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A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations

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Abstract. In this paper, a new simple shear deformation theory for bending analysis of functionally graded plates is developed. The present theory involves only three unknown and three governing equation as in the classical plate theory, but it is capable of accurately capturing shear deformation effects, instead of five as in the well-known first shear deformation theory and higher-order shear deformation theory. A shear correction factor is, therefore, not required. The material properties of the functionally graded plates are assumed to vary continuously through the thickness, according to a simple power law distribution of the volume fraction of the constituents. Equations of motion are obtained by utilizing the principle of virtual displacements and solved via Navier's procedure. The elastic foundation is modeled as two parameter elastic foundation. The results are verified with the known results in the literature. The influences played by transversal shear deformation, plate aspect ratio, side-to-thickness ratio, elastic foundation, and volume fraction distributions are studied. Verification studies show that the proposed theory is not only accurate and simple in solving the bending behaviour of functionally graded plates, but also comparable with the other higher-order shear deformation theories which contain more number of unknowns.

Keywords: a simple 3-unknown theory; bending; functionally graded plates, elastic foundation

1. Introduction

Functionally graded plates are widely used in the aerospace, aircrafts, automotive industry, marine and other structural applications because of advantageous features such as to eliminates the interface problems of conventional composite materials and thus the stress distribution becomes smooth (Li *et al.* 2008). In company with the increase in the application of functionally graded plates in engineering structures, a variety of plates theories have been developed to predict its behavior. A critical review of more recent works on the development of plates theories can be found in (Ghugal and Shimpi 2002, Sayad and Ghugal 2016). These plate theories can be divided into three following categories, classical plate theory (CPT), first-order shear deformation plate theory (FSDT) and higher-order plate theory (HSDT). The CPT ignores shear deformation effects and provides reasonable results for thin plates and gives acceptable results for functionally graded (FG) thin structures (plates) only (Abrate 2008, Arefi 2015, Pradhan and Chakraverty 2015a, Darilmaz 2015). However, it underestimates deflection and overestimates buckling load and frequency of moderately thick or thick plates (Ghugal and Shimpi 2002). The FSDT accounts for the transverse shear deformation effect by the way of linear variation of in-plane displacement through the thickness and gives acceptable results for moderately thick and thin plates, but needs a shear correction factor which is hard to find as it depends on the geometries, material properties and boundary conditions of each problem (Ferreira *et al.* 2009, Adda Bedia *et al.* 2015, Bellifa *et al.* 2016, Bouderba *et al.* 2016). A shear correction factor is required to compensate for the difference between actual stress state and assumed constant stress state (Castellazzi *et al.* 2013). Hosseini-Hashemi *et al.* (2010) studied the free vibration of moderately thick rectangular FG plates resting on elastic foundations. Yaghoobi and Yaghoobi (2013) investigated the buckling analysis of FG sandwich plates resting on an elastic foundation based on the first-order shear deformation plate theory and under thermo-mechanical loads. Meksi *et al.* (2015) studied the bending and the free vibration of FG plates using a novel simple first-order shear deformation plate theory based on neutral surface position and supported by either Winkler or Pasternak elastic foundations. Chen *et al.* (2017) studied the thermal buckling and vibration of FG sandwich plates, including the effects of transverse shear deformation and rotary inertia. The

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HSDTs account for shear deformation effects by higher-order variations of in-plane displacements or both in-plane and transverse displacements through the thickness, and do not required any shear correction factor and satisfy zero shear stress conditions at top and bottom surfaces of plates (Bourada *et al.* 2012, Bessaim *et al.* 2013, Ahouel *et al.* 2016, Ait Amar Meziane *et al.* 2014, Ait Atmane *et al.* 2015, Mahi *et al.* 2015, Ait Yahia *et al.* 2015, Attia *et al.* 2015, Belabed *et al.* 2014, Larbi Chaht *et al.* 2015, Belkorissat *et al.* 2015, Bounouara *et al.* 2016, Bousahla *et al.* 2016, Bennoun *et al.* 2016, Beldjelili *et al.* 2016, Draiche *et al.* 2016, Bellifa *et al.* 2017, Benchohra *et al.* 2017, El-Haina *et al.* 2017, Menasria *et al.* 2017, Meksi *et al.* 2017, Bouafia *et al.* 2017, Besseghier *et al.* 2017, Klouche *et al.* 2017, Zidi *et al.* 2017, Khetir *et al.* 2017). Zenkour (2006) studied the static behavior of a rectangular FG plate under simply supported condition and subjected to uniform transverse load based on the sinusoidal shear deformation theory. Pradhan and Chakraverty (2015b) expressed the trial functions as the linear combinations of simple algebraic polynomials to study free vibration of thick rectangular plates based on new inverse trigonometric shear deformation theories. Ait Atmane *et al.* (2010) analyzed free vibration of simply supported FG plates resting on a Winkler-Pasternak elastic foundation by a new hyperbolic shear deformation theory. Benyoucef *et al.* (2010) examined the static response of simply supported FG plates resting on an elastic foundation using a new hyperbolic displacement model. Bouderba *et al.* (2013) studied the thermo-mechanical bending response of FG plates resting on elastic foundations using a refined trigonometric shear deformation theory. Said *et al.* (2014) studies the bending response of functionally graded plates resting on a Winkler-Pasternak elastic foundation by employing the physical neutral surface concept. Taibi *et al.* (2015) presented a simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations. In (Li *et al.* 2016) a new refined plate theory for wave propagation analysis of simply supported functionally graded plate with only four unknown functions was developed. Bending behaviour of laminated composite flat panel under hygro-thermo-mechanical loading was presented by Kar *et al.* (2015) using higher-order plate theory (HSDT). Mehar and Panda (2016) investigated the non-linear bending behavior of functionally graded carbon nanotube reinforced composite (FG-CNTRC) flat panel under the thermo-mechanical load based on the higher order shear deformation theory. The same membrane analogy was later applied to the analyses of functionally graded carbon nanotube reinforced composite (FG-CNTRC) plates and shells under thermal and mechanical load (Mehar *et al.* 2016, Mehar and Panda 2017a, b, Mehar *et al.* 2017, Mahapatra *et al.* 2017). This work aims to develop a new simple shear deformation theory for the bending response of FG plates resting on a Winkler-Pasternak elastic foundation. The most interesting feature of this theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The proposed theory contains fewer unknowns and equations of motion than the first-order shear deformation theory. Indeed, unlike the previous mentioned theories, the number of variables in the present theory is same as that in the CPT. Equations of motion are obtained by utilizing the principle of virtual displacements. In this study, analytical of bending solutions are obtained for a simply supported isotropic and FG plate and accuracy is verified by comparing the obtained results with those reported in the literature.

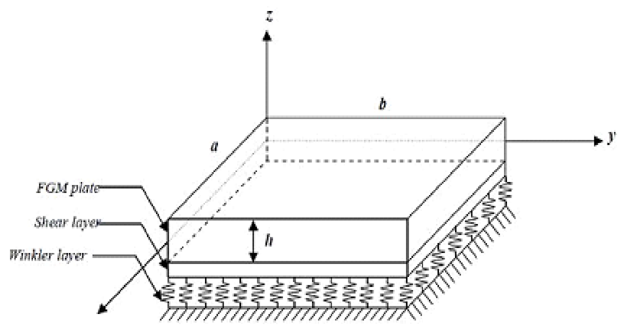

Fig. 1 Coordinate system and geometry for rectangular FG plates on Pasternak elastic foundation

2. Theoretical formulation

Consider a solid rectangular plate of length a, width b and thickness h made of functionally graded material with the coordinate system as shown in Fig. 1. It is assumed to be rested on a Winkler-Pasternak type elastic foundation with the Winkler stiffness of k_w and shear stiffness of k_s. The material properties of the FGM plate, such as Young's modulus E is assumed to be function of the volume fraction of constituent materials. Let the FG plate be subjected to a transverse load $q(x,y)$. Unlike the previous mentioned theories, the number of unknown functions involved in the present theory is only three as in CPT.

2.1 Kinematics of the present plate model

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows (Houari *et al*. 2016, Mouffoki *et al*. 2017)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} - \beta\ f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} - \beta\ f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u_0, v_0 and w_0 are three unknown displacement functions of midplane of the plate and β is a parameter of the present displacement model. $f(z)$ is a shape function representing the distribution of the transverse shear strains and shear stresses through the thickness of the plate and is given as (Nguyen *et al.* 2014, Nguyen *et al.* 2015)

$$f(z) = h\,\arctan\left(\frac{z}{h}\right) - \frac{16z^3}{15h^2} \tag{2}$$

The nonzero linear strains related to displacement field in Eq. (1) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} + \beta\, f(z)\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix},$$
$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \beta\, g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{3}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix},$$
$$\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^4 w_0}{\partial x^4}\\ -\dfrac{\partial^4 w_0}{\partial y^4}\\ -\dfrac{\partial^2(\nabla^2 w_0)}{\partial x\partial y}\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^3 w_0}{\partial y^3}\\ -\dfrac{\partial^3 w_0}{\partial x^3}\end{Bmatrix}, \tag{4}$$

and

$$g(z) = f'(z), \quad \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \tag{5}$$

2.2 Constitutive relations

The plate is graded from aluminium (bottom) to alumina (top). The mechanical properties of FGM are determined from the volume fraction of the material constituents. Young's modulus, E, is assumed to vary in the thickness direction based on the Voigt's rule over the whole range of the volume fraction. The effective material properties of FGM with two constituents can be expressed as (Fahsi *et al.* 2017, Sekkal *et al.* 2017, Tounsi *et al.* 2013, Boukhari *et al.* 2016, Bourada *et al.* 2015, Meradjah *et al.* 2015, Hamidi *et al.* 2015, Hebali *et al.* 2014, Zidi *et al.* 2014, Bousahla *et al.* 2014, Fekrar *et al.* 2014)

$$E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2}\right)^k \tag{6}$$

where (E_m) and (E_c) are the corresponding properties of the metal and ceramic, respectively, and k is the volume fraction exponent which takes values greater than or equal to zero. The value of k equals to zero represents a fully ceramic plate. The above power-law assumption reflects a simple rule of mixtures used to obtain the effective properties of the ceramic-metal plate.

The constitutive relations of a FG plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{7}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \nu\, C_{11} \tag{8a}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)}, \tag{8b}$$

2.3 Governing equations

The governing equations can be obtained using the principle of virtual displacements. The principle can be stated in the following form

$$\int_{-h/2}^{h/2}\int_{\Omega}\big[\sigma_x\delta\,\varepsilon_x + \sigma_y\delta\,\varepsilon_y + \tau_{xy}\delta\,\gamma_{xy} + \tau_{yz}\delta\,\gamma_{yz} + \tau_{xz}\delta\,\gamma_{xz}\big]d\Omega\,dz - \int_{\Omega}(q - f_e)dw d\Omega = 0 \tag{9}$$

Substituting Eqs. (3), (1) and (6) into Eq. (9) and integrating through the thickness of the plate, Eq. (9) can be rewritten as

$$\begin{aligned}&\int_{\Omega}\big[N_x\delta\,\varepsilon_x^0 + N_y\delta\,\varepsilon_y^0 + N_{xy}\delta\,\gamma_{xy}^0 + M_x\delta\,k_x\\ &+ M_y\delta\,k_y + M_{xy}\delta\,k_{xy} + \beta(S_x\delta\,\eta_x + S_y\delta\,\eta_y\\ &+ S_{xy}\delta\,\eta_{xy} + Q_{yz}\delta\,\gamma_{yz}^0 + Q_{xz}\delta\,\gamma_{xz}^0)\big]d\Omega\\ &- \int_{\Omega}(q - f_e)\delta\, w d\Omega = 0\end{aligned} \tag{10}$$

where Ω is the area of top surface and f_e is the density of reaction force of foundation. For the Pasternak foundation model

$$f_e = k_w w - k_{sx}\frac{\partial^2 w}{\partial x^2} - k_{sy}\frac{\partial^2 w}{\partial y^2} \tag{11}$$

where k_w is the modulus of subgrade reaction (elastic coefficient of the foundation) and k_{sx} and k_{sy} are the shear moduli of the subgrade (shear layer foundation stiffness). If foundation is homogeneous and isotropic, we will get k_{sx}= k_{sy}=k_s. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation.

In which the stress resultants N, M, S and Q are defined by

$$\left(N_i, M_i, S_i\right) = \int_{-h/2}^{h/2} (1, z, \beta f)(\sigma_i)\,dz\,, \quad (i = x, y, xy)$$
$$\text{and} \quad Q_i = \int_{-h/2}^{h/2} (\tau_i)\,\beta\, g(z)\,dz\,, \quad (i = xz, yz) \tag{12}$$

The governing equations can be obtained from Eq. (10) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0 and δw_0 zero separately. Thus one can obtain the equilibrium equations associated with the present shear deformation theory

$$\begin{aligned}
&\delta\, u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0\\
&\delta\, v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0\\
&\delta\, w_0: \ \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \beta\frac{\partial^4 S_x}{\partial x^4}\\
&\qquad + \beta\frac{\partial^4 S_{xy}}{\partial x^3 \partial y} + \beta\frac{\partial^4 S_{xy}}{\partial y^3 \partial x} + \beta\frac{\partial^4 S_y}{\partial y^4} - \beta\frac{\partial^3 Q_{xz}}{\partial x^3}\\
&\qquad - \beta\frac{\partial^3 Q_{yz}}{\partial y^3} - f_e + q = 0
\end{aligned} \tag{13}$$

2.4 Governing equations in terms of displacements

By substituting Eq. (3) into Eq. (6) and the subsequent results into Eq. (12), the stress resultants can be written as below

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & \beta B^s_{11} & \beta B^s_{12} & 0\\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & \beta B^s_{12} & \beta B^s_{22} & 0\\
0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & \beta B^s_{66}\\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & \beta D^s_{11} & \beta D^s_{12} & 0\\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & \beta D^s_{12} & \beta D^s_{22} & 0\\
0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & \beta D^s_{66}\\
\beta B^s_{11} & \beta B^s_{12} & 0 & \beta D^s_{11} & \beta D^s_{12} & 0 & \beta^2 H^s_{11} & \beta^2 H^s_{12} & 0\\
\beta B^s_{12} & \beta B^s_{22} & 0 & \beta D^s_{12} & \beta D^s_{22} & 0 & \beta^2 H^s_{12} & \beta^2 H^s_{22} & 0\\
0 & 0 & \beta B^s_{66} & 0 & 0 & \beta D^s_{66} & 0 & 0 & \beta^2 H^s_{66}
\end{bmatrix} \begin{Bmatrix} \varepsilon^0_x \\ \varepsilon^0_y \\ \gamma^0_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} \tag{14a}$$

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \beta^2 \begin{bmatrix} A^s_{55} & 0 \\ 0 & A^s_{44} \end{bmatrix} \begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} \tag{14b}$$

where

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B^s_{11} & D^s_{11} & H^s_{11} \\ A_{12} & B_{12} & D_{12} & B^s_{12} & D^s_{12} & H^s_{12} \\ A_{66} & B_{66} & D_{66} & B^s_{66} & D^s_{66} & H^s_{66} \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{15a}$$

$$\left(A_{22}, B_{22}, D_{22}, B^s_{22}, D^s_{22}, H^s_{22}\right) = \left(A_{11}, B_{11}, D_{11}, B^s_{11}, D^s_{11}, H^s_{11}\right) \tag{15b}$$

$$A^s_{44} = A^s_{55} = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{15c}$$

By substituting Eq. (14) into Eq. (13), the governing equations can be written in terms of generalized displacements (u_0, v_0 and w_0) as

$$\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y}\\
&- B_{11}\frac{\partial^3 w_0}{\partial x^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_0}{\partial x \partial y^2}\\
&- \beta\left(B^s_{66}\frac{\partial^5 w_0}{\partial x^3 \partial y^2} + \left(B^s_{12} + B^s_{66}\right)\frac{\partial^5 w_0}{\partial x \partial y^4} + B^s_{11}\frac{\partial^5 w_0}{\partial x^5}\right) = 0,
\end{aligned} \tag{16a}$$

$$\begin{aligned}
&A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y}\\
&- B_{22}\frac{\partial^3 w_0}{\partial y^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2 \partial y}\\
&- \beta\left(B^s_{66}\frac{\partial^5 w_0}{\partial x^2 \partial y^3} + \left(B^s_{12} + B^s_{66}\right)\frac{\partial^5 w_0}{\partial x^4 \partial y} + B^s_{22}\frac{\partial^5 w_0}{\partial y^5}\right) = 0,
\end{aligned} \tag{16b}$$

$$\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3} + \left(B_{12} + 2B_{66}\right)\frac{\partial^3 u_0}{\partial x \partial y^2} + \left(B_{12} + 2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2 \partial y}\\
&+ B_{22}\frac{\partial^3 v_0}{\partial y^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4}\\
&- 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w_0}{\partial y^4} + \beta\Bigg[B^s_{11}\frac{\partial^5 u_0}{\partial x^5}\\
&+ \left(B^s_{12} + B^s_{66}\right)\frac{\partial^5 u_0}{\partial x \partial y^4} + \left(B^s_{12} + B^s_{66}\right)\frac{\partial^5 v_0}{\partial x^4 \partial y}\\
&+ B^s_{22}\frac{\partial^5 v_0}{\partial y^5} + B^s_{66}\frac{\partial^5 v_0}{\partial x^3 \partial y^2} + B^s_{66}\frac{\partial^5 v_0}{\partial x^2 \partial y^3}\\
&- 2\, D^s_{11}\frac{\partial^6 w_0}{\partial x^6} - 2\left(D^s_{12} + 2D^s_{66}\right)\frac{\partial^6 w_0}{\partial x^2 \partial y^4}\\
&- 2\left(D^s_{12} + 2D^s_{66}\right)\frac{\partial^6 w_0}{\partial x^4 \partial y^2} - 2\, D^s_{22}\frac{\partial^6 w_0}{\partial y^6}\Bigg]\\
&- \beta^2\Bigg[H^s_{11}\frac{\partial^8 w_0}{\partial x^8} + 2\left(H^s_{12} + H^s_{66}\right)\frac{\partial^8 w_0}{\partial x^4 \partial y^4}\\
&+ H^s_{66}\frac{\partial^8 w_0}{\partial x^6 \partial y^2} + H^s_{66}\frac{\partial^8 w_0}{\partial x^2 \partial y^6} + H^s_{22}\frac{\partial^8 w_0}{\partial y^8}\\
&- A^s_{44}\frac{\partial^6 w_0}{\partial x^6} - A^s_{55}\frac{\partial^6 w_0}{\partial y^6}\Bigg] - f_e + q = 0
\end{aligned} \tag{16c}$$

3. Analytical solutions

The above governing equations are analytically solved for bending problems of a simply supported rectangular FG plate. Based on Navier solution procedure, the displacements are assumed as follows (Reddy 1984, Zenkour 2006)

$$\begin{Bmatrix} u_0(x,y) \\ v_0(x,y) \\ w_0(x,y) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\lambda\, x)\sin(\mu\, y) \\ V_{mn}\sin(\lambda\, x)\cos(\mu\, y) \\ W_{mn}\sin(\lambda\, x)\sin(\mu\, y) \end{Bmatrix} \tag{17}$$

where $\lambda=m\pi/a$, $\mu=n\pi/b$, (U_{mn}, V_{mn}, W_{mn}) are the unknown maximum displacement coefficients. The transverse load q is also expanded in the double-Fourier sine series as (Reddy 1984, Zenkour 2006)

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(\lambda\, x)\sin(\mu\, y) \tag{18}$$

The coefficients q_{mn} for the case of uniformly distributed load (UDL) are defined as follows

$$q_{mn} = \frac{16 q_0 a b}{\lambda\,\mu}, \quad (m,n=1,3,5,......) \tag{19}$$

where q_0 represents the intensity of the load at the plate centre.

For the case of a sinusoidally distributed load (SDL), we have

$$m=n=1 \text{ and } q_{11}=q_0 \tag{20}$$

Substituting Eqs. (17) and (18) into Eq. (16), the analytical solutions can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \end{Bmatrix} \tag{21}$$

where

$$\begin{aligned}
a_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
a_{12} &= -\lambda\,\mu\,(A_{12}+A_{66}) \\
a_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12}+2B_{66})\,\mu^2 \\
&\quad - \beta\left(B_{11}^s\lambda^4 + B_{12}^s\mu^4 + B_{66}^s\lambda^2\mu^2 + B_{66}^s\mu^4\right)] \\
a_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
a_{23} &= \mu\,[B_{22}\mu^2 + (B_{12}+2B_{66})\,\lambda^2 \\
&\quad - \beta\left(B_{22}^s\mu^4 + B_{12}^s\lambda^4 + B_{66}^s\lambda^2\mu^2 + B_{66}^s\lambda^4\right)] \\
a_{33} &= -D_{11}\lambda^4 - 2(D_{12}+2D_{66})\lambda^2\mu^2 \\
&\quad - \beta\left[D_{22}\mu^4 - 2(D_{11}^s\lambda^6 + D_{22}^s\mu^6)\right. \\
&\quad \left. - 2\left(\lambda^4\mu^2 + \lambda^2\mu^4\right)\left(D_{12}^s + 2D_{66}^s\right)\right] \\
&\quad - \beta^2\left[H_{11}^s\lambda^8 + H_{22}^s\mu^8 + 2\lambda^4\mu^4(H_{12}^s + H_{66}^s)\right. \\
&\quad \left. + \left(\lambda^6\mu^2 + \lambda^2\mu^6\right)H_{66}^s + A_{44}^s\lambda^6 + A_{55}^s\mu^6\right] \\
&\quad - k_w - k_{sx}\lambda^2 - k_{sy}\mu^2
\end{aligned} \tag{22}$$

One can easily obtain the value of the coefficient β in the same way as described by Mouffoki *et al.* (2017).

4. Numerical results

Table 1 Comparison of nondimensional deflection $\hat{w}$ of simply supported isotropic thin square plate under uniformly distributed load (a/h=100)

K_w	K_s	$\hat{w}$ Present	Benyoucef *et al*. (2010)	3D Huang *et al*. (2008)	Lam *et al*. (2000)
1	1	3.8000	3.8550	3.8546	3.853
	3^4	0 .7610	0.7630	0.7630	0.763
	5^4	0 .1153	0.1153	0.1153	0.115
3^4	1	3.1720	3.2108	3.2105	3.210
	3^4	0.7300	0.7317	0.7317	0.732
	5^4	0 .1145	0.1145	0.1145	0.115
5^4	1	1.4688	1.4765	1.4765	1.476
	3^4	0 .5693	0.5704	0.5704	0.570
	5^4	0.1094	0.1095	0.1095	0.109

In this section, various numerical examples are presented and discussed to verify the accuracy of the present theory in predicting the bending responses of simply supported isotropic and FG plates resting on elastic foundation. The FG plate is taken to be made of Titanium and Zirconia with the following material properties:

- Metal (Titanium, Ti-6Al-4V): E_m=66.2 GPa; v=1/3
- Ceramic (Zirconia, ZrO2): E_c=117 GPa; v=1/3.

The various non-dimensional parameters used are:

$$\hat{w} = \frac{10^3 D}{q_0 a^4} w\left(\frac{a}{2},\frac{b}{2},0\right), \quad \bar{w} = \frac{10^2 D}{q_0 a^4} w\left(\frac{a}{2},\frac{b}{2}\right),$$

$$\bar{\sigma}_x = \frac{1}{10^2 q_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right), \quad \bar{\tau}_{xy} = \frac{1}{10 q_0}\tau_{xy}\left(0,0,-\frac{h}{3}\right)$$

$$\bar{\tau}_{xz} = -\frac{1}{10 q_0}\tau_{xz}\left(0,\frac{b}{2},0\right), \quad K_w = k_w a^4/D,$$

$$K_s = k_s a^2/D. \quad \bar{z} = z/h$$

where $D=Eh^3/12(1-v^2)$ is a reference bending rigidity of the plate.

In order to validate the present formulations, numerical results for bending of a isotropic thin plate (k=1, a/h=100, v=0.3) are compared to that obtained by Lam *et al*. (2000) using Green's functions, the three-dimensional solutions given by Huang *et al.* (2008) and the hyperbolic shear deformation plate theory with five variable given by Benyoucef *et al.* (2010). The plate is assumed subjected to uniform load on the top surface and the results for the central deflection of the plate are given in Table 1.

For all values of foundation parameters K_w; K_s it can be seen that the results are in close agreement.

Table 2 shows the comparison of nondimensional deflections and stresses of simply supported $T_i\!-\!6Al\!-\!4V/Zr0_2$ rectangular plate on elastic foundation subjected to mechanical sinusoidal distributed load (a=10 h, b=2a, q_0=100). It can be seen that the results of present theory are in excellent agreement with those of (RSDPT) refined sinusoidal shear deformation plates theory with four variable only given by Bouderba *et al.* (2013) for all values of the volume fraction exponent ratio k and elastic foundation parameters K_w; K_s.

It can be observed that the non-dimensional deflection

Table 2 Effect of the volume fraction exponent and elastic foundation parameters on the dimensionless and stresses of an FGM rectangular plate under sinusoidal load. (a=10 h, b=2a, q_0=100)

k	K_0	K_1	theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
0	0	0	Present	0.65311	0.42224	0.87508	0.40152
			RSDPT[a]	0.68131	0.42424	0.86240	0.39400
	100	0	Present	0.39508	0.25542	0.52936	0.24289
			RSDPT[a]	0.40523	0.25233	0.51296	0.23435
	0	100	Present	0.083211	0.053797	0.11149	0.051157
			RSDPT[a]	0.083654	0.052093	0.10589	0.048377
	100	100	Present	0.076819	0.049665	0.102928	0.047227
			RSDPT[a]	0.077197	0.048071	0.097724	0.044643
0.5	100	100	Present	0.078404	0.047337	0.086092	0.040310
			RSDPT[a]	0.078729	0.045788	0.081728	0.038066
1	100	100	Present	0.079017	0.046427	0.076996	0.037089
			RSDPT[a]	0.079321	0.044892	0.073054	0.035023
2	100	100	Present	0.079467	0.046129	0.070882	0.034086
			RSDPT[a]	0.079758	0.044595	0.067185	0.032215
5	100	100	Present	0.079872	0.047319	0.067740	0.031644
			RSDPT[a]	0.080150	0.045736	0.064125	0.029922
∞	100	100	Present	0.080953	0.029613	0.061372	0.028160
			RSDPT[a]	0.081190	0.050559	0.058148	0.026565

[a]Taken from Bouderba *et al*. (2013)

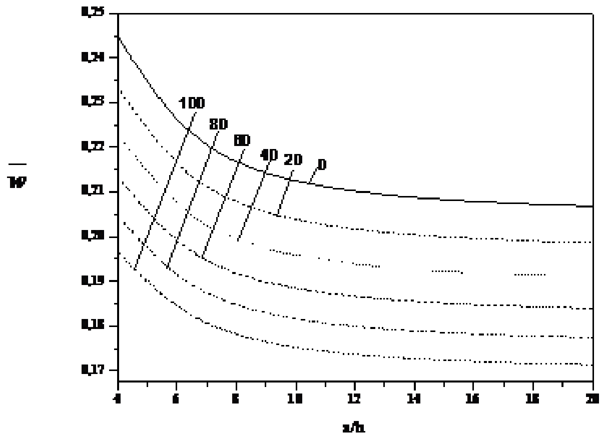

Fig. 2 Effect of Winkler modulus parameter K_w on the dimensionless center deflection ($\bar{w}$) of a square FG plate (k=2) for different side-to-thickness ratio a/h=10with K_s=10

and stresses are decreasing with the existence of the elastic foundations K_w; K_s. The inclusion of the Winkler foundation K_w parameter gives results more than those with the inclusion of Pasternak foundation parameters K_s. The deflection will increase as the volume fraction exponent k increases. The stresses are also influenced to the variation of the volume fraction exponent k, which means that the plate can be optimally design according to given working conditions by tailoring the graded material properties.

Figs. 2-9 show the effect of foundation stiffness on the dimensionless deflection, normal, shear and longitudinal tangential stress (k=2) in a square FG plate under a sinusoidally distributed load. They depict the variation of

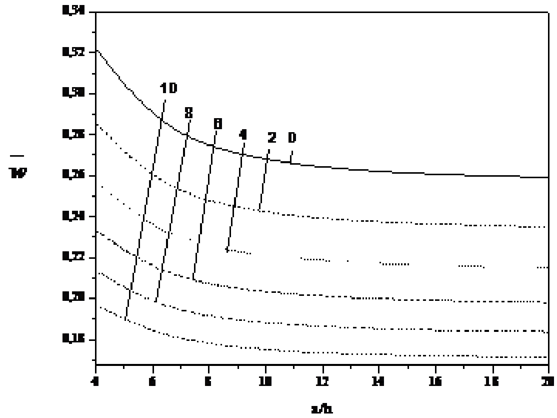

Fig. 3 Effect of Pasternak shear modulus parameter K_s on the dimensionless center deflection ($\bar{w}$) of a square FG plate (k=2) for different side-to-thickness ratio a/h=10 with K_w=100

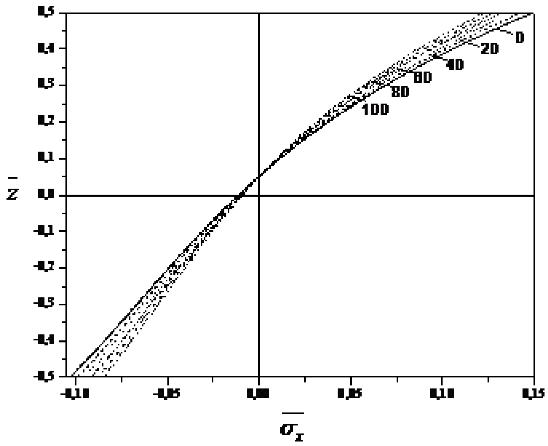

Fig. 4 Variation of dimensionless axial stress ($\bar{\sigma}_x$) through-the-thickness of a square FG plate (k=2) for different values of Winkler modulus parameter K_w with K_s=10 and a/h=10

the center deflection $\bar{w}$ with the side-to-thickness a/h and shows across the-thickness distributions of the shear stress $\bar{\tau}_{xz}$, the in-plane longitudinal normal stress $\bar{\sigma}_x$, and the longitudinal tangential stress $\bar{\tau}_{xy}$.

Figs. 2 and 3 show the effect of foundation stiffness and side-to-thickness ratio a/h on the dimensionless deflection of FG square plate (k=2).The deflection decreases with the increase of a/h ratios. It is maximum for the metallic plate and minimum for the ceramic plate. The deflections decrease gradually as either K_w or K_s increases. Decreases of deflection indicate that increasing the foundation stiffness will certainly enhance the deformation rigidity of the plate.

As plotted in Figs. 4 and 5, the in-plane longitudinal normal stress $\bar{\sigma}_x$ is compressive in the plate up to $\bar{z}$ =0.051, and then it becomes tensile. The maximum compressive stress occurs at a point on the bottom surface of FG plate, but the maximum tensile one at a point on the top surface. In addition, it can be seen that the elastic foundation has a significant effect on the maximum values

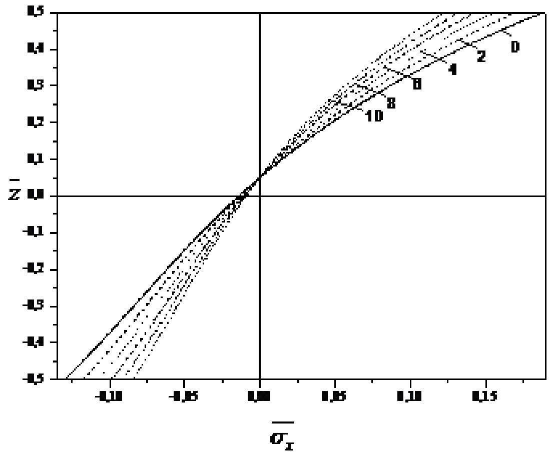

Fig. 5 Variation of dimensionless axial stress ($\bar{\sigma}_x$) through-the-thickness of a square FG plate (k=2) for different values of Pasternak shear modulus parameter K_s with K_w=100 and a/h=10

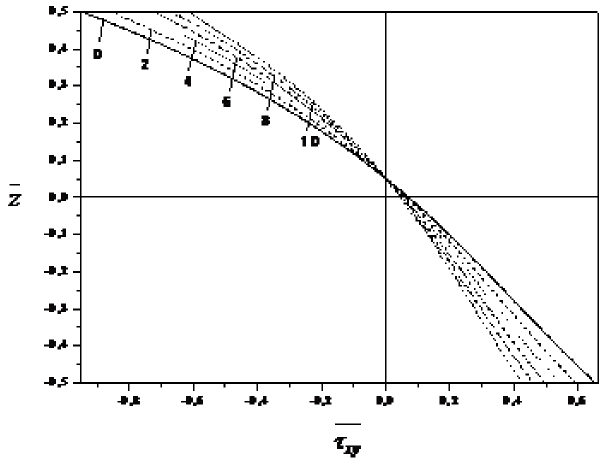

Fig. 7 Variation of dimensionless longitudinal tangential stress ($\bar{\tau}_{xy}$) through-the-thickness of a square FG plate (k=2) for different values of Pasternak shear modulus parameter K_s with K_w=100 and a/h=10

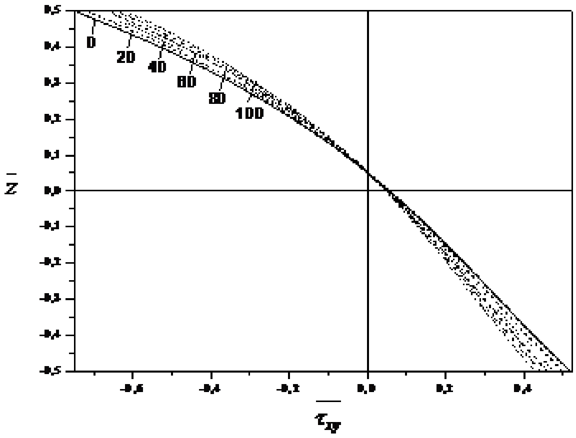

Fig. 6 Variation of dimensionless longitudinal tangential stress ($\bar{\tau}_{xy}$) through-the-thickness of a square FG plate (k=2) for different values of Winkler modulus parameter K_w with K_s=10 and a/h=10

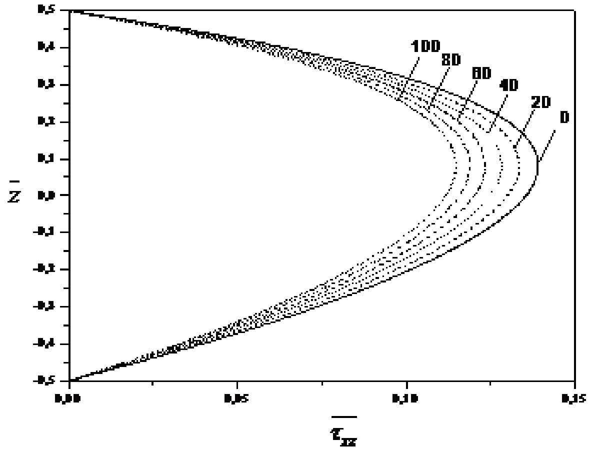

Fig. 8 Variation of dimensionless shear stress ($\bar{\tau}_{xz}$) through-the-thickness of a square FG plate (k=2) for different values of Winkler modulus parameter K_w with K_s=10 and a/h=10

of the axial stress, $\bar{\sigma}_x$.

Also, it is observed that the effect of Pasternak shears modulus parameter is more significant than Winkler modulus parameter and the axial stress $\bar{\sigma}_x$ increases gradually with decreasing K_w or K_s.

Figs. 6 and 7 depict the through-the-thickness distributions of the longitudinal tangential stress $\bar{\tau}_{xy}$ in the FG square. In this case, the tensile and compressive values of the longitudinal tangential stress, $\bar{\tau}_{xy}$, is maximum at a point on the bottom and top surfaces of the FG plate, respectively.

It is clear that the minimum value of zero for the axial stress $\bar{\sigma}_x$ and the longitudinal tangential stress $\bar{\tau}_{xy}$ occurs at $\bar{z}$ =0.051.

Figs. 8 and 9 show the distributions of the shear stresses in the square FG plate under sinusoidal distributed load. It is seen that the transverse shear stress $\bar{\tau}_{xz}$ are not parabolic in the FG plate and the stresses increases gradually as either

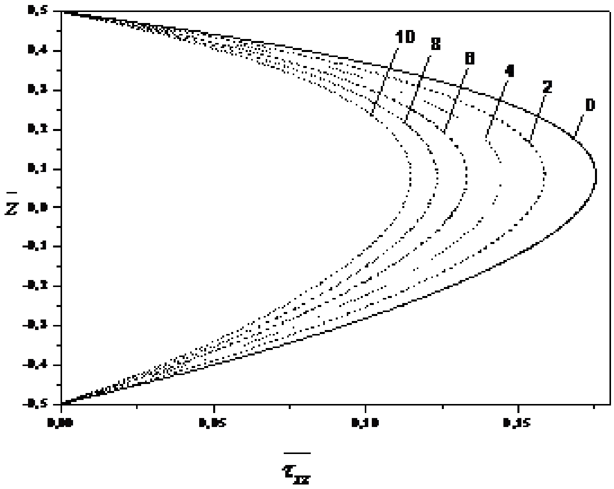

Fig. 9 Variation of dimensionless shear stress ($\bar{\tau}_{xz}$) through-the-thickness of a square FG plate (k=2) for different values of Pasternak shear modulus parameter K_s with K_w=100 and a/h=10

K_w or K_s decreases, which indicates that increased moduli of the elastic foundation can enhance the bending rigidity of the plate.

It is to be noted that the maximum value the transverse shear stress $\bar{\tau}_{xz}$ of occurs at $\bar{z}$ =0.1, not at the center of plate as in the homogeneous case.

5. Conclusions

The bending response of FG plates resting on a elastic foundation is developed using a new 3-unknowns shear deformation plates theory. The present theory has only three unknown and three governing equation as in the classical plate theory, but it is capable of accurately capturing shear deformation effects, instead of five as in the well-known higher-order shear deformation theory (HSDT).The theory gives parabolic distribution of transverse shear strains, and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. The gradation of properties through the thickness is assumed to be of the power law distribution of the volume fraction of the constituents. Results show that the proposed theory is not only accurate and simple in solving the bending behaviour of FG plates, but also comparable with the other higher-order shear deformation theories which contain more number of unknowns and so deserve special attention and offer potential for future research.

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