Steel and Composite Structures, *Vol. 26, No. 5 (2018) 557-572*
DOI: https://doi.org/10.12989/scs.2018.26.5.557

Bending analysis of advanced composite plates using a new quasi 3D plate theory

Tarek Houari [1], Aicha Bessaim [2,3,4], Mohammed Sid Ahmed Houari [*2,3,4], Mohamed Benguediab [1] and Abdelouahed Tounsi [3,5]

[1] *Département de Génie Mécanique, Faculté de Technologie, Université Sidi Bel Abbes, Algérie*
[2] *Département de Génie Civil, Faculté des Sciences et de la Technologie, Université Mustapha Stambouli Mascara, Algeria*
[3] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[4] *Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de génie civil, Algeria*
[5] *Laboratoire de Modélisation et Simulation Multi-échelle, Faculté des Sciences, Département de Physique, Université de Sidi Bel Abbes, Algeria*

(*Received April 16, 2017, Revised September 17, 2017, Accepted December 17, 2017*)

Abstract. In this paper, a refined higher-order shear deformation theory including the stretching effect is developed for the analysis of bending analysis of the simply supported functionally graded (FG) sandwich plates resting on elastic foundation. This theory has only five unknowns, which is even less than the other shear and normal deformation theories. The theory presented is variationally consistent, without the shear correction factor. The present one has a new displacement field which introduces undetermined integral variables. Equations of motion are obtained by utilizing the Hamilton's principles and solved via Navier's procedure. The convergence and the validation of the proposed theoretical numerical model are performed to demonstrate the efficacy of the model.

Keywords: FG sandwich plates; new plate theory; bending; stretching effect; analytical modeling

1. Introduction

Sandwich structures are one of the most functional forms of advanced composite structures developed by engineers. The main concept of sandwich composite structures is its high specific strength and bending stiffness to weight ratio (Belouettar *et al.* 2009). With its many advantages, composite sandwich structures have been widely used in a variety of engineering application including automotive, aerospace, mechanical, ships and other industrial applications (Vel *et al.* 2005). This composite material also draws a lot of interest in the construction industry and is now beginning to be in use for civil engineering projects as industrial buildings, vehicular bridges, solar power stations, nuclear reactor structures and petrochemical structures (Bennoun *et al.* 2016, Bounouara *et al.* 2016, Kolahchi *et al.* 2017a, b, Kolahchi 2017, El-Haina *et al.* 2017, Amar *et al.* 2017). Sandwich constructions consist of two outer strong layers and an inner relatively thick, lightweight core material (Vinson 2001). Sandwich structure has become even more attractive to the introduction of advanced composite materials for the face sheets like functionally graded ceramic–metal materials (Wang and Shen 2012). The considerable advantages offered by functionally graded materials (FGMs) over conventional materials are to eliminates the interface problems of conventional composite materials and thus the stress distribution becomes smooth (Li *et al.* 2008). Subsequently, a number of studies have been performed to analyze the static, vibration, and buckling of functionally graded structures due to the increased relevance of the FGMs structural components in the design of engineering structures (Tounsi *et al.* 2013, Kar and Panda 2014, 2015, Yaghoobi *et al.* 2014, Swaminathan and Naveenkumar 2014, Bousahla *et al.* 2014, Sofiyev and Kuruoglu 2015a, Akbaş 2015, Attia *et al.* 2015, Darilmaz 2015, Darilmaz *et al.* 2015, Bourada *et al.* 2015, Kolahchi *et al.* 2015, Mahi *et al.* 2015, Sofiyev and Osmancelebioglu 2017, Brischetto 2017, Benadouda *et al.* 2017, Khetir *et al.* 2017, Menasria *et al.* 2017). Plates supported elastic foundations are generally encountered in many engineering applications, such as bottom plates of hydraulic structures and surface plates of the airport (Ke-rang 1990). From the literature review, it is found that there are many studies of plates supported by elastic foundation and these studies have attracted the attention of many investigators. Al-Hosani *et al.* (1999) proposed a fundamental solution and boundary integral equations for thick Reissner plates resting on a Winkler elastic foundation by considering the effect of transverse normal stresses resulting from the foundation reaction on the plate surface. Darilmaz (2009) proposed a four-node hybrid stress element for analysing arbitrarily shaped plates resting on a two parameter elastic foundation. Baltacioglu *et al.* (2011) proposed a method of discrete

∗Corresponding author, Ph.D.,
E-mail: ms.houari@univ-mascara.dz; houarimsa@yahoo.fr

ISSN: 1229-9367 (Print), 1598-6233 (Online)

singular convolution for the large deflection analysis of laminated composite plates resting on nonlinear elastic foundations. A few researchers have utilized classical plate theory (CPT) to studies vibration and static behavior of thin functionally graded (FG) plates. He *et al*. (2001) reported the finite element formulation based on thin plate theory to control the shape and vibration of FGM plate with integrated piezoelectric sensors and actuators under mechanical load. Woo *et al*. (2006) provided an analytical solution for the nonlinear free vibration behavior of FG square thin plates using the von Karman theory. Abrate (2008) has presented result of free vibration of simply supported and clamped rectangular thin plates using the CPT. Zhang and Zhou (2008) studies the free vibration, buckling and on the basis of the physical neutral surface. The classical plate theory ignores the transverse shear strain and is suitable only to studies thin plates. However, it is not appropriate for the moderately thick and thick plates, which require that the transverse and normal strain should be taken into account. First-order shear deformation theory considers the transverse shear deformation effects and gives acceptable results for thick and thin plates, but needs a shear correction factor which is hard to find as it depends on the geometries, material properties and boundary conditions of each problem (Ferreira *et al*. 2009). Liu and Liew (1999) analyzed free vibration of rectangular plates with mixed boundary conditions using the first-order shear deformation theory (FSDT). The nonlinear static and dynamic responses of functionally graded ceramic–metal plates using the FSDT and the von Karman strain were examined by Praveen and Reddy (Praveen and Reddy 1998). Najafov *et al*. (2014) investigated the stability of exponentially graded (EG) cylindrical shells with shear stresses on a Pasternak foundation based on the first-order shear deformation theory (FSDT). Sofiyev *et al*. (2015b) studied the free vibration of sandwich cylindrical shells covered by functionally graded coatings and resting on the Pasternak elastic foundation considering combined influences of shear stresses and rotary inertia are examined. Sofiyev *et al*. (2016) analyzed the vibration and stability of axially loaded functionally graded (FG) sandwich cylindrical shells with and without shear stresses and rotary inertia resting Pasternak foundation. Based on the orthotropic Mindlin plate, Kolahchi *et al*. (2016) investigated the temperature-dependent nonlinear dynamic stability for a functionally graded CNT reinforced visco-plate resting on an orthotropic elastomeric foundation. Chen *et al*. (2017) investigated the thermal buckling and vibration of initially stressed sandwich plates with functionally graded material (FGM) face sheets, including the effects of transverse shear deformation and rotary inertia. The higher-order shear deformation theories (HSDTs) have been developed and do not require any shear correction factor. These theories include higher-order terms in the approximation of the in-plane displacement fields and satisfy zero shear stress conditions at top and bottom surfaces of plates. Reddy (2000) has presented result of the static behavior of functionally graded rectangular plates based on a third-order shear deformation plate theory. Cheng and Batra (2000) derived the field equations for a simply supported functionally graded plate by utilizing the first-order shear deformation theory or the third-order shear deformation theory and simplified them for a simply supported polygonal plate to that of an equivalent homogeneous Kirchhoff plate. Yaghoobi and Yaghoobi (2013) investigated the buckling analysis of symmetric sandwich plates with FG face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and under to mechanical, thermal and thermo-mechanical loads. Ferreira *et al*. (2006) studied the free vibration of functionally graded plates based on the first and the third-order shear deformation plate theories using the Mori–Tanaka homogenization method and the global collocation method with multiquadratic radial basis functions. Bellifa *et al*. (2016) studied the bending and the free vibration of functionally graded plates using a novel simple first-order shear deformation plate theory based on neutral surface position. Some studies on response of FG sandwich plates have been carried out using higher-order shear deformation theories. Zenkour (2005a, b) investigated bending, vibration and buckling problem of sandwich plates with FG faces and homogeneous hardcore using different shear deformation theories. Ait Amar Meziane *et al*. (2014) analyzed the buckling and free vibration of exponentially graded sandwich plates under various boundary conditions using an efficient and simple refined theory. Benyoucef *et al*. (2010) examined the static response of simply supported functionally graded plates resting on an elastic foundation using a new hyperbolic displacement model. Zenkour and Sobhy (2010) studied the critical buckling temperature for FGM sandwich plates using a sinusoidal shear deformation plate theory to derive the appropriate elastic stability equations. Ebrahimi and Habibi (2016) utilized the finite element method is to predict the deflection and vibration of porous FG plates made within the framework of the third order shear deformation plate theory. Recently, Tounsi and his co-workers (Hadji *et al*. 2011, Houari *et al*. 2011, Merdaci *et al*. 2011, Bouderba *et al*. 2013, Zidi *et al*. 2014, Ait Yahia *et al*. 2015, Boukhari *et al*. 2016, Hadj Henni *et al*. 2017) developed a new refined and robust plate theory for bending response, buckling, vibration and wave propagation of simply supported FG plate with only four unknown functions. In Houari *et al*. (2016) and Tounsi *et al*. (2016) a new simple shear deformation theory for the bending and free vibration response of FG plates with only three unknown functions was developed. As opposed to five or even greater numbers in the case of other higher shear deformation theories, this theory is variationally consistent, does not require shear correction factor, and accounts for parabolic distribution of the transverse shear strains, and satisfies the zero traction on the surfaces of the plate without using shear correction factor. Most recently, Tounsi and his co-workers (Hebali *et al*. 2016, Merdaci *et al*. 2016, Krenich *et al*. 2017, Meftah *et al*. 2017) developed another novel refined plate theory for mechanical behaviour of simply supported plate with only four unknown. These theories have a new displacement field which introduces undetermined integral variables. It should be noted that the thickness stretching effect is ignored in these new four-variable plate theories and other hear deformation plates theories. The importance of the thickness stretching effect in FG plates has been pointed and discussed out in the work of Carrera *et al*. (2011) using appropriate finite element approximations. Quasi-3D theories are higher order shear deformation theories (HSDTs) with higher-order variations through the thickness for the transverse displacement. There are many papers proposed in the literature concerned with investigation of the different behaviors of the FGM structures by using the quasi-3D theories. Kant and Swaminathan (2002) proposed a quasi-3D theory with all displacement components expanded as a cubic variation through the thickness for static response of laminated composite and sandwich plates. Neves *et al*. (2011) and Neves *et al*. (2012a, b) have presented an original hyperbolic sine shear deformation theory for the bending and free vibration analysis of FG plates. Houari *et al*. (2013) developed a theory for the thermoelastic bending analysis of FGM sandwich plates. Bessaim *et al*. (2013) presented a theory for the bending and free vibration analysis of sandwich plates. Belabed *et al*. (2014) have determined the bending and free vibration response for FG plate. Hebali *et al*. (2014) analyzed the static and free vibration analysis of functionally graded plates. Hamidi *et al*. (2015) have proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Draiche *et al*. (2016) have presented a refined theory with stretching effect for the flexure analysis of laminated composite plates. Sekkal *et al*. (2017) developed a new quasi-3D HSDT for buckling and vibration of FG plate. Recently, Abualnour *et al*. (2018) proposed a novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates. Benchohra *et al*. (2018) presented a new quasi-3D sinusoidal shear deformation theory for FG plates.

This paper aims to improve the plate theory developed by Tounsi and his co-workers (Hebali *et al*. 2016, Merdaci *et al*. 2016, Krenich *et al*. 2017, Meftah *et al*. 2017) by including the so-called stretching effect. Using the proposed theory, both static behavior of FGM sandwich plates resting on two-parameter elastic foundations are investigated. This theory has only five unknowns, which is even less than the other Quasi-3D theories. The most interesting feature of this theory is that it accounts for a hyperbolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The present one has a new displacement field which introduces undetermined integral variables. Analytical solutions are obtained for FGM sandwich plate, and accuracy is verified by comparing the obtained results with those reported in the literature.

2. Theoretical formulation

2.1 Geometrical configuration

A solid rectangular FG sandwich plate with uniform thickness with uniform thickness h, length a, and width b is considered in the present study (Fig. 1). The rectangular

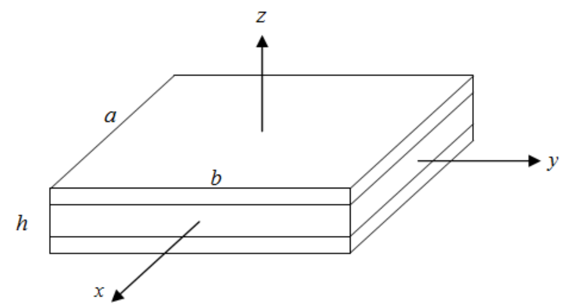


Fig. 1 Geometry of rectangular FGM sandwich plate with uniform thickness in the rectangular Cartesian coordinates

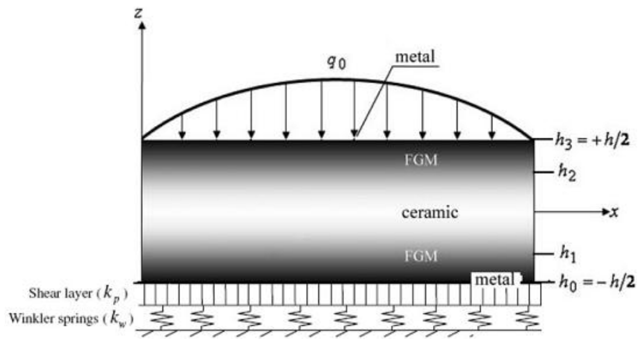


Fig. 2 The material variation along the thickness of the FGM sandwich plate

Cartesian coordinate system (x, y, z) is taken such that the x, y plane (z = 0) coincides with the mid-plane of the sandwich plate. The top and bottom faces of the plate are at $z = \pm h / 2$, and the edges of the plate are parallel to axes x and y. The sandwich plate is composed of three elastic layers, namely, “Layer 1”, “Layer 2”, and “Layer 3” from bottom to top of the plate. The vertical ordinates of the bottom, the two interfaces, and the top are denoted by $h_0 = -h / 2$, h_1, h_2, $h_3 = h / 2$, respectively. In order to study the effect of the thickness variation of the three layers on displacements and stresses, a simple notation is used in all the numerical examples considered. For example, a notation of 1-2-1 (i.e., bottom face-core-top face thickness) is used to represent that the top and bottom face sheets have same thickness, whereas the core thickness is twice the bottom/top thickness. Other cases, viz. 1-0-1, 1-1-1, 2-1-2, 3-1-3 and 2-2-1, are also considered to realize the various numerical examples.

The face layers of the sandwich plate are made of an isotropic material with material properties varying smoothly in the z direction only. The core layer is made of an isotropic homogeneous material, as shown in Fig. 2.

2.2 Materials proprieties

A simple power law in terms of the volume fraction of the ceramic phase is considered

$$V^{(1)} = \left(\frac{z - h_0}{h_1 - h_0} \right)^p, \quad z \in [h_0, h_1] \tag{1a}$$

$$V^{(2)} = 1, \quad z \in [h_1, h_2] \tag{1b}$$

$$V^{(3)} = \left(\frac{z - h_3}{h_2 - h_3} \right)^p, \quad z \in [h_2, h_3] \tag{1c}$$

where $V^{(n)}$, (n = 1, 2, 3) represents the volume fraction function of layer n; p is the volume fraction index ($0 \le p \le +\infty$), which control the material distribution in the thickness direction.

The effective material properties, like Young's modulus E, and Poisson's ratio ν, can be mathematically expressed by the rule of mixture as (Marur 1999, Bourada *et al*. 2011, Houari *et al*. 2013, Bellifa *et al*. 2017, Besseghier *et al*. 2017)

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{2}$$

where $P^{(n)}$ is the effective material property of FGM of layer n. P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, (n = 1, 2, 3). For simplicity, Poisson's ratio of plate is assumed to be constant in this study for that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus (Delale and Erdogan 1983).

2.3 Constitutive equations

For elastic and isotropic FGMs, the constitutive relations can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix}^{(n)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 \\ 0 & 0 & 0 & Q_{66} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix}^{(n)} \tag{3a}$$

and

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix}^{(n)} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}^{(n)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}^{(n)} \tag{3b}$$

where (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. Using the material properties defined in Eq. (3), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = Q_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{4a}$$

$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu\,(1-\nu)\,E(z)}{(1-2\nu)(1+\nu)}, \tag{4b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{4c}$$

Based on the thick plate theory and including the effect of transverse normal stress (thickness stretching effect), the basic assumptions for the displacement field of the plate can be described as

$$u(x,y,z) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{5a}$$

$$v(x,y,z) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{5b}$$

$$w(x,y,z) = w_0(x,y,t) + g(z)\varphi_z(x,y) \tag{5c}$$

The coefficients k_1 and k_2 depends on the geometry and the proposed theory of present study has a hyperbolic function in the form

$$f(z) = h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right) \tag{6}$$

It can be observed that the kinematic in Eq. (5) uses only five unknowns (u_0, v_0, w_0, θ and φ_z). Nonzero strains of the five variable plate model are expressed as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{7a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{7b}$$

$$\varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{7c}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{8a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\displaystyle\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\displaystyle\int \theta\,dy \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\displaystyle\int \theta\,dy + \dfrac{\partial \varphi_z}{\partial y} \\ k_1\displaystyle\int \theta\,dx + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix}, \qquad \varepsilon_z^0 = \varphi_z \tag{8b}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{8c}$$

It can be observed from Eq. (7) that the transverse shear strains (γ_{xz}, γ_{yz}) are equal to zero at the upper ($z = h / 2$) and lower ($z = -h / 2$) surfaces of the plate. A shear correction coefficient is, hence, not required.

The integrals used in the above equations shall be resolved by a Navier type procedure and can be expressed as follows

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \qquad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2\theta}{\partial x\partial y},$$
$$\int \theta\, dx = A'\frac{\partial\theta}{\partial x}, \qquad \int \theta\, dy = B'\frac{\partial\theta}{\partial y} \tag{9}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{10}$$

where α and β are defined in Eq. (21).

2.4 Governing equations

The principle of virtual displacements is utilized for the bending problem of FG sandwich plates. The principle of virtual work in the present case yields (Reddy 2002, Al-Basyouni *et al*. 2015, Zemri *et al*. 2015, Ahouel *et al*. 2016, Saidi *et al*. 2016, Mouffoki *et al*. 2017, Zidi *et al*. 2017, Hachemi *et al*. 2017)

$$\delta U = \int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]d\Omega\, dz - \int_{\Omega}(q - f_e)\delta w\, d\Omega = 0 \tag{11}$$

where f_e is the density of reaction force of foundation. For the Pasternak foundation model

$$f_e = k_w w - k_s^1\frac{\partial^2 w}{\partial x^2} - k_s^2\frac{\partial^2 w}{\partial y^2} \tag{12}$$

where k_w is the modulus of subgrade reaction (elastic coefficient of the foundation) and k_s^1 and k_s^2 are the shear moduli of the subgrade (shear layer foundation stiffness). If foundation is homogeneous and isotropic, we will get $k_s^1 = k_s^2 = k_s$. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation.

Substituting Eqs. (4) and (7) into Eq. (12) and integrating through the thickness of the plate, Eq. (12) can be rewritten as

$$\begin{aligned}&\int_{\Omega}\Big[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_z\delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0 \\ &+ M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s \\ &+ M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^0\Big]d\Omega \\ &- \int_{\Omega}(q - f_e)\delta w\, d\Omega = 0\end{aligned} \tag{13}$$

where Ω is the top surface and the stress resultants N, M, and S are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}(1, z, f)\sigma_i^{(n)}dz \qquad (i = x, y, xy);$$
$$N_z = \int_{h_{n-1}}^{h_n} g'(z)\sigma_z^{(n)}dz \tag{14a}$$

and

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{h_{n-1}}^{h_n} g\left(\tau_{xz}, \tau_{yz}\right)^{(n)}dz \tag{14b}$$

where h_n and h_{n-1} are the top and bottom z-coordinates of the nth layer.

The governing equations of equilibrium can be derived from Eq. (11) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_0, $\delta\theta$, and $\delta\varphi_z$ zero, separately. The following equations of motion of associated with the present shear deformation theory are obtained as

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
&\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - f_e + q = 0 \\
&\delta\theta: -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} \\
&+ k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial y} = 0 \\
&\delta\varphi_z: \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z + g(z)q = 0
\end{aligned} \tag{15}$$

Using Eq. (4) in Eq. (14), the stress resultants of a sandwich plate made up of three layers can be related to the total strains by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23} \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23} \\ 0 & 0 & B_{66} & 0 & 0 & D_{11} & 0 & 0 & D_{66}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \\ \varepsilon_z^0 \end{Bmatrix} \tag{16a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{16b}$$

where

$$\left(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right)= \int_{-h/2}^{h/2} C_{ij}\left(1, g^2(z), z, z^2, f(z), z\, f(z), f^2(z)\right)dz \tag{17a}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), g'(z)\right) g'(z)\, C_{ij}\, dz \tag{17b}$$

2.3 Equations of motion in terms of displacements

By substituting Eq. (16) into Eq. (15), the equilibrium equations can be expressed in terms of displacements (u_0, v_0, w_0, θ and φ_z) as

$$\begin{aligned} &A_{11} d_{11} u_0 + A_{66}\, d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 \\ &+ X_{13} d_1 \varphi_z - B_{11} d_{111} w_0 - \left(B_{12} + 2B_{66}\right) d_{122} w_0 \\ &+ \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{122}\theta \\ &+ \left(B_{11}^s k_1 + B_{12}^s\, k_2\right) d_1\theta = 0, \end{aligned} \tag{18a}$$

$$\begin{aligned} &A_{22}\, d_{22} v_0 + A_{66}\, d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 \\ &+ X_{23} d_2 \varphi_z - B_{22}\, d_{222} w_0 - \left(B_{12} + 2B_{66}\right) d_{112} w_0 \\ &+ \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{112}\theta \\ &+ \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2\theta = 0, \end{aligned} \tag{18b}$$

$$\begin{aligned} &B_{11}\, d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 \\ &+ B_{22}\, d_{222} v_0 + Y_{13}\, d_{11}\varphi_z + Y_{23}\, d_{22}\varphi_z - D_{11} d_{1111} w_0 \\ &- 2\left(D_{12} + 2D_{66}\right) d_{1122} w_0 - D_{22}\, d_{2222} w_0 \\ &+ \left(D_{11}^s\, k_1 + D_{12}^s\, k_2\right) d_{11}\theta \\ &+ 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122}\theta \\ &+ \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}\theta - f_e + q = 0 \end{aligned} \tag{18c}$$

$$\begin{aligned} &-\left(B_{11}^s k_1 + B_{12}^s k_2\right) d_1 u_0 - \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{122} u_0 \\ &- \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{112} v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right) d_2 v_0 \\ &- k_1 Y_{13}^s \theta_z - k_2 Y_{23}^s \theta_z + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11} w_0 \\ &+ 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} w_0 \\ &+ \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} w_0 \\ &- H_{11}^s\, k_1^2\, \theta - H_{22}^s\, k_2^2\, \theta - 2 H_{12}^s\, k_1\, k_2 \theta \\ &- \left(\left(k_1 A' + k_2 B'\right)^2 H_{66}^s\right) d_{1122}\theta + A_{44}^s \left(k_2 B'\right)^2 d_{22}\theta \\ &+ A_{55}^s \left(k_1\, A'\right)^2 d_{11}\theta + A_{44}^s \left(k_2 B'\right) d_{22}\varphi_z \\ &+ A_{55}^s \left(k_1 A'\right) d_{11}\varphi_z = 0 \end{aligned} \tag{18d}$$

$$\begin{aligned} &X_{13} d_1 u_0 + X_{23}\, d_2 u_0 + Z_{33}\varphi_z + Y_{13}\, d_{11} w_0 + Y_{23}\, d_{22} w_0 \\ &+ A_{44}^s \left(k_2 B'\right) d_{22}\theta + A_{55}^s \left(k_1\, A'\right) d_{11}\theta \\ &+ A_{44}^s\, d_{22}\varphi_z + A_{55}^s\, d_{11}\varphi_z = 0, \end{aligned} \tag{18e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \qquad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \qquad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \qquad (i, j, l, m = 1,2). \tag{19}$$

3. Analytical solution of simply supported FG sandwich plate

In this work, we are concerned with the exact solutions of Eq. (18) for a simply supported nanoplate. Using the Navier solution procedure, the following expressions of displacements (u_0, v_0, w_0, θ and φ_z) are taken

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn} \sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn} \sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn} \sin(\alpha\, x)\sin(\beta\, y) \\ Z_{mn} \sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{20}$$

where

$$\alpha = m\pi / a, \qquad \beta = n\pi / b \tag{21}$$

(U_{mn}, V_{mn}, W_{mn}, X_{mn}, Z_{mn}) are the unknown maximum displacement coefficients. The transverse load $q(x, y)$ is also expanded as

$$q(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{22}$$

The coefficients Q_{mn} are given below for some typical loads

$$Q_{mn} = \frac{4}{a\, b}\int_0^a \int_0^b q(x, y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, dxdy \tag{23}$$

For uniformly distributed load

$$Q_{mn} = \begin{cases} \dfrac{16 q_0}{mn\pi^2} & m, n = 1,3,5,\ldots \\ 0 & m, n = 2,4,6,\ldots \end{cases} \tag{24}$$

For sinusoidal distributed load; $Q_{mn} = q_0$ in which q_0 is the intensity of the load.

Substituting Eqs. (21) and (22) into Eq. (18), the analytical solutions can be determined by

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{13} & a_{23} & a_{33} & a_{34} & a_{35} \\ a_{14} & a_{24} & a_{34} & a_{44} & a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} \end{bmatrix}\right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Z_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

where

Table 1 Material properties used in the functionally graded sandwich plates

Properties	(Al/ZrO_2)		$(T_i$–6Al–4V/$ZrO_2)$	
	Zirconia	Aluminum	Zirconia	Titanium
E (GPa)	151	70	117	66.2
v	0.3	0.3	1/3	1/3

$$
\begin{aligned}
a_{11} &= -\left(A_{11}\alpha^2 + A_{66}\beta^2\right) \\
a_{12} &= -\alpha\beta\ \left(A_{12} + A_{66}\right) \\
a_{13} &= \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right) \\
a_{14} &= \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right)B_{66}^s\beta^2\right) \\
a_{15} &= X_{13}\alpha \\
a_{22} &= -\left(A_{66}\alpha^2 + A_{22}\beta^2\right) \\
a_{23} &= \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right) \\
a_{24} &= \beta\ \left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right)B_{66}^s\alpha^2\right) \\
a_{25} &= X_{23}\beta \\
a_{33} &= -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right) \\
&\quad - k_w - k_s^1\alpha^2 - k_s^2\beta^2 \\
a_{34} &= -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\,D_{66}^s\alpha^2\beta^2 \\
&\quad - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\
a_{35} &= -\left(Y_{13}\ \alpha^2 + Y_{23}\ \beta^2\right) \\
a_{44} &= -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s\alpha^2\beta^2 \\
&\quad - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1 A'\right)^2 A_{55}^s\alpha^2
\end{aligned} \tag{26}
$$

Table 2 Non dimensional center displacement $\bar{w}$ and non-dimensional axial stress $\overline{\sigma_x}$ of (Al/ZrO_2) FG sandwich square plate under sinusoidally distributed load (a/h = 10)

p	Theory	ε_z	$\bar{w}$				$\overline{\sigma_x}$			
			2-1-2	1-1-1	2-2-1	1-2-1	2-1-2	1-1-1	2-2-1	1-2-1
0	Neves *et al*. 2012c	$=0$	0.19610	0.19610	0.19610	0.19610	1.99470	1.99470	1.99460	1.99460
	Neves *et al*. 2012c	$\neq 0$	0.19490	0.19490	0.19490	0.19490	2.00660	2.00660	2.00650	2.00640
	Bessaim *et al*. 2013	$\neq 0$	0.19486	0.19486	0.19486	0.19486	1.99524	1.99524	1.99524	1.99524
	Akavci 2016	$=0$	0.19605	0.19605	0.19605	0.19605	1.99516	1.99516	1.99516	1.99516
	Akavci 2016	$\neq 0$	0.19466	0.19466	0.19466	0.19466	2.0730	2.0730	2.0730	2.0730
	Present	$=0$	0.19606	0.19606	0.19606	0.19606	1.99332	1.99332	1.99332	1.99332
	Present	$\neq 0$	0.19487	0.19487	0.19487	0.19487	1.99525	1.99525	1.99525	1.99525
1	Neves *et al*. 2012c	$=0$	0.30900	0.29490	0.28380	0.27400	1.47420	1.40670	1.30260	1.30640
	Neves *et al*. 2012c	$\neq 0$	0.30700	0.29290	0.28200	0.27220	1.48130	1.41370	1.30920	1.31330
	Bessaim *et al*. 2013	$\neq 0$	0.30430	0.29007	0.27874	0.26915	1.46131	1.39243	1.28274	1.29030
	Akavci 2016	$=0$	0.30627	0.29196	0.28083	0.27093	1.46322	1.39432	1.28879	1.29201
	Akavci 2016	$\neq 0$	0.30398	0.28977	0.27847	0.26891	1.52514	1.45397	1.34177	1.34783
	Present	$=0$	0.30635	0.29202	0.28085	0.27093	1.46214	1.39324	1.28769	1.29091
	Present	$\neq 0$	0.30431	0.29007	0.27875	0.26915	1.46132	1.39244	1.28272	1.29030
2	Neves *et al*. 2012c	$=0$	0.35420	0.33510	0.31860	0.30530	1.69200	1.60170	1.44760	1.45880
	Neves *et al*. 2012c	$\neq 0$	0.35190	0.33290	0.31640	0.30320	1.69940	1.60880	1.45430	1.46590
	Bessaim *et al*. 2013	$\neq 0$	0.35001	0.33068	0.31356	0.30060	1.68472	1.59170	1.42887	1.44497
	Akavci 2016	$=0$	0.35222	0.33282	0.31613	0.30261	1.68708	1.59420	1.43723	1.44736
	Akavci 2016	$\neq 0$	0.34957	0.33030	0.31319	0.30031	1.75757	1.66237	1.49644	1.51084
	Present	$=0$	0.35237	0.33294	0.31620	0.30262	1.68601	1.59310	1.43607	1.44622
	Present	$\neq 0$	0.35001	0.33068	0.31356	0.30060	1.68473	1.59171	1.42886	1.44497
10	Neves *et al*. 2012c	$=0$	0.40510	0.38680	0.36370	0.35030	1.93160	1.84850	1.63270	1.67610
	Neves *et al*. 2012c	$\neq 0$	0.40260	0.38430	0.36120	0.34800	1.93970	1.85590	1.63950	1.68320
	Bessaim *et al*. 2013	$\neq 0$	0.40153	0.38303	0.35885	0.34591	1.93266	1.84705	1.61792	1.66754
	Akavci 2016	$=0$	0.40390	0.38538	0.36204	0.34817	1.93451	1.84956	1.62871	1.67048
	Akavci 2016	$\neq 0$	0.40094	0.38248	0.35823	0.34549	2.01036	1.92481	1.69436	1.74262
	Present	$=0$	0.40426	0.38566	0.36221	0.34830	1.93354	1.84857	1.62753	1.66937
	Present	$\neq 0$	0.40153	0.38303	0.35885	0.34591	1.93266	1.84707	1.61792	1.66755

$$-\left(k_2 B^{'}\right)^2 A_{44}^s \beta^2$$

$$a_{45} = -k_1 A^{'} A_{55}^s \alpha^2 - k_2 B^{'} A_{44}^s \beta^2 + k_1 Y_{13}^s + k_2 Y_{23}^s \tag{26}$$

$$a_{55} = -\left(A_{55}^s \alpha^2 + A_{44}^s \beta^2 + Z_{33}\right)$$

4. Numerical results and discussions

In this section, the accuracy of the presented quasi-3D hyperbolic plate theory for the bending results of simply supported FG sandwich plates resting on elastic foundations and under sinusoidal loads is demonstrated by comparing the analytical solution with the existing results of quasi-3D and 2D shear theories in literature. In addition, the influences of shear deformation and thickness stretching on the bending response of the FGM sandwich plates are investigated. Typical values for metal and ceramics used in the FG sandwich plate The FG sandwich plates are made of (ZrO_2/Al) and (ZrO_2/T_i–6Al–4V), whose material properties are listed in Table 1.

For convenience, the following dimensionless forms are used

$$\bar{z} = \frac{z}{h}, \qquad \bar{w} = \frac{10hE_0}{a^2 q_0} w\left(\frac{a}{2}, \frac{b}{2}, \bar{z}\right),$$

$$\bar{\sigma}_x = \frac{10h^2}{a^2 q_0} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \bar{z}\right), \qquad \bar{\tau}_{xz} = \frac{h}{a q_0} \tau_{xz}\left(0, \frac{b}{2}, \bar{z}\right),$$

$$\hat{w} = \frac{10^2 D_c}{a^4 q_0} w\left(\frac{a}{2}, \frac{b}{2}, \bar{z}\right), \qquad \hat{\sigma}_x = \frac{1}{10^2 q_0} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \bar{z}\right),$$

$$\hat{\tau}_{xz} = \frac{1}{10 q_0} \tau_{xz}\left(0, \frac{b}{2}, \bar{z}\right), \qquad K_w = \frac{k_w a^4}{D_c}, \qquad K_s = \frac{k_s a^2}{D_c},$$

$$D_c = \frac{E_c h^3}{12(1-\nu^2)}, \qquad E_0 = 1 \text{ GPa}$$

The first example aims to verify the accuracy of the present theory in predicting the bending responses of FG sandwich plates. Table 2 contain the non-dimensional transverse displacement $\bar{w}$ of the mid-plane and non dimensional axial stresses $\overline{\sigma_x}$ of the FG sandwich plate made of (ZrO_2/Al) under sinusoidal loads for different skin-core-skin ratios and material parameter *k*. The obtained predictions are compared with the 2D and Quasi-3D solution of Neves *et al*. (2012c), Akavci (2016), and the quasi- 3D solutions of Bessaim *et al*. (2013). It should be noted that the quasi-3D solutions of Neves *et al*. (2012c), Akavci (2016) and Bessaim *et al*. (2013) are derived based on a hyperbolic variation of both in-plane and transverse displacements. The results of 2D solutions the present work, Neves *et al*. (2012c) and Akavci (2016) are provided to show the importance of including the thickness stretching effect. It can be seen that the dimensionless displacement and stresses predicted by the present new quasi-3D hyperbolic theory are in excellent agreement quasi-3D solutions, particularly with those reported by Bessaim *et al*.

Table 3 Nondimensional transverse shear stress $\bar{\tau}_{xz}$ for square (Al/ZrO_2) FG sandwich plate (a/h = 10)

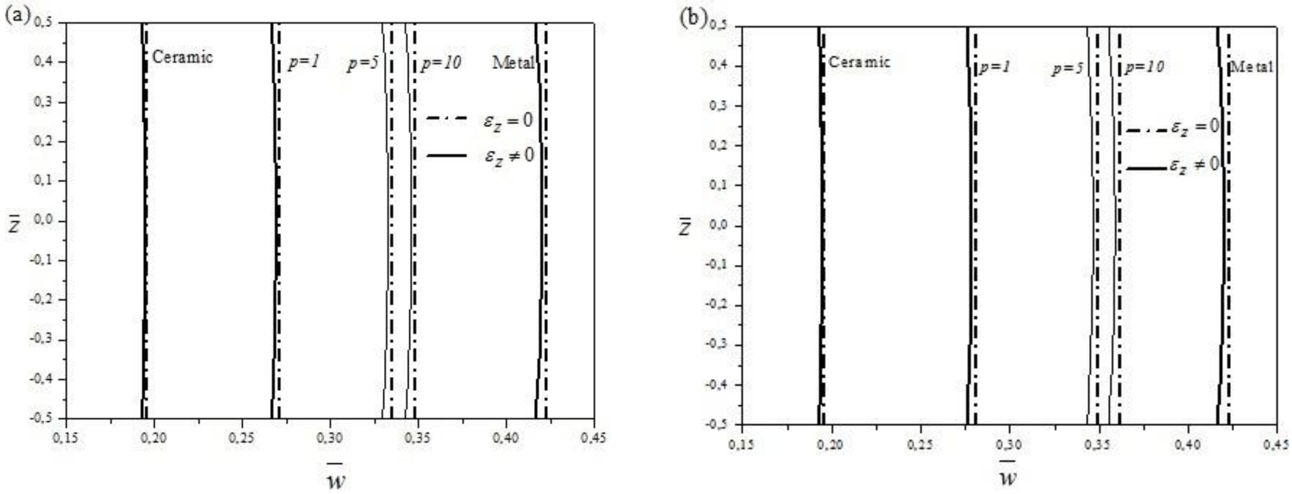
p	Theory	ε_z	$\bar{\tau}_{xz}$			
			2-1-2	1-1-1	2-2-1	1-2-1
0	Neves *et al*. 2012c	$=0$	0.2538	0.2459	0.2407	0.2358
	Neves *et al*. 2012c	$\neq 0$	0.2538	0.2461	0.2411	0.2363
	Bessaim *et al*. 2013	$\neq 0$	0.23794	0.23794	0.23794	0.23794
	Present	$=0$	0.23217	0.23217	0.23217	0.23217
	Present	$\neq 0$	0.23794	0.23794	0.23794	0.23794
1	Neves *et al*. 2012c	$=0$	0.2744	0.2640	0.2590	0.2489
	Neves *et al*. 2012c	$\neq 0$	0.2745	0.2643	0.2594	0.2496
	Bessaim *et al*. 2013	$\neq 0$	0.27050	0.26060	0.25890	0.25196
	Present	$=0$	0.26535	0.25533	0.25337	0.24635
	Present	$\neq 0$	0.27050	0.26060	0.25890	0.25196
2	Neves *et al*. 2012c	$=0$	0.2758	0.2664	0.2632	0.2515
	Neves *et al*. 2012c	$\neq 0$	0.2760	0.2668	0.2636	0.2523
	Bessaim *et al*. 2013	$\neq 0$	0.28792	0.27138	0.26885	0.25776
	Present	$=0$	0.28335	0.26661	0.26359	0.25241
	Present	$\neq 0$	0.28792	0.27138	0.26885	0.25776
10	Neves *et al*. 2012c	$=0$	0.2669	0.2635	0.2690	0.2559
	Neves *et al*. 2012c	$\neq 0$	0.2671	0.2639	0.2692	0.2568
	Bessaim *et al*. 2013	$\neq 0$	0.33210	0.29534	0.29036	0.26850
	Present	$=0$	0.32826	0.29167	0.28561	0.26392
	Present	$\neq 0$	0.33210	0.29534	0.29036	0.26850

Table 4 Non-dimensional center displacement $\hat{w}$ of FG (T_i–6Al–4V/ZrO_2) sandwich plate on elastic foundation under sinusoidally distributed load (a/h = 10, b = 2a)

Scheme	p	Theory	ε_z	(K_w, K_s)			
				0, 0	100, 0	0, 100	100, 100
1-0-1	0	Taibi *et al*. 2015	$= 0$	0,681308	0,405225	0,0836524	0,077194
		Akavci 2016	$\neq 0$	0,677195	0,404967	0.0728693	0,067958
		Present	$= 0$	0.681329	0.405232	0.0724390	0.067546
		Present	$\neq 0$	0.678245	0.405328	0.072876	0.067960
	0.5	Taibi *et al*. 2015	$= 0$	0,886739	0,469985	0,0861015	0,079275
		Akavci 2016	$\neq 0$	0.881167	0,470028	0.0747292	0,0695684
		Present	$= 0$	0.886715	0.469978	0.0742679	0.0691335
		Present	$\neq 0$	0.882541	0.470413	0.0747383	0.0695762
	2	Taibi *et al*. 2015	$= 0$	1,109938	0,526052	0,087816	0.080727
		Akavci 2016	$\neq 0$	1,10267	0.526445	0.0760263	0,0706912
		Present	$= 0$	1.10979	0.526019	0.0755397	0.0702342
		Present	$\neq 0$	1.104593	0.526872	0.0760337	0.0706977
3-1-3	0.5	Taibi *et al*. 2015	$= 0$	0,868596	0.464839	0.085927	0.079128
		Akavci 2016	$\neq 0$	0,86314	0.464849	0.0745969	0,0694537
		Present	$= 0$	0.868577	0.464833	0.0741382	0.0690211
		Present	$\neq 0$	0.864416	0.465262	0.0746036	0.0694619
	2	Taibi *et al*. 2015	$= 0$	1,08997	0,519461	0,0876306	0.0805702
		Akavci 2016	$\neq 0$	1.07386	0,519785	0.0758855	0,0705695
		Present	$= 0$	1.08085	0.519427	0.075402	0.0701153
		Present	$\neq 0$	1.075741	0.520220	0.0758944	0.0705772
2-1-2	0.5	Taibi *et al*. 2015	$= 0$	0,8604107	0,462484	0,0858464	0.079059
		Akavci 2016	$\neq 0$	0.855014	0,462481	0,0745356	0.0694005
		Present	$= 0$	0.8603967	0.462480	0.0740781	0.0689690
		Present	$\neq 0$	0.856336	0.462864	0.074545	0.0694089
	2	Taibi *et al*. 2015	$= 0$	1066384	0,516062	0,0875334	0.080488
		Akavci 2016	$\neq 0$	1,05934	0,516358	0,0758117	0,705056
		Present	$= 0$	1.066248	0.516031	0.0753303	0.070053
		Present	$\neq 0$	1.061188	0.516793	0.0758212	0.0705140
1-1-1	0.5	Taibi *et al*. 2015	$= 0$	0,838977	0,456219	00856283	0,0788745
		Akavci 2016	$\neq 0$	0,833746	0,456185	0.0743699	0.0692568
		Present	$= 0$	0.838979	0.456220	0.0739157	0.0688281
		Present	$\neq 0$	0.8350130	0.4565615	0.0743797	0.0692654
	2	Taibi *et al*. 2015	$= 0$	1,024387	0,506023	0,0872398	0,0872398
		Akavci 2016	$\neq 0$	1.01766	0.506246	0,0755889	0,0703128
		Present	$= 0$	1.024298	0.506002	0.0751129	0.0698651
		Present	$\neq 0$	1.019386	0.506675	0.0755999	0.0703225

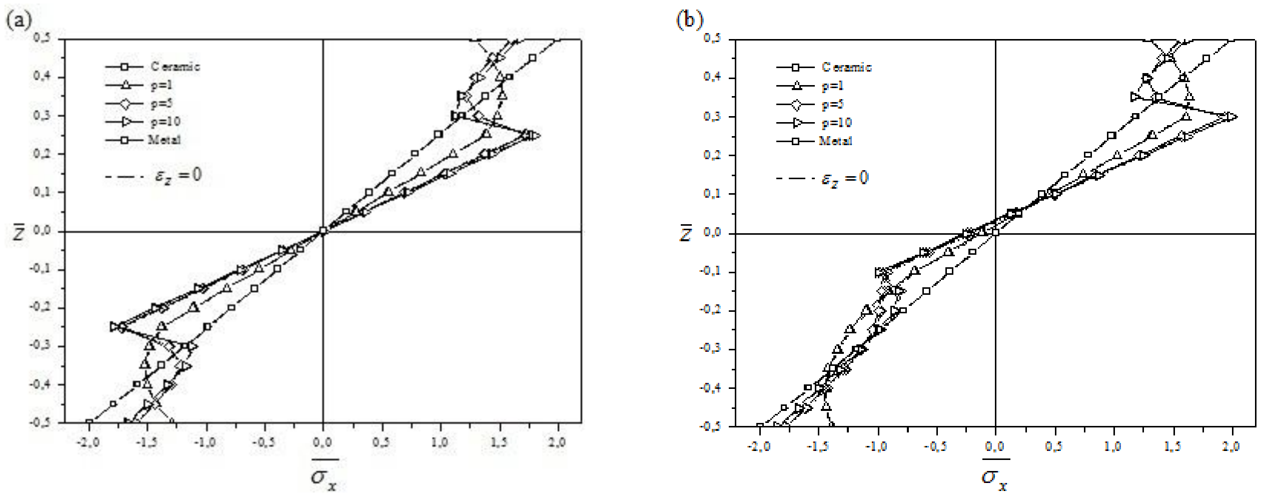
2013. Since the present quasi-3D theory and other quasi-3D theories include the thickness stretching effect, their solutions are very close to each other. However, the 2D solutions, which omits this effect, gives inaccurate prediction and slightly overestimates the deflection. Also, it can be observed from the Table 2 that the quasi-3D solutions, obtained lower transverse displacement and higher axial stress than the 2D solutions which eliminate the stretching effect and increasing value of the power-law exponent p increases the center displacement in all sequences. The difference between shear deformation theories is less significant when ε_z = 0 especially for fully ceramic plates (p = 0).

Table 4 presents values of transverse shear stress $\bar{\tau}_{xz}$ for p = 0, 1, 2, and 10 and different types of sandwich plates. The obtained results are compared with the different



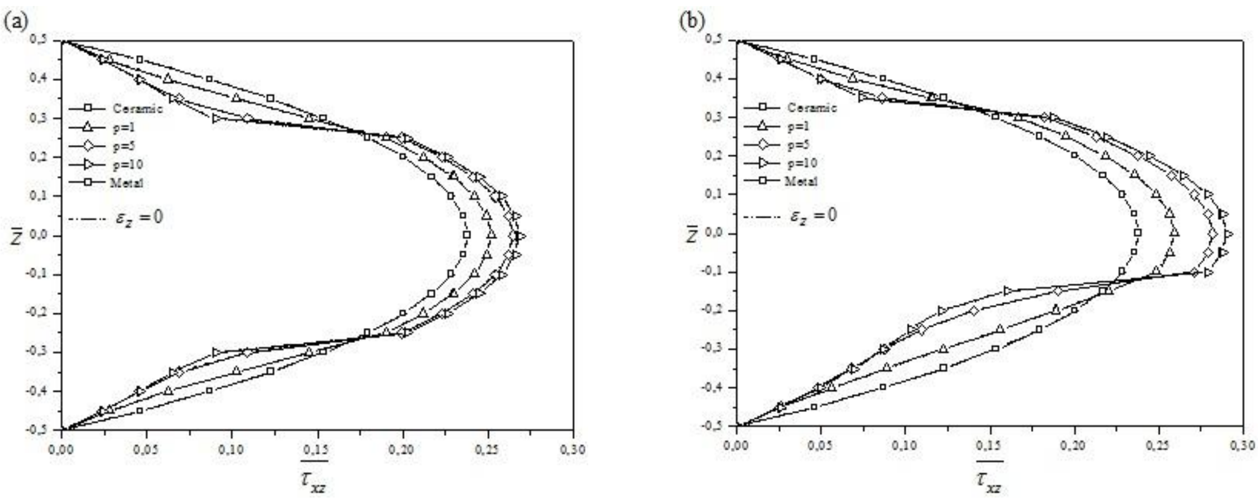
(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 3 The transverse displacement, $\bar{w}$, through the thickness of symmetric and unsymmetric sandwich square plates (a/h = 10)



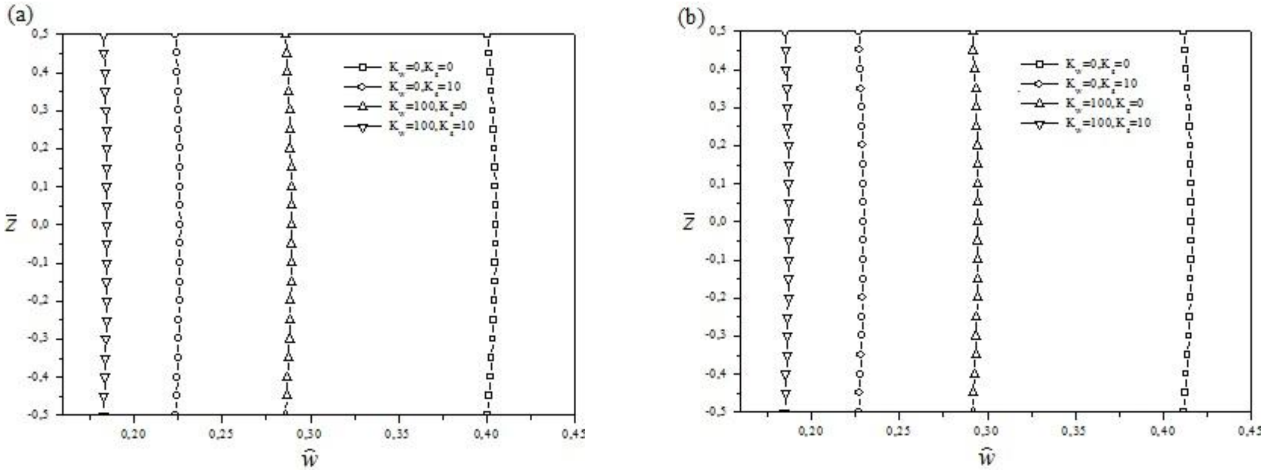
(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 4 The axial stress, $\bar{w}$, through the thickness of symmetric and unsymmetric sandwich square plates (a/h = 10)



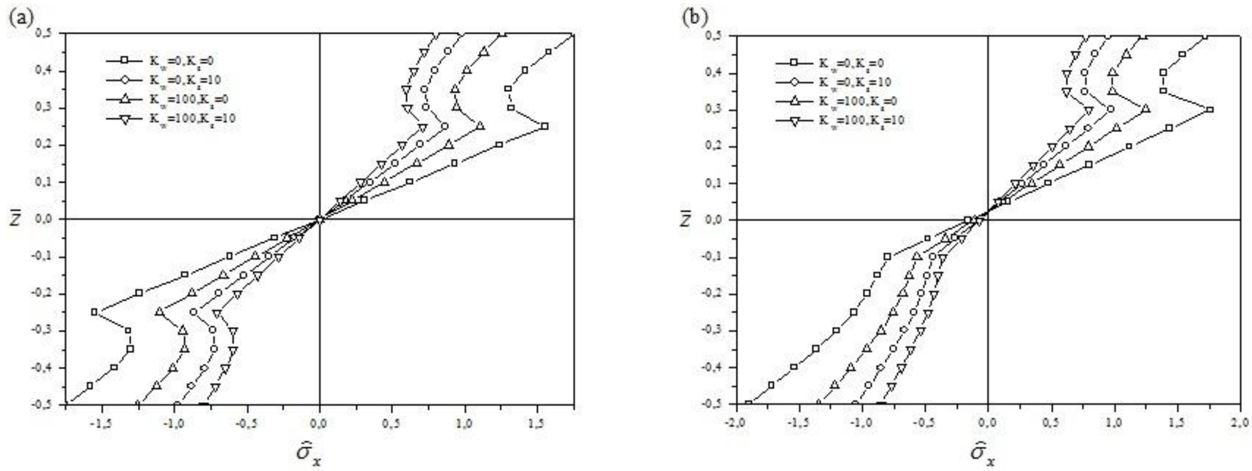
(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 5 The transverse shear stress, $\bar{\tau}_{xz}$, through the thickness of symmetric and unsymmetric sandwich square plates (a/h = 10)



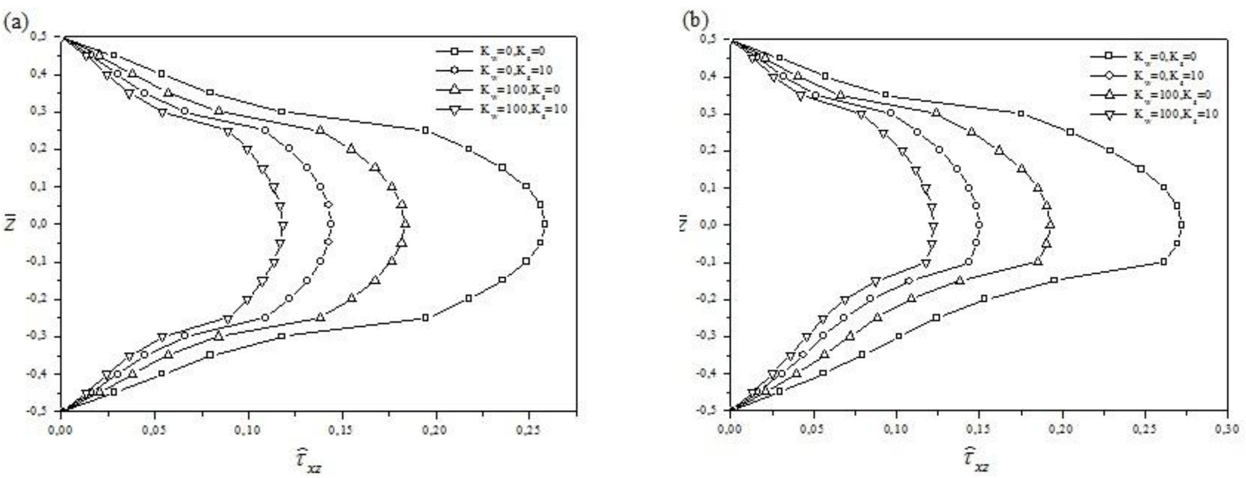
(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 6 Effect of Winkler and Pasternak modulus parameter on the dimensionless center deflection $\hat{w}$ through the thickness of a square FG sandwich plate (p = 5, a/h = 10)



(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 7 Effect of Winkler and Pasternak modulus parameter on the dimensionless axial stress $\hat{\sigma}_x$ through the thickness of a square FG sandwich plate (p = 5, a/h = 10)



(a) The (1-2-1) FG sandwich plate (b) The (2-2-1) FG sandwich plate

Fig. 8 Effect of Winkler and Pasternak modulus parameter on the dimensionless transverse shear stress $\hat{\tau}_{xz}$ through the thickness of a square FG sandwich plate (p = 5, a/h = 10) two (Neves *et al*. 2012c) and quasi-three dimensional theories (Neves *et al*. 2012c, Bessaim *et al*. 2013). There is a little difference between the results and this is due to the different approaches used to predict the response of the FG sandwich plate. But, in general, a good agreement between the results is found. The transverse shear stress $\bar{\tau}_{xz}$ increases as *p* increases.

Fig. 3 show the influence of the volume fraction index *p* on the variation of the out-of-plane displacement ($\overline{w}$) through the thickness direction for both symmetric and unsymmetric square FG sandwich plates with side-to-thickness ratio *a*/*h* = 10. The results are plotted by using both the present 2D and quasi-3D shear deformation theories. It can be seen that the transverse displacement ($\overline{w}$) of metal plates is larger than the corresponding one of ceramic plates and in general, the transverse displacement increases as the volume fraction index *p* increases.

Fig. 4 contain plot of the axial stress $\overline{\sigma_x}$ for various values of volume fraction index *p* through-the-thickness of rectangular FGM sandwich plate for both symmetric and unsymmetric square FG sandwich plates with side-to-thickness ratio *a*/*h* = 10. The maximum compressive stresses occur at a point on the top surface and the maximum tensile stresses occur, of course, at a point on the bottom surface of the FGM sandwich plate.

The homogeneous ceramic plate or metal plate yields the maximum compressive stresses at the bottom surface and the minimum tensile stresses at the top surface of the sandwich plate.

In Fig. 5 we have plotted the through-the-thickness distributions of the transverse shear stress the maximum value occurs at a point on the mid-plane of the plate for symmetric or homogeneous plates. It can be observed that the transverse shear stresses for non-symmetric FG sandwich plates are not parabolic and increasing the volume fraction index *p* leads to increase of the transverse shear stress in the skin of the plate which can increase the resistance of sandwich plates to face sheet debonding.

The second example to prove the validity of present Quasi-3D hyperbolic plate theory for bending response of a simply supported (T_i–6Al–4V/ZrO$_2$) FG sandwich plate resting on elastic foundation, the obtained numerical results presented in Table 4 and compared with the 2D solutions of Taibi *et al*. (2015) and the quasi-3D solutions of Akavci (2016). Additional results are plotted in Figs. 6–8 using the present Quasi-3D hyperbolic plate theory with ($\varepsilon_z \neq 0$).

Table 4 contain dimensionless center deflection $\hat{w}$ for an FG sandwich plate subjected to mechanical loads without elastic foundation or resting on one- or two-parameter elastic foundations for different values of volume fraction index *p* and several kinds of FG sandwich plates. It can be seen that the present results are in agreement with the published results for a simply supported FG sandwich plate on elastic foundation. As the volume fraction index *p* increases for FG plates, the deflection $\hat{w}$ will increase. Also, the inclusion of the Winkler foundation parameter gives results more than those with the inclusion of Pasternak foundation parameters. It can be shown that the deflections are decreasing with the existence of the elastic foundations. Since the quasi-3D model of the present formulation includes the thickness stretching effect, the non-dimensional center deflection $\hat{w}$ are slightly decreased with respect to other center deflection documented in Table 4 obtained by 2D solutions. Thus, the inclusion of thickness stretching effect makes the FG sandwich plates stiffer.

The effect of foundation stiffness and side-to-thickness ratio on the dimensionless deflection $\hat{w}$ for both symmetric and unsymmetric FG sandwich square plate (*k* = 5, *a*/*h* = 10) is shown in Fig. 6. It is seen from Fig. 6 that as the elastic foundation increase the center dimensionless deflection $\hat{w}$ of the FG sandwich plates decreases. Figs. 7 and 8 plot the influences of foundation stiffness on the dimensionless axial stress $\hat{\sigma}_x$ and the transverse shear stress $\hat{\tau}_{xz}$ through-the-thickness of the symmetric and unsymmetric FG sandwich square plate (*k* = 5, *a*/*h* = 10). It is seen from the figures that the axial normal and transverse shear stresses decrease gradually with the increasing value of foundation stiffness.

5. Conclusions

A novel Quasi-3D hyperbolic shear deformation plate theory is developed for bending response of FG sandwich plates resting on elastic foundation. By considering further simplifying suppositions to the existing Quasi-3D theory, with incorporation of an undetermined integral term, the present theory has only five unknowns, which is even less than the other shear and normal deformation theories, and hence, make this model simple and efficient to employ. Equations of motion are obtained by utilizing the Hamilton's principles and then are solved using Navier's procedure. The accuracy of the present work is ascertained by comparing it with existing solutions and excellent agreement was observed. Results show that the inclusion of thickness stretching effect ($\varepsilon_z \neq 0$) makes a nanoplates stiffer, and hence, leads to decrease of the transverse displacement. In conclusion, it can be said that the present theory is not only accurate but also simple in predicting displacements and stresses of FG sandwich plates without elastic foundation or resting on one- or two-parameter elastic foundations. The formulation lend sit self particularly well to study several problems related to the hygro-thermo-mechanical deformation of laminated and FG structures as is studied by Tounsi and his co-workers (Bouderba *et al*. 2016, Beldjelili *et al*. 2016, Bousahla *et al*. 2016, Chikh *et al*. 2017), also by including the size-dependent effect for analysis of mechanical behaviour of micro/structure (Adda Bedia *et al*. 2015, Belkorissat *et al*. 2015, Larbi Chaht *et al*. 2015, Kolahchi and Bidgoli 2016, Arani and Kolahchi 2016, Bouafia *et al*. 2017), which will be considered in the near future.

References

Abrate, S. (2008), "Functionally graded plates behave like homogeneous plates", *Compos. Part B: Eng.*, **39**(1), 151-158.

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Adda Bedia, W., Benzair, A., Semmah, A., Tounsi, A. and Mahmoud, S.R. (2015), "On the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity", *Brazil. J. Phys.*, **45**(2), 225-233.

Ahouel, M., Houari, M.S.A., AddaBedia, E.A. and Tounsi, A. (2016) "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct., Int. J.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Yahia, S., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.

Akavci, S.S. (2016), "Mechanical behavior of functionally graded sandwich plates on elastic foundation", *Compos. Part B: Eng.*, **96**, 136-152.

Akbaş, Ş.D. (2015), "Wave propagation of a functionally graded beam in thermal environments", *Steel Compos. Struct., Int. J.*, **19**(6), 1421-1447.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Al-Hosani, K., Fadhil, S. and El-Zafrany, A. (1999), "Fundamental solution and boundary element analysis of thick plates on Winkler foundation", *Comput. Struct.*, **70**(3), 325-336.

Amar, L.H.H., Kaci, A. and Tounsi, A. (2017), "On the size-dependent behavior of functionally graded micro-beams with porosities", *Struct. Eng. Mech., Int. J.*, **64**(5), 527-541.

Attia, A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct., Int. J.*, **18**(1), 187-212.

Arani, A.J. and Kolahchi, R. (2016), "Buckling analysis of embedded concrete columns armed with carbon nano-tubes", *Comput. Concrete, Int. J.*, **17**(5), 567-578.

Baltacıoğlu, A.K., Civalek, Ö., Akgöz, B. and Demir, F. (2011), "Large deflection analysis of laminated composite plates resting on nonlinear elastic foundations by the method of discrete singular convolution", *Int. J. Press. Ves. Pip.*, **88**(8), 290-300.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B: Eng.*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct., Int. J.*, **25**(3), 257-270.

Belouettar, S., Abbadi, A., Azari, Z., Belouettar, R. and Freres, P. (2009), "Experimental investigation of static and fatigue behaviour of composites honeycomb materials using four point bending tests", *Compos. Struct.*, **87**(3), 265-273.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), "An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities", *Earthq. Struct., Int. J.*, **13**(3), 255-265.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech., Int. J.*, **65**(1), 19-31.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five-variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Benyoucef, S., Mechab, I., Tounsi, A., Fekrar, A. and Atmane, H.A. (2010), "Bending of thick functionally graded plates resting on Winkler–Pasternak elastic foundations", *Mech. Compos. Mater.*, **46**(4), 425-434.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater.*, **15**(6), 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst., Int. J.*, **19**(6), 601 - 614.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **19**(2), 115-126.

Boukhari, A., Atmane, H.A., Tounsi, A., Adda, B. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech., Int. J.*, **58**(3), 397-422.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.

Bourada, M., Tounsi, A. and Houari, M.S.A. (2011), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech., Int. J.*, **60**(2), 313-335.

Brischetto, S. (2017), "A general exact elastic shell solution for bending analysis of functionally graded structures", *Compos. Struct.*, **175**, 70-85.

Carrera, E., Brischetto, S., Cinefra, M. and Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos. Part B: Eng.*, **42**(2), 123-133.

Chen, C.S., Liu, F.H. and Chen, W.R. (2017), "Vibration and stability of initially stressed sandwich plates with FGM face sheets in thermal environments", *Steel Compos. Struct., Int. J.*, **23**(3), 251-261.

Cheng, Z.Q. and Batra, R.C. (2000), "Deflection relationships between the homogeneous Kirchhoff plate theory and different functionally graded plate theories", *Arch. Mech.*, **52**(1), 143-158.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst., Int. J.*, **19**(3), 289-297.

Darilmaz, K. (2009), "An assumed-stress hybrid element for modeling of plates with shear deformations on elastic foundation", *Struct. Eng. Mech., Int. J.*, **33**(5), 573-588.

Darilmaz, K. (2015), "Vibration analysis of functionally graded material (FGM) grid systems", *Steel Compos. Struct., Int. J.*, **18**(2), 395-408.

Darilmaz, K., Aksoylu, M.G. and Durgun, Y. (2015), "Buckling analysis of functionally graded material grid systems", *Struct. Eng. Mech., Int. J.*, **54**(5), 877-890.

Delale, F. and Erdogan, F. (1983), "The crack problem for a non-homogeneous plane", *ASME J. Appl. Mech.*, **50**(3), 609-614.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng., Int. J.*, **11**(5), 671-690.

Ebrahimi, F. and Habibi, S. (2016), "Deflection and vibration analysis of higher-order shear deformable compositionally graded porous plate", *Steel Compos. Struct., Int. J.*, **20**(1), 205-225.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech., Int. J.*, **63**(5), 585-595.

Ferreira, A.J.M., Batra, R.C., Roque, C.M.C., Qian, L.F. and Jorge, R.M.N. (2006), "Natural frequencies of functionally graded plates by a meshless method", *Compos. Struct.*, **75**(1), 593-600.

Ferreira, A.J.M., Castro, L.M. and Bertoluzza, S. (2009), "A high order collocation method for the static and vibration analysis of composite plates using a first-order theory", *Compos. Struct.*, **89**(3), 424-432.

Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, M., Tounsi, A. and Mahmoud, S.R. (2017), "A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct., Int. J.*, **25**(6),717-726.

Hadj Henni, A., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., S.R. Mahmoud and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct., Int. J.*, **25**(6),693-704.

Hadji, L., Atmane, H.A., Tounsi, A., Mechab, I. and Adda Bedia, E. A. (2011), "Free vibration of functionally graded sandwich plates using four-variable refined plate theory", *Appl. Math. Mech..*, **32**(7), 925-942.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.

He, X.Q., Ng, T.Y., Sivashanker, S. and Liew, K.M. (2001), "Active control of FGM plates with integrated piezoelectric sensors and actuators", *Int. J. Solids. Struct.*, **38**(9), 1641-1655.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech.*, (*ASCE*), **140**(2), 374-383.

Hebali, H., Bakora, A., Tounsi, A. and Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(3),473-495.

Houari, M.S.A., Benyoucef, S., Mechab, I., Tounsi, A. and Adda Bedia, E.A. (2011), "Two-variable refined plate theory for thermoelastic bending analysis of functionally graded sandwich plates", *J. Therm. Stress.*, **34**(4), 315-334.

Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int. J. Mech. Sci.*, **76**, 102-111.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S. R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(2), 257-276.

Kant, T. and Swaminathan, K. (2002), "Analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory", *Compos. Struct.*, **56**(4), 329-344.

Kar, V.R. and Panda, S.K. (2014), "Large deformation bending analysis of functionally graded spherical shell using FEM", *Struct. Eng. Mech., Int. J.*, **53**(4), 661-679.

Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct., Int. J.*, **18**(3), 693-709.

Ke-rang, W. (1990), "Thick rectangular plates with free edges on elastic foundations", *Appl. Math. Mech. Eng. Ed.*, **11**(9), 869-879.

Kolahchi, R. (2017), "A comparative study on the bending, vibration and buckling of viscoelastic sandwich nano-plates based on different nonlocal theories using DC, HDQ and DQ methods", *Aerosp. Sci. Technol.*, **66**, 235-248.

Kolahchi, R. and Bidgoli, A.M. (2016), "Size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes", *Appl. Math. Mech. Engl. Ed.*, **37**, 265-274.

Kolahchi, R., Bidgoli, A.M.M. and Heydari, M.M. (2015), "Size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium", *Struct. Eng. Mech., Int. J.*, **55**(5), 1001-1014.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016), "Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium", *Compos. Struct.*, **150**, 255-265.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Oskouei, A.N. (2017a), "Visco-nonlocal-refined Zigzag theories for dynamic buckling of laminated nanoplates using differential cubature-Bolotin methods", *Thin-Wall. Struct.*, **113**, 162-169.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Nouri, A. (2017b), "Wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator based on refined zigzag theory", *Int. J. Mech. Sci.*, **130**, 534-545.

Khetir, H., Bouiadjra, M.B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech., Int. J.*, **64**(4), 391-402.

Krenich, F., Heireche, H., Houari, M.S.A. and Tounsi, A. (2017), "A novel nonlocal four variable plate theory for thermal stability of single- layered graphene sheets embedded in an elastic substrate medium", *Current Nanomater.*, **1**(3), 215-222.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Bég, O.A. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct., Int. J.*, **18**(2), 425-442.

Li, Q., Iu, V.P. and Kou, K.P. (2008), "Three-dimensional vibration analysis of functionally graded material sandwich plates", *J. Sound Vib.*, **311**(1), 498-515.

Liu. F.L. and Liew, K.M. (1999), "Analysis of vibrating thick rectangular plates with mixed boundary constraints using differential quadrature element method", *J. Sound Vib.*, **225**(5), 915-934.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Marur, P.R. (1999), "Fracture behaviour of functionally graded materials", Ph.D. Thesis; Auburn University, AL, USA.

Meftah, A., Bakora, A., Zaoui, F.Z., Tounsi, A. and Adda Bedia, E.A. (2017), "A non-polynomial four variable refined plate theory for free vibration of functionally graded thick rectangular plates on elastic foundation", *Steel Compos. Struct., Int. J.*, **23**(3), 317-330.

Merdaci, S., Tounsi, A., Houari, M.S.A., Mechab, I., Hebali, H. and Benyoucef, S. (2011), "Two new refined shear displacement models for functionally graded sandwich plates", *Arch. Appl. Mech.*, **81**(11), 1507-1522.

Merdaci, S., Tounsi, A. and Bakora, A. (2016), "A novel four variable refined plate theory for laminated composite plates", *Steel Compos. Struct., Int. J.*, **22**(4),713-732.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct., Int. J.*, **25**(2), 157-175.

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst., Int. J.*, **20**(3), 369-383.

Najafov, A.M., Sofiyev, A.H., Hui, D., Karaca, Z., Kalpakci, V. and Ozcelik, M. (2014), "Stability of EG cylindrical shells with shear stresses on a Pasternak foundation", *Steel Compos. Struct., Int. J.*, **17**(4), 453-470.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Roque, C.M.C., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2011), "Bending of FGM plates by a sinusoidal plate formulation and collocation with radial basis functions", *Mech. Res. Commun.*, **38**(5), 368-371.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Roque, C.M.C., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2012a), "A quasi-3D sinusoidal shear deformation theory for the static and free vibration analysis of functionally graded plates", *Compos. Part B: Eng.*, **43**(2), 711-725.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Cinefra, M., Roque, C.M.C., Jorge, R.M.N. and Soares, C.M.M. (2012b), "A quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *Compos. Struct.*, **94**(5), 1814-1825.

Neves, A.M.A., Ferreira, A.J., Carrera, E., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2012c), "Static analysis of functionally graded sandwich plates according to a hyperbolic theory considering Zig-Zag and warping effects", *Adv. Eng. Softw.*, **52**, 30-43.

Praveen, G.N. and Reddy, J.N. (1998), "Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates", *Int. J. Solids. Struct.*, **35**(33), 4457-4476.

Reddy, J.N. (2000), "Analysis of functionally graded plates", *Int. J. Numer. Methods. Eng.*, **47**(1-3), 663-684.

Reddy, J.N. (2002), *Energy Principles and Variational Methods in Applied Mechanics*, John Wiley & Sons Inc., New York, NY, USA.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng., Int. J.*, **11**(2), 289-307.

Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech., Int. J.*, **64**(6), 737-749.

Sofiyev, A.H. and Kuruoglu, N. (2015a), "Buckling of non-homogeneous orthotropic conical shells subjected to combined load", *Steel Compos. Struct., Int. J.*, **19**(1), 1-19.

Sofiyev, A.H., Hui, D., Najafov, A.M., Turkaslan, S., Dorofeyskaya, N. and Yuan, G.Q. (2015b), "Influences of shear stresses and rotary inertia on the vibration of functionally graded coated sandwich cylindrical shells resting on the Pasternak elastic foundation", *J. Sandw. Struct. Mater.*, **17**(6), 691-720.

Sofiyev, A.H., Hui, D., Valiyev, A.A., Kadioglu, F., Turkaslan, S., Yuan, G.Q., Kalpaksi, V. and Özdemir, A. (2016), "Effects of shear stresses and rotary inertia on the stability and vibration of sandwich cylindrical shells with FGM core surrounded by elastic medium", *Mech. Des. Struct. Mach.*, **44**(4), 384-404.

Sofiyev, A.H. and Osmancelebioglu, E. (2017), "The free vibration of sandwich truncated conical shells containing functionally graded layers within the shear deformation theory", *Compos. Part B: Eng.*, **120**, 197-211.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A-Solid.*, **47**, 349-361.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A. and Benyoucef, S. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech., Int. J.*, **60**(4), 547-565.

Vel, S.S., Caccese, V. and Zhao, H. (2005), "Elastic coupling effects in tapered sandwich panels with laminated anisotropic composite facings", *J. Compos. Mater.*, **39**(24), 2161-2183.

Vinson, J.R. (2001), "Sandwich structures", *Appl. Mech. Rev.*, **54**(3), 201-214.

Wang, Z.X. and Shen, H.S. (2012), "Nonlinear vibration and bending of sandwich plates with nanotube-reinforced composite face sheets", *Compos. Part B: Eng.*, **43**(2), 411-421.

Woo, J., Meguid, S.A. and Ong, L.S. (2006), "Nonlinear free vibration behavior of functionally graded plates", *J. Sound Vib.*, **289**(3), 595-611.

Yaghoobi, H. and Yaghoobi, P. (2013), "Buckling analysis of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions: An analytical approach", *Meccanica.*, **48**(8), 2019-2035.

Yaghoobi, H., Valipour, M.S., Fereidoon, A. and Khoshnevisrad, P. (2014), "Analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM", *Steel Compos. Struct., Int. J.*, **17**(5), 753-776.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: An assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech., Int. J.*, **54**(4), 693-710.

Zenkour, A.M. (2005a), “A comprehensive analysis of functionally graded sandwich plates: Part 1 - Deflection and stresses”, *Int. J. Solids Struct*., **42**(18-19), 5224-5242.

Zenkour, A.M. (2005b), “A comprehensive analysis of functionally graded sandwich plates: Part 2 - Buckling and free vibration”, *Int. J. Solids Struct*., **42**(18-19), 5243-5258.

Zenkour, A.M. and Sobhy, M. (2010), “Thermal buckling of various types of FGM sandwich plates”, *Compos. Struct*., **93**(1), 93-102.

Zhang, D.G. and Zhou, Y.H. (2008), “A theoretical analysis of FGM thin plates based on physical neutral surface”, *Comput. Mater. Sci*., **44**(2), 716-720.

Zidi, M., Tounsi, A., Houari, M.S.A. and Bég, O.A. (2014), “Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory”, *Aerosp. Sci. Technol*., **34**, 24-34.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017),”A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams”, *Struct. Eng. Mech., Int. J*., **64**(2), 145-153.

CC