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Large deflection analysis of a fiber reinforced composite beam

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Abstract. The objective of this work is to analyze large deflections of a fiber reinforced composite cantilever beam under point loads. In the solution of the problem, finite element method is used in conjunction with two dimensional (2-D) continuum model. It is known that large deflection problems are geometrically nonlinear problems. The considered non-linear problem is solved considering the total Lagrangian approach with Newton-Raphson iteration method. In the numerical results, the effects of the volume fraction and orientation angles of the fibre on the large deflections of the composite beam are examined and discussed. Also, the difference between the geometrically linear and nonlinear analysis of fiber reinforced composite beam is investigated in detail.

Keywords: large deflection analysis; fiber reinforced composite beam; total Lagragian; Finite Element Method

1. Introduction

Fiber reinforced composite structures have been used many engineering applications, such as aircrafts, space vehicles, automotive industries, defence industries and civil engineering applications because these structures have higher strength-weight ratios, more lightweight and ductile properties than classical materials. With the great advances in technology, the using of the fiber reinforced composite structures is growing in applications.

It is known that large deflection problems are geometrically nonlinear problems. In the literature, much more attention has been given to the linear analysis of composite beam structures. However, nonlinear studies of composite beams are has not been investigated broadly. In the open literature, studies of the nonlinear behavior of composite beams are as follows; Ghazavi and Gordaninejad (1989) studied geometrically nonlinear static of laminated bimodular composite beams by using mixed finite element model. Singh *et al*. (1992) investigated nonlinear static responses of laminated composite beam based on higher shear deformation theory and von Karman's nonlinear type. Pai and Nayfeh (1992) presented three-dimensional nonlinear dynamics of anisotropic composite beams with Von-Karman nonlinear type. Kim and Dugundji (1993) investigated large amplitude non-rotating free vibration of composite helicopter blades under large static deflection. Di Sciuva and Icardi (1995) investigated large deflection of anisotropic laminated composite beams with Timoshenko beam theory and von Karman nonlinear strain-displacement relations by using Euler method. Amada and Nagase analyzed large deflections of the functionally graded bamboo composites. Xie and Adams (1996) presented nonlinear finite element solution of the fiber-reinforced composite materials. Omidvar and Ghorbanpoor (1996) studied fiber-reinforced laminated composite beams by using finite element method with updated Langragian approach. Donthireddy and Chandrashekhara (1997) investigated thermoelastic nonlinear static and dynamic analysis of laminated beams by using finite element method. Fraternali and Bilotti (1997) analyzed nonlinear stress of laminated composite curved beams. Kolli and Chandrashekhara (1997) investigated nonlinear static and dynamic analysis of stiffened laminated composite plates by using Von-Karman nonlinear strain-displacement relations. Ganapathi *et al*. (1998) studied nonlinear vibration analysis of laminated composite curved beams. Patel (1999) examined nonlinear post-buckling and vibration of laminated composite orthotropic beams/columns resting on elastic foundation with Von-Karman's strain-displacement relations. Oliveira and Creus (2003) investigated flexure and buckling behaviors of thin-walled composite beams with nonlinear viscoelastic model. Valido and Cardoso (2003) developed a finite element model for optimal desing of laminated composite thin-walled beams with geometrically nonlinear effects. Machado (2007) studied nonlinear buckling and vibration of thin-walled composite beams. Vo and Lee (2009) presented geometrically nonlinear of thin-walled composite box beams with Von-Karman nonlinear nonlinearity. Cardoso *et al*. (2009) investigated geometrically nonlinear behavior of the laminated composite thin-walled beam structures with finite element solution. Emam and Nayfeh (2009) investigated post-buckling of the laminated composite beams with different boundary conditions. Malekzadeh and Vosoughi (2009) studied large amplitude free vibration of laminated composite beams resting on elastic foundation by using differential quadrature method. Salehi and Falahatgar (2010) studied geometrically non-linear of fiber-reinforced sector composite plates. Akgöz and Civalek (2011) and

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Civalek (2013) examined nonlinear vibration laminated plates resting on nonlinear-elastic foundation. Youzera *et al*. (2012) presented nonlinear dynamics of laminated composite beams with damping effect. Mareishi *et al*. (2014) investigated large deflection, nonlinear vibration and buckling of fiber reinforced composite beams with surface bonded piezoelectric material. Patel (2014) examined nonlinear static of laminated composite plates with the Green-Lagrange nonlinearity. Akbaş (2013a, 2014, 2015a, b, c) investigated geometrically nonlinear of cracked and functionally graded beams. Stoykov and Margenov (2014) studied Nonlinear vibrations of 3D laminated composite Timoshenko beams. Cunedioğlu and Beylergil (2014) examined vibration of laminated composite beams under thermal loading. Li and Qiao (2015), Shen *et al*. (2016, 2017), Li and Yang (2016) investigated nonlinear postbuckling analysis of composite laminated beams. Akbaş (2018a, b) investigated nonlinear displacements and post-buckling responses laminated composite beams. Mahi and Tounsi (2015) studied static and vibration of functionally graded, sandwich and laminated composite plates by using hyperbolic shear deformation theory. Draiche *et al*. (2016) investigated flexure analysis of laminated composite plate by using a refined theory with stretching effect. Chikh *et al*. (2017) investigated buckling of laminated plates under thermal loading with higher shear deformation theory. Kaci *et al*. (2018) examined post-buckling responses of composite beams by using shear deformable theory. Kurtaran (2015), Mororó *et al*. (2015), Pagani and Carrera (2017) analyzed large deflections of laminated composite beams. Benselama *et al*. (2015), Liu and Shu (2015), Topal (2017) investigated buckling behavior of composite laminate beams. Latifi *et al*. (2016), Ebrahimi and Hosseini (2017) presented nonlinear dynamics of laminated composite structures. Also, there are many nonlinear, vibration, buckling studies of other type composite structures such as functionally graded materials, sandwich, nano composites etc. in the literature (Tounsi *et al*. 2013, Akbaş and Kocatürk 2012, Zidci *et al*. 2014, Hamidi *et al*. 2015, Meziane *et al*. 2014, Abdelaziz *et al*. 2017, Akbaş 2013a, 2015d, e, 2016a, b, c, 2017a, b, c, d, e, f, g, 2018c, Beldjelili *et al*. 2016, Kocatürk and Akbaş 2010, 2011, 2012, 2013, El-Haina *et al*. 2017, Menasria *et al*. 2017, Attia *et al*. 2018, Belabed *et al*. 2018, Bouderba *et al*. 2013, Bourada *et al*. 2015, Hebali *et al*. 2014, Bennoun *et al*. 2016, Bousahla *et al*. 2014, Belabed *et al*. 2014), Abualnour *et al*. 2018, Bouafia *et al*. 2017, Benchohra *et al*. 2018, Bellifa *et al*. 2017, Zine *et al*. 2018, Boukhari *et al*. 2016, Yahia *et al*. 2015, Houari *et al*. 2016, Bellifa *et al*. 2016, Yazid *et al*. 2018, Bounouara *et al*. 2016, Youcef *et al*. 2018, Oucif *et al*. 2017, Alam and Al Riyami 2018, Argyridi and Sapountzakis (2016), Ge *et al*. 2018, Manthena *et al*. 2016).

In the most of the large deflection and nonlinear studies of composite beams, the Von-Karman strain displacement approximation is used. In the Von-Karman strain, full geometric non-linearity cannot be considered because of neglect of some components of strain, satisfactory results can be obtained only for large displacements but moderate rotations. In the open literature, nonlinear studies of composite beams with considering full geometric nonlinearity has not been investigated broadly.

In the present study, the large deflection static analysis of a fiber reinforced beam is investigated by using total Lagrangian finite element model of two dimensional (2-D) continuum in which full geometric nonlinearity can be considered as distinct from the studies by using Von-Karman nonlinearity. The main purpose of this paper is to fill this gap for fiber reinforced composite beams. In the numerical results, the effects of the volume fraction and orientation angles of the fibre on the large deflections of the fiber reinforced composite beam are investigated. Also, the difference between the geometrically linear and nonlinear analysis of fiber reinforced composite beam is investigated in detail.The distinctive feature of this study is large deflection analysis of fiber reinforced composite beams with full geometric non-linearity. However, the material nonlinearity and elasto-plastic behavior are not considered. It would be interesting to demonstrate the ability of the procedure through a wider campaign of investigations concerning elasto-plastic or material nonlinear analysis of fiber reinforced composite beams with geometrically nonlinearity.

2. Theory and formulation

A fiber reinforced composite cantilever beam of length L, width b, and height h, subjected to a transversal point load (F) at the right end of the beam with material or Lagrangian coordinate system (X, Y, Z) and with spatial or Euler coordinate system (x, y, z).

It is known that the large deflection is a geometrically nonlinear problem. In the nonlinear kinematic model of the beam for the large deflection problem, total Lagrangian approximation is used within the 2-D solid continuum model. In the solution of the nonlinear problem, finite element method is used for total Lagrangian kinematic model for an eight-node quadratic element.

In the solution of the nonlinear finite element of total Lagrangian formulations, small-step incremental approaches from known solutions with Newton-Raphson iteration method are used. In the Newton-Raphson solution for the problem, the applied load is divided by a suitable number of increments according to its value. After completing an iteration process, the previous accumulated

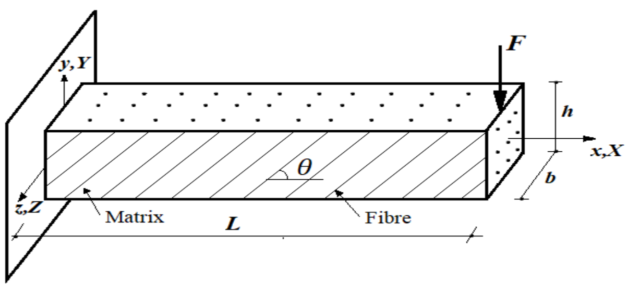


Fig. 1 A fiber reinforced composite cantilever beam subjected to a point load (F) at the right end of the beam

load is increased by a load increment. For n+1th load increment and ith iteration is obtained in the following form

$$d\boldsymbol{u}_n^i = (\boldsymbol{K}_T^i)^T \boldsymbol{R}_{n+1}^i \tag{1}$$

where $\boldsymbol{K}_T^i$ is the tangent stiffness matrix corresponding to a tangent direction at the ith iteration, $d\boldsymbol{u}_n^i$ is the solution increment vector at the ith iteration and n+1th load increment, $\boldsymbol{R}_{n+1}^i$ is the residual vector at the ith iteration and n+1th load increment. This iteration procedure is continued until the difference between two successive solution vectors is less than a selected tolerance criterion in Euclidean norm given by

$$\sqrt{\frac{[(d\boldsymbol{u}_n^{i+1} - d\boldsymbol{u}_n^i)^T (d\boldsymbol{u}_n^{i+1} - d\boldsymbol{u}_n^i)]^2}{[(d\boldsymbol{u}_n^{i+1})^T (d\boldsymbol{u}_n^{i+1})]^2}} \leq \xi_{tol} \tag{2}$$

A series of successive approximations gives

$$\boldsymbol{u}_{n+1}^{i+1}\ \boldsymbol{u}_{n+1}^i + d\boldsymbol{u}_{n+1}^i = \boldsymbol{u}_n + \Delta\boldsymbol{u}_n^i \tag{3}$$

where

$$\Delta\boldsymbol{u}_n^i = \sum_{k=1}^{i} d\boldsymbol{u}_n^k \tag{4}$$

The tangent stiffness matrix $\boldsymbol{K}_T^i$ and the residual vector $\boldsymbol{R}_{n+1}^i$ which are to be used in Eq. (1) at the ith iteration for the total Lagrangian finite element model of two dimensional continua for an eight-node quadratic element are given below

$$\begin{bmatrix} K^{11L} + K^{11NL} & K^{12L} \\ K^{21L} & K^{22L} + K^{22NL} \end{bmatrix}^i \begin{Bmatrix} \bar{u} \\ \bar{v} \end{Bmatrix}^i = \begin{Bmatrix} {}_0^2F^1 - {}_0^1F^1 \\ {}_0^2F^2 - {}_0^1F^2 \end{Bmatrix}^i \tag{5}$$

where

$$\begin{aligned} K_{ij}^{11L} = b \int_A \Bigg\{ & {}_0C_{11}\left(1+\frac{\partial u}{\partial X}\right)^2 \frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \\ & + {}_0C_{22}\left(\frac{\partial u}{\partial Y}\right)^2 \frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial Y} \\ & + {}_0C_{12}\left(1+\frac{\partial u}{\partial X}\right)\left(\frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \right. \\ & \left. + \frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial X}\right) \\ & + {}_0C_{66}\left[\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial \psi_i}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \psi_j}{\partial X}\right] \\ & \times \left[\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial \psi_j}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \psi_j}{\partial X}\right] dXdY \end{aligned} \tag{6a}$$

$$\begin{aligned} K_{ij}^{12L} = b \int_A \Bigg\{ & {}_0C_{11}\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial v}{\partial X}\frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \\ & + {}_0C_{22}\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial u}{\partial Y}\frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial Y} \\ & + {}_0C_{12}\left[\left(1+\frac{\partial u}{\partial X}\right)\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial Y} \right. \\ & \left. + \frac{\partial u}{\partial Y}\frac{\partial v}{\partial X}\ \frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial X}\right] \\ & + {}_0C_{66}\left[\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial \psi_i}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \psi_j}{\partial X}\right] \\ & \times \left[\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_j}{\partial Y} + \frac{\partial u}{\partial Y}\frac{\partial \psi_j}{\partial X}\right]\Bigg\} dXdY \\ & = K_{ij}^{21L} \end{aligned} \tag{6b}$$

$$\begin{aligned} K_{ij}^{22L} = b \int_A \Bigg\{ & {}_0C_{11}\left(\frac{\partial v}{\partial X}\right)^2 \frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \\ & + {}_0C_{22}\left(1+\frac{\partial v}{\partial Y}\right)^2 \frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial Y} \\ & + {}_0C_{12}\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial v}{\partial X}\left(\frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \right. \\ & \left. + \frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial X}\right) \\ & + {}_0C_{66}\left[\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_i}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \psi_j}{\partial Y}\right] \\ & \times \left[\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_j}{\partial X} + \frac{\partial v}{\partial X}\frac{\partial \psi_j}{\partial Y}\right]\Bigg\} dXdY \end{aligned} \tag{6c}$$

$$\begin{aligned} K_{ij}^{11NL} = K_{ij}^{22NL} = b \int_A \Bigg\{ & {}_0^1S_{11}\frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial X} \\ & + {}_0^1S_{12}\left(\frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial X} + \frac{\partial \psi_i}{\partial X}\frac{\partial \psi_j}{\partial Y}\right) \\ & + {}_0^1S_{22}\frac{\partial \psi_i}{\partial Y}\frac{\partial \psi_j}{\partial Y}^2\Bigg\} dXdY \end{aligned} \tag{6d}$$

$${}_0^2F_i^1 = b \int_A {}_0^2f_x\ \psi_i dXdY + \int_\Gamma {}_0^2t_x\ \psi_i ds \tag{6e}$$

$${}_0^2F_i^2 = b \int_A {}_0^2f_y\ \psi_i dXdY \int_\Gamma {}_0^2t_y\ \psi_i ds \tag{6f}$$

where ${}_0^2f_x$, ${}_0^2f_y$ are the body forces, ${}_0^2t_x$, ${}_0^2t_y$ are the surface forces in the x and y directions. u and v are displacements in the x and y directions, ψ indicates the shape functions.

$$\begin{aligned} {}_0^1F_i^1 = b \int_A \Bigg\{ & {}_0^1S_{11}\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial \psi_i}{\partial X} + {}_0^1S_{22}\frac{\partial u}{\partial Y}\frac{\partial \psi_i}{\partial Y} \\ & + {}_0^1S_{12}\left[\left(1+\frac{\partial u}{\partial X}\right)\frac{\partial \psi_i}{\partial Y} \right. \\ & \left. + \frac{\partial u}{\partial Y}\frac{\partial \psi_i}{\partial Y}\right]\Bigg\} dXdY \end{aligned} \tag{7a}$$

$$\begin{aligned} {}_0^1F_i^2 = b \int_A \Bigg\{ & {}_0^1S_{11}\frac{\partial v}{\partial X}\frac{\partial \psi_i}{\partial X} + {}_0^1S_{22}\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_i}{\partial Y} \\ & + {}_0^1S_{12}\left[\left(1+\frac{\partial v}{\partial Y}\right)\frac{\partial \psi_i}{\partial X} \right. \\ & \left. + \frac{\partial v}{\partial X}\frac{\partial \psi_i}{\partial Y}\right]\Bigg\} dXdY \end{aligned} \tag{7b}$$

The constitutive relation between the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor can be expressed as follows

$$
{}_0^1S = \begin{Bmatrix} {}_0^1S_{11} \\ {}_0^1S_{22} \\ {}_0^1S_{12} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} {}_0^1E_{11} \\ {}_0^1E_{22} \\ 2\,{}_0^1E_{12} \end{Bmatrix} \tag{8}
$$

where ${}_0^1S_{11}$, ${}_0^1S_{22}$, ${}_0^1S_{12}$ are the components of the second Piola-Kirchhoff stress tensor components in the initial configuration of the body, ${}_0^1E_{ij}$ are the components of the Green-Lagrange strain tensor, $\overline{Q}_{ij}$ are the transformed components of the reduced constitutive tensor in the initial configuration of the body. The transformed components of the reduced constitutive tensor for orthotropic material are as follows

$$
\begin{aligned}
\overline{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\cos^2 + Q_{12}(m^4 + n^4) \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})nm^3 \\
&\quad + (Q_{12} - Q_{22} + 2Q_{66})n^3m \\
\overline{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}m^4 \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3m \\
&\quad + (Q_{12} - Q_{22} + 2Q_{66})nm^3 \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 \\
&\quad + Q_{66}(n^4 + m^4)
\end{aligned} \tag{9}
$$

where $m = \cos\theta$ and $n = \sin\theta$, θ indicates the fiber orientation angle. $\overline{Q}_{ij}$ are the components of the reduced constitutive tensor for orthotropic material and their expressions are as follows

$$
\begin{gathered}
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \\
Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{21} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{66} = G_{12}
\end{gathered} \tag{10}
$$

where E_1 is the Young's modulus in the X direction, E_2 is the Young's modulus in the Y direction, ν_{12} and ν_{21} are Poisson's ratios and G_{12} is the shear modulus in XY plane. The gross mechanical properties of the composite materials are calculated by using the following expression (Vinson and Sierakowski 2002)

$$
\begin{aligned}
E_1 &= E_f\ V_f + E_m\left(1 - V_f\right), \\
E_2 &= E_m\ \left[\frac{E_f + E_m + \left(E_f - E_m\right)V_f}{E_f + E_m - \left(E_f - E_m\right)V_f}\right] \\
\nu_{12} &= \nu_f\ V_f + \nu_m\left(1 - V_f\right), \\
G_{12} &= G_m\ \left[\frac{G_f + G_m + \left(G_f - G_m\right)V_f}{G_f + G_m - \left(G_f - G_m\right)V_f}\right]
\end{aligned} \tag{11}
$$

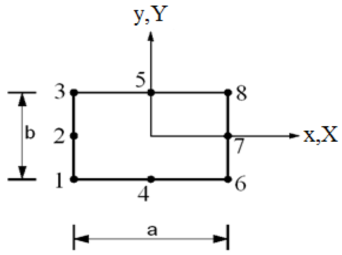


Fig. 2 Eight-node plane element

where f indicates the fibre and m indicates the matrix. V_f is the volume fraction of fiber. E, G and ν are the Young's modulus, the shear modulus and Poisson's ratio, respectively. The Green-Lagrange strain tensor is expressed in terms of displacements in the case of two-dimensional solid continuum as follows

$$
{}_0^1E = \begin{Bmatrix} {}_0^1E_{11} \\ {}_0^1E_{22} \\ 2\,{}_0^1E_{12} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial X} + \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial X}\right)^2 + \left(\dfrac{\partial v}{\partial X}\right)^2\right] \\ \dfrac{\partial v}{\partial Y} + \dfrac{1}{2}\left[\left(\dfrac{\partial u}{\partial Y}\right)^2 + \left(\dfrac{\partial v}{\partial Y}\right)^2\right] \\ \dfrac{\partial u}{\partial Y} + \dfrac{\partial v}{\partial X} + \dfrac{1}{2}\left[\dfrac{\partial u}{\partial X}\dfrac{\partial u}{\partial Y} + \dfrac{\partial v}{\partial X}\dfrac{\partial v}{\partial Y}\right] \end{Bmatrix} \tag{12}
$$

In the finite element model, eight-node plane element is used as shown in Fig. 2.

These total displacement fields and incremental displacement fields are interpolated as follows

$$
\{u\} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} \displaystyle\sum_{j=1}^{8} u_j\,\psi_j(x) \\ \displaystyle\sum_{j=1}^{8} v_j\,\psi_j(x) \end{Bmatrix} = [\Psi][\Delta] \tag{13}
$$

$$
\{\bar{u}\} = \begin{Bmatrix} \bar{u} \\ \bar{v} \end{Bmatrix} = \begin{Bmatrix} \displaystyle\sum_{j=1}^{8} \bar{u}_j\,\psi_j(x) \\ \displaystyle\sum_{j=1}^{8} \bar{v}_j\,\psi_j(x) \end{Bmatrix} = [\overline{\Psi}][du] \tag{14}
$$

where

$$
[\Psi] = \begin{bmatrix} \psi_1 & 0 & \psi_2 & 0 & \psi_3 & 0 & \psi_4 & 0 & \psi_5 & 0 & \psi_6 & 0 & \psi_7 & 0 & \psi_8 & 0 \\ 0 & \psi_1 & 0 & \psi_2 & 0 & 0 & \psi_4 & 0 & \psi_5 & 0 & \psi_6 & 0 & \psi_7 & 0 & \psi_8 & 0 \end{bmatrix} \tag{15}
$$

$$
\{\Delta\}^T = \begin{Bmatrix} u_1\ v_1\ u_2\ v_2\ u_3\ v_3\ u_4\ v_4 \\ u_5\ v_5\ u_6\ v_6\ u_7\ v_7\ u_8\ v_8 \end{Bmatrix} \tag{16}
$$

$$
\{du\}^T = \{\bar{u}_1\ \bar{v}_1\ \bar{u}_2\ \bar{v}_2\ \bar{u}_3\ \bar{v}_3\ \bar{u}_4\ \bar{v}_4\ \bar{u}_5\ \bar{v}_5\ \bar{u}_6\ \bar{v}_6\ \bar{u}_7\ \bar{v}_7\ \bar{u}_8\ \bar{v}_8\} \tag{17}
$$

Shape functions for an eight-node element are as follows

$$[\psi_1] = \left(X - \frac{a}{2}\right)\left(Y - \frac{b}{2}\right)\left(-\frac{1}{ab} - \frac{2X}{a^2 b} - \frac{2Y}{ab^2}\right)$$
$$[\psi_2] = \left(\frac{4}{ab^2}\right)\left(X - \frac{a}{2}\right)\left(Y + \frac{b}{2}\right)\left(Y - \frac{b}{2}\right)$$
$$[\psi_3] = \left(X - \frac{a}{2}\right)\left(Y + \frac{b}{2}\right)\left(\frac{1}{ab} + \frac{2X}{a^2 b} - \frac{2Y}{ab^2}\right)$$
$$[\psi_4] = \left(\frac{4}{ba^2}\right)\left(X - \frac{a}{2}\right)\left(X + \frac{a}{2}\right)\left(Y - \frac{b}{2}\right) \tag{18}$$
$$[\psi_5] = \left(\frac{4}{ba^2}\right)\left(X - \frac{a}{2}\right)\left(X + \frac{a}{2}\right)\left(Y + \frac{b}{2}\right)$$
$$[\psi_6] = \left(X + \frac{a}{2}\right)\left(Y - \frac{b}{2}\right)\left(\frac{1}{ab} - \frac{2X}{a^2 b} + \frac{2Y}{ab^2}\right)$$
$$[\psi_7] = \left(-\frac{4}{ab^2}\right)\left(X + \frac{a}{2}\right)\left(Y + \frac{b}{2}\right)\left(Y - \frac{b}{2}\right)$$
$$[\psi_8] = \left(X + \frac{a}{2}\right)\left(Y + \frac{b}{2}\right)\left(-\frac{1}{ab} + \frac{2X}{a^2 b} + \frac{2Y}{ab^2}\right)$$

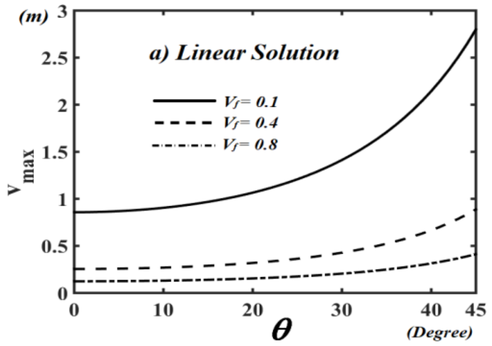
3. Numerical results

In the numerical examples, large deflections, namely geometrically nonlinear deflections of the cantilever fiber reinforced beam are calculated and presented for different the volume fraction and orientation angles of the fibre. Also, the difference between geometrically linear and nonlinear results are presented and discussed. Using the conventional assembly procedure for the finite elements, the tangent stiffness matrix and the residual vector are obtained from the element stiffness matrices and residual vectors in the total Lagrangian sense for finite element model of 2-D solid continuum. After that, the solution process outlined in the preceding section is used to obtain the solution for the problem of concern. In obtaining the numerical results, graphs and solution of the nonlinear finite element model, MATLAB program is used. Numerical calculations of the integrals seen in the rigidity matrices will be performed by using five-point Gauss rule.

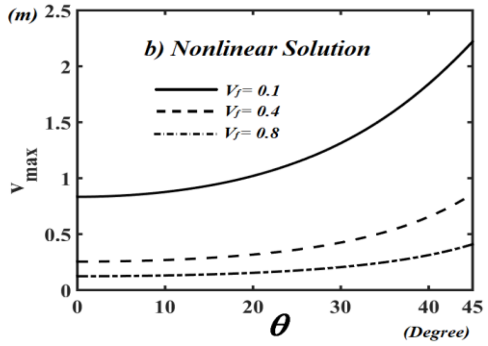
In the numerical examples, as the composite material of the beam, the graphite fibre-reinforced polyamide composite is selected. The material properties of the graphite fibre-reinforced polyamide composite are as follows (Krawczuk *et al*. 1997, Kısa 2004) with matrix and fibre properties values; E_m = 2.756 GPa, E_f = 275.6 GPa, G_m = 1.036 GPa, G_f = 114.8 GPa, ν_m = 0.33, ν_f = 0.2. The geometry properties of the beam are considered as follows: b = 0.3 m, h = 0.3 m and L = 5 m.

In Fig. 3, the relationship between the fiber orientation angles (θ) and the maximum vertical displacements (at the free end of the beam) is plotted for different values of the volume fraction of fiber V_f for the point load F = 200 kN. It is observed from Fig. 3 that with increasing the fiber orientation angles, the deflections of the composite beam increase significantly in both linear and nonlinear solutions. The equivalent Young's modulus and bending rigidity decrease according to the Eq. (9) with increasing the fiber orientation angles. As a result, the strength of the beam decreases and the deflections increase naturally. In smaller values of the volume fraction of fiber (V_f), the deflections fast increase with increasing the fiber orientation angle in contrast with higher values of the volume fraction of fiber. Also, it is seen from figure 3 that the difference among the results of V_f increase considerably with increasing the fiber orientation angles (θ). It shows that the fiber orientation angles is very effective for mechanical behavior of fiber reinforced composite beams and the results of the volume fraction of fiber.

In Fig. 4, the maximum vertical displacements versus load rising are presented for different the fiber orientation angles (θ) for V_f = 0.1 in both linear and nonlinear solutions. As seen from figure 4, the results of the linear solution are bigger than the results of the nonlinear solution in all values of the θ. With increase the fiber orientation angle, the displacements increase significantly. Also, it is seen from figure 4 that the difference between the linear and nonlinear solution increase with increase the fiber orientation angle. In higher values of the fiber orientation angle, the nonlinear displacements converge. The results of the figure 4 show that the fiber orientation angle is very effective for the difference between the linear and nonlinear solution of the fiber reinforced composite beams. This situation can be clearly seen in Fig. 5. In Fig. 5, the effects of the fiber orientation angle on the linear and nonlinear deflected shapes of the fiber reinforced composite beam are shown for F = 1000 kN and V_f = 0.5. The fiber orientation

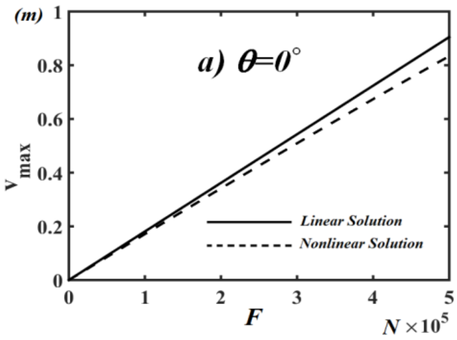


(a) For linear solution

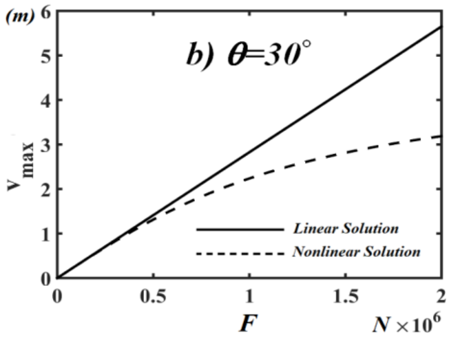


(b) For nonlinear solution

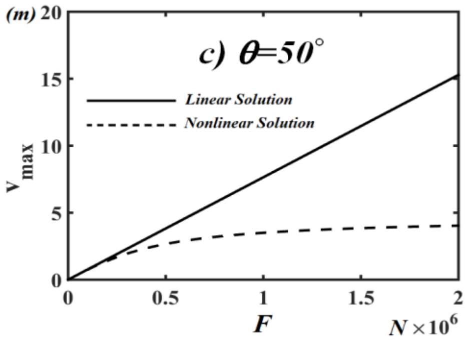
Fig. 3 The relationship between fiber orientation angles (θ) and maximum deflections ($v_{\max}$) for linear and nonlinear solution for different values of the volume fraction of fiber V_f



(a) For θ = 0°

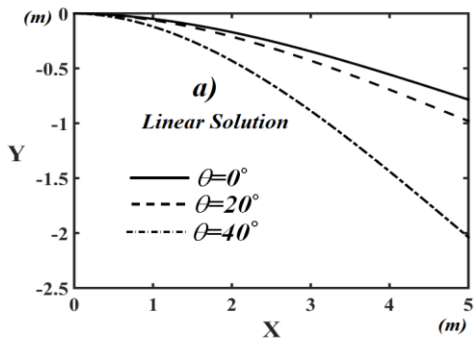


(b) For θ = 30°

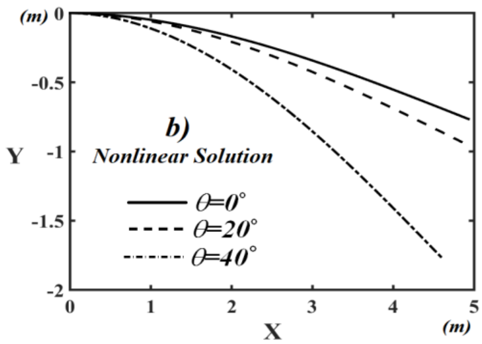


(c) For θ = 50°

Fig. 4 Load- maximum deflections curves for different values of the fiber orientation angles (θ) for linear solution and nonlinear solution



(a) For linear solution

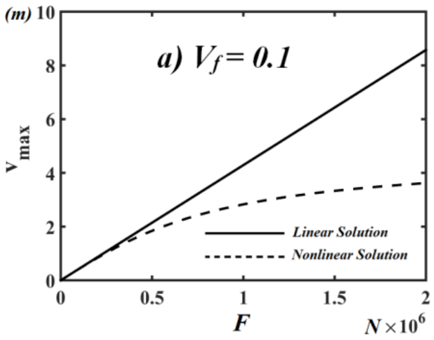


(b) For nonlinear solution

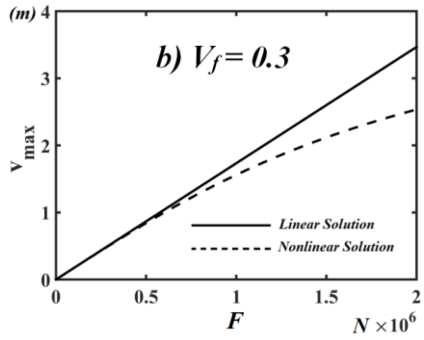
Fig. 5 The effect of the fiber orientation angles (θ) on the deflected shape of the composite beam for (a) Linear solution; (b) Nonlinear solution

angle changes the deflections of the fiber reinforced composite beam considerably. There is very big difference between the linear and nonlinear results, especially in higher values of θ. Hence, the nonlinear solution must be considered in the higher values of load and the fiber orientation angle for getting more realistic results.

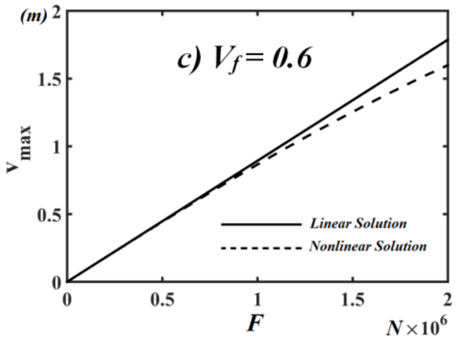
Fig. 6 shows the effects of the volume fraction of fiber V_f on the on the linear and nonlinear deflections of the fiber reinforced beam for θ = 40° in the load-displacement relation graphs. In order to more learn the effects of the volume fraction of fiber V_f, the deflected shapes are plotted for different values of V_f for θ = 20° and F = 100 kN in Fig. 7. It is observed from Fig. 6 that increasing the volume fraction of fiber V_f leads to decreasing the displacements. The difference between the linear and nonlinear solution decreases significantly with increase the the volume fraction of fiber. There is a significant difference between the geometrically linear case and nonlinear case in smaller values of the fiber orientation angle and higher values of the load. It shows that nonlinear theory must be considered in the large displacements problems for higher load values and smaller values of the fiber orientation angle. Otherwise, linear theory fails to satisfy large displacement problems. The effect of the volume fraction of fiber V_f can be seen clearly in Fig. 7. It is observed from Fig. 7 that the volume fractions of fiber play important role on the static response of the beam. With increasing the the volume fraction of fiber, the deflections increase considerably as the load is constant.



(a) For V_f = 0.1

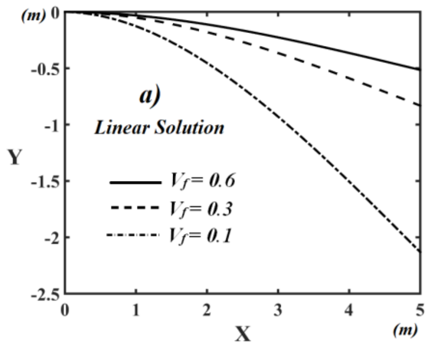


(b) For V_f = 0.3

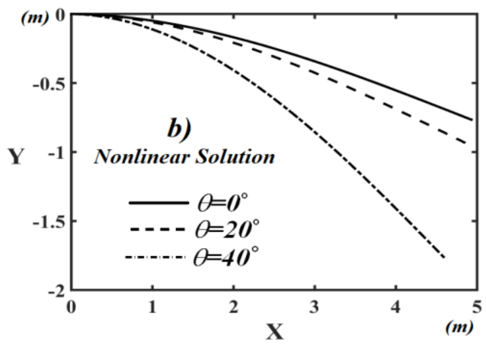


(c) For V_f = 0.6

Fig. 6 Load- maximum deflections curves for different values of the volume fraction of fiber V_f for linear solution and nonlinear solution



(a) For linear solution



(b) For nonlinear solution

Fig. 7 The effect of the volume fraction of fiber V_f on the deflected shape of the composite beam for (a) Linear solution; (b) Nonlinear solution

4. Conclusions

Large deflection static analysis of a cantilever fiber reinforced composite beam is investigated. In the solution of the problem, total Lagrangian finite element is used in the 2-D solid continuum model within Newton-Raphson iteration method. In the numerical results, the effects of the volume fraction and orientation angles of the fibre on the linear and nonlinear deflections of the fiber reinforced composite beam are investigated and discussed.

It is concluded from numerical results that the fibber orientation angles and the volume fraction of fiber play important role on the large deflection behaviour of the fiber reinforced composite beams. The fibber orientation angle is very effective to change the deflections and the nonlinear static responses. In higher load and fibber orientation angle values, there are significant differences of the analysis results for the linear and nonlinear. The volume fractions of fiber plays important role on the large deflection behavior of the fiber reinforced composite beams. With decrease values of the volume fractions of fiber, the displacements and the difference between of linear and nonlinear increase. In higher values of load and fibber orientation angle and in smaller values of the volume fractions of fiber, the nonlinear theory must be must be considered for large deflection problems of the fiber reinforced composite beams.

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