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Free vibrations of fluid conveying microbeams under non-ideal boundary conditions

Duygu Atcı* and Süleyman Murat Bağdatlı [b]

Department of Mechanical Engineering, Manisa Celal Bayar University, 45140 Yunusemre, Manisa, Turkey

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Abstract. In this study, vibration analysis of fluid conveying microbeams under non-ideal boundary conditions (BCs) is performed. The objective of the present paper is to describe the effects of non-ideal BCs on linear vibrations of fluid conveying microbeams. Non-ideal BCs are modeled as a linear combination of ideal clamped and ideal simply supported boundary conditions by using the weighting factor (k). Non-ideal clamped and non-ideal simply supported beams are both considered to show the effects of BCs. Equations of motion of the beam under the effect of moving fluid are obtained by using Hamilton principle. Method of multiple scales which is one of the perturbation techniques is applied to the governing linear equation of motion. Approximate solutions of the linear equation are obtained and the effects of system parameters and non-ideal BCs on natural frequencies are presented. Results indicate that, natural frequencies of fluid conveying microbeam changed significantly by varying the weighting factor k. This change is more remarkable for clamped microbeams rather than simply supported ones.

Keywords: microsystems; vibration; non-ideal boundary conditions; perturbation methods

1. Introduction

There are many practical applications of fluid conveying beams/pipes/tubes in engineering and industrial fields. Dynamics of these systems should be analyzed by considering the structure-fluid interaction. There are many pioneering studies which are investigated dynamics, mechanics and applications of fluid conveying beams/pipes/tubes. Paidoussis and Li (1993) wrote a remarkable review for the dynamics of fluid conveying systems. Dynamics of cantilevered pipes conveying fluid are also given by Paidoussis *et al*. (2007). Ibrahim (2010, 2011) presented an overview of mechanics of fluid conveying pipes. Pipes having a single or more spring support along its length are considered and governing equations of linear and non-linear problems are given in mentioned overview. Vibrations of pipes conveying fluid are studied by Ni *et al*. (2011). A semi-analytical method, differential transformation method is used to obtain natural frequencies and critical fluid flow velocities. Another application is presented by Wang *et al*. (2013a). Numerical analysis based on finite element method is applied to pipes conveying fluid to obtain vibration characteristics.

It is known that axially moving beams show similar dynamic behaviors with fluid-conveying beams. In previous studies, axially moving beams with multiple supports are investigated by Bağdatlı *et al*. (2013). Also, Ding and Chen (2011) studied natural frequencies of non-linear vibrations of axially moving beams. Bağdatlı and Uslu (2015) studied the axially moving string under non-ideal boundary conditions. Non-linear vibrations of spring-supported axially moving string (Kesimli *et al*. 2015) and multiple supported axially accelerating flexible beam (Kural and Ozkaya 2012) are studied.

One of the most contemporary applications of fluid conveying systems are micro-scale systems which are characterized as micro-electromechanical systems (MEMS). Micro systems are sized from 1 μm to 1 mm. They are used in heat and mass transfer operations through their high performance, matter and energy saving properties. Besides, fluid conveying micro-channels are used to cool micro-scale electronic systems through their high flux properties. In last decades, MEMS found application fields as micro surgery, micro injectors, micro heat exchangers and Lab-on-Chip applications.

Due to the recent technological developments in science and engineering, the problems of micro-scale systems become more remarkable. Previous studies showed that micro size effects should be considered to have accurate solutions. Many experimental studies presented the size dependent vibration characteristics of micro-scale systems; Fleck *et al*. (1994), Ma and Clarke (1995), Stolken and Evans (1998), Chong and Lam (1999), Lam *et al*. (2003) and McFarland and Colton (2005). These studies demonstrated that dynamic behaviors of micro-scale systems become different from results of classical continuum theory of large length scale systems. In mechanical problems of small length scale systems, it is needed to use a proper approach rather than classical continuum mechanical theories. Therefore, in order to consider the size effects, new elasticity theories are developed. Firstly, the classical couple stress theory is presented by Mindlin (1964), Toupin (1962), Mindlin and

*Corresponding author, Ph.D. Candidate,
E-mail: duygu.comen@cbu.edu.tr
[a] Ph.D. Candidate
[b] Ph.D., Assistant Professor

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Tiersten (1962) for linear elastic materials. In the mentioned theory, in addition to equilibriums of forces and moments, the concept equilibrium of the couple is suggested. Couple is the loading which forces the material particle to rotate. It is needed to use higher order equilibrium relations to consider couple stress. Afterwards, modified couple stress theory is developed by Yang *et al*. (2002). It is shown that the couple stress tensor in classical couple stress theory is symmetric and an internal material length scale parameter (l) is needed to capture micro size effects. Park and Gao (2006) applied modified couple stress theory to Euler-Bernoulli beam model. It is shown that the difference between the numerical results of the new model and classical beam model become remarkable when the thickness of the beam is decreased. The results of two models approach each other when the thickness of the beam is increased. In modified couple stress theory, deflections of a cantilever beam are calculated smaller than classical theory. Besides, it is found that modified couple stress theory verifies the results of experimental studies of bending tests of micro-scale systems. Kong *et al*. (2008) investigated the natural frequencies of Euler-Bernoulli microbeam. It is indicated that the natural frequencies of modified couple stress theory is larger than the classical theory. Timoshenko beam model is studied by Ma *et al*. (2008). The static bending and free vibration problems are solved.

There are recent studies which are subjected to applications of modified couple stress theory. Size-dependent vibration characteristics of fluid conveying micro-tubes are studied by Wang (2010). It is shown that natural frequencies decrease when the internal fluid velocity is increased. Natural frequencies are calculated as larger than classical beam theory. Free vibration analysis of micro-pipe conveying fluid by wave method is performed by Baohui *et al*. (2012). Flexural vibrations of micro-scale pipes conveying fluid by considering the size effects of micro-flow and micro-structure are examined by Wang *et al*. (2013b). Free vibrations of axially functionally graded tapered microbeams based on modified couple stress theory are given by Akgoz and Civalek (2013b). Also, buckling analysis of linearly tapered micro columns are given by Akgoz and Civalek (2013a). Another application of the new theory is performed by Zeighampour and Beni (2014). Double-walled carbon nanotube conveying fluid is subjected. It is shown that the effects of system parameters are stronger than classical beam theory. Size-dependent vibrations of a micro-beam conveying fluid and resting on an elastic foundation are presented by Kural and Ozkaya (2015). The comparisons of modified couple stress theory and classical beam theory are included. Fluid conveying functionally graded microshells are subjected to a study performed by Ansari *et al*. (2015). Vibration and instability analysis are given by using modified couple stress theory. Yin *et al*. (2011) used the strain gradient theory for micro pipes conveying fluid. Recently, size dependent stability analysis of cantilever micro-pipes is performed by Hosseini and Bahaadini (2016). Modified strain gradient theory which is a combination of modified couple stress and classical theories is used. The results of the combined theory give greater natural frequencies than that performed by other two theories. Akgoz and Civalek (2015b) applied the modified strain gradient elasticity theory to a non-homogenous microbeam under Winkler foundation. A new microstructure dependent shear deformable beam model is presented by Akgoz and Civalek (2015a). Additionally, size dependent models of nano structures are studied by many researchers. Wang (2012) presented the vibration analysis of nanotubes conveying fluid based on gradient elasticity theory. Besides, nonlocal strain gradient theory is applied to microtubes and size dependent effects on critical flow velocity is investigated by Li *et al*. (2016). Akgoz and Civalek (2016) used strain gradient theory for bending analysis of embedded carbon nanotubes resting on an elastic foundation. Also, Yin *et al*. (2011) applied the strain gradient beam model to microscale pipes conveying fluid. Vibration and stability analysis are given in the mentioned study. It is shown that greater natural frequencies and higher critical flow velocities are obtained by strain gradient theory when it is compared to results of classical theory.

In most general studies, boundary conditions are assumed as ideal supports. Physical conditions of the support are ignored and BCs are supposed to be flawless. However it is difficult to reach flawless boundary conditions due to disorders in the structure of the system. As an example, an ideal simply support enables sloping while preventing displacements at the support points. A non-ideal simply support can carry moment in low orders. Similarly, an ideal clamped supported disables slope and carries moment. However a non-ideal clamped support is able to enable slope and/or displacement at the support point. As a result, non-ideal boundary conditions require new mathematical models to give accurate solutions of the systems. Non-ideal BCs are subjected to various studies in recent years. Pakdemirli and Boyacı (2001), studied non-ideal boundary conditions for stretched beam. Effect of non-ideal boundary conditions on the vibrations of continuous systems and non-linear vibrations of a simply supported beam with a non-ideal support in between are also studied by Pakdemirli and Boyacı (2002, 2003). Effects of non-ideal boundary conditions on vibrations of micro-beams are examined by Ekici and Boyacı (2007). Besides, nonlinear vibrations and stability analysis of axially moving strings having non-ideal boundary conditions are given (Yurddas *et al*. 2012, 2013). A new mathematical model for non-ideal boundary conditions is proposed by Lee (2013). Free vibration analysis of Euler Bernoulli and Timoshenko beams with non-ideal clamped boundary conditions is carried out. In order to determine the effect of non-ideal BCs, a weighting factor (k) is presented. Bağdatlı and Uslu (2015) applied this mathematical model to free vibrations of axially moving string. Finally, vibrations of fluid conveying microbeams under the effects of non-ideal boundary conditions are studied by Atcı and Bağdatlı (2017).

The objective of this paper is to show the effects of non-ideal boundary conditions on free vibration characteristics of fluid-conveying microbeams. Non-ideal BCs are modelled as linear combination of ideal clamped and ideal simply supported boundary conditions through the weighting factor k. Modified couple stress theory is used to capture the micro size effects of the fluid conveying micro beam. Equations of motion of the system are obtained by using Hamilton's principle. Method of multiple scales, a perturbation technique is applied to governing non-dimensional equations of motion. Numerical solutions of the linear problem are given for different boundary condition cases. Natural frequencies are plotted and the results are discussed to show the effects of non-ideal boundary conditions and other system parameters.

2. Equations of motion

It is needed to express kinetic and potential energies of the system to generate the mathematical model and to obtain equations of the motion of fluid conveying micro beam which is shown in Figs. 1(a)-(b).

Equations of motion are in the non-linear form due to the elongations on the beam during the vibration. This effect appears in the potential energy expression. It is assumed that the velocity of the fluid in the beam is harmonically changing around a constant value.

$$v^* = v_0^* + \varepsilon v_1^* \sin \Omega_1 t^* \tag{1}$$

where ε is a small order parameter, v_0 is the average fluid velocity and v_1 is the amplitude. Here Ω_1 is the changing frequency of the velocity and the superscript $(\ )^*$ indicates that the parameters are dimensional.

Kinetic and potential energies of the system that consist of microbeam and the inner fluid are expressed respectively as

$$\begin{aligned} K &= \frac{1}{2}\rho_b A_b \int_0^L \left\{ \dot{w}^{*2} + \dot{u}^{*2} \right\} dx \\ &+ \frac{1}{2}\rho_f A_f \int_0^L \left\{ \begin{aligned} &\left(\dot{w}^* + w^{*\prime} v^*\right)^2 \\ &+\left(v^* + \dot{u}^* + u^{*\prime} v^*\right)^2 \end{aligned} \right\} dx^* \end{aligned} \tag{2}$$

$$\begin{aligned} V &= \frac{1}{2} E A_b \int_0^L \left(u^{*\prime} + \frac{1}{2} w^{*\prime 2} \right)^2 dx^* \\ &+ \frac{1}{2} EI \int_0^L w^{*\prime\prime 2} dx^* + \int_0^L N \left(u^{*\prime} + \frac{1}{2} w^{*\prime 2} \right) dx^* \\ &+ \frac{1}{2} G A_b l^2 \int_0^L w^{*\prime\prime 2} dx^* \end{aligned} \tag{3}$$

Axial displacement which occurs during the motion of the microbeam is described by $u^*(x, t)$ and transverse displacement is denoted by $w^*(x, t)$.

A_b and A_f are cross-sectional areas of the microbeam and the moving fluid, respectively. ρ_b and ρ_f are densities of the micro beam and inner fluid. E is the elasticity modulus, G is the shear modulus, I is the area moment of inertia and L is the length of the micro-beam. The first term of the Eq. (3) is the effect of the longitudinal elongation of the beam during transverse vibration. The second term is bending and the third term is the axial tension force determined by N. The last term is the shear effect of the beam which is derived by modified couple stress theory (Yang *et al*. 2002, Park and Gao 2006). Material length scale parameter l is offered to capture the size effects of the microbeam. Numerical values of material length scale parameter are obtained experimentally in previous studies and it is determined that material length scale parameter is unique for each material (Lam *et al*. 2003).

Hamilton principle is indicates that

$$\int_{t_1}^{t_2} (\delta K - \delta V) dt = 0 \tag{4}$$

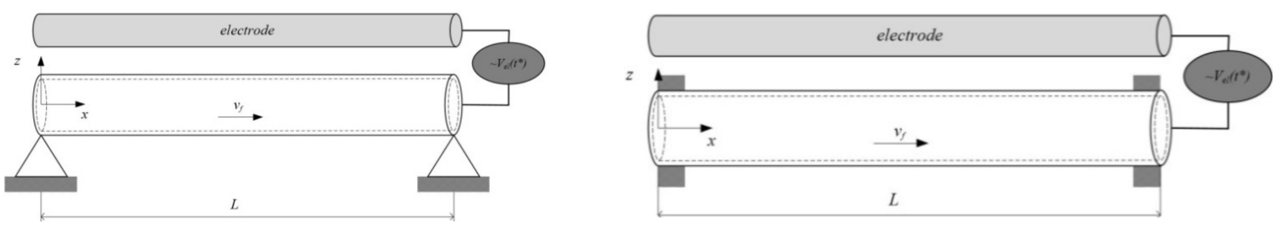
Non-linear equations of motion are obtained by substituting Eqs. (2)-(3) into the Eq. (4). Equations are written in non-dimensional form as follows

$$\begin{aligned} &\ddot{w} + 2\beta v \dot{w}' + (\beta v^2 - 1) w'' + \beta \dot{v} w' \\ &-V_1^2 \left[\left(u' + \frac{d^2}{2L^2} w'^2 \right) w' \right]' \\ &+\left(v_f^{\,2} + \gamma^2 \right) w^{iv} = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} &\ddot{u} + 2\beta v \dot{u}' + \beta v^2 u'' + \beta \dot{v} u' \\ &+\beta \dot{v} - V_1^2 \left[u' + \frac{d^2}{2L^2} w'^2 \right]' = 0 \end{aligned} \tag{6}$$

Eqs. (5)-(6) are independent from geometric and material properties of the structure. Dimensionless parameters of the system are described as

$$\begin{aligned} &w = \frac{w^*}{d} \qquad u = \frac{u^*}{L} \qquad x = \frac{x^*}{L} \qquad v = \frac{v^*}{\varphi} \qquad t = \frac{t^*}{T} \\ &\rho_b A_b = m_b \qquad \rho_f A_f = m_f \qquad T = L\sqrt{\frac{m_b + m_f}{N}} \end{aligned} \tag{7}$$



(a) Wind speed profile (b) Wind direction profile

Fig. 1 ANN model output training data for upstream typhoon wind field coming from N direction with exponent 0.22

$$\beta = \frac{m_f}{m_b + m_f} \quad \varphi = \sqrt{\frac{N}{m_b + m_f}} \quad V_1 = \sqrt{\frac{EA_b}{N}}$$
$$\overline{\alpha}_2 = \frac{EA_b}{N}\frac{d^2}{L^2} \quad v_f^{\ 2} = \frac{EI}{NL^2} \quad \gamma^2 = \frac{GA_b l^2}{NL^2} \tag{7}$$

where m_b and m_f are the masses of the micro beam and inner fluid, respectively. t is time and T is dimensionless time parameter. β is called fullness of the beam and described as the ratio between the fluid mass and total mass of the microbeam. Here, v_f is dimensionless beam parameter. γ is dimensionless microbeam parameter which is obtained corresponds to micro size effects of the beam. α_2 is beam elasticity parameter. This parameter is described as $\overline{\alpha}_2 = \varepsilon\alpha_2$ in small order. Here ε is a small order perturbation parameter.

It should be noted that longitudinal vibrations move significantly faster than transverse vibrations which means V_1 >> 1 (Chakraborty *et al*. 1998, Thurman and Mote 1969). Due to this reason, the terms excluding last term in Eq. (6) are neglected. When Eq. (6) is integrated and substituted into Eq. (5), equation of motion is obtained in the form

$$\ddot{w} + \beta\left(2v\dot{w}' + \dot{v}w'\right) + \left(\beta v^2 - 1\right)w'' + \left(v_f^{\ 2} + \gamma^2\right)w^{iv} = \overline{\alpha}_2\left\{\frac{1}{2}\int_0^1 w'^2 dx\right\}w'' \tag{8}$$

3. Non-ideal boundary conditions

Generally in mechanical problems, BCs are considered as ideal supports. Physical conditions of the connecting point are neglected. Especially for small size systems, even the small order variations on BCs become important. In this study, fluid conveying microbeam is considered to operate under non-ideal BCs. Mathematical model of non-ideal BCs isconsidered as a linear combination of ideal simply support and ideal clamped support (Lee 2013). In general form, non-dimensional non-ideal BCs are presented as

$$\begin{aligned} w(0) &= 0 \quad kw''(0) - (1-k)w'(0) = 0 \\ w(L) &= 0 \quad kw''(L) + (1-k)w'(L) = 0 \end{aligned} \tag{9}$$

where k is the weighting factor which describes the ratio between ideal clamped and ideal simply supported boundary conditions. Ideal clamped boundary condition is obtained when k = 0 in Eq. (9). When k = 1, ideal simply supported boundary condition is obtained. The weighting factor is taken very close to zero for non-ideal clamped support and close to 1 for non-ideal simply supported BCs.

4. Perturbation analysis

Method of multiple scales which is one of the perturbation techniques (Nayfeh 1981) is used to obtain approximate solutions of the equation of motion. Time scale is divided into the slow time scale $T_0 = t$ and fast time scale, $T_1 = \varepsilon t$. Differentiations with respect to slow and fast time scales are $\partial/\partial t = D_0 + \varepsilon D_1$ and $\partial^2/\partial t^2 = D_0^2 + 2\varepsilon D_0 D_1$ where $D_n = \partial/\partial T_n$. The expansion of the transverse displacement is assumed as

$$w(x,t:\varepsilon) = w_0(x,T_0,T_1) + \varepsilon w_1(x,T_0,T_1) + O(\varepsilon^2) \tag{10}$$

Expansion of two terms which is given above and derivatives with respect to new time scales are applied to the governing equation of motion which is given in Eq. (8). Equations of motion in Order (1) and Order (ε) are obtained as follows

$$Order(1): D_0^{\ 2}w_0 + 2\beta v_0 D_0 w_0' + \left(\beta v_0^{\ 2} - 1\right)w_0'' + \left(v_f^{\ 2} + \gamma^2\right)w_o^{\ iv} = 0 \tag{11}$$

$$\begin{aligned} Order(\varepsilon): &D_0^{\ 2}w_1 + 2\beta v_0 D_0 w_1' + \left(\beta v_0^{\ 2} - 1\right)w'' \\ &+ \left(v_f^{\ 2} + \gamma^2\right)w_1^{\ iv} = -2D_0D_1w_0 - 2\beta v_0 D_1 w_0' \\ &-2\beta v_1 \sin\Omega_1 T_0 D_0 w_0' - \beta v_1 \Omega_1 \cos\Omega_1 T_0 w_0' \\ &-2\beta v_0 v_1 \sin\Omega_1 T_0 w_0'' + \alpha_2\left(\frac{1}{2}\int_0^1 w_0'^2 dx\right)w_0'' \end{aligned} \tag{12}$$

4.1 Linear problem

The first order equation given in Eq. (11) forms the linear problem. The solution of the problem is written in the complex form

$$w_0\left(x,T_0,T_1\right) = A\left(T_1\right)e^{i\omega T_0}Y\left(x\right) + \overline{A}\left(T_1\right)e^{-i\omega T_0}\overline{Y}\left(x\right) \tag{13}$$

A is the complex amplitude. Substituting Eq. (13) into the Eq. (11)

$$\left(v_f^{\ 2} + \gamma^2\right)Y^{iv} + \left(\beta v_0^{\ 2} - 1\right)Y'' + 2i\omega\beta v_0 Y' - \omega^2 Y = 0 \tag{14}$$

Solution of $Y(x)$ is offered as

$$Y\left(x\right) = c_1 e^{ir_1 x} + c_2 e^{ir_2 x} + c_3 e^{ir_3 x} + c_4 e^{ir_4 x} \tag{15}$$

Equation of dispersion is achieved when Eq. (15) is substituted into the Eq. (14)

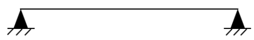
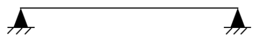
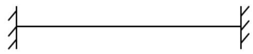
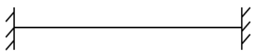
$$r_n^{\ 4}\left(v_f^{\ 2} + \gamma^2\right) - r_n^{\ 2}\left(\beta v_0^{\ 2} - 1\right) - r_n 2\omega\beta v_0 - \omega^2 = 0 \tag{16}$$

where n = 1, 2, 3, 4.

5. Numerical solutions

Free vibrations of fluid conveying microbeams are subjected to this paper. The objective of this study is to

Table 1 Boundary conditions of described four different cases

Case(1)	$0.9 \le k \le 1$	Non-ideal simply supported-ideal simply supported $Y(0)=0 \quad kY''(0)-(1-k)Y'(0)=0$ $Y(1)=0 \quad Y''(1)=0$	
Case(2)	$0.9 \le k \le 1$	Non-ideal simply supported at both ends $Y(0)=0 \quad kY''(0)-(1-k)Y'(0)=0$ $Y(1)=0 \quad kY''(1)+(1-k)Y'(1)=0$	
Case(3)	$0 \le k \le 0.1$	Non-ideal clamped-ideal clamped $Y(0)=0 \quad kY''(0)-(1-k)Y'(0)=0$ $Y(1)=0 \quad Y'(1)=0$	
Case(4)	$0 \le k \le 0.1$	Non-ideal clamped at both ends $Y(0)=0 \quad kY''(0)-(1-k)Y'(0)=0$ $Y(1)=0 \quad kY''(1)+(1-k)Y'(1)=0$	

investigate the effects of non-ideal BCs on natural frequencies of fluid conveying microbeams. Four different boundary condition cases are considered for this purpose. Numerical results are obtained for these cases and the effects of different boundary conditions on the system are presented. In Case (1), the microbeam is non-ideal simply supported at the left hand side and ideal simply supported at the right hand side. In Case (2), the microbeam is under non-ideal simply supported boundary conditions at both sides. In Case (3), the beam is non-ideal clamped at the left hand side and ideal clamped at the right hand side. In Case (4), both ends of the beam are non-ideal clamped.

In Table 1, the most general boundary condition expression which is given in Eq. (9) is specified for four different boundary cases of fluid-conveying micro-beam.

It is needed to specify them material length scale parameter (l) of the microbeam according to modified couple stress theory. It is presented as follows (Lam *et al.* 2003)

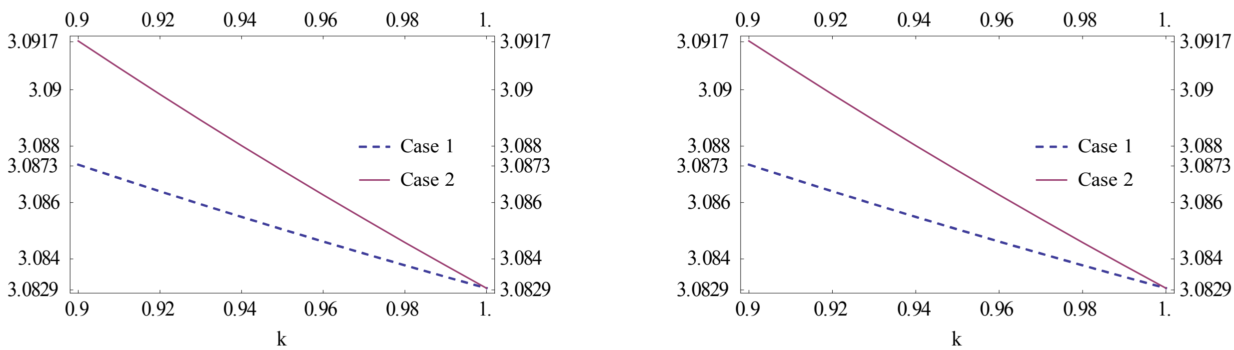
$$l = \frac{b_h}{\sqrt{3(1-\nu)}} \tag{17}$$

where b_h is material higher-order bending parameter and ν is Poisson's ratio. In this study steel microbeams are considered. Material length scale parameter is distinctive for each kind of material. For steel, b_h is given as 10 μm (Ellis and Smith 1968) and ν = 0.3. Microbeam is designed with 25 μm outer and 20 μm inner diameters. Beam parameter v_f and microbeam parameter γ are calculated for steel microbeam.

In order to obtain natural frequencies of fluid conveying microbeam, r_1, r_2, r_3 and r_4 in Eq. (16) are calculated numerically. If the BCs which are given in Table (1) are substituted into the Eq. (15) one obtains

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

$$\begin{aligned} &a_{11}=1,\ a_{12}=1, a_{13}=1, a_{14}=1 \\ &a_{21}=(1-k)ir_1+kr_1^2,\ a_{22}=(1-k)ir_2+kr_2^2 \\ &a_{23}=(1-k)ir_3+kr_3^2,\ a_{24}=(1-k)ir_4+kr_4^2 \\ &a_{31}=e^{ir_1},\ a_{32}=e^{ir_2},\ a_{33}=e^{ir_3},\ a_{34}=e^{ir_4} \\ &a_{41}=(1-k)ir_1e^{ir_1}-kr_1^2e^{ir_1} \\ &a_{42}=(1-k)ir_2e^{ir_2}-kr_2^2e^{ir_2} \end{aligned} \tag{18}$$



(a) v_0 = 0.5, 1st mode (b) v_0 = 0.5, 2nd mode

Fig. 2 Natural frequencies varying with the weighting factor for a simply supported microbeam (1st mode, F = 1, v_0 = 1.0, v_1 = 1.0, v_f = 0.1, β = 0.5, α_2 = 1)

$$a_{43} = (1-k)ir_3e^{ir_3} - kr_3^{\,2}e^{ir_3}$$
$$a_{44} = (1-k)ir_4e^{ir_4} - kr_4^{\,2}e^{ir_4} \tag{18}$$

Numerically calculated r_n are substituted into the coefficient matrix to obtain the natural frequencies omega's that makes the determinant value of the matrix zero. This procedure is repeated to introduce the effects of weighting factor k and other system parameters on natural frequencies.

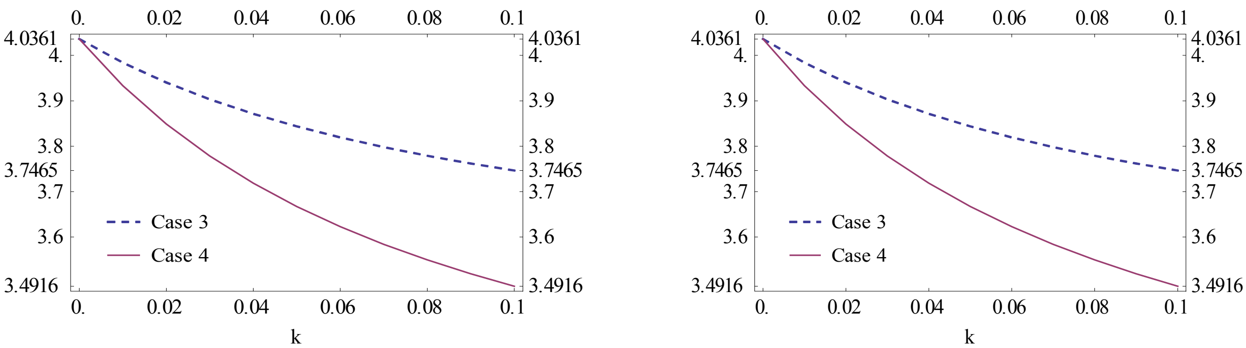
In Figs. 2(a)-(b), natural frequencies varying with the weighting factor k are given for a simply supported microbeam. The first mode and the second modes of vibration are plotted in Figs. 2(a)-(b). Natural frequencies for $k = 1$ (ideal simply supported) are $\omega_1 = 3.0829$ and $\omega_2 = 7.3594$ for the first and the second modes respectively. In Fig. 2(a), it is seen that the first mode natural frequency of Case (2) increases by 0.28% while k decreases from 1 to 0.9. Similarly the second mode natural frequency increases by 0.2% as shown in Fig. 2(b). It should be noticed that this change remains at low values for Case (1) which has non-ideal boundary condition at one side and ideal boundary condition at the other.

In Figs. 3(a)-(b), natural frequencies varying with the weighting factor k are given for clamped microbeam. The first mode and the second mode of vibrations are plotted. Natural frequencies for $k = 0$ (ideal clamped) are $\omega_1 = 4.0361$ and $\omega_2 = 9.3907$ in the first and the second modes respectively. The first mode natural frequency of Case (4) decreases about 15.6% while k increases from 0 to 0.1. It is seen from Fig. 3(b) that the second mode natural frequency decreases about 15.5%. This change remains at low values for Case (3). It should be noticed that non-ideal BCs significantly affect the natural frequencies of clamped microbeams rather than simply supported ones.

In Figs. 4(a)-(b), natural frequencies varying with weighting factor and fluid velocity are shown. Fluid velocitiesare v_0 = 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0. The first mode vibrations of Case (2) and Case (4) show the effects of non-ideal boundary conditions explicitly. Natural frequencies of Cases (2) and (4) decreased with the increasing velocity of the inner fluid. Also, the effect of non-ideal BCs is more noticeable on clamped microbeams than simply supported microbeams

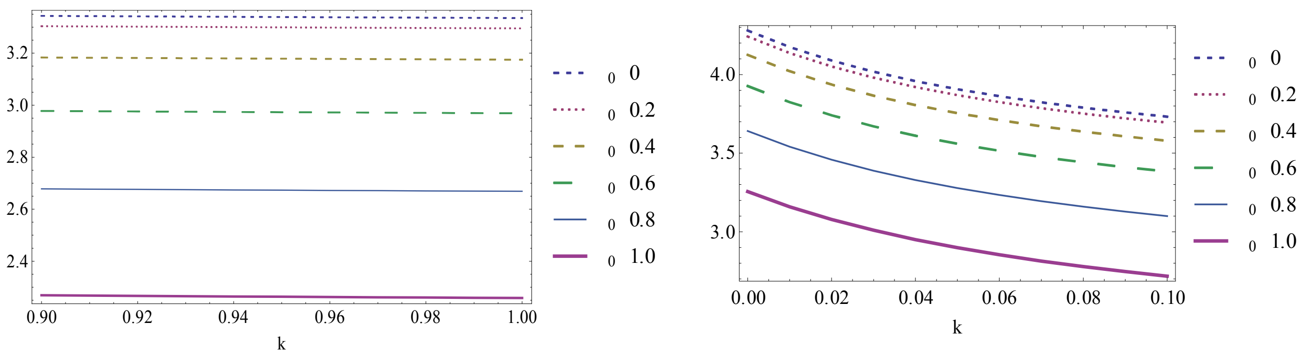
Comparison of Case (1) and (2) is given in Fig. 5(a). Also, Case (3) and (4) are shown in Fig. 5(b). Weighting factor k is chosen as k = 0.9 and k = 0.1 for non-ideal simply supported and clamped microbeams respectively. Natural frequencies are presented with respect to the fluid velocity.

In Figs. 5(a)-(b), ideal simply supported microbeam, microbeams with non-ideal supported at one side and non-ideal supported at both sides are compared. In particular, the effect of non-ideal BCs on clamped microbeams is seen obviously. This effect remains at low values for simply supported microbeams. It is seen clearly from Fig. 5(b) that the difference between natural frequencies decreases while the weighting factor increases. Namely, when the boundary conditions of the beam distinct from its ideal structure,



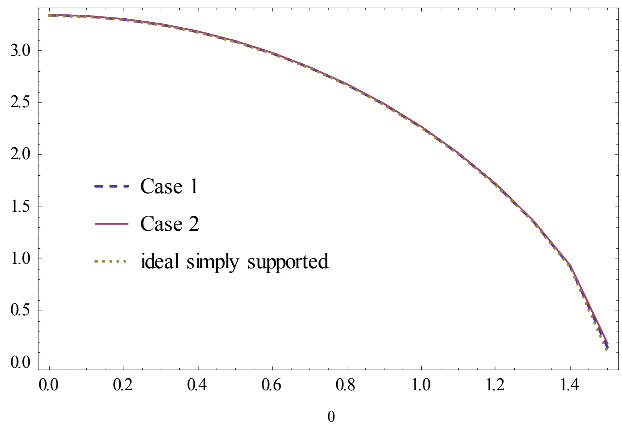
(a) $v_0 = 0.5$, 1st mode (b) $v_0 = 0.5$, 2nd mode

Fig. 3 Natural frequencies varying with the weighting factor for a clamped microbeam (1st mode, $F = 1$, $v_0 = 1.0$, $v_f = 0.1$, $\beta = 0.5$, $\alpha_2 = 1$)

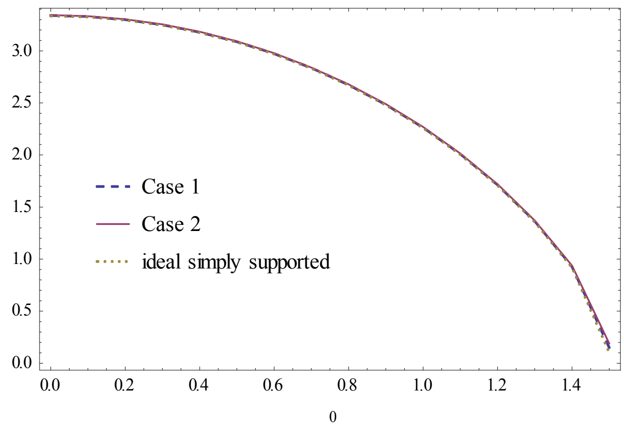


(a) Case (2) (b) Case (4)

Fig. 4 Effects of varying fluid velocities on natural vibrations of simply supported and clamped microbeams

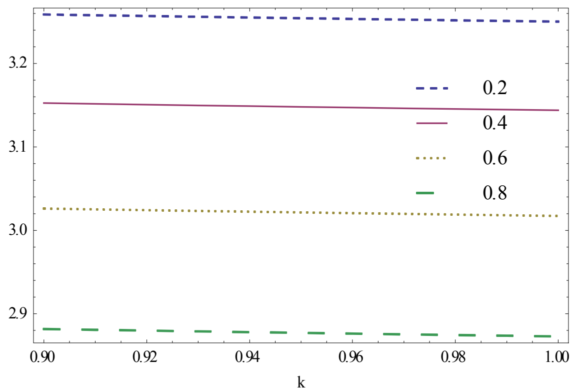


(a) $k = 0.9$, 1st mode

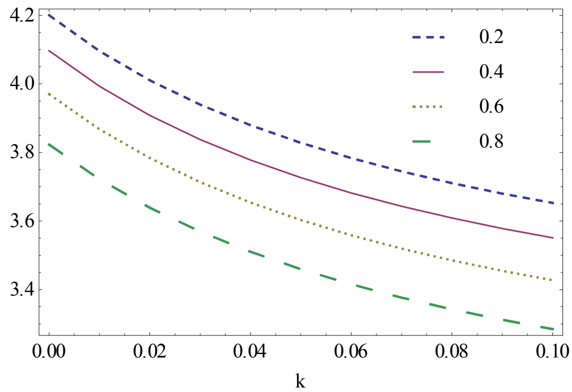


(b) $k = 0.1$, 1st mode

Fig. 5 Natural frequencies of simply supported and clamped microbeams varying with fluid velocity



(a)



(b)

Fig. 6 Natural frequencies of simply supported and clamped microbeams varying with the ratio of fullness of the beam

the effect of fluid velocity on the natural frequency becomes smaller.

In Figs. 6(a)-(b), natural frequencies of simply supported and clamped microbeams are compared in terms of the fullness ratio, β. Case (2) and Case (4) are chosen to show the effects of non-ideal BCs. It is seen that natural frequencies of the beam decrease by the increasing β.

6. Conclusions

In this study, free vibration analysis of fluid conveying microbeams under non-ideal boundary conditions is presented. Non-ideal BCs are modeled as linear combination of ideal simply supported and clamped BCs. The weighting factor k is described for this purpose. A new boundary condition model is applied to the fluid conveying microbeam system and the results of natural frequencies affected by non-ideal boundaries are presented.

Four different boundary condition cases are considered to see the effects of non-ideal BCs. As it is expected, when both sides of the beam are non-ideal supported, natural frequencies differ from the values of ideal supported microbeams which have the weighting factors $k = 0$ (ideal clamped) and $k = 1$ (ideal simply supported). The natural frequencies of clamped supported beam decreased and the natural frequencies of simply supported beam increased. In Case (2), non-ideal simply supported beam is modeled. The first mode natural frequency of the beam increased by 0.28% when the second mode frequency increased by 0.2%. This change remains at low levels in Case (1), which is figured as non-ideal support at the left side and ideal support at the right side. In Case (4), non-ideal clamped microbeam is modeled. The first mode natural frequency decreased about 15.6% and the second mode frequency decreased by 15.5%. In Case (3), which is non-ideal clamped on the left side and ideal clamped on the right side, natural frequencies do not differ significantly. It is understood that non-ideal BCs highly affect clamped microbeams rather than simply supported ones.

The change of natural frequencies caused by non-ideal BCs becomes smaller as the fluid velocity v_0 increases. Additionally, effect of the fullness ratio of microbeam is investigated and the effect of the fluid amount in the beam is shown. It is seen that natural frequencies of microbeam increase while the fullness ratio β is increasing.

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