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Bending of FGM rectangular plates resting on non-uniform elastic foundations in thermal environment using an accurate theory

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Abstract. This article presents the bending analysis of FGM rectangular plates resting on non-uniform elastic foundations in thermal environment. Theoretical formulations are based on a recently developed refined shear deformation theory. The displacement field of the present theory is chosen based on nonlinear variations in the in-plane displacements through the thickness of the plate. The present theory satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without using shear correction factor. Unlike the conventional trigonometric shear deformation theory, the present refined shear deformation theory contains only four unknowns as against five in case of other shear deformation theories. The material properties of the functionally graded plates are assumed to vary continuously through the thickness, according to a simple power law distribution of the volume fraction of the constituents. The elastic foundation is modeled as non-uniform foundation. The results of the shear deformation theories are compared together. Numerical examples cover the effects of the gradient index, plate aspect ratio, side-to-thickness ratio and elastic foundation parameters on the thermo-mechanical behavior of functionally graded plates. Numerical results show that the present theory can archive accuracy comparable to the existing higher order shear deformation theories that contain more number of unknowns.

Keywords: refined plate theory; thermal environment; FGM; elastic foundations

1. Introduction

Functionally graded materials (FGMs) are considered as one of the modern generation of composite materials. The advantage of using advanced functionally graded materials is that they can survive in high thermal gradient environment, while maintaining their structural integrity. The concept of the FGM was proposed in 1980 by Japanese material scientists, as documented well in Ref. (Koizumi 1993). A typical FGM is made from a mixture of a ceramic and a metal. The history of the FGM as well as its applications can be found in the report by Jha *et al.* (2013). Having this structure, makes the FGM proper for some applications like reactor shells, turbines, building structures and many other engineering applications. Most of the studies on FGM have been restricted to thermal stress analysis, thermal buckling, fracture mechanics and optimization.

Plates supported by elastic foundations have been widely adopted by many researchers to model various engineering problems during the past decades. To describe the interactions of the plate and foundation as more appropriate as possible, scientists have proposed various kinds of foundation models, Ref. (Kerr 1964). The simplest model for the elastic foundation is the Winkler model (1867), which regards the foundation as a series of separated springs without coupling effects between each other, resulting in the disadvantage of discontinuous deflection on the interacted surface of the plate. This was later improved by Pasternak (1954) who took account of the interactions between the separated springs in the Winkler model by introducing a new dependent parameter. From then on, the Pasternak model was widely used to describe the mechanical behavior of structure-foundation interactions (Han and Liew 1997, Omurtag *et al.* 1997, Matsunaga 2000, Filipich and Rosales 2002, Zhou *et al.* 2004, Zenkour 2009, Benyoucef *et al.* 2010, Kiani *et al.* 2011, Thai and Choi 2011, Behravan Rad 2012, Behravan Rad and Shariyat 2013, Bouderba *et al.* 2013, Sobhy 2013, Khalfi *et al.* 2014, Liang *et al.* 2014, Yaghoobi and Fereidoon 2014, Yaghoobi *et al.* 2014, Ait Amar Meziane *et al.* 2014, Bakora and Tounsi 2015, Meksi *et al.* 2015, Tebboune *et al.* 2015, Hamidi *et al.* 2015, Bounouara *et al.* 2016, Ait Atmane *et al.* 2016, Yazid *et al.* 2018).

Various studies on FGM materials under thermo-mechanical environment are found in the literature. Praveen and Reddy (1998) carried out thermo-elastic analysis of FG plates. They investigated the static and dynamic response of the FGM plates by varying the volume fraction of the ceramic and metallic constituents using the simple power-law distribution. Reddy and Cheng (2001) studied the three-dimensional distribution of displacement and stresses of smart FG plates. Review on various investigations of FGM including thermo-mechanical studies are found in Birman and Byrd (2007). In general, the behavior of functionally graded (FG) plates/shells under mechanical and thermal loadings can be predicted using either three-dimensional (3D) elasticity theory or equivalent- single-layer (ESL)

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theories (Thai and Kim 2015). The simplest ESL model is the classical plate theory (CPT), also known as Kirchhoff theory (1850), which ignores both shear and normal deformation effects. The next theory in the hierarchy of ESL models is the first-order shear deformation theory (FSDT) developed by Mindlin (1951). The FSDT accounts for the shear deformation effect by the way of a linear variation of in-plane displacements through the thickness. A shear correction factor is therefore required (Youcef *et al*. 2018, Bellifa *et al*. 2016, Bouderba *et al*. 2016, Al-Basyouni *et al*. 2015). To avoid the use of the shear correction factor, higher-order shear deformation theories (HSDTs) were introduced (Mahi *et al*. 2015, Houari *et al*. 2016, Boukhari *et al*. 2016, Benadouda *et al*. 2017, Sekkal *et al*. 2017a, Zidi *et al*. 2017, Hachemi *et al*. 2017, Bellifa *et al*. 2017a, Belabed *et al*. 2018). In principle, the theories developed by this mean can be made as accurate as desired by including a sufficient number of terms in the series. Among the HSDTs, the third-order shear deformation theory (TSDT) of Reddy (1984) is the most widely used one due to its simplicity and accuracy. A review of shear deformation theories for isotropic and laminated plates was carried out by Ghugal and Shimpi (2002) and Khandan *et al*. (2012). A comprehensive review of various analytical and numerical models for predicting the bending, buckling and vibration responses of FG plates under mechanical and thermal loadings was recently carried out by Swaminathan *et al*. (2015). Using the concept of FGM many works have been also published to examine the mechanical responses of the functionally graded composite material reinforced with/without carbon nanotube (Mehar *et al*. 2016, 2017a, b, c, d, Mahapatra *et al*. 2017a, b, Bellifa *et al*. 2017b, Mehar and Panda 2016a, b, c and 2017a, b, c, Zemri *et al*. 2015, Kar and Panda 2015a, b and 2016a, b, Ahouel *et al*. 2016, Taibi *et al*. 2015, Belkorissat *et al*. 2015, Meradjah *et al*. 2015, Kar *et al*. 2015, Bakhadda *et al*. 2018, Kaci *et al*. 2018, Meksi *et al*. 2018, Zine *et al*. 2018). In addition, the thermal effect on composite structures is investigated recently by several researchers (Menasria *et al*. 2017, El-Haina *et al*. 2017, Fahsi *et al*. 2017, Chikh *et al*. 2017, Mouffoki *et al*. 2017, Khetir *et al*. 2017, Besseghier *et al*. 2017, Klouche *et al*. 2017, Bousahla *et al*. 2016, Beldjelili *et al*. 2016, Mahapatra *et al*. 2016, Attia *et al*. 2015, Zidi *et al*. 2014).

This work presents a bending analysis of power law functionally graded material (P-FGM) rectangular plates resting on non-uniform elastic foundations in thermal environment by using a simple and an efficient refined shear deformation theory (RSDT with two models: refined trigonometric shear deformation theory (RTSDT) and refined parabolic shear deformation theory (RPSDT)). The proposed theories contain fewer unknowns and equations of motion than the first order shear deformation theory, but satisfy the equilibrium conditions at the top and bottom surfaces of the plate without using any shear correction factors. The displacement field of the proposed theory is chosen based on nonlinear variation in the in-plane displacements through the thickness. The partition of the transverse displacement into the bending and shear components leads to a reduction in the number of equations of motion, and consequently, makes the new theory much more amenable to implementation. The material properties of the functionally graded plates are assumed to vary continuously through the thickness, according to a simple power law distribution of the volume fraction of the constituents. The elastic foundation is modeled as non-uniform foundation. The accuracy of obtained solutions is verified by comparing the present results with those predicted by solutions available in the literature.

2. Theoretical formulation

Consider a functionally graded plate of thickness h, side length a in the x-direction, and b in the y-direction resting on nonlinear elastic foundations as shown in Fig. 1.

2.1 Basic assumptions and Kinematics

The assumptions of the present theory are as follows:

- The transverse displacements are partitioned into bending and shear components.
- The in-plane displacement is partitioned into extension, bending, and shear components.
- The bending parts of the in-plane displacements are similar to those given by classical plate theory (CPT).
- The shear parts of the in-plane displacements give rise to the nonlinear variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate.

Based on the assumptions made in the preceding section, the displacement field can be obtained

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} - \psi(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} - \psi(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{1}$$

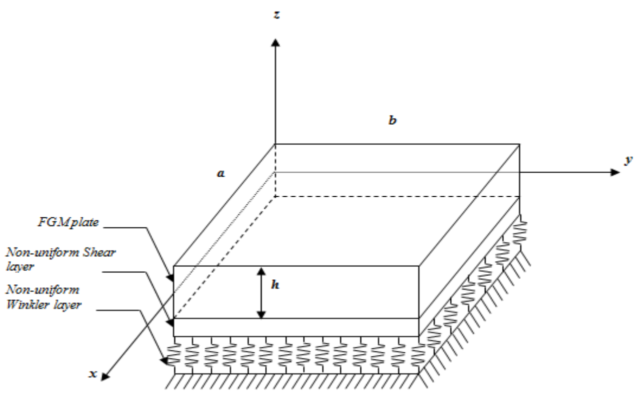
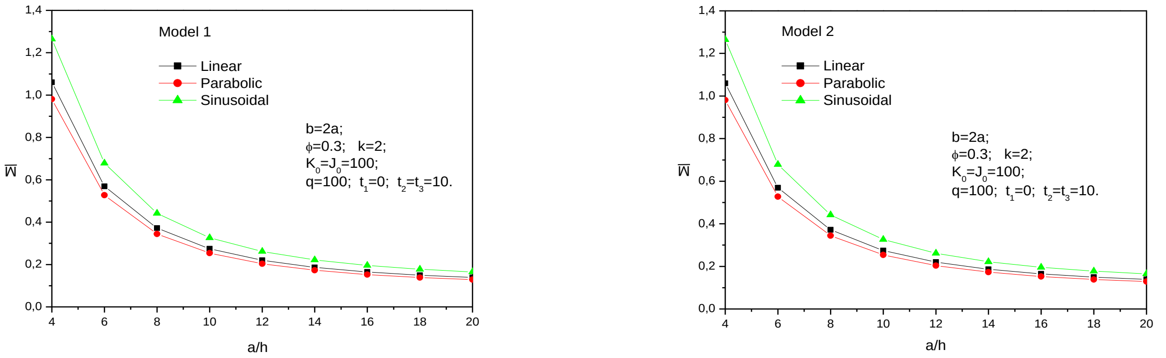


Fig. 1 Coordinate system and geometry for rectangular FG plates on non-uniform elastic foundation

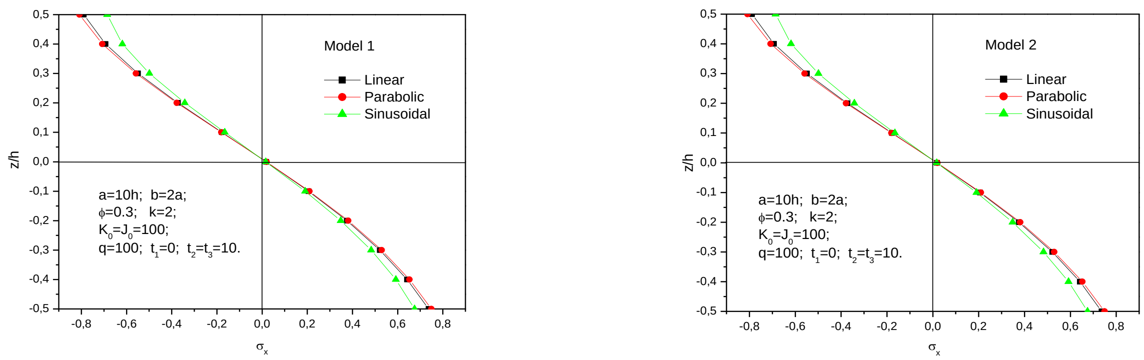


(a) Model 1 (b) Model 2

Fig. 2 ANN model effect of type elastic foundation on the dimensionless center deflection $(\bar{w})$ of a rectangular P-FG plate (k = 2) for different side-to-thickness ratio a/h

Table 1 Different shear shape strain functions

Model	$\psi(z)$ function
Ambartsumian (1958)	$\psi(z)=\frac{z}{2}\left(\frac{h^2}{4}-\frac{z^2}{3}\right)$
Kaczkowski (1968), Panc (1975), Reissner (1975)	$\psi(z)=\frac{5z}{4}\left(1-\frac{4z^2}{3h^2}\right)$
Levinson (1980), Murthy (1981) and Reddy (1984)	$\psi(z)=z\left(1-\frac{4z^2}{3h^2}\right)$
Touratier (1991)	$\psi(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$
Soldatos (1992)	$\psi(z)=h\sinh\left(\frac{z}{h}\right)-z\cosh\left(\frac{1}{2}\right)$
Karama *et al*. (2003) and Aydogdu (2009)	$\psi(z)=ze-2\left(\frac{z}{h}\right)^2=z\alpha 2\left(\frac{z}{h}\right)^{-2}/\ln\alpha, \quad \forall\alpha\succ 0.$
Present « model 1 » Tounsi *et al*. (2013)	$\psi(z)=z-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$
Present « model 2 » Ait Yahia *et al*. (2015)	$\psi(z)=\left(\frac{4z^3}{3h^2}\right)$



(a) Model 1 (b) Model 2

Fig. 3 Variation of dimensionless axial stress $(\bar{\sigma}_x)$ through-the-thickness of a rectangular P-FG plate (k = 2) for different type of elastic foundations where u, v, $w$are displacements in the x, y, z directions, u_0, v_0 and w_b, w_s are mid-plane displacements and $\psi(z)$ is a shape function that represents the distribution of the transverse shear strain and stress through the thickness, as presented in Table 1.

The kinematic relations can be obtained as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+\psi(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\xi(z)\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{2}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},$$
$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix} \tag{3a}$$

and

$$\xi(z)=1-\frac{d\psi(z)}{dz} \tag{3b}$$

2.2 Constitutive equations

The plate is subjected to a sinusoidally distributed load $Q(x, y)$ and a temperature field $T(x, y, z)$. The material properties P of the FG plate, such as Young's modulus E, Poisson's ratio v, and thermal expansion coefficient α are given according the formulation

$$P(z)=P_M+\left(P_C-P_M\right)\left(\frac{1}{2}+\frac{z}{h}\right)^k \tag{4}$$

where P_C and P_M are the corresponding properties of the ceramic and metal, respectively, and k is the volume fraction exponent which takes values greater than or equal to zero.

The linear constitutive relations are

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0\\ C_{12}&C_{22}&0\\ 0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x-\alpha\,\Delta T\\ \varepsilon_y-\alpha\,\Delta T\\ \gamma_{xy}\end{Bmatrix}\quad \text{and}\quad \begin{Bmatrix}\tau_{yz}\\ \tau_{zx}\end{Bmatrix}=\begin{bmatrix}C_{44}&0\\ 0&C_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} \tag{5}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. Using the material properties defined in Eq. (4), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11}=C_{22}=\frac{E(z)}{1-\nu^2}, \tag{6a}$$

$$C_{12}=\frac{\nu\,E(z)}{1-\nu^2}, \tag{6b}$$

$$C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)}, \tag{6c}$$

where $\Delta T = T - T_0$ in which T_0 is the reference temperature.

The applied temperature distribution $T(x, y, z)$ through the thickness are assumed, respectively, to be

$$T(x,y,z)=T_1(x,y)+\frac{z}{h}T_2(x,y)+\frac{1}{\pi}\sin\left(\frac{\pi z}{h}\right)T_3(x,y), \tag{7}$$

2.3 Governing equations

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The principle of virtual work in the present case yields (Ait Atmane *et al*. 2015)

$$\int_{-h/2}^{h/2}\int_{\Omega}\begin{bmatrix}\sigma_x\delta\,\varepsilon_x+\sigma_y\delta\,\varepsilon_y+\tau_{xy}\delta\,\gamma_{xy}+\tau_{yz}\delta\,\gamma_{yz}\\ +\tau_{xz}\delta\,\gamma_{xz}\end{bmatrix}d\Omega\,dz - \int_{\Omega}(Q-f_e)\delta\,w d\Omega=0 \tag{8}$$

where Ω is the top surface, and f_e is the density of reaction force of foundation. For the Pasternak foundation model (see Behravan Rad 2012, Attia *et al*. 2018)

$$f_e=K_1(x)w-K_2(x)\nabla^2 w \tag{9a}$$

and

$$K_1(x)=\frac{K_0h^3}{a^4}\begin{cases}1-\phi\dfrac{x}{a} & \textit{Linear},\\ 1-\phi(\dfrac{x}{a})^2 & \textit{Parabolic},\ ,\\ 1-\phi\sin(\pi\dfrac{x}{a}) & \textit{Sinusoidal},\end{cases}\qquad K_2(x)=\frac{J_0h^3}{a^2}\begin{cases}1-\phi\dfrac{x}{a} & \textit{Linear},\\ 1-\phi(\dfrac{x}{a})^2 & \textit{Parabolic},\\ 1-\phi\sin(\pi\dfrac{x}{a}) & \textit{Sinusoidal},\end{cases} \tag{9b}$$

where K_0, J_0 are a constant and ϕ is a varied parameter. K_1 is the Winkler foundation stiffness and K_2 is the effect of the shear interactions of the vertical elements, and ∇^2 is the

Laplace operator in x and y. Note that, if $\phi = 0$, the elastic foundation becomes Pasternak foundation and if the shear layer foundation stiffness is neglected, the Pasternak foundation becomes the Winkler foundation, the foundation is homogeneous and isotropic.

Substituting Eqs. (2) and (5) into Eq. (8) and integrating through the thickness of the plate, Eq. (8) can be rewritten as

$$\int_{\Omega} \begin{bmatrix} N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \varepsilon_{xy}^0 \\ + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s \\ + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^s \end{bmatrix} d\Omega - \int_{\Omega} (Q - f_e)(\delta w_b + \delta w_s) d\Omega \tag{10}$$

The stress resultants N, M, and S are defined by

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} 1 \\ z \\ \psi(z) \end{Bmatrix} dz, \tag{11a}$$

and

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) \xi(z) dz. \tag{11b}$$

Substituting Eq. (5) into Eq. (11) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, S = A^s \gamma \tag{12}$$

where

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t, \quad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t, \tag{13a}$$

$$N^T = \left\{N_x^T, N_y^T, 0\right\}^t, M^{bT} = \left\{M_x^{bT}, M_y^{bT}, 0\right\}^t, \quad M^{sT} = \left\{M_x^{sT}, M_y^{sT}, 0\right\}^t, \tag{13b}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{13c}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{13d}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^S & D_{22}^S & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{13e}$$

$$S = \left\{S_{yz}^s, S_{xz}^s\right\}^t, \gamma = \left\{\gamma_{yz}, \gamma_{xz}\right\}^t, A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \tag{13f}$$

where A_{ij}, B_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11} \left(1, z, z^2, \psi(z), z\,\psi(z), \psi^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{14a}$$

and

$$\begin{pmatrix} A_{22}, B_{22}, D_{22}, \\ B_{22}^s, D_{22}^s, H_{22}^s \end{pmatrix} = \begin{pmatrix} A_{11}, B_{11}, D_{11}, \\ B_{11}^s, D_{11}^s, H_{11}^s \end{pmatrix}, C_{11} = \frac{E(z)}{1-\nu^2} \tag{14b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} \frac{E(z)}{2(1+\nu)} [\xi(z)]^2 dz, \tag{14c}$$

The stress and moment resultants, $N_x^T = N_y^T$, $M_x^{bT} = M_y^{bT}$, $M_x^{sT} = M_y^{sT}$ due to thermal loading are defined respectively by

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_x^{sT} \end{Bmatrix} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu} \alpha(z) T \begin{Bmatrix} 1 \\ z \\ \psi(z) \end{Bmatrix} dz, \tag{15}$$

The governing equations of equilibrium can be derived from Eq. (10) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_b and δw_s zero separately.

Thus one can obtain the equilibrium equations associated with the present shear deformation theory

$$\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \tag{16a}$$

$$\delta w_b: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - f_e + Q = 0, \quad \delta w_s: \ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - f_e + Q = 0 \tag{16b}$$

Substituting Eq. (12) into Eq. (16), we obtain the following equation

$$\begin{aligned}&A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + (A_{12} + A_{66})d_{12}v_0\\&- B_{11}d_{111}w_b - (B_{12} + 2B_{66})d_{122}w_b\\&- (B^s_{12} + 2B^s_{66})d_{122}w_s - B^s_{11}d_{111}w_s = f_1,\end{aligned} \tag{17a}$$

$$\begin{aligned}&A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + (A_{12} + A_{66})d_{12}u_0 - B_{22}d_{222}w_b\\&- (B_{12} + 2B_{66})d_{112}w_b - (B^s_{12} + 2B^s_{66})d_{112}w_s\\&- B^s_{22}d_{222}w_s = f_2,\end{aligned} \tag{17b}$$

$$\begin{aligned}&B_{11}d_{111}u_0 + (B_{12} + 2B_{66})d_{122}u_0\\&+ (B_{12} + 2B_{66})d_{112}v_0 + B_{22}d_{222}v_0 - D_{11}d_{1111}w_b\\&- 2(D_{12} + 2D_{66})d_{1122}w_b - D_{22}d_{2222}w_b\\&- D^s_{11}d_{1111}w_s - 2(D^s_{12} + 2D^s_{66})d_{1122}w_s\\&- D^s_{22}d_{2222}w_s = f_3\end{aligned} \tag{17c}$$

$$\begin{aligned}&B^s_{11}d_{111}u_0 + (B^s_{12} + 2B^s_{66})d_{122}u_0\\&+ (B^s_{12} + 2B^s_{66})d_{112}v_0 + B^s_{22}d_{222}v_0 - D^s_{11}d_{1111}w_b\\&- 2(D^s_{12} + 2D^s_{66})d_{1122}w_b - D^s_{22}d_{2222}w_b\\&- H^s_{11}d_{1111}w_s - 2(H^s_{12} + 2H^s_{66})d_{1122}w_s\\&- H^s_{22}d_{2222}w_s + A^s_{55}d_{11}w_s + A^s_{44}d_{22}w_s = f_4\end{aligned} \tag{17d}$$

where $\{f\} = \{f_1, f_2, f_3, f_4\}^t$ is a generalized force vector, d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned}&d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\\&d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, d_i = \frac{\partial}{\partial x_i}, (i, j, l, m = 1,2).\end{aligned} \tag{18}$$

The components of the generalized force vector $\{f\}$are given by

$$f_1 = \frac{\partial N^T_x}{\partial x}, \qquad f_2 = \frac{\partial N^T_y}{\partial y}, \tag{19a}$$

$$\begin{aligned}&f_3 = f_e + Q - \frac{\partial^2 M^{bT}_x}{\partial x^2} - \frac{\partial^2 M^{bT}_y}{\partial y^2},\\&f_4 = f_e + Q - \frac{\partial^2 M^{sT}_x}{\partial x^2} - \frac{\partial^2 M^{sT}_y}{\partial y^2}\end{aligned} \tag{19b}$$

3. Exact solutions for FG plates

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eq. (17) for a simply supported FG plate. To solve this problem, Navier assumed that the transverse mechanical and temperature loads, Q and T_i in the form of a in the double Fourier series as

$$\begin{Bmatrix} Q \\ T_i \end{Bmatrix} = \begin{Bmatrix} Q_0 \\ t_i \end{Bmatrix} \sin(\lambda x)\sin(\mu y), (i = 1, 2, 3) \tag{20}$$

where $\lambda = \pi / a$, $\mu = \pi / b$, Q_0 and t_i are constants.

Following the Navier solution procedure, we assume the following solution form for u_0, v_0, w_b and w_s that satisfies the boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \begin{Bmatrix} U\cos(\lambda x)\sin(\mu y) \\ V\sin(\lambda x)\cos(\mu y) \\ W_b\sin(\lambda x)\sin(\mu y) \\ W_s\sin(\lambda x)\sin(\mu y) \end{Bmatrix}, \tag{21}$$

where U, V, W_b, and W_s are arbitrary parameters to be determined subjected to the condition that the solution in Eq. (21) satisfies governing Eqs. (17). One obtains the following operator equation

$$[S]\{\Delta\} = \{F\}, \tag{22}$$

where $\{\Delta\} = \{U, V, W_b, W_s\}^t$ and $[S]$ is the symmetric matrix given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \tag{23}$$

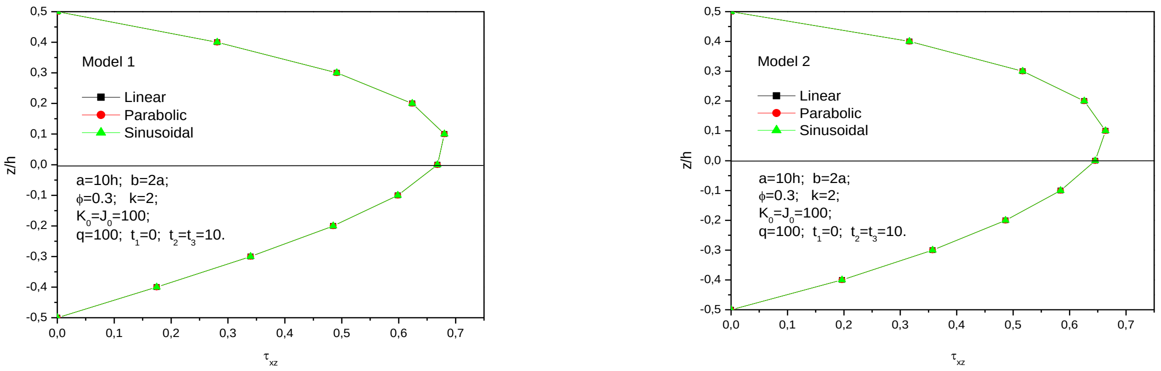
in which

$$\begin{aligned}S_{11} &= -(A_{11}\lambda^2 + A_{66}\mu^2),\\S_{12} &= -\lambda\,\mu\,(A_{12} + A_{66}),\\S_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2],\\S_{14} &= \lambda\,[B^s_{11}\lambda^2 + (B^s_{12} + 2B^s_{66})\,\mu^2],\end{aligned} \tag{24a}$$

$$\begin{aligned}S_{22} &= -(A_{66}\lambda^2 + A_{22}\mu^2),\\S_{23} &= \mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2],\\S_{24} &= \mu\,[(B^s_{12} + 2B^s_{66})\,\lambda^2 + B^s_{22}\mu^2],\\S_{33} &= -\begin{pmatrix} D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 \\ + K_1 + K_2\lambda^2 + K_2\mu^2 \end{pmatrix},\\S_{34} &= -\begin{pmatrix} D^s_{11}\lambda^4 + 2(D^s_{12} + 2D^s_{66})\lambda^2\,\mu^2 \\ + D^s_{22}\,\mu^4 + K_1 + K_2\lambda^2 + K_2\mu^2 \end{pmatrix},\\S_{44} &= -\begin{pmatrix} H^s_{11}\lambda^4 + 2(H^s_{11} + 2H^s_{66})\lambda^2\mu^2 + H^s_{22}\mu^4 \\ + A^s_{55}\lambda^2 + A^s_{44}\mu^2 + K_1 + K_2\lambda^2 + K_2\mu^2 \end{pmatrix}\end{aligned} \tag{24b}$$

The components of the generalized force vector $\{F\} = \{F_1, F_2, F_3, F_4\}^t$ are given by

$$\begin{aligned}F_1 &= \lambda\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right),\\F_2 &= \mu\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right),\\F_3 &= -Q_0 - h(\lambda^2 + \mu^2)\left(B^T t_1 + D^T t_2 + {}^aD^T t_3\right),\\F_4 &= -Q_0 - h(\lambda^2 + \mu^2)\left({}^sB^T t_1 + {}^sD^T t_2 + {}^sF^T t_3\right),\end{aligned} \tag{25}$$



(a) Model 1 (b) Model 2

Fig. 4 Variation of dimensionless shear stress $(\bar{\tau}_{xz})$ through-the-thickness of a rectangular P-FG plate (k = 2) for different types of elastic foundation

where

$$\left\{A^T, B^T, D^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\left\{1, \bar{z}, \bar{z}^2\right\} dz, \tag{26a}$$

$$\left\{{}^aB^T, {}^aD^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\bar{\varsigma}(z)\left\{1, \bar{z}\right\} dz, \tag{26b}$$

$$\left\{{}^sB^T, {}^sD^T, {}^sF^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\bar{\psi}(z)\left\{1, \bar{z}, \bar{\varsigma}(z)\right\} dz \tag{26c}$$

in which

$$\bar{z} = z/h, \bar{\psi}(z) = \psi(z)/h \ and \ \bar{\varsigma}(z) = \frac{1}{\pi}\sin\left(\frac{\pi z}{h}\right)$$

4. Results and discussion

In this section, numerical examples are presented and discussed for verifying the accuracy of the present theory in bending analysis of P-FGM rectangular plates resting on non-uniform elastic foundations in thermal environment. Comparisons are made with various plate theories available in the literature. The description of various displacement models is given in Table 2.

The P-FGM plate is taken to be made of Titanium and Zirconia with the following material properties "Table 3":

Table 2 Displacement models

Model	Theory	Unknown functions
CPT	Classical plate theory	3
FSDT	First-order shear deformation theory (Reissner 1945 and Mindlin 1951)	5
PSDT	Parabolic shear deformation theory (Reddy 2000)	5
TSDT	Trigonometric shear deformation theory (Zenkour 2009)	5
Present	Present refined shear deformation theory	4

Table 3 Material properties used in the FG plate (see Tounsi *et al* 2013)

Properties	Metal Titanium (Ti-6Al-4V)	Ceramic Zirconia (ZrO2)
E (GPa)	66.2	117
$\alpha\left(10^{-6}/C^{\circ}\right)$	10.3	7.11
ν	1/3	1/3

The reference temperature is taken by T_0 = 25℃ (room temperature). Numerical results are presented in terms of non-dimensional stresses and deflection. The various non-dimensional parameters used are

- Center deflection

$$\bar{w} = \frac{10^2 D}{a^4 q_0} w\left(\frac{a}{2}, \frac{b}{2}\right),$$

- Axial stress

$$\bar{\sigma}_x = \frac{1}{10^2 q_0}\sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right),$$

- Longitudinal shear stress

$$\bar{\tau}_{xy} = \frac{1}{10 q_0}\tau_{xy}\left(0, 0, \frac{-h}{3}\right),$$

- Transversal shear stress

$$\bar{\tau}_{xz} = -\frac{1}{10 q_0}\tau_{xz}\left(0, \frac{b}{2}, 0\right)$$

- Thickness coordinate

$$\bar{z} = z/h, \qquad K_0 = \frac{a^4 K_1}{D};$$

$$J_0 = \frac{a^2 K_2}{D} = \frac{b^2 K_2}{D}, \qquad D = \frac{h^3 E_C}{12\left(1-\nu^2\right)}$$

Table 4 Effect of the volume fraction exponent and linear elastic foundation parameters on the dimensionless and stresses of an P-FG rectangular plate (a = 10h, b = 2a, q_0 = 100, t = 0, ϕ = 0.3)

k	K_0	J = 0	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
0	0	0	Model 1	0.68131	0.42424	0.86240	-0.39400
			Model 2	0.68135	0.42408	0.86251	-0.38181
			PSDT	0.68134	0.42408	0.86253	-0.38180
			TSDT	0.68131	0.42424	0.86240	-0.39400
			FSDT	0.68135	0.42148	0.86459	-0.30558
			CPT	0.65704	0.42148	0.86459	–
	100	0	Model 1	0.43145	0.26782	0.51296	-0.23434
			Model 2	0.43147	0.26771	0.51299	-0.22708
			PSDT	0.43148	0.26769	0.51299	-0.22708
			TSDT	0.43145	0.26779	0.51296	-0.23434
			FSDT	0.43147	0.26602	0.51422	-0.18175
			CPT	0.42159	0.26964	0.52179	–
	0	100	Model 1	0.09633	0.05895	0.10590	-0.04839
			Model 2	0.09632	0.05892	0.10590	-0.04688
			PSDT	0.09632	0.05889	0.10590	-0.04687
			TSDT	0.09632	0.05892	0.10589	-0.04837
			FSDT	0.09632	0.05851	0.10615	-0.03751
			CPT	0.09583	0.06042	0.10958	–
	100	100	Model 1	0.08903	0.05446	0.09772	-0.04464
			Model 2	0.08904	0.05444	0.09772	-0.04325
			PSDT	0.08904	0.05441	0.09772	-0.04325
			TSDT	0.08904	0.05444	0.09772	-0.04464
			FSDT	0.08903	0.05406	0.09795	-0.03462
			CPT	0.08860	0.05584	0.10116	–
1	100	100	Model 1	0.09187	0.05092	0.07305	-0.03503
			Model 2	0.09187	0.05090	0.07306	-0.03393
			PSDT	0.09187	0.05086	0.07306	-0.03393
			TSDT	0.09187	0.05089	0.07305	-0.03502
			FSDT	0.09187	0.05052	0.07321	-0.02716
			Model 1	0.09187	0.05092	0.07305	-0.03503
5	100	100	Model 1	0.09245	0.05060	0.06718	-0.03220
			Model 2	0.09245	0.05057	0.06718	-0.03117
			PSDT	0.09245	0.05055	0.06719	-0.03117
			TSDT	0.09246	0.05057	0.06718	-0.03221
			FSDT	0.09245	0.05022	0.06738	-0.02434
5	100	100	Model 1	0.09299	0.05195	0.06412	-0.02991
			Model 2	0.09298	0.05192	0.06413	-0.02891
			PSDT	0.09299	0.05189	0.06413	-0.02892
			TSDT	0.09299	0.05191	0.06412	-0.02992
			FSDT	0.09298	0.05160	0.06440	-0.02205
∞	100	100	Model 1	0.09439	0.05761	0.05815	-0.02657
			Model 2	0.09439	0.05758	0.05815	-0.02575
			PSDT	0.09439	0.05756	0.05815	-0.02574
			TSDT	0.09439	0.05758	0.05814	-0.02656
			FSDT	0.09439	0.05719	0.05829	-0.02060

Numerical results are tabulated in Tables 4-6 and plotted in Figs 2-4 using the present refined shear deformation theory (RSDT "model 1 and model 2"). We note that the shear correction factor is taken k = 5/6 in FSDT.

The correlation between the present refined shear deformation theory (RSDT) and different higher-order and first-order shear deformation theories and classical plate theory is illustrated in Tables 4-5.

These Tables give also the effects of the volume fraction exponent ratio k and type of elastic foundation parameters on the dimensionless deflection and stresses of FG rectangular plate. Table 4 gives the effects of the volume fraction exponent ratio k and linear elastic foundation parameters on the dimensionless displacements and stresses of P-FG rectangular plate subjected to a mechanical load. It can be shown that the deflection and stresses are decreasing with the existence of the elastic foundations. The inclusion of the Winkler foundation parameter gives results more than those with the inclusion of Pasternak foundation parameters. As the volume fraction exponent increases for P-FG plates, the deflection will increase. The stresses are also sensitive to the variation of k.

Tables 5 and 6 present similar results as those given in Table 4 including the effect of the temperature field. The obtained results are compared with those predicted by FSDT, TSDT and PSDT. An excellent agreement is obtained between the present theory and TSDT (Zenkour 2009) and PSDT (Reddy 2000) for all values of power law index k and with or without the presence of the elastic foundation. It is important to observe that the stresses for a

Table 5 Effect of the volume fraction exponent and linear elastic foundation parameters on the dimensionless and stresses of an P-FG rectangular plate models (a = 10h, b = 2a, q_0 = 100, t = 0, ϕ = 0.3)

k	K_0	$J = 0$	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
0	0	0	Model 1	2.1762	0.19592	-0.49747	-0.38826
			Model 2	2.1762	0.19572	-0.49735	-0.37679
			PSDT	2.1982	0.19106	-0.46854	-0.37714
			TSDT	2.1762	0.19592	-0.49742	-0.38826
	100	0	Model 1	1.3781	-0.30378	-1.6140	0.12170
			Model 2	1.3783	-0.30358	-1.6136	0.11741
			PSDT	1.3921	-0.31344	-1.5960	0.12204
			TSDT	1.3781	-0.30378	-1.6139	0.12172
	0	100	Model 1	0.30772	-0.97078	-2.9140	0.71573
			Model 2	0.30769	-0.97058	-2.9140	0.69301
			PSDT	0.31080	-0.98704	-2.9098	0.70349
			TSDT	0.30770	-0.97098	-2.9138	0.71573
	100	100	Model 1	0.28438	-0.98518	-2.9399	0.72762
			Model 2	0.28443	-0.98488	-2.9400	0.70459
			PSDT	0.28729	-1.0016	-2.9362	0.71507
			TSDT	0.28443	-0.98538	-2.9397	0.72767
1	100	100	Model 1	0.23888	-0.77448	-2.3876	0.54619
			Model 2	0.23889	-0.77421	-2.3876	0.52864
			PSDT	0.24161	-0.79081	-2.3854	0.53861
			TSDT	0.23889	-0.77461	-2.3872	0.54615
5	100	100	Model 1	0.23604	-0.68577	-2.3165	0.52365
			Model 2	0.23604	-0.68551	-2.3167	0.50576
			PSDT	0.23872	-0.70196	-2.3152	0.51525
			TSDT	0.23605	-0.68590	-2.3167	0.52354
5	100	100	Model 1	0.24427	-0.59074	-2.4012	0.54892
			Model 2	0.24425	-0.59060	-2.4015	0.52928
			PSDT	0.24692	-0.60679	-2.4000	0.53858
			TSDT	0.24426	-0.59086	-2.4010	0.54889
∞	100	100	Model 1	0.26417	-0.50004	-2.5031	0.63896
			Model 2	0.26413	-0.49952	-2.5031	0.61868
			PSDT	0.26667	-0.51623	-2.5006	0.62782
			TSDT	0.26417	-0.50018	-2.5029	0.63900

Table 6 Effects of side-to-thickness ratio and type of elastic foundation parameters on the dimensionless deflection of an P-FG rectangular plate (q_0 = 100, t = 10, b = 2a, ϕ = 0.3)

k	K_0	J = 0	Theory	Elastic foundation	a/h = 5	10	20	50
0	0	0	Model 1	Parabolic	6.72940	2.17620	1.03690	0.71780
				Sinusoidal	6.72940	2.17620	1.03690	0.71780
			Model 2	Parabolic	6.73010	2.17620	1.03690	0.71780
				Sinusoidal	6.73010	2.17620	1.03690	0.71780
	100	100	Model 1	Parabolic	0.74686	0.26413	0.12889	0.08982
				Sinusoidal	0.95261	0.33594	0.16377	0.11411
			Model 2	Parabolic	0.74663	0.26416	0.12889	0.08982
				Sinusoidal	0.95267	0.33594	0.16377	0.11410
1	0	0	Model 1	Parabolic	6.69020	2.32030	1.22610	0.91966
				Sinusoidal	6.69020	2.32030	1.22610	0.91966
			Model 2	Parabolic	6.69130	2.32030	1.22610	0.91966
				Sinusoidal	6.69130	2.32030	1.22610	0.91966
	100	100	Model 1	Parabolic	0.58343	0.22139	0.11982	0.09048
				Sinusoidal	0.74998	0.28384	0.15352	0.11590
			Model 2	Parabolic	0.58345	0.22138	0.11982	0.09048
				Sinusoidal	0.74998	0.28384	0.15352	0.11590
3	0	0	Model 1	Parabolic	7.22810	2.51560	1.33420	1.00310
				Sinusoidal	7.22810	2.51560	1.33420	1.00310
			Model 2	Parabolic	7.23010	2.51570	1.33420	1.00310
				Sinusoidal	7.23010	2.51570	1.33420	1.00310
	100	100	Model 1	Parabolic	0.57515	0.22098	0.12032	0.09116
				Sinusoidal	0.74096	0.28401	0.15454	0.11704
			Model 2	Parabolic	0.57504	0.22096	0.12032	0.09116
				Sinusoidal	0.74089	0.28394	0.15453	0.11705
5	0	0	Model 1	Parabolic	7.68080	2.65230	1.39090	1.03740
				Sinusoidal	7.68080	2.65230	1.39090	1.03740
			Model 2	Parabolic	7.68300	2.65250	1.39090	1.03740
				Sinusoidal	7.68300	2.65250	1.39090	1.03740
	100	100	Model 1	Parabolic	0.59136	0.22618	0.12189	0.09161
				Sinusoidal	0.76235	0.29086	0.15663	0.11772
			Model 2	Parabolic	0.59154	0.22618	0.12189	0.09161
				Sinusoidal	0.76271	0.29090	0.15662	0.11772
∞	0	0	Model 1	Parabolic	9.98890	3.36980	1.71350	1.24960
				Sinusoidal	9.98890	3.36980	1.71350	1.24960
			Model 2	Parabolic	9.99000	3.36990	1.71350	1.24960
				Sinusoidal	9.99000	3.36990	1.71350	1.24960
	100	100	Model 1	Parabolic	0.65904	0.24430	0.12738	0.09355
				Sinusoidal	0.85277	0.31548	0.16439	0.12072
			Model 2	Parabolic	0.65877	0.24432	0.12738	0.09355
				Sinusoidal	0.85240	0.31546	0.16438	0.12072

fully ceramic plate are not the same as that for a fully metal plate with elastic foundations. This is because the plate here is affected with the inclusion of the temperature field.

From results presented in Tables 4 to 5, it should be noted that the unknown function in present theory is four, while the unknown function in FSDT, PSDT and TSDT is five. It can be concluded that the present theory is not only accurate but also simple in bending analysis of P-FGM rectangular plates resting on non-uniform elastic foundations in thermal environment.

Table 6 gives the effects of side-to-thickness ratio and the type of elastic foundation parameters on the dimensionless deflection of P-FG rectangular plate under thermo-mechanical loads using the present refined shear deformation theory (RSDT “model 1 and model 2”). It is clear that the center deflection decreases as the side-to-thickness ratio a/h increases. In addition, all displacements are decreasing with the existence of the elastic foundations. The inclusion of the Winkler foundation parameter gives results more than those with the inclusion of Pasternak foundation parameters.

The effect of foundation stiffness and type of elastic foundation and side-to-thickness ratio on the dimensionless deflection of P-FG rectangular plate (k = 2) is shown in Fig. 2. It can be seen that the increase of side-to-thickness ratio a/h leads to a decrease of the center deflection of the P-FG plate. The axial stress, $\bar{\sigma}_x$, are plotted in Fig. 3. It can be seen that the maximum compressive stresses occur at a point near the top surface and the maximum tensile stresses occur, of course, at a point near the bottom surface of the P-FG plate.

Finally, Fig. 4 depict the through-the-thickness distributions of the shear stress $\bar{\tau}_{xz}$ in the FG rectangular plates under the thermal loads (q_0 = 100, t_1 = 0 and t_2 = t_3 = 10). The volume fraction exponent of the P-FG plate is taken as k = 2. The maximum values of $\bar{\tau}_{xz}$ occur at $\bar{z} \cong$ 0.1 of the P-FG plate, not at the plate center as in the homogeneous case.

5. Conclusions

A refined shear deformation plate theory is used in bending analysis of FGM rectangular plates resting on non-uniform elastic foundations in thermal environment. This theory contains only four unknown functions and then four governing equations are only obtained. Moreover, it does not need a shear correction factor. The theory satisfies the zero traction boundary conditions at the plate’s surfaces. The accuracy of the suggested theory (model 1:RTSDT,model 2: RPSDT) is verified by making some comparisons between the results obtained by the CPT, FSDT, TSDT and PSDT theories and the exact published ones. The results of the shear deformation theories are compared together.

- The gradients in material properties play an important role in determining the response of the FG plates.
- However, the inclusion of the foundation parameters may give displacements and stresses with higher magnitudes.
- Based on Navier’s type solution, the equations of motion are solved analytically.
- The mixture of the ceramic and metal with continuously varying volume fraction can eliminate interface problems of sandwich plates and thus the stresses distributions are smooth.
- All comparison studies demonstrated that the deflections and stresses obtained using the present refined theory (with four unknowns) and other higher-order shear deformation theories (five unknowns) are almost identical.

In addition, unlike any other theory, the theory presented gives rise to only four governing equations resulting in considerably lower computational effort when compared with the other higher-order theories reported in the literature having more number of governing equations.

Hence, it can be said that the proposed theory is accurate and simple in bending analysis of FGM rectangular plates resting on non-uniform elastic foundations in thermal environment. Finally, this work can be extended in future by considering the case of various boundary conditions and the stretching effect (Abualnour *et al*. 2018, Benchohra *et al*. 2018, Bouhadra *et al*. 2018, Bouafia *et al*. 2017, Sekkal *et al*. 2017b, Abdelaziz *et al*. 2017, Bennoun *et al*. 2016, Draiche *et al*. 2016, Bourada *et al*. 2015, Larbi Chaht *et al*. 2015, Bousahla *et al*. 2014, Fekrar *et al*. 2014, Hebali *et al*. 2014, Ait Amar Meziane *et al*. 2014, Belabed *et al*. 2014, Bessaim *et al*. 2013).

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