = \left[\tau_{yz}\left(\frac{a^2}{\alpha_{xx}T_2h^2E_{22}}\right)\right.$$

It is observed that the value of transverse shear stress $(\bar{\tau}_{yz})$ is obtained as zero at the top and bottom of the plate without any use of shear correction factor for the considered theories (SHSDT, SSDT, ITSDT, ICSDT).

The effect of change in material anisotropy (E_1/E_2) on two layered anti-symmetric cross ply $[0^0/90^0]$ plate (a/h = 10) under linearly variable sinusoidal temperature field (T_2 = 100, $T_1 = T_3 = 0$, $q = 0$) is illustrated in Fig. 6. The considered higher order deformation theories predict an increase in non-dimensional deflection with the increase in E_1/E_2 ratio. In this case, the SESDT, ITSDT and ICSDT are more accurate as compared to SSDT, TSDT and SHSDT.

In order to investigate the deflection response of cross ply plates under linearly varying uniform temperature field, four layered symmetric $[0^0/90^0/90^0/0^0]$ and anti-symmetric $[0^0/90^0/0^0/90^0]$ cross ply plates are taken into consideration. The relation defined in Eq. (23) is used for the determination of the non-dimensional deflection.

The effect of span to thickness ratio on maximum transverse deflection for four layered symmetric cross ply

Table 6 Effect of thickness on dimensionless deflection of symmetric cross ply [$0^0/90^0/90^0/0^0$] square plate subjected to uniform temperature field (α_{yy}/α_{xx} = 3)

a/h	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	2.2376	2.2551	2.2504	2.2505	2.2572	2.2576
6.25	2.1416	2.1637	2.1548	2.1549	2.1641	2.1641
10	1.9787	1.9963	1.9879	1.9880	1.9957	1.9954
12.5	1.9265	1.9399	1.9334	1.9334	1.9394	1.9391
20	1.8601	1.8666	1.8633	1.8634	1.8663	1.8661
25	1.8429	1.8473	1.8450	1.8451	1.8471	1.8470
50	1.8184	1.8196	1.8190	1.8190	1.8195	1.8195
100	1.8120	1.8123	1.8121	1.8121	1.8122	1.8122

Table 7 Effect of thickness on dimensionless deflection of anti-symmetric cross ply [$0^0/90^0/0^0/90^0$] square plate sunder uniform temperature field (α_{yy}/α_{xx} = 3)

a/h	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	2.7589	2.7571	2.7574	2.7574	2.7569	2.7569
6.25	2.7675	2.7656	2.7661	2.7661	2.7655	2.7655
10	2.7864	2.7848	2.7854	2.7854	2.7847	2.7848
12.5	2.7939	2.7926	2.7931	2.7931	2.7926	2.7926
20	2.8048	2.8041	2.8044	2.8044	2.8041	2.8041
25	2.8079	2.8074	2.8076	2.8076	2.8074	2.8075
50	2.8126	2.8125	2.8125	2.8125	2.8125	2.8125
100	2.8139	2.8139	2.8139	2.8139	2.8139	2.8139

[$0^0/90^0/90^0/0^0$] square plate subjected to linearly varying uniform temperature field (T_2 = 100, T_1 = T_3 = 0, q = 0) is depicted in Table 6. It is observed that there is a notable increase in non-dimensional deflection of the plate under uniform temperature field as compared to sinusoidal temperature field. The deformation in the plate (a/h = 10) subjected to uniform temperature field is approximately 48% higher than the plate subjected to sinusoidal temperature field. The non-dimensional deflection decreases with increase in span to thickness ratio under uniform temperature field as under sinusoidal temperature field. In this case, ITSDT, SESDT, and ICSDT yield relatively accurate results.

Table 7 presents the variation in maximum transverse deflection for four layered anti-symmetric cross ply [$0^0/90^0/0^0/90^0$] square plate with respect to span-thickness ratio of the plate. The plate is subjected to linearly varying uniform temperature field (T_2 = 100, T_1 = T_3 = 0, q = 0) linearly varying across the thickness. It is observed that unlike symmetric cross ply plates, the non-dimensional deflection increases with increase in span to thickness ratio under uniform temperature field.

Further, the multilayered symmetric cross ply plates are considered and the transverse dimensionless deflection due to uniformly distributed thermal load (T_2 = 100, T_1 = T_3 = 0, q = 0) is examined using ICSDT. The analysis of three [$0^0/90^0/0^0$], four [$0^0/90^0/90^0/0^0$], five [$0^0/90^0/0^0/90^0/0^0$] and six [$0^0/90^0/0^0/0^0/90^0/0^0$] layered plates aligned in symmetric cross ply lamination is carried out. The results are obtained by varying span to thickness ratio (a/h) and are plotted in Fig. 7. It is observed that the dimensionless deflection increases as the number of layers increase.

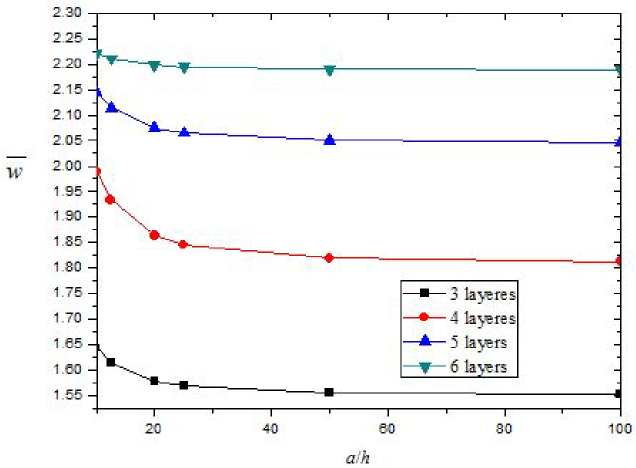

Fig. 7 Variation in dimensionless deflection for three, four, five and six layered symmetric cross ply plates with change in span to thickness ratio subjected to uniform temperature load

3.2 Thermo-mechanical analysis of simply supported cross ply laminates under combined mechanical and linearly varying temperature loading

In order to analyse cross ply laminated plates under combined thermal and mechanical loading in terms of non-dimensional deflection, an orthotropic plate [0^0], two layered anti-symmetric [$0^0/90^0$] and three layered symmetric [$0^0/90^0/0^0$] plates subjected to combined sinusoidal mechanical loading and linearly varying sinusoidal thermal load ($T_1 = T_3 = 0$, $q = 100$, $T_2 = 100$) are considered. The relation for non-dimensional transverse deflection ($\overline{w}$) as defined by Reddy and Hsu (1980) is as follows

$$\overline{w} = w\left(\left(\frac{q_0 a^4}{h^3\lambda}\right) + \left(\frac{\alpha_{xx}\overline{T}_2 a^2}{10h}\right)\right)^{-1}$$
$$\lambda = \left(\frac{\pi^4}{12}\right)\left(4G_{12} + \frac{\left(E_1 + \left(1+\nu_{12}\right)E_2\right)}{1-\nu_{12}\nu_{21}}\right) \tag{24}$$

The results are obtained for the considered plates using the considered theories and are compared with the existing results given by FSDT (Reddy and Hsu 1980), HSDT (Zenkour 2004) and SPT (Zenkour 2004).

The variation of non-dimensional deflection for orthotropic plate [0^0], two layered anti-symmetric [$0^0/90^0$] and three layered symmetric [$0^0/90^0/0^0$] plate with change in span to thickness ratio subjected to combined sinusoidal thermal and sinusoidal mechanical loading are shown in Tables 8-10 respectively. All the considered higher order theories (HSDT, SPT, SHSDT, SESDT, SSDT, TSDT, ICSDT, and ITSDT) have good agreement of the results with FSDT for thin cross ply plates. However, for thick cross ply plates, SPT, SHSDT, SSDT and TSDT under-predicts the non-dimensional deflection compared to FSDT and HSDT.

Further, a four layered symmetric [$0^0/90^0/90^0/0^0$] cross ply plate subjected to combined uniformly distributed mechanical and linearly variable uniform temperature load ($T_1 = T_3 = 0$, $q = 100$, $T_2 = 100$) is considered. The effect of span to thickness ratio on maximum transverse deflection for the considered plate is shown in Table 11. The non-dimensional deflection decreases with increase in span to thickness ratio.

In order to observe the effect of change in aspect ratio on dimensionless deflection for four layered anti-symmetric cross ply [$0^0/90^0/0^0/90^0$] plate, Fig. 8 is plotted. The plate is subjected to combined uniformly distributed mechanical and linearly variable uniform temperature load ($T_1 = T_3 = 0$, $q = 100$, $T_2 = 100$). The results are given using SSDT, ICSDT and ITSDT for $a/h = 10$. It is observed that the non-dimensional deflection increases with increase in aspect ratio of the plate.

Table 8 Effect of thickness on dimensionless deflection of orthotropic square plate under sinusoidal temperature field and sinusoidal transverse loading ($T_2 = 100$, $q = 100$, $\alpha_{yy}/\alpha_{xx} = 3$, $\alpha_{xx} = 10^{-6}$)

a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	2.8332	2.7769	2.7654	2.7947	2.6844	2.7926	2.7925	2.7243	2.7359
6.25	2.1868	2.1631	2.1575	2.1669	2.1093	2.1696	2.1695	2.1326	2.1393
10	1.4671	1.4634	1.4621	1.4614	1.4451	1.4644	1.4643	1.4531	1.4553
12.5	1.2973	1.2958	1.2951	1.294	1.2845	1.2962	1.2962	1.2895	1.2908
20	1.1150	1.1113	1.1111	1.1103	1.107	1.1113	1.1113	1.1089	1.1094
25	1.0683	1.0682	1.0681	1.0676	1.0655	1.0682	1.0682	1.0667	1.067
50	1.0105	1.0105	1.0105	1.0104	1.0099	1.0105	1.0105	1.0102	1.0102
100	0.9962	0.9961	0.9960	0.9960	0.9959	0.9961	0.9961	0.9960	0.9960

Table 9 Effect of thickness on dimensionless deflection of two layered anti symmetric cross ply [$0^0/90^0$] square plate subjected to sinusoidal temperature field and sinusoidal transverse loading ($T_2 = 100$, $q = 100$, $\alpha_{yy}/\alpha_{xx} = 3$, $\alpha_{xx} = 10^{-6}$)

a/h	FSDT (Reddy and Hsu, 1980)	HSDT (Zenkour, 2004)	SPT (Zenkour, 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	4.0415	3.812	3.7821	3.8915	3.6469	3.8323	3.8317	3.7011	3.719
6.25	3.4666	3.3273	3.3090	3.3736	3.2231	3.3375	3.3371	3.2567	3.2678
10	2.8438	2.7927	2.7859	2.8088	2.7527	2.7954	2.7953	2.7653	2.7695
12.5	27001	2.6679	2.6636	2.6779	2.6424	2.6695	2.6694	2.6504	2.653
20	2.5443	2.5321	2.5304	2.5358	2.5221	2.5326	2.5326	2.5252	2.5263
25	2.5083	2.5006	2.4996	2.503	2.4943	2.5009	2.5009	2.4962	2.4969
50	2.4597	2.4586	2.4584	2.4592	2.4571	2.4587	2.4587	2.4576	2.4577
100	2.44541	2.4481	2.4481	2.4483	2.4478	2.4482	2.4482	2.4479	2.4479

Table 10 Effect of thickness on dimensionless deflection of three layered symmetric cross ply [$0^0/90^0/0^0$] square plate subjected to sinusoidal temperature field and sinusoidal transverse loading (MP1, T_2 = 100, q = 100, α_{yy}/α_{xx} = 3, α_{xx} = 10^{-6})

a/h	FSDT (Reddy and Hsu 1980)	HSDT (Zenkour 2004)	SPT (Zenkour 2004)	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	3.0377	3.2948	3.3238	3.1977	3.3967	3.3126	3.3133	3.3987	3.3923
6.25	2.9983	2.5394	2.5637	2.4598	2.629	2.5465	2.547	2.6231	2.6165
10	1.5384	1.6366	1.6493	1.596	1.6858	1.6373	1.6376	1.6794	1.6754
12.5	1.3451	1.4115	1.4202	1.3839	1.4456	1.4116	1.4118	1.4408	1.4379
20	1.1312	1.1587	1.1624	1.1471	1.1733	1.1586	1.1587	1.1711	1.1698
25	1.0811	1.0989	11013	1.0914	1.1085	1.0988	1.0989	1.107	1.1062
50	1.0138	1.0183	1.0189	1.0164	1.0208	1.0183	1.0183	1.0204	1.0202
100	0.9973	0.9980	0.9982	0.9975	0.9986	0.9880	0.998	0.9985	0.9985

Table 11 Effect of thickness on dimensionless deflection of four layer symmetric cross ply [$0^0/90^0/90^0/0^0$] square plate subjected to uniform temperature field and uniform transverse loading (T_2 = 100, q = 100, α_{yy}/α_{xx} = 3, α_{xx} = 10^{-6})

a/h	SHSDT	SESDT	TSDT	SSDT	ICSDT	ITSDT
5	5.4189	5.4146	5.3035	5.3047	5.4189	5.4145
6.25	4.1943	4.1975	4.1000	4.1009	4.1943	4.1897
10	2.7126	2.7174	2.6627	2.6631	2.7126	2.7097
12.5	2.3338	2.3376	2.2991	2.2994	2.3338	2.3318
20	1.9004	1.9022	1.8854	1.8855	1.9004	1.8995
25	1.7963	1.7975	1.7865	1.7866	1.7963	1.7957
50	1.6548	1.6551	1.6523	1.6523	1.6548	1.6546
100	1.6189	1.6190	1.6183	1.6183	1.6189	1.6189

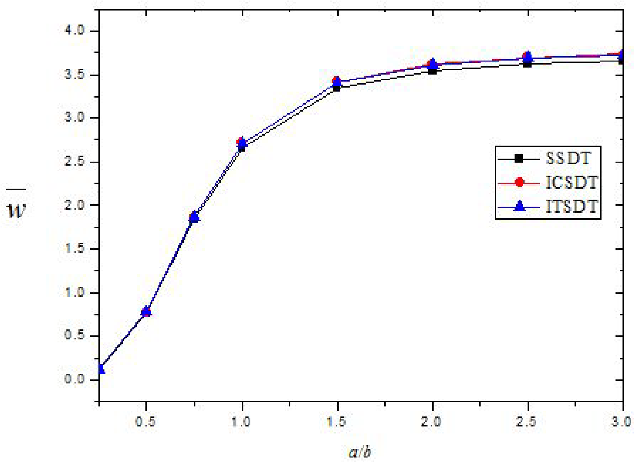

Fig. 8 Effect of aspect ratio and lamination on maximum dimensionless deflection of simply supported anti-symmetric cross ply [$0^0/90^0/0^0/90^0$] square plate subjected to uniform temperature field and uniform transverse loading (T_2 = 100, q = 100, α_{yy}/α_{xx} = 3, α_{xx} = 10^{-6}, a/h = 10)

3.3 Thermo-mechanical analysis of cross ply laminates under non-linearly varying temperature field

In order to investigate the deflection response of cross ply plates under non-linearly varying uniform temperature field, four layered symmetric [$0^0/90^0/90^0/0^0$] and anti-symmetric [$0^0/90^0/0^0/90^0$] cross ply plates are taken into consideration. The relation defined in Eq. (23) is used for the formulation of non-dimensional deflection. The plate is subjected to non-linearly varying temperature field (T_1 = 0, q = 0, T_3 = T_2 = 100). The results for variation in non-dimensional deflection with respect to change in thickness are presented in Figs. 9 and 10. The results are compared with the results of SPT (Zenkour 2004). The SESDT, ICSDT and SSDT predicts better results as compared to other ESL theories for both the considered laminated plates. It may be noted that for symmetric cross ply plate, non dimensional deflection decreases with increase in span to thickness ratio. However, for anti-symmetric cross ply plate, non-dimensional deflection increases with increase in span to thickness ratio.

Further, the effect of non-linear temperature field (T_3) on the dimensionless deflection is examined for a four layered symmetric laminated plate [$0^0/90^0/90^0/0^0$]. Fig. 11 shows the variation in dimensionless deflection due to extent of

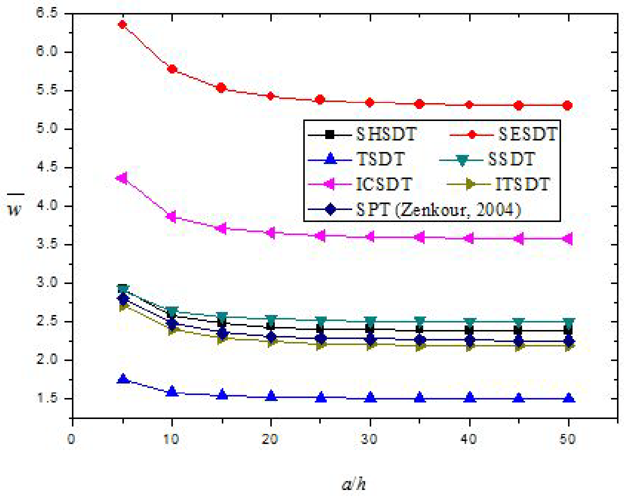

Fig. 9 Variation in dimensionless deflection with thickness for symmetric $[0^0/90^0/90^0/0^0]$ cross ply plate ($T_1 = 0$, $q = 0$, $T_3 = T_2 = 100$)

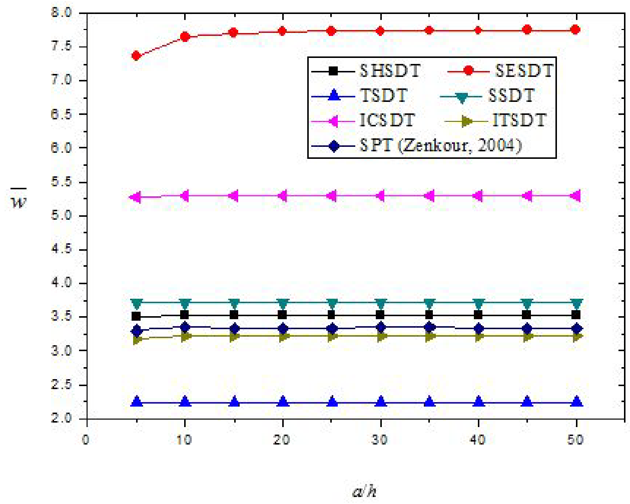

Fig. 10 Variation in dimensionless deflection with thickness for anti-symmetric $[0^0/90^0/0^0/90^0]$ cross ply plate ($T_1 = 0$, $q = 0$, $T_3 = T_2 = 100$)

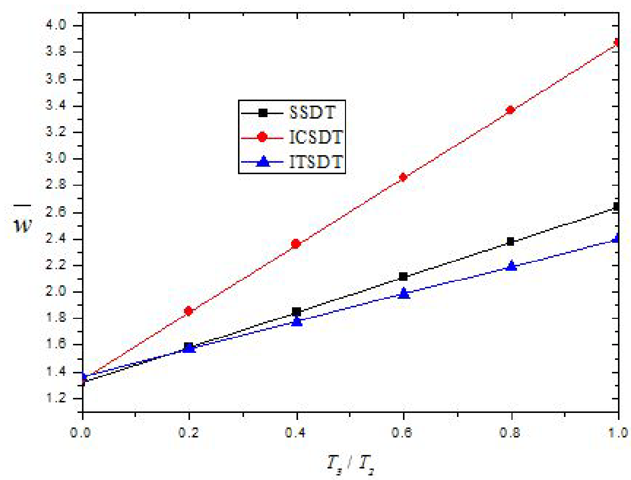

Fig. 11 Effect of nonlinear temperature field ratio (T_3/T_2) on non-dimensional deflection for symmetric $[0^0/90^0/90^0/0^0]$ cross ply plate ($a/h = 10$)

nonlinear temperature field ratio (T_3/T_2). The linear increment in deflection is observed due to increment in the extent of non-linear temperature field ratio (T_3/T_2). However the rate of the increment is different for different theories due to difference of shear strain function ($f(z)$).

4. Conclusions

The thermo-mechanical response characteristics of laminated composite plates subjected to linearly and non-linearly distributed temperature field and in the presence and absence of mechanical loads are evaluated. The laminated composite plates are modeled in the framework of non-polynomial shear deformation theories. The performance of six recently developed shear deformation theories is assessed quantitatively and their applicability and accuracy are ascertained. The principle of virtual work is adopted to yield the governing equations and these equations are solved in the closed form using Navier solution for simply supported cross ply plates. The response is obtained in the form of transverse deformation and the results are compared with existing results. The effects of parameters such as span-thickness ratio, lamination sequence, loading conditions, aspect ratio, and material anisotropy ratio are examined and the following conclusions are observed:

- It is observed that the performance of the theories designated as SESDT, ICSDT, and ITSDT for the prediction of thermo-mechanical response characteristics is better as compared to SHSDT, TSDT, and SSDT.
- The non-dimensional deflection decreases with increase in span-thickness ratio for the symmetric cross ply plate subjected to sinusoidal as well as uniform temperature field. However, the non-dimensional deflection increases with increase in span-thickness ratio for the anti-symmetric cross ply plate. It is also observed that the deformation of the plate subjected to uniform temperature field is higher than the plate subjected to sinusoidal temperature field.
- The response of the plate subjected to non-linear temperature field is dependent significantly on the choice of shear deformation theory since the non-linear temperature fields are assumed in accordance with the shear deformation theory. It is observed that the extent of non-linear temperature field with respect to linear temperature field influences the deformation behaviour. The increase in the non-linear to linear temperature ratio (T_3/T_2) increases the transverse deformation linearly. However, the rate of this increment is dependent on the shear strain function employed.

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CC

Appendix A

$$\overline{R_{11}^{C}} = A_{11}\alpha^{2} + A_{66}\beta^{2}, \overline{R_{12}^{C}} = \left(A_{12}+A_{66}\right)\alpha\,\beta$$

$$\overline{R_{13}^{C}}=-\left(B_{11}\alpha^{2}+B_{12}\beta^{2}+2B_{66}\alpha\,\beta\right)\alpha$$

$$\overline{R_{14}^{C}}=\left(\left(\Omega B_{11}+E_{11}\right)\alpha^{2}+\left(\Omega B_{66}+E_{66}\right)\beta^{2}\right)$$

$$\overline{R_{15}^{C}}=\left(\Omega B_{12}+E_{12}+\Omega B_{66}+E_{66}\right)\alpha\,\beta$$

$$\overline{R_{22}^{C}}=A_{66}\alpha^{2}+A_{22}\beta^{2}$$

$$\overline{R_{23}^{C}}=-\left(B_{22}\beta^{2}+B_{12}\alpha^{2}+2B_{66}\alpha^{2}\right)\beta$$

$$\overline{R_{24}^{C}}=\left(\Omega B_{66}+E_{66}+\Omega B_{12}+E_{12}\right)\alpha\,\beta$$

$$\overline{R_{25}^{C}}=\left(\left(\Omega B_{66}+E_{66}\right)\alpha^{2}+\left(\Omega B_{22}+E_{22}\right)\beta^{2}\right)$$

$$\overline{R_{33}^{C}}=\left(D_{11}\alpha^{4}+\left(2D_{12}+4D_{66}\right)\alpha^{2}\beta^{2}+D_{22}\beta^{4}\right)$$

$$\overline{R_{34}^{C}}=-\begin{pmatrix}\left(\Omega D_{11}+F_{11}\right)\alpha^{2}+\left(\Omega D_{12}+F_{12}\right)\beta^{2}\\ +2\left(\Omega D_{66}+F_{66}\right)\beta^{2}\end{pmatrix}\alpha$$

$$\overline{R_{35}^{C}}=-\begin{pmatrix}\left(\Omega D_{12}+F_{12}\right)\alpha^{2}+\left(\Omega D_{22}+F_{22}\right)\beta^{2}\\ +2\left(\Omega D_{66}+F_{66}\right)\alpha^{2}\end{pmatrix}\beta$$

$$\begin{aligned}\overline{R_{44}^{C}}=\Omega\left(\Omega D_{11}+F_{11}\right)\alpha^{2}+\left(\Omega F_{11}+H_{11}\right)\alpha^{2}\\ +\Omega\left(\Omega D_{66}+F_{66}\right)\beta^{2}\\ +\left(\Omega F_{66}+H_{66}\right)\beta^{2}+\Omega^{2}A_{55}+2\Omega K_{55}+L_{55}\end{aligned}$$

$$\overline{R_{45}^{C}}=\begin{pmatrix}\Omega\left(\Omega D_{12}+F_{12}\right)+\left(\Omega F_{12}+H_{12}\right)\\ +\Omega\left(\Omega D_{66}+F_{66}\right)+\left(\Omega F_{66}+H_{66}\right)\end{pmatrix}\alpha\,\beta$$

$$\begin{aligned}\overline{R_{55}^{C}} = \Omega\left(\Omega D_{22}+F_{22}\right)\beta^{2}\\ +\left(\Omega F_{22}+H_{22}\right)\beta^{2}+\Omega\left(\Omega D_{66}+F_{66}\right)\alpha^{2}\\ +\left(\Omega F_{66}+H_{66}\right)\alpha^{2}+\Omega^{2}A_{44}+2\Omega K_{44}+L_{44}\end{aligned}$$