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An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions

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Abstract. In this research, a simple hyperbolic shear deformation theory is developed and applied for the bending, vibration and buckling of powerly graded material (PGM) sandwich plate with various boundary conditions. The displacement field of the present model is selected based on a hyperbolic variation in the in-plane displacements across the plate's thickness. By splitting the deflection into the bending and shear parts, the number of unknowns and equations of motion of the present formulation is reduced and hence makes them simple to use. Equations of motion are obtained from Hamilton's principle. Numerical results for the natural frequencies, deflections and critical buckling loads of several types of powerly graded sandwich plates under various boundary conditions are presented. The accuracy of the present formulation is demonstrated by comparing the computed results with those available in the literature. As conclusion, this theory is as accurate as other theories available in the literature and so it becomes more attractive due to smaller number of unknowns.

Keywords: functionally graded materials; sandwich plates; hyperbolic plate theory; various boundary conditions; free vibration; buckling loads; bending

1. Introduction

Sandwich plates have received considerable attention in many engineering applications such as aerospace, automobile, and shipbuilding due to their high strength and stiffness, low weight and durability. These plates are generally manufactured form three homogeneous layers, two face sheets adhesively bonded to the core. However, the sudden variation in material characteristics within the interface between different materials can lead to face sheet/core delamination, which is a dangerous problem in sandwich construction. To improve the resistance of sandwich structures to such type of failure, the concept of a functionally graded material (FGM) is being actively applied in sandwich plate design. Nowadays, FGM suits the specific demand in different engineering applications especially for high temperature environment applications of heat exchanger tubes, thermal barrier coating for turbine blades, thermoelectric generators, furnace linings, electrically insulated metal ceramic joints, space/aerospace industries, automotive applications, and biomedical area etc (Koizumi 1993, Suresh and Mortensen 1998, Miyamoto *et al*. 1999, Kirigulige *et al*. 2005, Pollien *et al*. 2005, Shahistha *et al*. 2014, Yaghoobi *et al*. 2014, Kar and Panda 2015, Bouguenina *et al*. 2015, Bennai *et al*. 2015, Hadji and AddaBedia 2015, LarbiChaht *et al*. 2015, AitAtmane *et al*. 2015, Celebi *et al*. 2016, Darabi and Vosoughi 2016, Bounouara *et al*. 2016, Ahouel *et al*. 2016, Ebrahimi and Habibi 2016, Ebrahimi and Jafari 2016, Madani *et al*. 2016, Benferhat *et al*. 2016, Ebrahimi and Shafiei 2016, GhorbanpourArani *et al*. 2016, Turan *et al*. 2016, Zidi *et al*. 2017, Rahmani *et al*. 2017, Bouafia *et al*. 2017).

With the increase use of FG sandwich plates, understanding their mechanical behaviors becomes an essential task. Three-dimensional finite element simulations for investigating low velocity impact response of sandwich panels with a FG core were conducted by Etemadi *et al*. (2009). Anderson (2003) proposed an analytical 3D elasticity solution method for a sandwich composite with a FG core subjected to transverse loading by a rigid spherical indentor. An exact thermoelasticity solution for a 2D sandwich structures with FG coating was established by Shodja *et al*. (2007). Natarajan and Manickam (2012) investigated the bending and free vibration response of functionally graded (FG) sandwich plates using higher-order shear deformation theories (HSDT). Xiang *et al* (2013) studied the dynamic behavior of FG sandwich plates by employing an nth-order shear deformation theory and a meshless method. Sobhy (2013) examined the buckling and free vibration of FG sandwich plates by utilizing various HSDTs. In a number of recent articles-see (Bourada *et al*. 2012, Tounsi *et al*. 2013, Bourada *et al*. 2016, Laoufi *et al*.

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2016, Draiche *et al.* 2016, El-Hassar *et al.* 2016, Javed *et al.* 2016, Chikh *et al.* 2017, Menasria *et al.* 2017, Khetir *et al.* 2017)-a new simple and robust plate theory for mechanical behavior and buckling of simply supported FGM sandwich and composite plate with only four or five unknown functions has been developed. Neves *et al.* (2012) studied the bending behaviour of FG sandwich plates according to a hyperbolic theory considering Zig-Zag and warping effects. Bessaim *et al.* (2013) developed anew higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets. Houari *et al* (2013) studied the thermoelastic bending of FG sandwich plates using a new higher order shear and normal deformation theory. Thai *et al.* (2014) analysed a functionally graded sandwich plates using a new first-order shear deformation theory. Nguyen *et al.* (2014) presented a new inverse trigonometric shear deformation theory for isotropic and functionally graded sandwich plates. Ait Amar Meziane *et al* (2014) developed a new refined plate theory to the vibration and buckling of exponentially graded sandwich plate resting on elastic foundations under various boundary conditions. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for FG plates. Swaminathan and Naveenkumar (2014) present an analytical formulations and solutions for the stability analysis of simply supported FG sandwich plates based on two higher-order refined computational models. Taibi *et al.* (2015) proposed a simple shear deformation theory for thermo-mechanical behaviour of FG sandwich plates on elastic foundations. Mahi *et al* (2015) proposed a novel hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Recently, Hamidi *et al.* (2015) developed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending response of FG sandwich plates. Bakora and Tounsi (2015) examined the thermo-mechanical post-buckling behavior of thick FG plates resting on elastic foundations.Nguyen (2015) presented a higher-order hyperbolic shear deformation plate model for analysis of functionally graded materials. Bellifa *et al.* (2017) proposed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Meksi *et al.* (2017) presented an analytical solution for bending, buckling and vibration responses of FGM sandwich plates. AitAtmane *et al.* (2017) discussed the effect of thickness stretching and porosity on mechanical response of a FG beams resting on elastic foundations. Baseri *et al.* (2016) presented an analytical solution for buckling of embedded laminated plates based on higher order shear deformation plate theory. Bennoun *et al.* (2016) proposed a novel five variable refined plate theory for vibration analysis of FG sandwich plates. Chikh *et al.* (2016) investigated the thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. Benbakhti *et al.* (2016) presented a new five unknown quasi-3D type HSDT for thermo-mechanical bending analysis of FGM sandwich plates. Benahmed *et al.* (2017) proposed a novel quasi-3D hyperbolic shear deformation theory for FG thick rectangular plates on elastic foundation. Benchohra *et al.* (2017) developed also a new quasi-3D sinusoidal shear deformation theory for FG plates. Klouche *et al.* (2017) presented an original single variable shear deformation theory for buckling analysis of thick isotropic plates. El-Haina *et al.* (2017) given a simple analytical approach for thermal buckling of thick FG sandwich plates. Fahsi *et al.* (2017) proposed a four variable refined *n*th-order shear deformation theory for mechanical and thermal buckling analysis of FG plates.

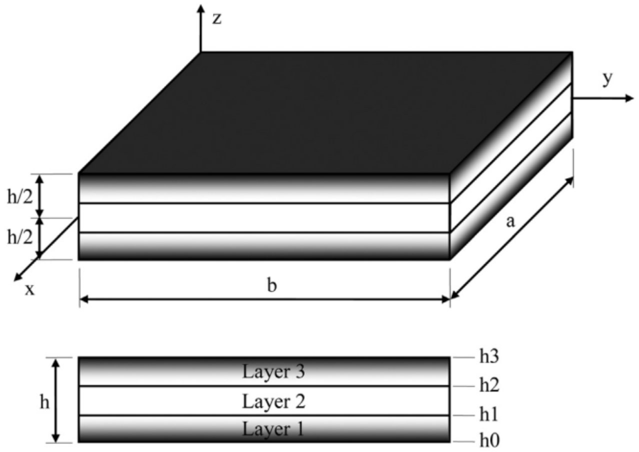

Fig. 1 Geometry and coordinates of FG sandwich plates

The present work deals with the analytical formulations and solutions for the bending, buckling and vibration analyses of FG sandwich plates composed of a powerly functionally graded face sheets and an isotropic homogeneous core. To achieve this objective, a simple hyperbolic shear deformation theory is presented and applied for sandwich plate with various boundary conditions. The displacement field is expressed with only 4 unknowns, which is even less than the first order shear deformation theory (FSDT) and do not require shear correction factor (Adda Bedia *et al.* 2015, Meksi *et al.* 2015, Bellifa *et al.* 2016, Bouderba *et al.* 2016). Equations of motion are obtained from Hamilton's principle. Analytical solutions for sandwich plates under various boundary conditions are determined. Numerical examples are illustrated to check the accuracy of the present formulation in predicting the bending, buckling and vibration behaviors of powerly graded sandwich plates.

2. Problem formulation

In this work, a rectangular powerly graded sandwich plate with a uniform thickness is considered. The sandwich plate is composed of three microscopically heterogeneous layers, with reference to rectangular coordinates (x, y, z) as plotted in Fig. 1. The top and bottom faces of the plate are at $z=\pm h/2$, and the edges of the plate are parallel to the x and y axes.

The sandwich plate is composed of three elastic layers, namely, "Layer 1", "Layer 2", and "Layer 3" from the uppermost surface to the lowest surface of the plate. The vertical ordinates of the base, the two interfaces, and the top are denoted by $-h_0=h/2$, h_1, h_2, $h_3=h/2$, respectively. For brevity, the ratio of the thickness of each layer from the base to the top is denoted by the combination of three numbers, i.e., "1-0-1", "2-1-2" and so on.

The volume fraction of the sandwich plate faces is assumed to vary according to a simple power law function of z while that of the core equals unity, and they are given as (Bousahla *et al*. 2014, Bourada *et al*. 2015, Zidi *et al*. 2014, Hebali *et al*. 2014, Fekrar *et al*. 2014, Bouderba *et al*. 2013, Hadji *et al*. 2016, Barka *et al*. 2016, Hebali *et al*. 2016, Houari *et al*. 2016, Besseghier *et al*. 2017)

$$P(z)=(P_c-P_m)V^{(n)}+P_m \tag{1}$$

where P denotes the effective material characteristic such as Young's modulus E, Poisson's ratio v, and mass density ρ; subscripts c and m indicate the ceramic and metal phases, respectively; and V is the volume fraction of the ceramic phase defined by

$$V^{(1)}(z)=\left(\frac{z-h_0}{h_1-h_0}\right)^k \quad \text{for} \quad h_0\le z\le h_1 \tag{2a}$$

$$V^{(2)}(z)=1 \quad \text{for} \quad h_1\le z\le h_2 \tag{2b}$$

$$V^{(1)}(z)=\left(\frac{z-h_0}{h_1-h_0}\right)^k \quad \text{for} \quad h_0\le z\le h_1 \tag{2c}$$

where k is the inhomogeneity parameter which takes values greater than or equal to zero. It is noted that the core is independent of the value of k which is fully ceramic.

2.1 Kinematics and constitutive equations

The displacement field of the present formulation is modeled based on the following assumptions: (1) The transverse displacement is splitted into both bending and shear components; (2) the axial displacements are divided into three components, namely: extension, bending and shear parts; (3) the bending parts of the axial displacements are identical to those expressed by CPT; and (4) the shear parts of the axial displacements give rise to the hyperbolic variations of shear strains and hence to shear stresses across the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be determined (AitYahia *et al*. 2015, Attia *et al*. 2015, Belkorissat *et al*. 2015, Beldjelili *et al*. 2016, Boukhari *et al*. 2016, Barati and Shahverdi 2016, Bousahla *et al*. 2016, Becheri *et al*. 2016)

$$\begin{aligned}
u(x,y,z,t)&=u_0(x,y,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x}\\
v(x,y,z,t)&=v_0(x,y,t)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y}\\
w(x,y,z,t)&=w_b(x,y,t)+w_s(x,y,t)
\end{aligned} \tag{3}$$

where u_0 and v_0 indicate the displacements along the x and y coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively. The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate, thus a shear correction factor is not required. In this study, the shape function is considered as follows

$$f(z)=\frac{h\sinh(10z/h)}{10\cosh(5)-h/100} \tag{4}$$

The displacement model (3) leads to the following kinematic relations

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \tag{5}$$

Where

$$\begin{gathered}
\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},\\
\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix},
\end{gathered} \tag{6}$$

and

$$g(z)=1-\frac{df(z)}{dz} \tag{7}$$

The linear constitutive relations of a powerly graded sandwich plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}^{(n)}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\ C_{12}&C_{22}&0&0&0\\ 0&0&C_{44}&0&0\\ 0&0&0&C_{55}&0\\ 0&0&0&0&C_{66}\end{bmatrix}^{(n)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{8}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The elastic constants C_{ij} are defined as

$$\begin{gathered}
C_{11}^{(n)}=C_{22}^{(n)}=\frac{E^{(n)}(z)}{1-\nu^2},\quad C_{12}^{(n)}=\nu C_{11}^{(n)},\\
C_{44}^{(n)}=C_{55}^{(n)}=C_{66}^{(n)}=\frac{E^{(n)}(z)}{2(1+\nu)},
\end{gathered} \tag{9}$$

2.2 Equations of motion

In this work, the analysis of bending, buckling and free vibration of powerly graded sandwich plate is performed using Hamilton's principle. The principle can be expressed in an analytical form as follows

$$0=\int_0^T(\delta U+\delta V-\delta K)dt \tag{10}$$

where δU is the variation of strain energy; δV is the variation of work done by the external forces; and δK is the variation of kinetic energy.

The strain energy expression is given as follows

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]dV\\&=\int_A\Big[N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta k_x^b+M_y^b\delta k_y^b+M_{xy}^b\delta k_{xy}^b\\&\quad+M_x^s\delta k_x^s+M_y^s\delta k_y^s+M_{xy}^s\delta k_{xy}^s+S_{yz}^s\delta\gamma_{yz}^s+S_{xz}^s\delta\gamma_{xz}^s\Big]dA=0\end{aligned} \tag{11}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{Bmatrix}N_x, & N_y, & N_{xy}\\ M_x^b, & M_y^b, & M_{xy}^b\\ M_x^s, & M_y^s, & M_{xy}^s\end{Bmatrix}=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\left(\sigma_x,\sigma_y,\tau_{xy}\right)^{(n)}\begin{Bmatrix}1\\ z\\ f(z)\end{Bmatrix}dz, \tag{12a}$$

$$\left(S_{xz}^s,S_{yz}^s\right)=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\left(\tau_{xz},\tau_{yz}\right)^{(n)}g(z)dz. \tag{12b}$$

where h_{n-1} and h_n are the top and bottom z-coordinates of the nth layer.

Substituting Eq. (8) into Eq. (12) and integrating through the thickness of the plate, the stress resultants are expressed as

$$\begin{Bmatrix}N\\ M^b\\ M^s\end{Bmatrix}=\begin{bmatrix}A & B & B^s\\ B & D & D^s\\ B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\ k^b\\ k^s\end{Bmatrix},\quad S=A^s\gamma, \tag{13}$$

In which

$$N=\{N_x,N_y,N_{xy}\}^t,\qquad M^b=\{M_x^b,M_y^b,M_{xy}^b\}^t,\qquad M^s=\{M_x^s,M_y^s,M_{xy}^s\}^t, \tag{14a}$$

$$\varepsilon=\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\}^t,\qquad k^b=\{k_x^b,k_y^b,k_{xy}^b\}^t,\qquad k^s=\{k_x^s,k_y^s,k_{xy}^s\}^t, \tag{14b}$$

$$A=\begin{bmatrix}A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix},\quad B=\begin{bmatrix}B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix},\quad D=\begin{bmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix}, \tag{14c}$$

$$B^s=\begin{bmatrix}B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix},\quad D^s=\begin{bmatrix}D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix},\quad H^s=\begin{bmatrix}H_{11}^s & H_{12}^s & 0\\ H_{12}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix}, \tag{14d}$$

$$S=\{S_{xz}^s,S_{yz}^s\}^t,\qquad \gamma=\{\gamma_{xz}^s,\gamma_{yz}^s\}^t,\qquad A^s=\begin{bmatrix}A_{44}^s & 0\\ 0 & A_{55}^s\end{bmatrix}, \tag{14e}$$

and stiffness components are given as

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}C_{11}^{(n)}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\ \nu^{(n)}\\ \dfrac{1-\nu^{(n)}}{2}\end{Bmatrix}dz, \tag{15a}$$

$$\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right) \tag{15b}$$

$$A_{44}^s=A_{55}^s=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}C_{44}^{(n)}[g(z)]^2dz, \tag{15c}$$

The variation of work done by the applied loads can be expressed as

$$\delta V=-\int_A(P+q)\delta\left(w_b+w_s\right)dA \tag{16}$$

With

$$P=\left[P_x^0\frac{\partial^2(w_b+w_s)}{\partial x^2}+P_y^0\frac{\partial^2(w_b+w_s)}{\partial y^2}+2P_{xy}^0\frac{\partial^2(w_b+w_s)}{\partial x\partial y}\right] \tag{17}$$

where q is the transversely load and (P_x^0, P_y^0, P_{xy}^0) are the in-plane loads.

The variation of kinetic energy of the plate can be written as

$$\begin{aligned}\delta K&=\int_V\left[\dot u\delta\dot u+\dot v\delta\dot v+\dot w\delta\dot w\right]\rho(z)\,dV\\&=\int_A\Big\{I_0\left[\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+(\dot w_b+\dot w_s)(\delta\dot w_b+\delta\dot w_s)\right]\\&\quad-I_1\left(\dot u_0\frac{\partial\delta\dot w_b}{\partial x}+\frac{\partial\dot w_b}{\partial x}\delta\dot u_0+\dot v_0\frac{\partial\delta\dot w_b}{\partial y}+\frac{\partial\dot w_b}{\partial y}\delta\dot v_0\right)\\&\quad-J_1\left(\dot u_0\frac{\partial\delta\dot w_s}{\partial x}+\frac{\partial\dot w_s}{\partial x}\delta\dot u_0+\dot v_0\frac{\partial\delta\dot w_s}{\partial y}+\frac{\partial\dot w_s}{\partial y}\delta\dot v_0\right)\\&\quad+I_2\left(\frac{\partial\dot w_b}{\partial x}\frac{\partial\delta\dot w_b}{\partial x}+\frac{\partial\dot w_b}{\partial y}\frac{\partial\delta\dot w_b}{\partial y}\right)+K_2\left(\frac{\partial\dot w_s}{\partial x}\frac{\partial\delta\dot w_s}{\partial x}+\frac{\partial\dot w_s}{\partial y}\frac{\partial\delta\dot w_s}{\partial y}\right)\\&\quad+J_2\left(\frac{\partial\dot w_b}{\partial x}\frac{\partial\delta\dot w_s}{\partial x}+\frac{\partial\dot w_s}{\partial x}\frac{\partial\delta\dot w_b}{\partial x}+\frac{\partial\dot w_b}{\partial y}\frac{\partial\delta\dot w_s}{\partial y}+\frac{\partial\dot w_s}{\partial y}\frac{\partial\delta\dot w_b}{\partial y}\right)\Big\}dA\end{aligned} \tag{18}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz \tag{19}$$

By substituting Eqs. (18), (16) and (11) into Eq. (10), using Hamilton's principle, and collecting the coefficients of (δu_0, δv_0, δw_b and δw_s) after taking the required integration by parts, the following equations of motion of the plate are obtained

$$\delta u_0: \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_b}{\partial x}-J_1\frac{\partial \ddot{w}_s}{\partial x}$$

$$\delta v_0: \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial \ddot{w}_b}{\partial y}-J_1\frac{\partial \ddot{w}_s}{\partial y}$$

$$\delta w_b: \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}-P-q= \tag{20}$$

$$I_0(\ddot{w}_b+\ddot{w}_s)+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_b-J_2\nabla^2\ddot{w}_s$$

$$\delta w_s: \frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}-P-q=$$

$$I_0(\ddot{w}_b+\ddot{w}_s)+J_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s$$

The equations of motion of the present theory can be expressed in terms of displacements (u_0, v_0, w_b, w_s) by replacing Eq. (14) into Eq. (21) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}d_{22}u_0+(A_{12}+A_{66})d_{12}v_0\\&-B_{11}d_{111}w_b-(B_{12}+2B_{66})d_{122}w_b\\&-(B_{12}^s+2B_{66}^s)d_{122}w_s-B_{11}^s d_{111}w_s\\&=I_0\ddot{u}_0-I_1d_1\ddot{w}_b-J_1d_1\ddot{w}_s,\end{aligned}\tag{21a}$$

$$\begin{aligned}&A_{22}d_{22}v_0+A_{66}d_{11}v_0+(A_{12}+A_{66})d_{12}u_0\\&-B_{22}d_{222}w_b-(B_{12}+2B_{66})d_{112}w_b\\&-(B_{12}^s+2B_{66}^s)d_{112}w_s-B_{22}^s d_{222}w_s\\&=I_0\ddot{v}_0-I_1d_2\ddot{w}_b-J_1d_2\ddot{w}_s,\end{aligned}\tag{21b}$$

$$\begin{aligned}&B_{11}d_{111}u_0+(B_{12}+2B_{66})d_{122}u_0+(B_{12}+2B_{66})d_{112}v_0\\&+B_{22}d_{222}v_0-D_{11}d_{1111}w_b-2(D_{12}+2D_{66})d_{1122}w_b\\&-D_{22}d_{2222}w_b-D_{11}^s d_{1111}w_s-2(D_{12}^s+2D_{66}^s)d_{1122}w_s\\&-D_{22}^s d_{2222}w_s-P-q=I_0(\ddot{w}_b+\ddot{w}_s)+I_1(d_1\ddot{u}_0+d_2\ddot{v}_0)\\&-I_2(d_{11}\ddot{w}_b+d_{22}\ddot{w}_b)-J_2(d_{11}\ddot{w}_s+d_{22}\ddot{w}_s)\end{aligned}\tag{21c}$$

$$\begin{aligned}&B_{11}^s d_{111}u_0+(B_{12}^s+2B_{66}^s)d_{122}u_0+(B_{12}^s+2B_{66}^s)d_{112}v_0\\&+B_{22}^s d_{222}v_0-D_{11}^s d_{1111}w_b-2(D_{12}^s+2D_{66}^s)d_{1122}w_b\\&-D_{22}^s d_{2222}w_b-H_{11}^s d_{1111}w_s-2(H_{12}^s+2H_{66}^s)d_{1122}w_s\\&-H_{22}^s d_{2222}w_s+A_{44}^s d_{11}w_s+A_{55}^s d_{22}w_s-P-q=I_0(\ddot{w}_b+\ddot{w}_s)\\&+J_1(d_1\ddot{u}_0+d_2\ddot{v}_0)-J_2(d_{11}\ddot{w}_b+d_{22}\ddot{w}_b)-K_2(d_{11}\ddot{w}_s+d_{22}\ddot{w}_s)\end{aligned}\tag{21d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},$$
$$d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i},\quad (i,j,l,m=1,2). \tag{22}$$

3. Exact solutions for EGMs sandwich plates

The exact solution of Eq. (21) for the powerly graded sandwich plate under various boundary conditions are determined in this section. The boundary conditions for an arbitrary edge with simply supported and clamped edge conditions are:

Table 1 The admissible functions $X_m(x)$ and $Y_n(y)$

Boundary conditions			The functions X_m and Y_n	
	At x=0, y=0	At x=a, y=b	$X_m(x)$	$Y_n(y)$
SSSS	$X_m(0)=X_m''(0)=0$ $Y_n(0)=Y_n''(0)=0$	$X_m(a)=X_m''(a)=0$ $Y_n(b)=Y_n''(b)=0$	$\sin(\lambda x)$	$\sin(\mu y)$
CSCS	$X_m(0)=X_m'(0)=0$	$X_m(a)=X_m'(a)=0$	$\sin^2(\lambda x)$	$\sin(\mu y)$
CCCC	$Y_n(0)=Y_n''(0)=0$ $X_m(0)=X_m'(0)=0$	$Y_n(b)=Y_n''(b)=0$ $X_m(a)=X_m'(a)=0$	$\sin^2(\lambda x)$	$\sin^2(\mu y)$
FCFC	$Y_n(0)=Y_n''(0)=0$ $X_m''(0)=X_m'''(0)=0$	$Y_n(b)=Y_n'(b)=0$ $X_m''(a)=X_m'''(a)=0$	$\cos^2(\lambda x)$ $[\sin^2(\lambda x)+1]$	$\sin^2(\mu y)$

()’ Denotes the derivative with respect to the corresponding coordinates.

• Clamped (C)

$$\begin{aligned}u_0=v_0=w_b=\partial w_b/\partial x=\partial w_b/\partial y=w_s\\=\partial w_s/\partial x=\partial w_s/\partial y=0\\ \text{at } x=0,a \quad\text{and}\quad y=0,b\end{aligned}\tag{23}$$

• and simply supported (S)

$$v_0=w_b=\partial w_b/\partial y=w_s=\partial w_s/\partial y=0 \text{ at } x=0,a \tag{24a}$$

$$u_0=w_b=\partial w_b/\partial x=w_s=\partial w_s/\partial x=0 \text{ at } y=0,b \tag{24b}$$

The following representation for the displacement quantities, that satisfy the above boundary conditions, is appropriate in the case of our problem

$$\begin{Bmatrix}u_0\\v_0\\w_b\\w_s\end{Bmatrix}=\begin{Bmatrix}U_{mn}\dfrac{\partial X_m(x)}{\partial x}Y_n(y)e^{i\omega t}\\ V_{mn}X_m(x)\dfrac{\partial Y_n(y)}{\partial y}e^{i\omega t}\\ W_{bmn}X_m(x)Y_n(y)e^{i\omega t}\\ W_{smn}X_m(x)Y_n(y)e^{i\omega t}\end{Bmatrix}\tag{25}$$

where U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters and $\omega=\omega_{mn}$ denotes the eigenfrequency associated with (m, n) [th]eigenmode. The functions $X_m(x)$ and $Y_n(y)$ are suggested by Sobhy (2013) to satisfy at least the geometric boundary conditions given in Eqs. (23) and (24), and represent approximate shapes of the deflected surface of the plate. These functions, for the different cases of boundary conditions, are listed in Table 1.

The transversely load q is also chosen as

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Q_{mn}\sin(\lambda x)\sin(\mu y) \tag{26}$$

where the coefficients Q_{mn} are given below for certain typical loads

$$Q_{mn}=\begin{cases}q_0 & \text{for sinusoidal loads}\\ \dfrac{16q_0}{mn\pi^2} & \text{for uniform loads}\end{cases}\tag{27}$$

with $\alpha=m\pi/a$ and $\beta=n\pi/b$.

Substituting expressions (26) and (25) into the governing Eqs. (21) and multiplying each equation by the corresponding eigenfunction then integrating over the domain of solution, we can obtain, after some mathematical manipulations, the following equations

$$\left\{\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33}-\beta\overline{P} & S_{34}-\beta\overline{P} \\ S_{41} & S_{42} & S_{43}-\beta\overline{P} & S_{44}-\beta\overline{P} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}\right\} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ f_q \\ f_q \end{Bmatrix} \tag{28}$$

In wich

$$\begin{aligned}
S_{11} &= A_{11}\alpha_{12} + A_{66}\alpha_8 \\
S_{12} &= \left(A_{12} + A_{66}\right)\alpha_8 \\
S_{13} &= -B_{11}\alpha_{12} - \left(B_{12} + 2B_{66}\right)\alpha_8 \\
S_{14} &= -\left(B_{12}^s + 2B_{66}^s\right)\alpha_8 - B_{11}^s\alpha_{12} \\
S_{21} &= \left(A_{12} + A_{66}\right)\alpha_{10} \\
S_{22} &= A_{22}\alpha_4 + A_{66}\alpha_{10} \\
S_{23} &= -B_{22}\alpha_4 - \left(B_{12} + 2B_{66}\right)\alpha_{10} \\
S_{24} &= -(B_{12}^s + 2B_{66}^s)\alpha_{10} - B_{22}^s\alpha_4 \\
S_{31} &= B_{11}\alpha_{13} + (B_{12} + 2B_{66})\alpha_{11} \\
S_{32} &= (B_{12} + 2B_{66})\alpha_{11} + B_{22}\alpha_5 \\
S_{33} &= -D_{11}\alpha_{13} - 2(D_{12} + 2D_{66})\alpha_{11} - D_{22}\alpha_5 \\
S_{34} &= -D_{11}^s\alpha_{13} - 2(D_{12}^s + 2D_{66}^s)\alpha_{11} - D_{22}^s\alpha_5 \\
S_{41} &= B_{11}^s\alpha_{13} + (B_{12}^s + 2B_{66}^s)\alpha_{11} \\
S_{42} &= (B_{12}^s + 2B_{66}^s)\alpha_{11} + B_{22}^s\alpha_5 \\
S_{43} &= -D_{11}^s\alpha_{13} - 2(D_{12}^s + 2D_{66}^s)\alpha_{11} - D_{225}^s \\
S_{44} &= -H_{11}^s\alpha_{13} - 2(H_{12}^s + 2H_{66}^s)\alpha_{11} - H_{22}^s\alpha_5 + \left(A_{44}^s\right)\alpha_9 + \left(A_{55}^s\right)\alpha_3 \\
\overline{P} &= P_x^0 \\
\xi &= P_y^0 / P_x^0
\end{aligned} \tag{29a}$$

And

$$\begin{aligned}
m_{11} &= -I_0\alpha_6 \\
m_{13} &= I_1\alpha_6 \\
m_{14} &= J_1\alpha_6 \\
m_{22} &= -I_0\alpha_2 \\
m_{23} &= I_1\alpha_2 \\
m_{24} &= J_1\alpha_2 \\
m_{31} &= -I_1\alpha_9
\end{aligned} \quad \text{and} \quad \begin{aligned}
m_{32} &= -I_1\alpha_3 \\
m_{33} &= -I_0\alpha_1 + I_2\left(\alpha_3 + \alpha_9\right) \\
m_{34} &= -I_0\alpha_1 + J_2\left(\alpha_3 + \alpha_9\right) = m_{43} \\
m_{41} &= -J_1\alpha_9 \\
m_{42} &= -J_1\alpha_3 \\
m_{44} &= -I_0\alpha_1 + K_2\left(\alpha_3 + \alpha_9\right)
\end{aligned} \tag{29b}$$

With

$$\begin{aligned}
&\beta = \xi\alpha_3 + \alpha_9 \\
&\left(\alpha_1, \quad \alpha_3, \quad \alpha_5\right) = \int_0^b\int_0^a \left(X_m Y_n, \quad X_m Y_n'', \quad X_m Y_n''''\right) X_m Y_n \, dxdy \\
&\left(\alpha_2, \quad \alpha_4, \quad \alpha_{10}\right) = \int_0^b\int_0^a \left(X_m Y_n', \quad X_m Y_n''', \quad X_m'' Y_n'\right) X_m Y_n' \, dxdy \\
&\left(\alpha_6, \quad \alpha_8, \quad \alpha_{12}\right) = \int_0^b\int_0^a \left(X_m' Y_n, \quad X_m' Y_n'', \quad X_m''' Y_n\right) X_m' Y_n \, dxdy \\
&\left(\alpha_7, \quad \alpha_9, \quad \alpha_{11}, \quad \alpha_{13}\right) = \\
&\int_0^b\int_0^a \left(X_m' Y_n', \quad X_m'' Y_n, \quad X_m'' Y_n'', \quad X_m'''' Y_n\right) X_m Y_n \, dxdy
\end{aligned} \tag{29c}$$

$$f_q = \int_0^b\int_0^a Q_{mn} \sin\lambda x \sin\mu y \sin\lambda x \sin\mu y \, dxdy$$

Table 2 Dimensionless deflection $\overline{w}$ of square plates (a/h=10)

Boundary conditions	k	Method	Scheme 1-0-1	2-1-2	1-1-1	2-2-1	1-0-1
SSSS	0	FSDT[(a)]	0.2961	0.2961	0.2961	0.2961	0.2961
		Present	0.2956	0.2956	0.2956	0.2956	0.2956
	0.5	FSDT[(a)]	0.5229	0.4849	0.4564	0.4371	0.4178
		Present	0.5227	0.4846	0.4560	0.4366	0.4172
	1	FSDT[(a)]	0.7455	0.6594	0.5956	0.5541	0.5130
		Present	0.7454	0.6593	0.5954	0.5537	0.5124
	2	FSDT[(a)]	1.0846	0.9256	0.8011	0.7205	0.6433
		Present	1.0839	0.9254	0.8009	0.7200	0.6427
	5	FSDT[(a)]	1.4576	1.2714	1.0782	0.9385	0.8139
		Present	1.4519	1.2678	1.0767	0.9367	0.8131
	10	FSDT[(a)]	1.5609	1.4143	1.2109	1.0434	0.9011
		Present	1.5519	1.4053	1.2070	1.0392	0.8998
CSCS	0	FSDT[(a)]	0.1841	0.1841	0.1841	0.1841	0.1841
		Present	0.1836	0.1836	0.1836	0.1836	0.1836
	0.5	FSDT[(a)]	0.3208	0.2975	0.2803	0.2688	0.2571
		Present	0.3205	0.2972	0.2799	0.2682	0.2565
	1	FSDT[(a)]	0.4547	0.4021	0.3636	0.3389	0.3141
		Present	0.4546	0.4020	0.3634	0.3384	0.3134
	2	FSDT[(a)]	0.6593	0.5617	0.4865	0.4385	0.3920
		Present	0.6586	0.5615	0.4863	0.4379	0.3913
	5	FSDT[(a)]	0.8900	0.7712	0.6529	0.5697	0.4940
		Present	0.8835	0.7670	0.6513	0.5676	0.4931
	10	FSDT[(a)]	0.9595	0.8606	0.7339	0.6338	0.5464
		Present	0.9492	0.8503	0.7294	0.6290	0.5448
CCCC	0	FSDT[(a)]	0.1612	0.1612	0.1612	0.1612	0.1612
		Present	0.1606	0.1606	0.1606	0.1606	0.1606
	0.5	FSDT[(a)]	0.2780	0.2579	0.2431	0.2333	0.2233
		Present	0.2777	0.2576	0.2427	0.2327	0.2226
	1	FSDT[(a)]	0.3923	0.3469	0.3140	0.2930	0.2718
		Present	0.3922	0.3468	0.3137	0.2924	0.2710
	2	FSDT[(a)]	0.5674	0.4828	0.4184	0.3777	0.3380
		Present	0.5666	0.4825	0.4182	0.3770	0.3371
	5	FSDT[(a)]	0.7685	0.6626	0.5603	0.4897	0.4247
		Present	0.7610	0.6577	0.5584	0.4873	0.4236
	10	FSDT[(a)]	0.8327	0.7412	0.6302	0.5452	0.4693
		Present	0.8208	0.7292	0.6249	0.5396	0.4676
FCFC	0	FSDT[(a)]	0.1043	0.1043	0.1043	0.1043	0.1043
		Present	0.1038	0.1038	0.1038	0.1038	0.1038
	0.5	FSDT[(a)]	0.1786	0.1657	0.1563	0.1501	0.1437
		Present	0.1784	0.1655	0.1560	0.1496	0.1432
	1	FSDT[(a)]	0.2513	0.2222	0.2012	0.1879	0.1744
		Present	0.2512	0.2221	0.2010	0.1875	0.1739
	2	FSDT[(a)]	0.3628	0.3084	0.2674	0.2416	0.2164
		Present	0.3622	0.3082	0.2672	0.2411	0.2158
	5	FSDT[(a)]	0.4925	0.4232	0.3575	0.3129	0.2713
		Present	0.4868	0.4195	0.3561	0.3110	0.2705
	10	FSDT[(a)]	0.5355	0.4742	0.4023	0.3484	0.2997
		Present	0.5265	0.4651	0.3983	0.3442	0.2984

[(a)]Taken from Thai *et al*. (2014)

The non-trivial solution is obtained when the determinant of Eq. (28) equals zero. For the free vibration

Table 3 Dimensionless buckling load $\overline{N}$ of square plates (ξ=1, *a/h*=10)

Boundary conditions	*k*	Method	Scheme 1-0-1	2-1-2	1-1-1	2-2-1	1-0-1
SSSS	0	FSDT[a]	6.5022	6.5022	6.5022	6.5022	6.5022
		Present	6.5118	6.5118	6.5118	6.5118	6.5118
	0.5	FSDT[a]	3.6817	3.9702	4.2181	4.4047	4.6081
		Present	3.6831	3.9721	4.2211	4.4091	4.6138
	1	FSDT[a]	2.5824	2.9193	3.2320	3.4742	3.7528
		Present	2.5825	2.9196	3.2332	3.4768	3.7568
	2	FSDT[a]	1.7749	2.0798	2.4032	2.6719	2.9926
		Present	1.7759	2.0801	2.4035	2.6736	2.9953
	5	FSDT[a]	1.3208	1.5114	1.7855	2.0512	2.3652
		Present	1.3258	1.5184	1.7878	2.0551	2.3675
	10	FSDT[a]	1.2333	1.3612	1.5897	1.8450	2.1364
		Present	1.2404	1.3698	1.5949	1.8524	2.1394
CSCS	0	FSDT[a]	11.9477	11.9477	11.9477	11.9477	11.9477
		Present	11.9802	11.9802	11.9802	11.9802	11.9802
	0.5	FSDT[a]	6.8587	7.3942	7.8489	8.1861	8.5573
		Present	6.8638	7.4010	7.8597	8.2012	8.5771
	1	FSDT[a]	4.8390	5.4712	6.0504	6.4925	7.0048
		Present	4.8397	5.4721	6.0545	6.5015	7.0191
	2	FSDT[a]	3.3370	3.9170	4.5225	5.0176	5.6129
		Present	3.3405	3.9183	4.5240	5.0239	5.6226
	5	FSDT[a]	2.4721	2.8529	3.3697	3.8622	4.4536
		Present	2.4901	2.8683	3.3779	3.8763	4.4619
	10	FSDT[a]	2.2930	2.5565	2.9978	3.4713	4.0269
		Present	2.3177	2.5873	3.0162	3.4975	4.0378
CCCC	0	FSDT[a]	15.9226	15.9226	15.9226	15.9226	15.9226
		Present	15.9805	15.9805	15.9805	15.9805	15.9805
	0.5	FSDT[a]	9.2338	9.9529	10.5578	11.0011	11.4933
		Present	9.2431	9.9653	10.5774	11.0286	11.5292
	1	FSDT[a]	6.5434	7.3990	8.1753	8.7612	9.4443
		Present	6.5447	7.4008	8.1830	8.7777	9.4705
	2	FSDT[a]	4.5236	5.3169	6.1354	6.7961	7.5952
		Present	4.5302	5.3195	6.1381	6.8077	7.6130
	5	FSDT[a]	3.3400	3.8738	4.5813	5.2417	6.0445
		Present	3.3730	3.9025	4.5965	5.2677	6.0598
	10	FSDT[a]	3.0825	3.4629	4.0732	4.7084	5.4696
		Present	3.1273	3.5198	4.1073	4.7567	5.4897
FCFC	0	FSDT[a]	18.6047	18.6047	18.6047	18.6047	18.6047
		Present	18.6842	18.6842	18.6842	18.6842	18.6842
	0.5	FSDT[a]	10.8640	11.7085	12.4145	12.9276	13.5006
		Present	10.8770	11.7258	12.4418	12.9656	13.5503
	1	FSDT[a]	7.7220	8.7323	9.6429	10.3246	11.1229
		Present	7.7238	8.7349	9.6536	10.3476	11.1593
	2	FSDT[a]	5.3477	6.2913	7.2569	8.0294	8.9676
		Present	5.3569	6.2949	7.2608	8.0457	8.9924
	5	FSDT[a]	3.9393	4.5849	5.4268	6.2015	7.1514
		Present	3.9854	4.6251	5.4482	6.2380	7.1729
	10	FSDT[a]	3.6230	4.0915	4.8230	5.5683	6.4748
		Present	3.6852	4.1712	4.8709	5.6360	6.5030

[a] Taken from Thai *et al*. (2014)

Table 4 Dimensionless fundamental frequency $\overline{\omega}$ of square plates (*a/h*=10)

Boundary conditions	*k*	Method	Scheme 1-0-1	2-1-2	1-1-1	2-2-1	1-0-1
SSSS	0	FSDT[a]	1.8244	1.8244	1.8244	1.8244	1.8244
		Present	1.8257	1.8257	1.8257	1.8257	1.8257
	0.5	FSDT[a]	1.4442	1.4841	1.5192	1.5471	1.5745
		Present	1.4447	1.4846	1.5199	1.5480	1.5756
	1	FSDT[a]	1.2429	1.3000	1.3533	1.3956	1.4393
		Present	1.2434	1.3004	1.3538	1.3963	1.4402
	2	FSDT[a]	1.0605	1.1218	1.1882	1.2436	1.3023
		Present	1.0613	1.1224	1.1886	1.2443	1.3031
	5	FSDT[a]	0.9431	0.9796	1.0435	1.1077	1.1735
		Present	0.9455	0.9815	1.0445	1.1091	1.1744
	10	FSDT[a]	0.9246	0.9390	0.9932	1.0587	1.1223
		Present	0.9279	0.9424	0.9952	1.0611	1.1234
CSCS	0	FSDT[a]	2.6701	2.6701	2.6701	2.6701	2.6701
		Present	2.6735	2.6735	2.6735	2.6735	2.6735
	0.5	FSDT[a]	2.1277	2.1862	2.2371	2.2768	2.3162
		Present	2.1289	2.1876	2.2388	2.2791	2.3190
	1	FSDT[a]	1.8365	1.9209	1.9986	2.0593	2.1226
		Present	1.8372	1.9216	1.9996	2.0610	2.1250
	2	FSDT[a]	1.5694	1.6616	1.5792	1.8394	1.9251
		Present	1.5710	1.6625	1.7600	1.8410	1.9271
	5	FSDT[a]	1.3927	1.4512	1.5471	1.6405	1.7380
		Present	1.3985	1.4558	1.5495	1.6440	1.7400
	10	FSDT[a]	1.3610	1.3889	1.4720	1.5672	1.6629
		Present	1.3691	1.3978	1.4771	1.5736	1.6656
CCCC	0	FSDT[a]	3.2936	3.2936	3.2936	3.2936	3.2936
		Present	3.2993	3.2993	3.2993	3.2993	3.2993
	0.5	FSDT[a]	2.6376	2.7099	2.7719	2.8199	2.8679
		Present	2.6394	2.7119	2.7748	2.8236	2.8724
	1	FSDT[a]	2.2814	2.3864	2.4818	2.5556	2.6330
		Present	2.2823	2.3873	2.4835	2.5584	2.6369
	2	FSDT[a]	1.9520	2.0680	2.1889	2.2868	2.3923
		Present	1.9543	2.0692	2.1900	2.2893	2.3954
	5	FSDT[a]	1.7293	1.8064	1.9269	2.0415	2.1629
		Present	1.7387	1.8138	1.9308	2.0471	2.1661
	10	FSDT[a]	1.6858	1.7268	1.8329	1.9497	2.0703
		Present	1.6990	1.7414	1.8411	1.9602	2.0746
FCFC	0	FSDT[a]	3.4688	3.4688	3.4688	3.4688	3.4688
		Present	3.4759	3.4759	3.4759	3.4759	3.4759
	0.5	FSDT[a]	2.7872	2.8634	2.9284	2.9781	3.0282
		Present	2.7894	2.8659	2.9318	2.9827	3.0338
	1	FSDT[a]	2.4144	2.5256	2.6258	2.7027	2.7838
		Present	2.4155	2.5266	2.6278	2.7061	2.7885
	2	FSDT[a]	2.0675	2.1914	2.3190	2.4215	2.5323
		Present	2.0703	2.1928	2.3203	2.4245	2.5362
	5	FSDT[a]	1.8296	1.9145	2.0430	2.1632	2.2918
		Present	1.8412	1.9236	2.0477	2.1701	2.2957
	10	FSDT[a]	1.7806	1.8285	1.9429	2.0656	2.1942
		Present	1.7968	1.8467	1.9531	2.0785	2.1995

[a] Taken from Thai *et al*. (2014)

problem, we have $P_x^0 = P_y^0 = P_{xy}^0 = f_q = 0$. While for the buckling analysis, we put $\omega = P_{xy}^0 = f_q = 0$; $P_x^0 = \overline{P}$ and $P_y^0 = \xi \overline{P}$, i.e., $\xi = P_y^0 / P_x^0$. and for the bending analysis, we put $P_x^0 = P_y^0 = P_{xy}^0 = \omega = 0$.

4. Numerical results and discussions

In this section, some numerical examples are exposed and discussed to check the accuracy of the present formulation and examine the impacts of the inhomogeneity parameter, thickness ratio of layers, i.e., scheme, transverse shear deformation and boundary conditions on deflection, critical buckling load and natural frequency of FG sandwich plates.

The combination of materials consists of aluminum and alumina with the following material properties:

- Ceramic (Alumina, Al_2O_3): E_c=380 GPa; ν=0.3; ρ_c=3800 kg/m.
- Metal (Aluminium, Al): E_m=70 GPa; ν=0.3; ρ_m=2707 kg/m^3.

The employed non-dimensional quantities are

$$\bar{w} = \frac{10E_c h^3}{q_0 a^4} w\left(\frac{a}{2},\frac{b}{2}\right), \bar{\omega} = \frac{\omega a^2}{h}\sqrt{\rho_0 / E_0}\ ,$$
$$\bar{N} = \frac{\bar{P}\, a^2}{100\, E_0 h^3}\ , \quad E_0 = 1\text{GPa}\ , \quad \rho_0 = 1\text{kg/m}^3 \tag{30}$$

Tables 2 to 4 provide the nondimensionalized values of the transverse deflections $\bar{w}$, buckling load $\bar{N}$ and natural frequencies $\bar{\omega}$ of various types of powerly graded sandwich plates under various boundary conditions. The results are compared with those obtained using FSDT developed by Thai *et al*. (2014). Good agreement is achieved between the present results obtained by using the present simple hyperbolic shear deformation theory and those of Thai *et al*. (2014). It is remarked that the stiffer and softer plates correspond to the FCFC and SSSS ones, respectively. With the increase of the inhomogeneity parameter *k*, the plate becomes softer and hence, leads to a reduction of both the frequency and buckling load and an increase of deflection. This due to the fact that when the parameter increases the plate tends to be metallic.

In Figs. 2-4, the variations of deflection, critical buckling load and fundamental natural frequency of FG sandwich square plates versus the inhomogeneity parameter *k* are presented, respectively. Different layer configurations are employed for multi-layered FGM plates. The thickness ratio of the plate is considered equal to 10. It can be observed that increasing the inhomogeneity parameter *k* leads to increase in deflection (Fig. 2) and a reduction of critical buckling load (Fig. 3) and natural frequency (Fig. 4). This behavior can be attributed to the fact that higher inhomogeneity parameter *k* corresponds to lower volume fraction of the ceramic phase. Thus, increasing the inhomogeneity parameter makes the plate softer because of the high portion of metal in comparison with the ceramic part, and consequently, results in an increase in deflection and a reduction of both buckling load and natural frequency. It is observed from results that the hardest and softest plates correspond to the (1-2-1) and (1-0-1) schemes, respectively. Such behavior is due to the fact that the (1-2-1) and (1-0-1) FG sandwich plates correspond to the highest and lowest volume fractions of the ceramic phase, and thus makes them become the hardest and softest ones. In addition, it can be seen form Figs. 2-4, that when clamped boundary

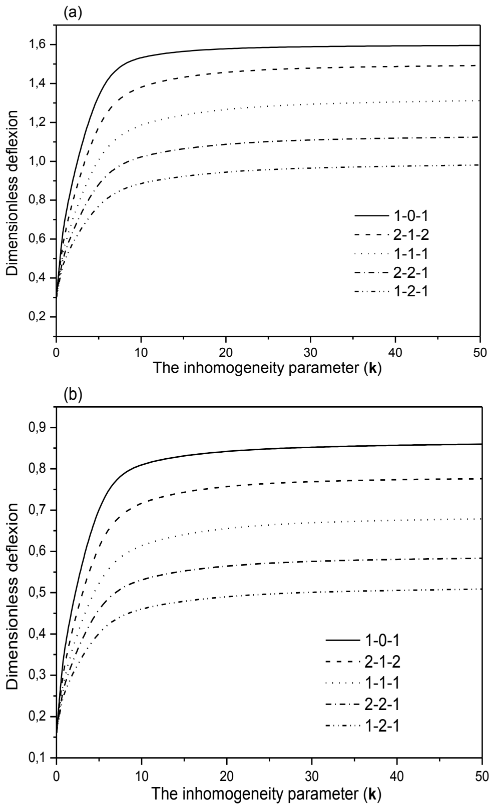

Fig. 2 Effect of the inhomogeneity parameter (*k*) on dimensionless deflection ($\bar{w}$) of square FG sandwich plates (*a/h*=10): (a) simply supported plate; (b) clamped plate

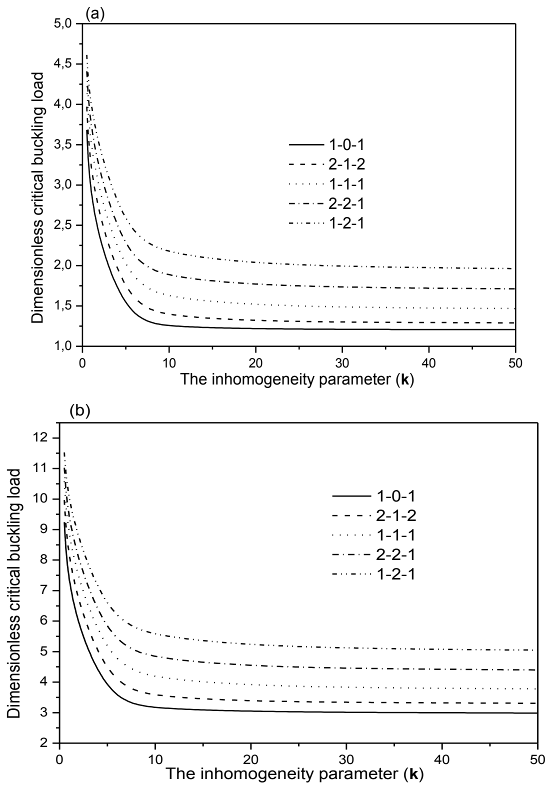

Fig. 3 Effect of the inhomogeneity parameter (*k*) on dimensionless critical buckling load ($\bar{N}$) of square FG sandwich plates (*a/h*=10) under biaxial compression: (a) simply supported plate; (b) clamped plate

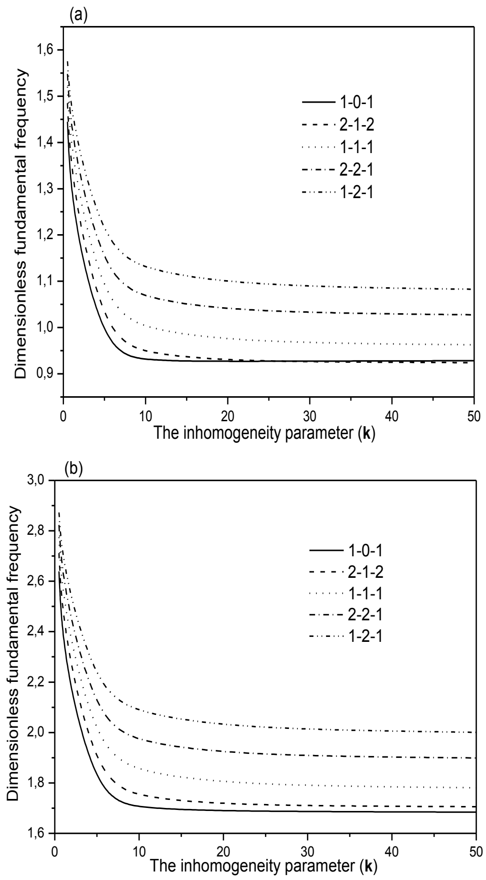

Fig. 4 Effect of the inhomogeneity parameter (*k*) on dimensionless frequency ($\bar{\varpi}$) of square FG sandwich plates (*a/h*=10): (a) simply supported plate; (b) clamped plate

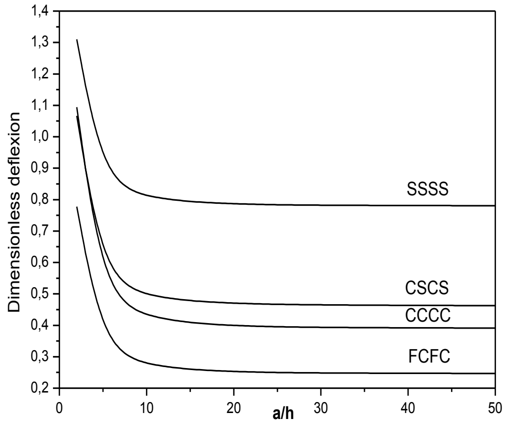

Fig. 5 Effect of boundary conditions on dimensionless deflection $\bar{w}$ of (1-1-1) FG sandwich square plates (*k*=2)

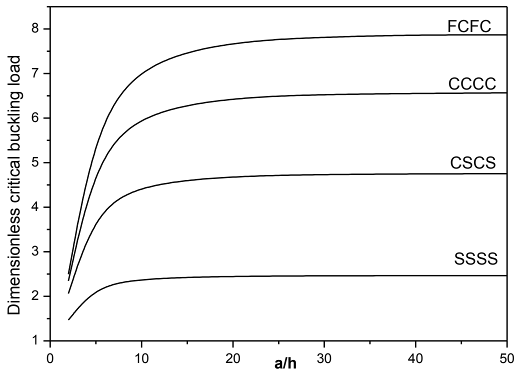

Fig. 6 Effect of boundary conditions on dimensionless critical buckling load $\bar{N}$ of (1-1-1) FG sandwich square plates (*k*=2) under biaxial compression

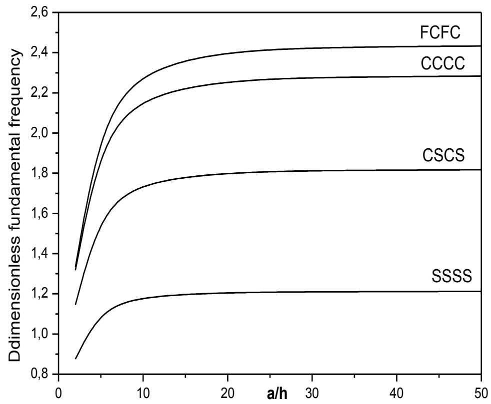

Fig. 7 Effect of boundary conditions on dimensionless frequency $\bar{\varpi}$ of (1-1-1) FG sandwich square plates (*k*=2)

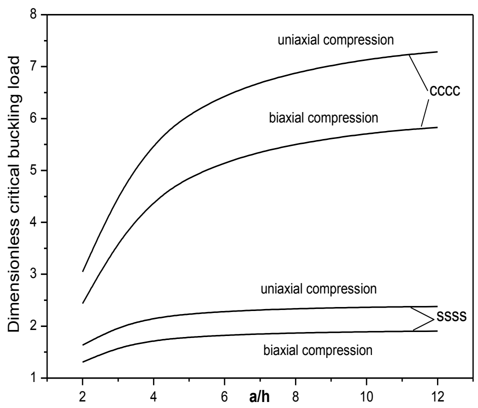

Fig. 8 Comparison of dimensionless critical buckling load $\bar{N}$ of (1-2-1) FG sandwich clamped and simply supported (*a/h*=10, *k*=2)

conditions (CCCC) are considered, the plate becomes stiffer; this has led to a reduction of the deflection (Fig. 2(b)) and increased critical buckling load (Fig. 3(b)) and natural frequency (Fig. 4(b)).

Figs. 5-7 demonstrate the effect of boundary conditions on deflection, buckling load and natural frequency of FG sandwich plates. It is observed from this investigation that the hardest and softest plates correspond to the FCFC and SSSS ones, respectively.

Fig. 8 shows the effect of the parameter ξ on the critical buckling loads $\bar{N}$. As expected, the uniaxial buckling load (ξ=0) is greater than the biaxial one (ξ=1).

5. Conclusions

This work presents a bending, buckling and free vibration analysis of FG sandwich plates composed of FG face sheets and an isotropic homogeneous core by employing a simple hyperbolic shear deformation theory with 4 unknowns. Different cases of boundary conditions are considered in the present investigation. The results obtained by the present formulation are compared with other results available in literature. The following conclusions may be drawn from the present study:

- The obtained results are in good agreement with those found in literature.
- The vibration frequencies and buckling loads for FG sandwich plates are generally lower than the corresponding values for homogeneous ceramic plates, while the deflections are higher than those of homogeneous ceramic plates.
- The vibration frequencies and buckling loads increase as the side-to-thickness ratio increases, while the deflections decrease.
- The vibration frequencies and buckling loads for simply supported powerly graded sandwich plates are lower than those for free and clamped powerly graded sandwich plates.
- The deflections for simply supported powerly graded sandwich plates are higher than those for free and clamped powerly graded sandwich plates.
- The critical buckling load for the plate under biaxial compression is lower than the plate under uniaxial compression.

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