Steel and Composite Structures, *Vol. 27, No. 5 (2018) 599-611*
DOI: https://doi.org/10.12989/scs.2018.27.5.599

A novel four-unknown quasi-3D shear deformation theory for functionally graded plates

Nabil Hebbar [1], Mohamed Bourada [*1], Mohamed Sekkal [1,2],
Abdelouahed Tounsi [1,2,3] and S.R. Mahmoud [4]

[1] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[2] *Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria*
[3] *Civil and Environmental Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia*
[4] *Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia*

(*Received March 30, 2017, Revised January 16, 2018, Accepted March 29, 2018*)

Abstract. In this article a four unknown quasi-3D shear deformation theory for the bending analysis of functionally graded (FG) plates is developed. The advantage of this theory is that, in addition to introducing the thickness stretching impact ($\varepsilon_z \neq 0$), the displacement field is modeled with only four variables, which is even less than the first order shear deformation theory (FSDT). The principle of virtual work is utilized to determine the governing equations. The obtained numerical results from the proposed theory are compared with the CPT, FSDT, and other quasi-3D HSDTs.

Keywords: quasi 3D theory; bending; functionally graded plate

1. Introduction

Functionally graded materials (FGMs) are a type of advanced composite materials which was initially proposed in Japan (Bever and Duwez 1972, Koizumi 1993). The important feature of this type of composites is the continuity along a desired direction within a structural element such as shell, plate or beam (Matsunaga 2008, Hosseini-Hashemi *et al*. 2010, Reddy 2011, Eltaher *et al*. 2013, Swaminathan and Naveenkumar 2014, Bousahla *et al*. 2014, Yaghoobi *et al*. 2014, Kar and Panda 2015, Darılmaz 2015, Ait Atmane *et al*. 2015, Akbaş 2015, Al-Basyouni *et al*. 2015, Kolahchi *et al*. 2015, Pradhan and Chakraverty 2015, Meradjah *et al*. 2015, Bourada *et al*. 2015, Akbaş 2016, Celebi *et al*. 2016, Bellifa *et al*. 2016, Ghorbanpour Arani *et al*. 2016, Ahouel *et al*. 2016, Raminnea *et al*. 2016, Bellifa *et al*. 2017a, Benadouda *et al*. 2017, Sekkal *et al*. 2017a, Rahmani *et al*. 2017, Aldousari 2017, Bouafia *et al*. 2017, Bakhadda *et al*. 2018).

Some kinds of conventional composites suffer in continuity within the thickness direction; such discontinuity can be adjusted by a gradual and smooth variation of mechanical properties within the thickness of the structural element as in FGMs. Moreover, FGMs allow us to obtain high thermal and toughness mechanical characteristics, as a result of mixing for example ceramic and metal (Tounsi *et al*. 2013, Bouderba *et al*. 2013 and 2016, Zidi *et al*. 2014, Taibi *et al*. 2015, Hamidi *et al*. 2015, Beldjelili *et al*. 2016, Bousahla *et al*. 2016, El-Haina *et al*. 2017, Khetir *et al*. 2017, Menasria *et al*. 2017, Mouffoki *et al*. 2017).

General observations on FGMs can be consulted in the review article by Jha *et al*. (2013). However, a detailed review that focuses on the stress, dynamic and buckling investigation of FG plates was presented by Swaminathan *et al*. (2015). From this article a very large list of studies on shear deformation theories of plates made of FGMs can be found. Both analytical and numerical formulation of the shear deformation plate theories under several types of loads were indicated but without considering the mathematical implication of the employed methodologies and contributions.

The first non-polynomial HSDT was proposed by Levy (1877) and after more than one century the sinusoidal HSDT were investigated and evaluated by Stein (1986). Currently, this HSDT was extensively employed by Touratier (1991), Vidal and Polit (2008, 2009, 2011, 2013), Ghugal (2010), Baseri *et al*. (2016), etc. Then, Soldatos (1992) developed a hyperbolic shear strain shape function; Karama *et al*. (2003) proposed an exponential expansion, etc. Others non-polynomial shear strain shape functions are presented also in the article by Viola *et al*. (2013a, b). Kolahchi and Bidgoli (2016) employed a sinusoidal beam model for dynamic instability of single-walled carbon nanotubes. Arani and Kolahchi (2016) investigated buckling behavior of embedded concrete columns armed with carbon nanotubes. Kolahchi *et al*. (2016a) used differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories. Bilouei *et al*. (2016) examined the stability of concrete columns retrofitted with nano-fiber reinforced polymer. Madani *et al*. (2016) proposed a differential cubature method for vibration

∗Corresponding author, Professor,
E-mail: med_bourada@yahoo.fr

http://www.techno-press.org/?journal=scs&subpage=6
ISSN: 1229-9367 (Print), 1598-6233 (Online)

analysis of embedded FG-CNT-reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions. Kolahchi *et al*. (2016b) analyzed the dynamic stability of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium. Kolahchi *et al*. (2017) presented visco-nonlocal-refined Zigzag models for dynamic buckling of laminated nanoplates using differential cubature-Bolotin methods. Zamanian *et al*. (2017) discussed the agglomeration effects on the buckling behavior of embedded concrete columns reinforced with SiO_2 nano-particles.

A refined and generalized self-consistent model was presented by Bian *et al*. (2005). This is an extension of Soldatos' HSDT (Soldatos 1992) to examine the cylindrical bending response of FG plates. Mantari *et al*. (2012a, b, c, 2014), and Mantari and Granados (2015a, b) proposed several refined non-polynomial HSDTs to investigate the bending, dynamic and thermoelasticity problems of classical composites and FG plates. Mahi *et al*. (2015) developed a new hyperbolic HSDT with five unknowns without including the thickness stretching effect to investigate the static and dynamic analysis of isotropic, functionally graded, sandwich and laminated composite plates. Nami and Janghorban (2013) analyzed the bending response of rectangular nanoplates using trigonometric shear deformation theory based on nonlocal elasticity theory. Akavci (2014) presented an efficient hyperbolic HSDT for free vibration of FG thick rectangular plates on elastic foundation. Hebali *et al*. (2014) proposed a quasi-3D hyperbolic HSDT for static and vibration analysis of FG plates. Ahmed (2014) studied the post-buckling behavior of sandwich beams with functionally graded faces using a consistent higher order theory. Belabed *et al*. (2014) developed an efficient and simple HSDT for bending and dynamic analyses of FG plates. The number of variables and governing equations were reduced to five instead of six by splitting the transverse displacement into bending, shear and thickness stretching parts. Akavci (2016) proposed a new quasi-3D hyperbolic HSDT for analyzing bending stresses, natural frequencies and buckling loads of FG sandwich plates. Mantari (2016) used a generalized non-polynomial quasi-3D shear deformation theory for advanced composite plates. Liu *et al*. (2017) presented an analysis of FG plates by a simple locking-free quasi-3D hyperbolic plate isogeometric method. Bennoun *et al*. (2016) proposed a novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. Abdelaziz *et al*. (2017) developed an efficient hyperbolic shear deformation theory for bending, buckling and dynamic of FG sandwich plates with various boundary conditions. Hachemi *et al*. exposed a new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations (2017). Sekkal *et al*. (2017b) presented a new quasi-3D HSDT for buckling and vibration of FG plate. Abualnour *et al*. (2018) proposed a novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates. Other works can be also consulted in the articles by Attia *et al*. (2015 and 2018), Belabed *et al*. (2018), Benchohra *et al*. (2018), Bouhadra *et al*. (2018) and Meksi *et al*. (2018).

In this work, a novel generation of 4-unknown quasi-3D shear deformation theory is proposed. The advantage of this theory is that, in addition to incorporating the thickness stretching influence ($\varepsilon_z \neq 0$), the displacement field is modeled with only 4 unknowns, which is even less than the FSDT and do not need shear correction factor. Thus, the novelty of this work is the use of 4-unknown quasi-3D shear deformation for bending analysis of FG plates, resulting in considerably lower computational effort when compared with the other higher-order theories reported in the literature having more number of unknown functions. The principle of virtual work is utilized to obtain the governing equations. Analytical results from the novel theory are compared with the CPT, FSDT, and other quasi-3D HSDTs. This theory is as accurate as other quasy-3D HSDTs with higher number of variables and so deserves attention.

2. Theoretical formulation

A rectangular plate of uniform thickness *h*, length *a*, and width *b*, made of a FGM is presented in Fig. 1.

The rectangular Cartesian coordinate system *x*, *y*, *z*, has the plane *xy* at $z = 0$, coinciding with the mid-surface of the plate. The material characteristics can change through the thickness according to the function $V(z)$ as given in the following equation

$$P(z) = \begin{cases} P_b V(z) & V(z) = e^{p\left(\frac{z}{h}+\frac{1}{2}\right)}, \quad \text{Case 1 (exponentially graded)} \\ (P_t - P_b)V(z) + P_b & V(z) = \left(\frac{z}{h}+\frac{1}{2}\right)^p, \quad \text{Case 2 (powerly graded)} \end{cases} \tag{1}$$

where P_t and P_b present the property of the top and bottom faces of the plate, respectively, and *p* is the exponent that specifies the material distribution profile within the thickness. In this work, for example, the Young's modulus, *E*, and shear modulus, *G*, change depending on the case

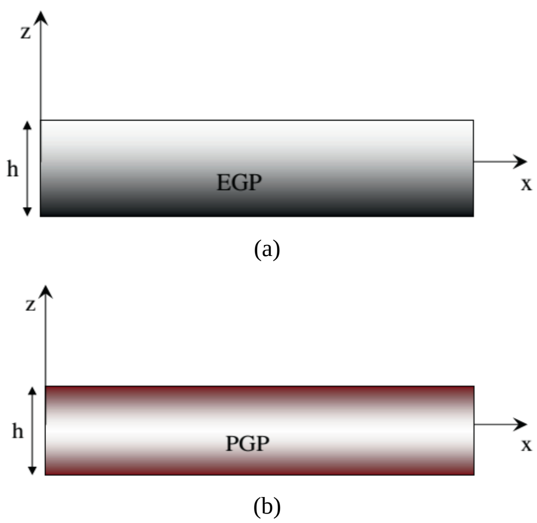

Fig. 1 Geometry of a functionally graded plate

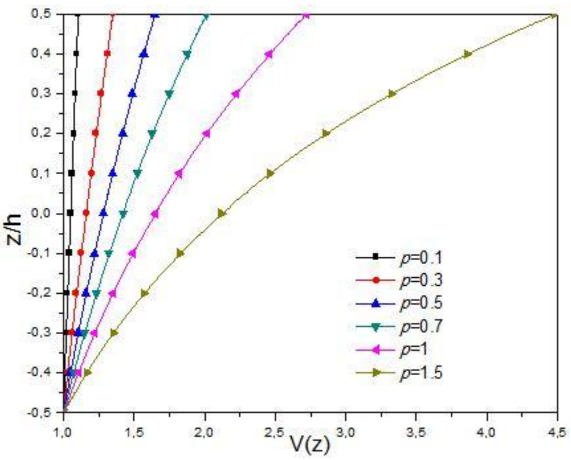

Fig. 2 Exponentially graded function $V(z)$ along the thickness of an EGP for different values of the material parameter p

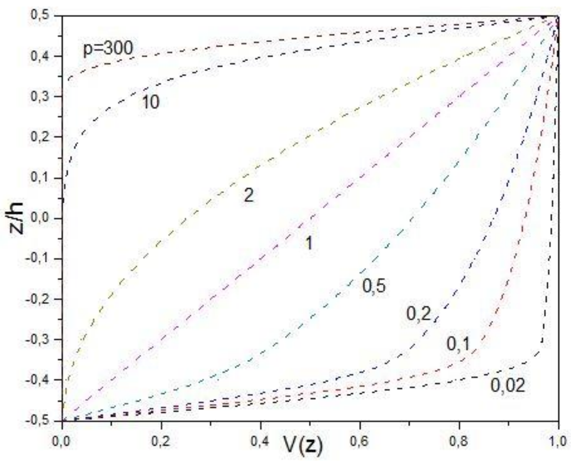

Fig. 3 Powerly graded function $V(z)$ along the thickness of an PGP for different values of the material parameter p

problem according to Eq. (1), and the Poisson ratio, $v$considered to be constant. Fig. 2 indicates the exponential function $V(\bar{z} = z/h)$ along the thickness of an exponentially graded plate (EGP) for different values of the material parameter p. While Fig. 3, indicates the corresponding function for powerly graded plates (PGP).

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point $(x, y, \pm h/2)$ on the top and bottom surfaces of the plate, is expressed as follows (Besseghier *et al*. 2017, Fahsi *et al*. 2017, Zine *et al*. 2018)

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{2a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{2b}$$

$$w(x,y,z) = w_0(x,y) + g(z)\theta(x,y) \tag{2c}$$

The coefficients k_1 and k_2 depends on the geometry. It can be seen that the kinematic in Eq. (2) introduces only four unknowns (u_0, v_0, w_0 and θ).

In this work, the shape function is taken based on the hyperbolic function given by Mantari and Guedes Soares (2014) as

$$\begin{aligned} f(z) &= \frac{4h}{5}\sinh\left(\frac{5}{4h}z\right) + z\left[-\cosh\left(\frac{5}{8}\right) + \frac{3}{20}\cos\left(\frac{5}{8}\right)\right] \\ g(z) &= -\frac{3}{20}\cos\left(\frac{5}{4h}z\right) \end{aligned} \tag{3}$$

The strain-displacement expressions, based on this formulation, are given as follows

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \\ &\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = f'(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + g(z)\begin{Bmatrix} \gamma_{yz}^1 \\ \gamma_{xz}^1 \end{Bmatrix}, \qquad \varepsilon_z = g'(z)\,\varepsilon_z^0 \end{aligned} \tag{4}$$

Where

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \\ &\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\displaystyle\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\int \theta\,dy \end{Bmatrix} \end{aligned} \tag{5a}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\displaystyle\int \theta\,dy \\ k_1\displaystyle\int \theta\,dx \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz}^1 \\ \gamma_{xz}^1 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial \theta}{\partial x} \end{Bmatrix}, \qquad \varepsilon_z^0 = \theta \tag{5b}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{5c}$$

The integrals presented in the above equations shall be resolved by a Navier type method and can be expressed as follows

$$\begin{aligned} &\frac{\partial}{\partial y}\int \theta\,dx = A'\frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta\,dy = B'\frac{\partial^2 \theta}{\partial x \partial y}, \\ &\int \theta\,dx = A'\frac{\partial \theta}{\partial x}, \qquad \int \theta\,dy = B'\frac{\partial \theta}{\partial y} \end{aligned} \tag{6}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, $A'B'$, k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \qquad B' = -\frac{1}{\beta^2}, \qquad k_1 = -\alpha^2, \qquad k_2 = -\beta^2 \tag{7}$$

where α and β are defined in expression (20).

The linear constitutive relations are given below

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{8}$$

where C_{ij} are the three-dimensional elastic constants defined by

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{9a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu\, E(z)}{(1-2\nu)(1+\nu)}, \tag{9b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{9c}$$

Considering the static version of the principle of virtual work, the following relations can be obtained (Fekrar *et al*. 2014, Ait Amar Meziane *et al*. 2014, Larbi Chaht *et al*. 2015, Ait Yahia *et al*. 2015, Belkorissat *et al*. 2015, Zemri *et al*. 2015, Boukhari *et al*. 2016, Houari *et al*. 2016, Bounouara *et al*. 2016, Draiche *et al*. 2016, Zidi *et al*. 2017, Bellifa *et al*. 2017b, Chikh *et al*. 2017, Klouche *et al*. 2017, Kaci *et al*. 2018, Yazid *et al*. 2018, Youcef *et al*. 2018, Mokhtar *et al*. 2018)

$$0 = \left[\int_{-h/2}^{h/2} \left\{ \int_{\Omega} \begin{bmatrix} \sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \sigma_z \delta\, \varepsilon_z \\ + \tau_{xy} \delta\, \gamma_{xy} + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz} \end{bmatrix} dxdy \right\} dz \right] - \left[\int_{\Omega} q \delta\, w dxdy \right] \tag{10}$$

$$0 = \int_{\Omega} \begin{bmatrix} N_x \delta\, \varepsilon_x^0 + N_y \delta\, \varepsilon_y^0 + N_z \delta\, \varepsilon_z^0 + N_{xy} \delta\, \gamma_{xy}^0 \\ + M_x^b \delta\, k_x^b + M_y^b \delta\, k_y^b + M_{xy}^b \delta\, k_{xy}^b + M_x^s \delta\, k_x^s \\ + M_y^s \delta\, k_y^s + M_{xy}^s \delta\, k_{xy}^s + Q_{yz}^s \delta\, \gamma_{yz}^0 + S_{yz}^s \delta\, \gamma_{yz}^1 \\ + Q_{xz}^s \delta\, \gamma_{xz}^0 + S_{xz}^s \delta\, \gamma_{xz}^1 - q\delta w \end{bmatrix} dxdy \tag{11}$$

Where Ω is the top surface and the stress resultants N, M, S and Q are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz,$$

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz, \ \left(i = x, y, xy\right); \tag{12a}$$

$$N_z = \int_{-h/2}^{h/2} g'(z)\sigma_z dz$$

and

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz,$$
$$\left(Q_{xz}^s, Q_{yz}^s\right) = \int_{-h/2}^{h/2} f'\left(\tau_{xz}, \tau_{yz}\right) dz \tag{12b}$$

The static version of the governing equations are obtained from Eq. (11) by integrating the displacement gradients by parts and setting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$ to zero separately. The generalized equations obtained are as follows

$$\begin{aligned} &\delta\, u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta\, v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta\, w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = -q \\ &\delta\, \theta : -\mathrm{k}_1 A' \frac{\partial^2 M_x^s}{\partial x^2} - \mathrm{k}_2 B' \frac{\partial^2 M_y^s}{\partial y^2} - \left(k_1 A' + k_2 B'\right) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - N_z \\ &+ \mathrm{k}_1 A' \frac{\partial Q_{xz}^s}{\partial x} + \mathrm{k}_2 B' \frac{\partial Q_{yz}^s}{\partial y} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = \frac{3}{20}\cos\left(\frac{5}{8}\right) q \end{aligned} \tag{13}$$

Substituting Eq. (5) into Eq. (8) and the subsequent results into Eqs. (12), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \theta \begin{Bmatrix} L \\ L^a \\ R \end{Bmatrix},$$
$$\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} F^s & X^s \\ X^s & A^s \end{bmatrix} \begin{Bmatrix} \gamma^0 \\ \gamma^1 \end{Bmatrix} \tag{14a}$$

$$N_z = L(\varepsilon_x^0 + \varepsilon_y^0) + L^a(k_x^b + k_y^b) + R(k_x^s + k_y^s) + R^a \varepsilon_z^0 \tag{14b}$$

in which

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, \qquad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t,$$
$$M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t \tag{15a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \qquad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t,$$
$$k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t \tag{15b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},$$
$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{15c}$$

$$B^s=\begin{bmatrix}B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix},\qquad D^s=\begin{bmatrix}D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix},\qquad H^s=\begin{bmatrix}H_{11}^s & H_{12}^s & 0\\ H_{12}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix} \tag{15d}$$

$$Q=\{Q_{xz}^s,Q_{yz}^s\}^t,\qquad S=\{S_{xz}^s,S_{yz}^s\}^t,\qquad \gamma^0=\{\gamma_{xz}^0,\gamma_{yz}^0\}^t,\qquad \gamma^1=\{\gamma_{xz}^1,\gamma_{yz}^1\}^t \tag{15e}$$

$$F^s=\begin{bmatrix}F_{55}^s & 0\\ 0 & F_{44}^s\end{bmatrix},\quad X^s=\begin{bmatrix}X_{55}^s & 0\\ 0 & X_{44}^s\end{bmatrix},\quad A^s=\begin{bmatrix}A_{55}^s & 0\\ 0 & A_{44}^s\end{bmatrix} \tag{15f}$$

$$\begin{Bmatrix}L\\ L^a\\ R\\ R^a\end{Bmatrix}=\int_z \lambda(z)\begin{Bmatrix}1\\ z\\ f(z)\\ g'(z)\dfrac{1-\nu}{\nu}\end{Bmatrix}g'(z)dz \tag{15g}$$

and stiffness components are given as

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\int_z \lambda(z)\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}\dfrac{1-\nu}{\nu}\\ \nu\\ \dfrac{1-2\nu}{2\nu}\end{Bmatrix}dz \tag{16a}$$

$$\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right) \tag{16b}$$

$$\left(F_{44}^s,X_{44}^s,A_{44}^s\right)=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left([f'(z)]^2,f'(z)g(z),g^2(z)\right)dz \tag{16c}$$

$$\left(F_{55}^s,X_{55}^s,A_{55}^s\right)=\left(F_{44}^s,X_{44}^s,A_{44}^s\right) \tag{16d}$$

Introducing Eq. (14) into Eq. (13), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0-B_{11}d_{111}w_0\\&-\left(B_{12}+2B_{66}\right)d_{122}w_0+\left(B_{66}^s\left(k_1A'+k_2B'\right)+B_{12}^sk_2B'\right)d_{122}\theta\\&+B_{11}^sk_1A'd_{111}\theta+Ld_1\theta=0,\end{aligned} \tag{17a}$$

$$\begin{aligned}&A_{22}d_{22}v_0+A_{66}d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}d_{222}w_0\\&-\left(B_{12}+2B_{66}\right)d_{112}w_0+\left(B_{66}^s\left(k_1A'+k_2B'\right)+B_{12}^sk_1A'\right)d_{112}\theta\\&+B_{22}^sk_2B'd_{222}\theta+Ld_2\theta=0,\end{aligned} \tag{17b}$$

$$\begin{aligned}&B_{11}d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0\\&+B_{22}d_{222}v_0-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\&-D_{22}d_{2222}w_0+D_{11}^sk_1A'd_{1111}\theta+\\&\left((D_{12}^s+2D_{66}^s)\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+D_{22}^sk_2B'd_{2222}\theta\\&+L^a(d_{11}\theta+d_{22}\theta)+q=0\end{aligned} \tag{17c}$$

$$\begin{aligned}&-k_1A'B_{11}^sd_{111}u_0-\left(B_{12}^sk_2B'+B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{122}u_0\\&-\left(B_{22}^sk_1A'+B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{112}v_0-B_{22}^sk_2B'd_{222}v_0\\&+D_{11}^sk_1A'd_{1111}w_0+\left((D_{12}^s+2D_{66}^s)\left(k_1A'+k_2B'\right)\right)d_{1122}w_0\\&+D_{22}^sk_2B'd_{2222}w_0-H_{11}^s(k_1A')^2d_{1111}\theta-H_{22}^s(k_2B')^2d_{2222}\theta\\&-\left(2H_{12}^sk_1k_2A'B'+\left(k_1A'+k_2B'\right)^2H_{66}^s\right)d_{1122}\theta\\&+\left((k_1A')^2F_{55}^s+2k_1A'X_{55}^s+A_{55}^s\right)d_{11}\theta\\&+\left((k_2B')^2F_{44}^s+2k_2B'X_{44}^s+A_{44}^s\right)d_{22}\theta\\&-2R(k_1A'd_{11}\theta+k_2B'd_{11}\theta)-L(d_1u_0+d_2v_0)\\&+L^a(d_{11}w_0+d_{22}w_0)-R^a\theta-\frac{3}{20}\cos\left(\frac{5}{8}\right)q_m=0\end{aligned} \tag{17d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_i=\frac{\partial}{\partial x_i},\qquad d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\qquad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\qquad d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m}\qquad (i,j,l,m=1,2). \tag{18}$$

The Navier solution method is employed to determine the analytical solutions for which the displacement variables are written as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions.

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\ V_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\ W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\ X_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \tag{19}$$

with

$$\alpha=m\pi/a\qquad\text{and}\qquad\beta=n\pi/b \tag{20}$$

The transverse load q is also expanded in the double-Fourier sine series as

$$Q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q\sin(\alpha\,x)\sin(\beta\,y) \tag{21}$$

Substituting Eq. (19) into Eq. (17), the following problem is obtained

$$\left(\begin{bmatrix}S_{11} & S_{12} & S_{13} & S_{14}\\ S_{12} & S_{22} & S_{23} & S_{24}\\ S_{13} & S_{23} & S_{33} & S_{34}\\ S_{14} & S_{24} & S_{34} & S_{44}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ q\\ -\dfrac{3}{20}\cos\left(\dfrac{5}{8}\right)q\end{Bmatrix} \tag{22}$$

Table 1 Non-dimensional center deflection $\bar{w}(a/2, b/2, 0)$ for various EGPs (a/h = 2)

b/a	Theory	p = 0.1	p = 0.5	p = 1	p = 1.5
6	3-D (Zenkour 2007)	1.638	1.352	1.059	0.826
	Mantari and Guedes Soares (2014)	1.658	1.354	1.045	0.802
	Mantari and Guedes Soares (2013)	1.637	1.336	1.033	0.794
	Mantari and Guedes Soares (2012d)	1.735	1.418	1.100	0.850
	TPT (Zenkour 2007)	1.629	1.331	1.028	0.791
	HPT (Zenkour 2007)	1.548	1.265	0.980	0.756
	Present	1.658	1.354	1.045	0.802
1	3-D (Zenkour 2007)	1.638	1.352	1.059	0.826
	Mantari and Guedes Soares (2014)	1.658	1.354	1.045	0.820
	Mantari and Guedes Soares (2013)	1.637	1.336	1.033	0.794
	Mantari and Guedes Soares (2012d)	1.735	1.418	1.100	0.850
	TPT (Zenkour 2007)	1.629	1.331	1.028	0.791
	HPT (Zenkour 2007)	1.548	1.265	0.980	0.756
	Present	1.658	1.354	1.045	0.802

Where

$$\begin{aligned}
&S_{11} = A_{11}\alpha^2 + A_{66}\beta^2, \qquad S_{12} = \alpha\beta\left(A_{12} + A_{66}\right),\\
&S_{13} = -\alpha\left(B_{11}\alpha^2 + (B_{12} + 2B_{66})\beta^2\right),\\
&S_{14} = \alpha\begin{pmatrix}(k_2B'B_{12}^s + \left(k_1A' + k_2B'\right)B_{66}^s)\beta^2\\ + k_1A'B_{11}^s\alpha^2 - L\end{pmatrix},\\
&S_{22} = A_{66}\alpha^2 + A_{22}\beta^2,\\
&S_{23} = -\beta\left(B_{22}\beta^2 + (B_{12} + 2B_{66})\alpha^2\right),\\
&S_{24} = \beta\begin{pmatrix}(k_1A'B_{12}^s + \left(k_1A' + k_2B'\right)B_{66}^s)\alpha^2\\ + k_2B'B_{22}^s\beta^2 - L\end{pmatrix},\\
&S_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\\
&S_{34} = -k_1A'D_{11}^s\alpha^4 - 2D_{12}^s k_2B'\alpha^2\beta^2\\
&\quad - 2D_{66}^s(k_1A' + k_2B')\alpha^2\beta^2\\
&\quad - k_2B'D_{22}^s\beta^4 + L^a(\alpha^2 + \beta^2),\\
&S_{44} = (k_1A')^2H_{11}^s\alpha^4 + (2k_1k_2A'B'H_{12}^s\\
&\quad + (k_1A' + k_2B')^2H_{66}^s)\alpha^2\beta^2\\
&\quad + \left((k_1A')^2F_{55}^s + 2k_1A'X_{55}^s + A_{55}^s\right)\alpha^2\\
&\quad + (k_2B')^2H_{22}^s\beta^4 + R^a -\\
&\quad 2R(k_1A'\alpha^2 + k_2B'\beta^2)\\
&\quad + \left((k_2B')^2F_{44}^s + 2k_2B'X_{44}^s + A_{44}^s\right)\beta^2
\end{aligned} \tag{23}$$

3. Numerical results and discussion

In this section, the bending analysis of FG plates is presented. For this end, various FG plates with different geometry and material characteristics are here studied in order to assess the accuracy of the proposed theory.

Fig. 4 presents the non-dimensional variation of maximum deflection within the plate thickness (b/a = 1, 2, 3, 4, a/h = 4, p = 0.1). It can be seen that the maximum deflection is highly influenced by the aspect ratio (b/a).

For example, FG plates with elastic properties varying exponentially in z, as reported by Zenkour (2007); FG plates with elastic properties powerly graded along the thickness direction z, as indicated by Zenkour (2006) and accurately solved by Carrera *et al*. (2008).

3.1 Exponentially graded plates

The bending analysis is carried out by utilizing aluminum (bottom, Al) graded exponentially within the thickness of a rectangular plate (see Fig. 1(a)). The material characteristics used for calculating the numerical results are

$$E_b = 70\,\text{GPa}, \qquad \nu_b = 0.3 \tag{24}$$

The following non-dimensional quantities are employed

$$\begin{gathered}
\bar{w} = w\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{10E_bh^3}{q_0a^4},\\
\bar{\sigma}_x = \sigma_x\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{h^2}{q_0a^2}, \qquad \bar{\sigma}_y = \sigma_y\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{h^2}{q_0a^2},\\
\bar{\tau}_{xz} = \tau_{xz}\left(0, \frac{b}{2}, z\right)\frac{h}{q_0a}, \qquad \bar{z} = \frac{z}{h}
\end{gathered} \tag{25}$$

The bending results in this section are compared with: (a) the 3D exact solutions and a HSDT with stretching influence by Zenkour (2007); (b) a developed quasi-3D HSDT with 6 unknowns (Mantari and Guedes Soares, 2013); and a recent quasi-3D HSDT proposed by Mantari and Guedes Soares (2014).

Tables 1-5 show results of non-dimensional maximum deflection, normal stresses and shears stresses. It is found from the examination of these tables that the computed results are in good agreements with the published ones.

Fig. 5 plots the non-dimensional normal stresses ($\bar{\sigma}_x$)

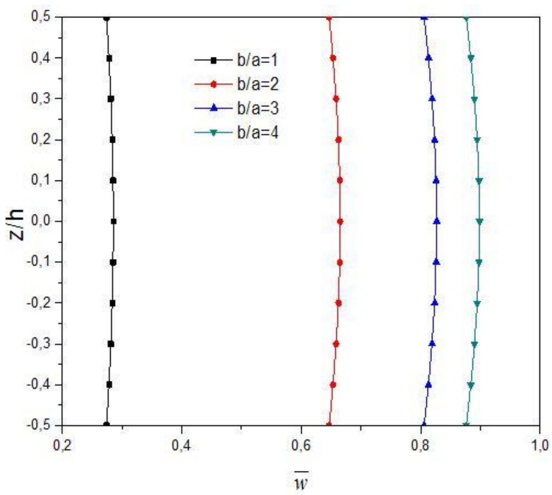

Fig. 4 Variation of non-dimensional displacement, $\bar{w}$ (a/2, b/2, z), through the thickness of a thick EGP (a/h = 4 and p = 0.5).

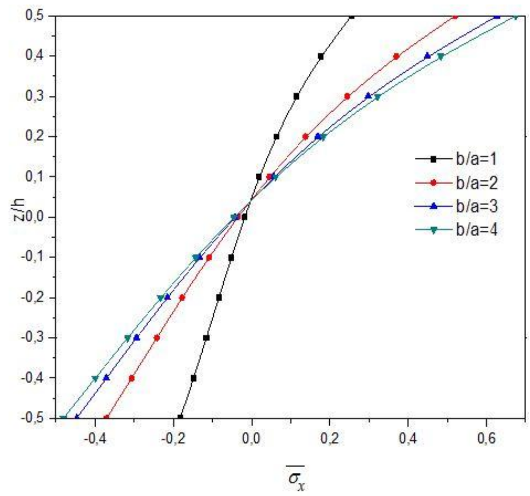

Fig. 5 Variation of non-dimensional normal stress, $\bar{\sigma}_x$ (a/2, b/2, z), through the thickness of a thick EGP (a/h = 4 and p = 0.5)

Table 2 Non-dimensional normal stresses $\bar{\sigma}_y$ (a/2, b/2, h/2) for various EGPs (a/h = 4)

b/a	Theory	p = 0.1	p = 0.5	p = 1	p = 1.5
6	3-D (Zenkour 2007)	0.206	0.231	0.266	0.309
	Mantari and Guedes Soares (2014)	0.218	0.247	0.289	0.337
	Mantari and Guedes Soares (2013)	0.213	0.239	0.280	0.329
	Mantari and Guedes Soares (2012d)	0.201	0.230	0.271	0.319
	TPT (Zenkour 2007)	0.237	0.268	0.314	0.370
	HPT (Zenkour 2007)	0.282	0.322	0.380	0.448
	Present	0.219	0.247	0.286	0.333
1	3-D (Zenkour 2007)	0.217	0.247	0.290	0.340
	Mantari and Guedes Soares (2014)	0.225	0.256	0.302	0.359
	Mantari and Guedes Soares (2013)	0.224	0.255	0.301	0.356
	Mantari and Guedes Soares (2012d)	0.216	0.248	0.293	0.345
	TPT (Zenkour 2007)	0.235	0.268	0.317	0.374
	HPT (Zenkour 2007)	0.241	0.276	0.326	0.385
	Present	0.224	0.255	0.300	0.352

Table 3 Non-dimensional center deflection $\bar{w}$(a/2, b/2, 0) for various EGPs (a/h = 10)

b/a	Theory	p = 0.1	p = 0.5	p = 1	p = 1.5	p = 2	p = 2.5	p = 3
6	Ref[(a)]	1.034	0.845	0.655	0.507	0.391	0.302	0.232
	Ref[(b)]	1.035	0.846	0.656	0.507	0.391	0.302	0.232
	Ref[(c)]	1.039	0.852	0.667	0.524	0.412	0.323	0.254
	TPT[(b)]	1.032	0.844	0.654	0.505	0.39	0.301	0.231
	Present	1.034	0.845	0.655	0.507	0.391	0.302	0.232
1	Ref[(a)]	0.279	0.228	0.177	0.137	0.106	0.081	0.063
	Ref[(b)]	0.280	0.229	0.177	0.137	0.106	0.081	0.063
	Ref[(c)]	0.282	0.231	0.181	0.142	0.111	0.087	0.068
	TPT[(b)]	0.279	0.228	0.177	0.137	0.105	0.081	0.062
	Present	0.279	0.228	0.177	0.137	0.106	0.081	0.063

[(a)] Mantari and Guedes Soares (2014); [(b)] Mantari and Guedes Soares (2013);
[(c)] Mantari and Guedes Soares (2012d)

Table 4 Non-dimensional normal stresses $\bar{\sigma}_y$ (a/2, b/2, h/2) for various EGPs (a/h = 10)

b/a	Theory	p = 0.1	p = 0.5	p = 1	p = 1.5	p = 2	p = 2.5	p = 3
6	Ref[(a)]	0.607	0.693	0.817	0.960	1.127	1.322	1.549
	Ref[(b)]	0.601	0.686	0.808	0.951	1.118	1.312	1.539
	Ref[(c)]	0.603	0.688	0.811	0.954	1.120	1.315	1.542
	TPT[(b)]	0.627	0.717	0.845	0.993	1.165	1.364	1.593
	Present	0.607	0.693	0.816	0.960	1.127	1.322	1.549
1	Ref[(a)]	0.210	0.239	0.281	0.329	0.387	0.455	0.534
	Ref[(b)]	0.206	0.234	0.275	0.324	0.382	0.451	0.532
	Ref[(c)]	0.206	0.235	0.277	0.326	0.385	0.450	0.528
	TPT[(b)]	0.220	0.250	0.294	0.346	0.407	0.477	0.560
	Present	0.210	0.238	0.280	0.329	0.386	0.454	0.534

[(a)] Mantari and Guedes Soares (2014); [(b)] Mantari and Guedes Soares (2013);
[(c)] Mantari and Guedes Soares (2012d)

Table 5 Non-dimensional shear stresses $\bar{\tau}_{xz}$ (0, b/2, 0) for various EGPs (a/h = 10)

b/a	Theory	p = 0.1	p = 0.5	p = 1	p = 1.5	p = 2	p = 2.5	p = 3
6	Ref[(a)]	0.607	0.693	0.817	0.960	1.127	1.322	1.549
	Ref[(b)]	0.601	0.686	0.808	0.951	1.118	1.312	1.539
	Ref[(c)]	0.603	0.688	0.811	0.954	1.120	1.315	1.542
	TPT[(b)]	0.627	0.717	0.845	0.993	1.165	1.364	1.593
	Present	0.607	0.693	0.816	0.960	1.127	1.322	1.549
1	Ref[(a)]	0.210	0.239	0.281	0.329	0.387	0.455	0.534
	Ref[(b)]	0.206	0.234	0.275	0.324	0.382	0.451	0.532
	Ref[(c)]	0.206	0.235	0.277	0.326	0.385	0.450	0.528
	TPT[(b)]	0.220	0.250	0.294	0.346	0.407	0.477	0.560
	Present	0.210	0.238	0.280	0.329	0.386	0.454	0.534

[(a)] Mantari and Guedes Soares (2014); [(b)] Mantari and Guedes Soares (2013);
[(c)] Mantari and Guedes Soares (2012d)

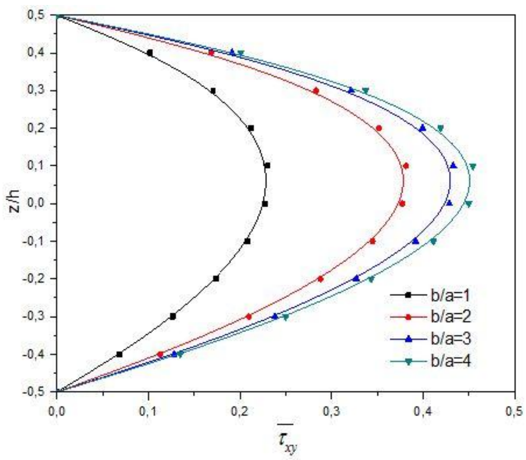

Fig. 6 Variation of non-dimensional shear stress, $\bar{\tau}_{xz}$ (0, b/2, z), through the thickness of a thick EGP (a/h = 4 and p = 0.5)

within the plate thickness. Again, the computed stress $\bar{\sigma}_x$ is influenced by the aspect ratio (b/a).

Finally, Fig. 6 plots the non-dimensional shear stresses ($\bar{\tau}_{xz}$) variation through the plate thickness.

It can be observed that by increasing the aspect ratio (b/a), the non-dimensional shear stresses are increased.

3.2 Powerly graded plates

A square plate fabricated by metal and ceramic powerly graded within its thickness is presented in Fig. 1(b). In fact, the Young modulus varying in thickness direction of the FG plate according to rule of mixtures is shown in Fig. 3.

$$E_b = 70\,\text{GPa}, \qquad E_t = 380\,\text{GPa}, \qquad \nu_b = \nu_t = 0.3 \tag{26}$$

The following non-dimensional quantities are employed

$$\begin{aligned} \bar{u} &= u\left(0, \frac{b}{2}, z\right)\frac{100E_t h^3}{qa^4}, & \bar{w} &= w\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{10E_t h^3}{qa^4}, \\ \bar{\sigma}_x &= \sigma_x\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{h}{qa}, & \bar{\sigma}_y &= \sigma_y\left(\frac{a}{2}, \frac{b}{2}, z\right)\frac{h}{qa}, \end{aligned} \tag{27}$$

Table 6 Non-dimensional deflection and normal stresses of square PGPs

p	Theory	$\bar{w}$ (a/2, b/2, 0)			$\bar{\sigma}_x$ (a/2, b/2, h/3)		
		a/h = 4	a/h = 10	a/h = 100	a/h = 4	a/h = 10	a/h = 100
1	Carrera *et al*. (2008)	0.717	0.588	0.563	0.622	1.506	14.969
	Mantari and Guedes Soares (2014)	0.693	0.568	0.546	0.588	1.459	14.496
	Neves *et al*. (2011)	0.700	0.585	0.562	0.593	1.495	14.969
	FSDT (Carrera *et al*., 2011)	0.729	0.589	0.563	0.806	2.015	20.150
	CPT (Carrera *et al*., 2011)	0.562	0.562	0.562	0.806	2.015	20.150
	Present	0.693	0.569	0.546	0.576	1.457	14.482
4	Carrera *et al*. (2008)	1.159	0.882	0.829	0.488	1.197	11.923
	Mantari and Guedes Soares (2014)	1.092	0.841	0.793	0.434	1.116	11.326
	Neves *et al*. (2011)	1.118	0.875	0.829	0.440	1.178	11.932
	FSDT (Carrera *et al*. 2011)	1.113	0.874	0.829	0.642	1.605	16.049
	CPT (Carrera *et al*. 2011)	0.828	0.828	0.828	0.642	1.605	16.049
	Present	1.092	0.841	0.793	0.417	1.115	11.310
10	Carrera *et al*. (2008)	1.375	1.007	0.936	0.370	0.897	8.908
	Mantari and Guedes Soares (2014)	1.305	0.979	0.914	0.323	0.836	8.527
	Neves *et al*. (2011)	1.349	0.875	0.829	0.323	1.178	11.932
	FSDT (Carrera *et al*. 2011)	1.318	0.997	0.936	0.480	1.199	11.990
	CPT (Carrera *et al*. 2011)	0.935	0.935	0.935	0.480	1.199	11.990
	Present	1.322	0.978	0.914	0.306	0.836	8.518

Table 7 Non-dimensional displacements and stresses of square PGPs

p	Theory	$\bar{u}$ (–h/4)	$\bar{u}$ (–h/6)	$\bar{w}$ (0)	$\bar{\sigma}_x$ (h/2)	$\bar{\sigma}_x$ (h/3)	$\bar{\tau}_{yz}$ (h/6)	$\bar{\tau}_{xz}$ (0)	$\bar{\tau}_{xy}$ (–h/3)
1	Ref[(a)]	0.584	0.444	0.568	3.13	1.459	0.299	0.275	0.562
	Ref[(b)]	0.663	0.509	0.589	3.087	1.489	0.262	0.246	0.611
	Present	0.585	0.445	0.569	3.124	1.457	0.291	0.266	0.563
2	Ref[(a)]	0.808	0.629	0.722	3.635	1.344	0.277	0.222	0.494
	Ref[(b)]	0.928	0.731	0.757	3.609	1.395	0.276	0.227	0.544
	Present	0.809	0.630	0.722	3.630	1.342	0.269	0.214	0.495
3	Ref[(a)]	0.907	0.71	0.798	3.876	1.214	0.244	0.185	0.503
	Ref[(b)]	1.045	0.827	0.838	3.874	1.275	0.272	0.211	0.553
	Present	0.908	0.711	0.797	3.870	1.213	0.236	0.178	0.503
5	Ref[(a)]	0.971	0.756	0.872	4.213	1.043	0.195	0.158	0.529
	Ref[(b)]	1.116	0.879	0.912	4.249	1.103	0.243	0.202	0.576
	Present	0.972	0.757	0.871	4.209	1.042	0.189	0.152	0.530
10	Ref[(a)]	1.001	0.762	0.979	5.031	0.836	0.162	0.171	0.554
	Ref[(b)]	1.137	0.876	1.009	5.089	0.878	0.204	0.22	0.589
	Present	1.002	0.762	0.978	5.028	0.836	0.157	0.164	0.554

$$\bar{\tau}_{xy} = \tau_{xy}(0,0,z)\frac{h}{qa}, \qquad \bar{\tau}_{yz} = \tau_{yz}\left(\frac{a}{2},0,z\right)\frac{h}{qa},$$
$$\bar{\tau}_{xz} = \tau_{xz}\left(0,\frac{b}{2},z\right)\frac{h}{qa} \tag{27}$$

Table 6 shows results of non-dimensional deflection and normal stress results at the specified position (see Eq. (27)) for simply supported homogenous square plate under to bi-sinusoidal distributed load, p = {1, 4, 10} and a/h = 10. Good results are achieved by the proposed model compared with the ones given by Carrera (2008), Mantari and Guedes Soares (2014), Neves *et al*. (2011) and both FSDT and CPT (Carrera *et al*. 2011).

Table 7 gives results of non-dimensional displacements ($\bar{u}$ and $\bar{w}$) and normal, in plane shear, and transverse shear stresses results at the specified position (see Eq. (27)) for simply supported homogenous square plate under to bi-sinusoidal distributed load, p = {1, 2, 3, 5, 10} and a/h = 10. Good results are achieved by the proposed model compared with the ones given by Mantari and Guedes Soares (2014) and Zenkour (2006). It should be indicated that the theory presented by Zenkour (2006) employs 5 unknowns without considering the thickness stretching influence.

The through thickness variations of displacements and stresses are also illustrated in Fig. 7 for square plates with a/h = 10.

4. Conclusions

A novel quasi-3D HSDT with only 4 variables and stretching influences is presented in this work. The governing equations are obtained from the principle of virtual displacements. Analytical solutions are determined for simply supported rectangular plates. By considering further simplifying suppositions to the quasi-3D theory of Zenkour (2007), the number of variables of the novel quasi-3D is diminished by one, and hence, makes the novel theory simple and efficient to utilize. Numerical results demonstrate that these suppositions have a minimal influence on the accuracy of the results for the examined problems. Therefore, it can be deduced that the novel quasi-3D theory is not only accurate but also simple in predicting the bending response of FG plates.

References

Abdelaziz, H.H., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct., Int. J.*, **25**(6), 693-704.

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Ahmed, A. (2014), "Post buckling analysis of sandwich beams with functionally graded faces using a consistent higher order theory", *Int. J. Civil Struct. Environ.*, **4**(2), 59-64.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct., Int. J.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct., Int. J.*, **19**(2), 369-384.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.

Akavci, S.S. (2014), "An efficient shear deformation theory for free vibration of functionally graded thick rectangular plates on elastic foundation", *Compos. Struct.*, **108**, 667-676.

Akavci, S.S. (2016), "Mechanical behavior of functionally graded sandwich plates on elastic foundation", *Compos. Part B*, **96**, 136-152.

Akbaş, Ş.D. (2015), "Wave propagation of a functionally graded beam in thermal environments", *Steel Compos. Struct., Int. J.*, **19**(6), 1421-1447.

Akbaş, Ş.D. (2016), "Forced vibration responses of functionally graded viscoelastic beams under thermal environment", *Int. J. Innov. Res. Sci. Eng. Technol.*, **5**(12), 36-46.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Aldousari, S.M. (2017), "Bending analysis of different material distributions of functionally graded beam", *Appl. Phys. A*, **123**, 296.

Arani, A.J. and Kolahchi, R. (2016), "Buckling analysis of embedded concrete columns armed with carbon nanotubes", *Comput. Concrete, Int. J.*, **17**(5), 567-578.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct., Int. J.*, **18**(1), 187-212.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech., Int. J.*, **65**(4), 453-464.

Bakhadda, B., Bachir Bouiadjra, M., Bourada, F., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation", *Wind Struct., Int. J.*, (Accepted).

Baseri, V., Jafari, G.S. and Kolahchi, R. (2016), "Analytical solution for buckling of embedded laminated plates based on higher order shear deformation plate theory", *Steel Compos. Struct., Int. J.*, **21**(4), 883-919.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Brazil. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017a), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct., Int. J.*, **25**(3), 257-270.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017b), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech., Int. J.*, **62**(6), 695-702.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), "An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities", *Earthq. Struct., Int. J.*, **13**(3), 255-265.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech., Int. J.*, **65**(1), 19-31.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst., Int. J.*, **19**(6), 601-614.

Bever, M.B. and Duwez, P.E. (1972), "Gradients in composite materials", *Mater. Sci. Eng.*, **10**, 1-8.

Bian, Z.G., Chen, W.Q., Lim, C.W. and Zhang, N. (2005), "Analytical solutions for single- and multi-span functionally graded plates in cylindrical bending", *Int. J. Solids Struct.*, **42**(24-25), 6433-6456.

Bilouei, B.S., Kolahchi, R. and Bidgoli, M.R. (2016), "Buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP)", *Comput. Concrete, Int. J.*, **18**(5), 1053-1063.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech., Int. J.*, **58**(3), 397-422.

Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S. and Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching in advanced composite plates", *Struct. Eng. Mech., Int. J.*, (Accepted).

Boukhari, A., Ait Atmane, H., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech., Int. J.*, **60**(2), 313-335.

Carrera, E., Brischetto, S. and Robaldo, A. (2008), "Variable kinematic model for the analysis of functionally graded material plates", *AIAA J.*, **46**, 194-203.

Carrera, E., Brischetto, S., Cinefra, M. and Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos.: Part B*, **42**, 123-133.

Celebi, K., Yarimpabuc, D. and Keles, I. (2016), "A unified method for stresses in FGM sphere with exponentially-varying properties", *Struct. Eng. Mech., Int. J.*, **57**(5), 823-835.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst., Int. J.*, **19**(3), 289-297.

Darılmaz, K. (2015), "Vibration analysis of functionally graded material (FGM) grid systems", *Steel Compos. Struct., Int. J.*, **18**(2), 395-408.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng., Int. J.*, **11**(5), 671-690.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech.*, **63**(5), 585-595.

Eltaher, M.A., Alshorbagy, A.E. and Mahmoud, F.F. (2013), "Determination of neutral axis position and its effect on natural frequencies of functionally graded macro/nanobeams", *Compos. Struct.*, **99**, 193-201.

Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng., Int. J.*, **13**(3), 385-410.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**, 795-810.

Ghorbanpour Arani, A., Cheraghbak, A. and Kolahchi, R. (2016), "Dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory", *Struct. Eng. Mech., Int. J.*, **60**(3), 489-505.

Ghugal, Y.M. (2010), "A static flexure of thick isotropic plates using trigonometric shear deformation theory", *J. Solid Mech.*, **2**, 79-90.

Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, M., Tounsi, A. and Mahmoud, S.R. (2017), " A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct., Int. J.*, **25**(6), 717-726.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**, 374-383.

Hosseini-Hashemi, S., Taher, H.R.D., Akhavan, H. and Omidi, M. (2010), "Free vibration of functionally graded rectangular plates using first-order shear deformation plate theory", *Appl. Math. Model.*, **34**(5), 1276-1291.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(2), 257-276.

Jha, D.K., Tarun, K. and Singh, R.K. (2013), "A critical review of recent research on functionally graded plates", *Compos. Struct.*, **96**, 833-849.

Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech., Int. J.*, **65**(5), 621-631.

Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct., Int. J.*, **18**(3), 693-709.

Karama, M., Afaq, K.S. and Mistou, S. (2003), "Mechanical behavior of laminated composite beam by the new multilayered laminated Composite Structures model with transverse shear stress continuity", *Int. J. Solids Struct.*, **40**(6), 1525-1546.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech., Int. J.*, **64**(4), 391-402.

Klouche, F., Darcherif, L., Sekkal, M., Tounsi, A. and Mahmoud, S.R. (2017), "An original single variable shear deformation theory for buckling analysis of thick isotropic plates", *Struct. Eng. Mech., Int. J.*, **63**(4), 439-446.

Koizumi, M. (1993), "The concept of FGM, Ceramic transactions", *Funct. Grad. Mater.*, **34**, 3-10.

Kolahchi, R., Bidgoli, A.M. (2016), "Size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes", *Appl. Math. Mech. -Engl. Ed.*, **37**(2), 265-274.

Kolahchi, R., Bidgoli, A.M.M. and Heydari, M.M. (2015), "Size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium", *Struct. Eng. Mech., Int. J.*, **55**(5), 1001-1014.

Kolahchi, R., Hosseini, H. and Esmailpour, M. (2016a), "Differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories", *Compos. Struct.*, **157**, 174-186.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016b), "Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium", *Compos. Struct.*, **150**, 255-265.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Oskouei, A.N. (2017), "Visco-nonlocal-refined Zigzag theories for dynamic buckling of laminated nanoplates using differential cubature-Bolotin methods", *Thin-Wall. Struct.*, **113**, 162-169.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel. Compos. Struct., Int. J.*, **18**(2), 425-442.

Levy, M. (1877), "Memoire sur la theorie des plaques elastique planes ", *J. Math. Pures Appl.*, **30**, 219-306.

Liu, S., Yu, T., Bui, T.Q., Yin, S., Thai, D.K. and Tanaka, S. (2017), "Analysis of functionally graded plates by a simple locking-free quasi-3D hyperbolic plate isogeometric method", *Compos. Part B: Eng.*, **120**, 182-196.

Madani, H., Hosseini, H. and Shokravi, M. (2016), "Differential cubature method for vibration analysis of embedded FG-CNT-reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions", *Steel Compos. Struct., Int. J.*, **22**(4), 889-913.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**, 2489-2508.

Mantari, J.L. (2016), "General recommendations to develop 4-unknowns quasi-3D HSDTs to study FGMs", *Aerosp. Sci. Technol.*, **58**, 559-570.

Mantari, J.L. and Guedes Soares, C. (2012d), "Bending analysis of thick exponentially graded plates using a new trigonometric higher order shear deformation theory", *Compos. Struct.*, **94**, 1991-2000.

Mantari, J.L. and Guedes Soares, C. (2013), "A novel higher-order shear deformation theory with stretching effect for functionally graded plates", *Compos.: Part B*, **45**, 268-281.

Mantari, J.L. and Guedes Soares, C. (2014), "Four-unknown quasi-3D shear deformation theory for advanced composite plates", *Compos. Struct.*, **109**, 231-239.

Mantari, J.L. and Granados, E.V. (2015a), "Thermoelastic analysis of advanced sandwich plates based on a new quasi-3D hybrid type HSDT with 5 unknowns", *Compos. Part B*, **69**, 317-334.

Mantari, J.L. and Granados, E.V. (2015b), "Thermoelastic behavior of advanced composite sandwich plates by using a new 6 unknown quasi-3D hybrid type HSDT", *Compos. Struct.*, **126**, 132-144.

Mantari, J.L., Oktem, A.S. and Guedes Soares, C. (2012a), "A new higher order shear deformation theory for sandwich and composite laminated plates", *Compos., Part B*, **43**, 1489-1499.

Mantari, J.L., Oktem, A.S. and Guedes Soares, C. (2012b), "Bending and free vibration analysis of isotropic and multilayered plates and shells by using a new accurate higher-order shear deformation theory", *Compos. Part B*, **43**, 3348-3360.

Mantari, J.L., Oktem, A.S. and Guedes Soares, C. (2012c), "Bending response of functionally graded plates by using a new higher order shear deformation theory", *Compos. Struct.*, **94**(2), 714-723.

Mantari, J.L., Granados, E.V., Hinostroza, M.A. and Guedes Soares, C. (2014), "Modelling advanced composite plates resting on elastic foundation by using a quasi-3D hybrid type HSDT", *Compos. Struct.*, **118**, 455-471.

Matsunaga, H. (2008), "Free vibration and stability of functionally graded plates according to a 2-D higher-order deformation theory", *Compos. Struct.*, **82**(4), 499-512.

Meksi, R., Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct.Mater.*, 1099636217698443.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel. Compos. Struct., Int. J.*, **25**(2), 157-175.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(3), 793-809.

Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst., Int. J.*, (Accepted).

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst., Int. J.*, **20**(3), 369-383.

Nami, M.R. and Janghorban, M. (2013), "Static analysis of rectangular nanoplates using Trigonometric shear deformation theory based on nonlocal elasticity theory", *Beilstein J. Nanotechnol.*, **4**, 968-973.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Roque, C.M.C., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2011), "Bending of FGM plates by a sinusoidal plate formulation and collocation with radial basis functions", *Mech. Res. Commun.*, **38**, 368-371.

Pradhan, K.K. and Chakraverty, S. (2015), "Free vibration of functionally graded thin elliptic plates with various edge supports", *Struct. Eng. Mech., Int. J.*, **53**(2), 337-354.

Rahmani, O., Refaeinejad, V. and Hosseini, S.A.H. (2017), "Assessment of various nonlocal higher order theories for the bending and buckling behavior of functionally graded nanobeams", *Steel Compos. Struct., Int. J.*, **23**(3), 339-350.

Raminnea, M., Biglari, H. and Vakili Tahami, F. (2016), "Nonlinear higher order Reddy theory for temperature-dependent vibration and instability of embedded functionally graded pipes conveying fluid-nanoparticle mixture", *Struct. Eng. Mech., Int. J.*, **59**(1), 153-186.

Reddy, J.N. (2011), "A general nonlinear third-order theory of functionally graded plates", *Int. J. Aeros. Lightw. Struct.*, **1**(1), 1-21.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017a), "A novel and simple higher order shear deformation theory for stability and vibration of functionally graded sandwich plate", *Steel Compos. Struct., Int. J.*, **25**(4), 389-401.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017b), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech., Int. J.*, **64**(6), 737-749.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous mono-clinic plates", *Acta Mech.*, **94**, 195-220.

Stein, M. (1986), "Nonlinear theory for plates and shells including the effects of transverse shearing", *AIAA J.*, **24** (9), 1537-1544.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Solid.*, **47**, 349-361.

Swaminathan, K., Naveenkumar, D.T., Zenkour, A.M. and Carrera, E. (2015), "Stress vibration and buckling analyses of FGM plates—a state-of-the-art review", *Compos. Struct.*, **120**, 10-31.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Touratier, M. (1991), "An efficient standard plate theory", *Int. J. Eng. Sci.*, **29**(8), 901-916.

Vidal, P. and Polit, O. (2008), "A family of sinus finite elements for the analysis of rectangular laminated beams", *Compos. Struct.*, **84**, 56-72.

Vidal, P. and Polit, O. (2009), "Assessment of the refined sinus model for the non-linear analysis of composite beams", *Compos. Struct.*, **87**, 370-381.

Vidal, P. and Polit, O. (2011), "A sine finite element using a zig-zag function for the analysis of laminated composite beams", *Compos. Part B*, **42**, 1671-1682.

Vidal, P. and Polit, O. (2013), "A refined sinus plate finite element for laminated and sandwich structures under mechanical and thermomechanical loads", *Comput. Methods Appl. Mech. Eng.*, **253**, 396-412.

Viola, E., Tornabene, F. and Fantuzzi, N. (2013a), "General higher-order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels", *Compos. Struct.*, **95**, 639-666.

Viola, E., Tornabene, F. and Fantuzzi, N. (2013b), "Static analysis of completely doubly-curved laminated shells and panels using general higher-order shear deformation theories", *Compos. Struct.*, **101**, 59-93.

Yaghoobi, H., Valipour, M.S., Fereidoon, A. and Khoshnevisrad, P. (2014), "Analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM", *Steel Compos. Struct., Int. J.*, **17**(5), 753-776.

Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Houari, M.S.A. (2018), "A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium", *Smart Struct. Syst., Int. J.*, **21**(1), 15-25.

Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A. and Tounsi, A. (2018), "Dynamic analysis of nanoscale beams including surface stress effects", *Smart Struct. Syst., Int. J.*, **21**(1), 65-74.

Zamanian, M., Kolahchi, R. and Bidgoli, M.R. (2017), "Agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2 nano-particles", *Wind Struct., Int. J.*, **24**(1), 43-57.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech., Int. J.*, **54**(4), 693-710.

Zenkour, A.M. (2006), "Generalized shear deformation theory for bending analysis of functionally graded plates", *Appl. Math. Model*, **30**, 67-84.

Zenkour, A.M. (2007), "Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate", *Appl. Math. Model*, **77**, 197-214.

Zidi, M., Tounsi, A., Houari, M.S.A. and Bég, O.A. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Tech.*, **34**, 24-34.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech., Int. J.*, **64**(2), 145-153.

Zine, A., Tounsi, A., Draiche, K., Sekkal, M. and Mahmoud, S.R. (2018), "A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells", *Steel Compos. Struct., Int. J.*, **26**(2), 125-137.

CC