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Analytical solution of a two-dimensional thermoelastic problem subjected to laser pulse

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Abstract. In this article, the problem of a two-dimensional thermoelastic half-space are studied using mathematical methods under the purview of the generalized thermoelastic theory with one relaxation time is studied. The surface of the half-space is taken to be thermally insulated and traction free. Accordingly, the variations of physical quantities due to by laser pulse given by the heat input. The nonhomogeneous governing equations have been written in the form of a vector-matrix differential equation, which is then solved by the eigenvalue approach. The analytical solutions are obtained for the temperature, the components of displacement and stresses. The resulting quantities are depicted graphically for different values of thermal relaxation time. The result provides a motivation to investigate the effect of the thermal relaxation time on the physical quantities.

Keywords: Eigenvalue approach; exact solution; Lord and Shulman theory; relaxation time; laser pulse

1. Introduction

The classical uncoupled theory of thermoelasticity (CT) was modified by Biot (1956) to eliminate the paradox that elastic changes have no effect on the temperature. Contrary to physical observations, however, the heat equations for both theories predict infinite speeds of propagation for heat waves. The first of such modeling is the extended thermoelasticity theory (LS) of Lord and Shulman (1967) who introduced the concept of thermal relaxation time into the classical Fourier law of heat conduction. Subsequently, modifying the stress versus strain relationship as well as the entropy relationship with relaxation time, Green and Lindsay (1972) proposed the temperature rate dependent thermoelasticity (GL) theory. The theory was extended for anisotropic body by Dhaliwal and Sherief (1980). During the second half of twentieth century, a large amount of work has been devoted to solving thermoelastic problems. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using numerical and analytical methods (Othman and Abbas 2012, 2014, Abbas and Zenkour 2013, 2014, Kumar and

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Rupender 2010, Ezzat 2011, Ezzat and El-Karamany 2003, Ezzat and Awad 2010, Zenkour and Abouelregal 2015a, b, Bouchafa *et al*. 2015, Isavand *et al*. 2015, Saadatfar and Aghaie-Khafri 2015, Kakar and Kakar 2014, Abbas and Kumar 2016).

In the case of ultra-short laser heating, the elastic waves propagate long distance in structures, they provide an efficient means to characterize properties of engineering structures such as the elastic, thermal properties. Very rapid thermal processes, under the action of an ultra-short laser pulse, are interesting from the standpoint of thermoelasticity since they require an analysis of the coupled temperature and deformation fields. The irradiation of the surface of a solid by pulsed laser light generates wave motion in the solid material. This waves have many applications which use in the measurement of dimensional properties. For instance, plate thickness can be accurately measured by the “time-of-flight”technique of a pulsed wave. The waves are not audible to the human ear, because the dominant frequencies of the generated wave motion are generally above 2×10^4 Hz, and are therefore called “ultrasonic waves”. In the case of ultra-short pulsed laser heating, two effects become important.

Deresiewicz (1975) investigated the propagation of waves under plain strain state in thermoelastic plates. Mallik and Kanoria (2008) studied the effect of spatially varying heat source in two dimensional generalized thermoelastic problem for a transversely isotropic thick plate. Verma and Hasebe (1999) investigated the propagation of thermoelastic waves in infinite plates under Green and Naghdi type II (without energy dissipation). Ezzat and Youssef (2005) established a model of the equations of generalized magneto-thermoelasticity in a perfectly conducting medium. Agarwal (1978, 1979a, b) studied the wave propagation in generalized thermoelaticity and electromagneto-thermoelastic medium. Abbas and Youssef (2015) investigated the effect of fractional order in two-dimensional generalized thermoelastic porous material.

In this work, the analytical solutions for generalized thermoelastic interaction on a half-space with one relaxation time is developed. When the surface of the half-space is quiescent first, the basic equations of the mathematical model is presented. The eigenvalue approach is used to obtain the solutions of non-dimensional nonhomogeneous equations. The analytical solutions are obtained for the temperature, the components of displacement and stresses. The comparison among the theories i.e., Lord and Shulman’s (LS) and the classically coupled thermoelastic (CT) theory is presented graphically. The results further show that the analytical scheme can overcome mathematical problems to analyze these problems

2. The governing equation

Following Lord and Shulman (1967), the basic equations for an isotropic, homogeneous, elastic medium in the context of generalized thermoelastic theory in the absence of body forces can be considered by the following:

The equations of heat conduction

$$\left(K_{ij}T_{,j}\right)_{,i}=\left(1+\tau_o\frac{\partial}{\partial t}\right)\left(\rho c_e\frac{\partial T}{\partial t}+\gamma T_0\frac{\partial e_{ii}}{\partial t}-Q\right) \tag{1}$$

In the case of absence of body force, the equations of motion can be expressed by

$$\sigma_{ij,j}=\rho\frac{\partial^2 u_i}{\partial t^2}. \tag{2}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e_{kk} - \gamma T]\delta_{ij}, \tag{3}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

where λ, μ are the Lame's constants, ρ is the density of the medium, c_e is the specific heat at constant strain. The variable $T = T^* - T_0$ is the temperature increment of the material, T_0 is the reference temperature, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, t is the time, τ_o is the thermal relaxation time, δ_{ii} is the Kronecker symbol, K is the thermal conductivity. The tensor e_{ij} are the components of strain tensor, σ_{ij} are the components of stress tensor, u_i are the components of displacement vector and Q is the internal heat source. Now, we suppose an elastic, homogeneous, isotropic, generalized thermoelastic half space initially at uniform temperature T_0 and we will use the Cartesian co-ordinates (x, y, z) with origin on the surface $x = 0$ and y-axis directed vertically into the medium. The region $x > 0$ is occupied by the elastic solid. For two dimensional problem, the components of displacement vector and temperature can be taken in the following form

$$u = u_x = u(x,y,t),\ v = u_y = v(x,y,t),\ w = u_z = 0,\ T = T(x,y,t) \tag{5}$$

Thus, the above equations can be take the form

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma T, \tag{6}$$

$$\sigma_{yy} = (\lambda + 2\mu)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} - \gamma T, \tag{7}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{8}$$

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} - \gamma\frac{\partial T}{\partial x}, \tag{9}$$

$$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} - \gamma\frac{\partial T}{\partial y}, \tag{10}$$

$$\rho c_e\left(\frac{\partial T}{\partial t} + \tau_o\frac{\partial^2 T}{\partial t^2}\right) = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \gamma T_0\left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(1 + \tau_o\frac{\partial}{\partial t}\right)Q(x,y,t). \tag{11}$$

For our convenience, the following dimensionless physical quantities and notations are used

$$(x',y',u',v')=\frac{\eta}{c_1}(x,y,u,v),\ Q'=\frac{\gamma}{\rho K\eta^2}Q,\ c_1^2=\frac{\lambda+2\mu}{\rho},\ T'=\frac{\gamma T}{\rho c_1^2},$$
$$\left(\sigma'_{xx},\sigma'_{xy},\sigma'_{yy}\right)=\frac{1}{\rho c_1^2}\left(\sigma_{xx},\sigma_{xy},\sigma_{yy}\right),\ \left(t',\tau'_o\right)=\eta\left(t,\tau_o\right),\ \eta=\frac{\rho c_e c_1^2}{k}. \tag{12}$$

In terms of these dimensionless form of variables in Eq. (12), the above Eqs. (6)-(11) can be taken the following forms (for convenience, the dashed has been dropped)

$$\sigma_{xx}=\frac{\partial u}{\partial x}+m_3\frac{\partial v}{\partial y}-T, \tag{13}$$

$$\sigma_{yy}=\frac{\partial v}{\partial y}+m_3\frac{\partial u}{\partial x}-T, \tag{14}$$

$$\sigma_{xy}=m_1\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{15}$$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+m_1\frac{\partial^2 u}{\partial y^2}+m_2\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial T}{\partial x}, \tag{16}$$

$$\frac{\partial^2 v}{\partial t^2}=\frac{\partial^2 v}{\partial y^2}+m_1\frac{\partial^2 v}{\partial x^2}+m_2\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial T}{\partial y}, \tag{17}$$

$$\frac{\partial T}{\partial t}+\tau_o\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}-\varepsilon\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+Q+\tau_o\frac{\partial Q}{\partial t}, \tag{18}$$

where $m_1=\dfrac{\mu}{\lambda+2\mu}$, $m_2=\dfrac{\lambda+\mu}{\lambda+2\mu}$, $m_3=\dfrac{\lambda}{\lambda+2\mu}$, $\varepsilon=\dfrac{T_0\gamma^2}{\rho^2 c_1^2 c_e}$.

The surface of half-space is illuminated by laser pulse given by the heat input (Al-Qahtani and Datta 2008)

$$Q(x,y,t)=I_0 f(t)g(y)h(x), \tag{19}$$

where the temporal profile $f(t)$ is presented as $f(t)=\dfrac{t}{t_p^2}e^{-\frac{t}{t_p}}$, the pulse is also assumed to have Gaussian profile in y as $g(y)=\dfrac{1}{2\pi r^2}e^{\frac{-y^2}{r^2}}$ and as a function of the depth x, the heat deposition due to the laser pulse is assumed to decay exponentially within the solid $h(x)=\gamma^* e^{-\gamma^* x}$, where $\mathbf{I}_0$ is the energy absorbed, t_p is the pulse rise time, r is the beam radius, γ^* is the absorption depth of heating energy. A schematic representation of the pulse is shown in Fig. 1.

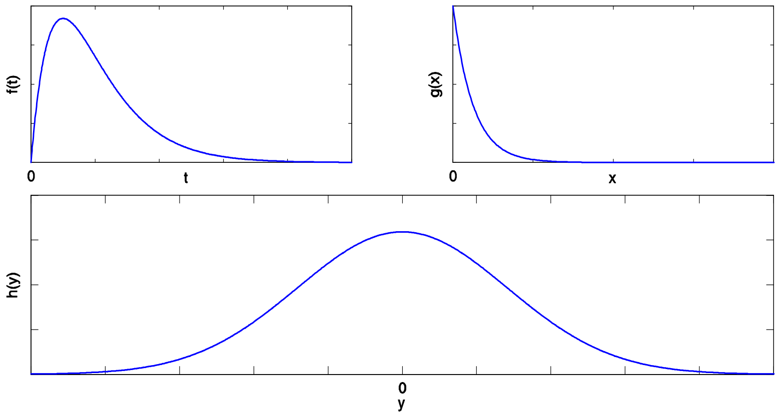

Fig. 1 Temporal and spatial profile of the pulse

Now, the initial conditions of the problem are assumed to be homogeneous. Thus, in our consideration, the boundary conditions can be given as

$$\sigma_{xx}(0,y,t)=0,\ \ \sigma_{xy}(0,y,t)=0,\ \frac{\partial T(0,y,t)}{\partial x}=0 \tag{20}$$

3. Solution of the problem

In terms of normal modes, the solution of the considered physical variable can be decomposed by the form

$$[u,v,T](x,y,t)=[u^*,v^*,T^*](x)e^{(\omega t+iqy)}, \tag{21}$$

where u*(x), v*(x) and T*(x) are the field quantities amplitudes, for *y*–direction q refer to the wave number, ω is a complex constant, *i* is the imaginary unit.

Thus, the Eqs. (16)-(18), take the form

$$\frac{d^2u^*}{dx^2}=b_{31}u^*+b_{45}\frac{dv^*}{dx}+b_{46}\frac{dT^*}{dx}, \tag{22}$$

$$\frac{d^2v^*}{dx^2}=b_{52}v^*+b_{53}T^*+b_{54}\frac{du^*}{dx}, \tag{23}$$

$$\frac{d^2T^*}{dx^2}=b_{62}v^*+b_{63}T^*+b_{64}\frac{du^*}{dx}+g(y,t)e^{-\gamma^* x}, \tag{24}$$

where $b_{41}=\omega^2+q^2m_1$, $b_{45}=-iqm_2$, $b_{46}=1$, $b_{52}=\frac{1}{m_1}\left(\omega^2+q^2\right)$, $b_{53}=\frac{iq}{m_1}$, $b_{54}=\frac{-iqm_2}{m_1}$, $b_{62}=iq\varepsilon\omega$,

$b_{63} = q^2 + \omega\,(1 + \tau_o\,\omega)$, $b_{64} = \omega\,(1 + \tau_o\,\omega)\,\varepsilon$, $g(y,t) = -\dfrac{I_0\gamma^*}{2\pi r^2 t_p^2}\left(1+\tau_0(1/t-1/t_p)\right)t\,e^{\left(-\frac{y^2}{r^2}-\frac{t}{t_p}-\omega t-iqy\right)}$.

Now, let us solve the nonhomogeneous coupled differential Eqs. (22), (23) and (24) using the eigenvalue approach. The vector-matrix of the Eqs. (22)-(24) can be expressed as the following form

$$\frac{dV}{dx} = BV + g e^{-\gamma^* x}, \tag{25}$$

Now, as in Das *et al*. (1997) and Abbas (2014a, b, 2015c) we can apply the eigenvalue approach to solve the Eq. (27). Then, the matrix B characteristic equation can be expressed as:

The general solutions V of the nonhomogeneous system Eq. (25) are the sum of the complementary solution V_c of the homogeneous equations and a particular solution V_p of the nonhomogeneous system. Now, the solutions of homogeneous system obtain by using the eigenvalue approach which proposed by Abbas (2015b). The matrix B has the characteristic equation in the form

$$R^6 - S_1 R^4 + S_2 R^2 + S_3 = 0 \tag{26}$$

with

$$S_1 = b_{41} + b_{52} + b_{45}b_{54} + b_{63} + b_{46}b_{64},$$

$$S_2 = b_{41}b_{52} - b_{53}b_{62} - b_{46}b_{54}b_{62} + b_{41}b_{63} + b_{52}b_{63} + b_{45}b_{54}b_{63} + b_{46}b_{52}b_{64} - b_{45}b_{53}b_{64}, \quad -$$

$$S_3 = b_{41}b_{53}b_{62} - b_{41}b_{52}b_{63}.$$

The characteristic Eq. (26) have six roots which written in the form

$$R = \pm R_1,\ R = \pm R_2,\ R = \pm R_3. \tag{27}$$

The eigenvector $\vec{Y} = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6]^T$, corresponding to eigenvalue R can be calculated as

$$Y_1 = b_{46}R\left(b_{52} - R^2\right) - b_{45}b_{53}R, \tag{28}$$

$$Y_2 = b_{53}\left(b_{41} - R^2\right) - b_{46}b_{54}R^2, \tag{29}$$

$$Y_3 = b_{41}\left(R^2 - b_{52}\right) + R^2\left(b_{52} + b_{45}b_{54} - R^2\right), \tag{30}$$

$$Y_4 = RY_1,\ Y_5 = RY_2, Y_6 = RY_3. \tag{31}$$

Now, we can easily calculate the eigenvector $\vec{Y}_j$, corresponding to eigenvalue R_j, j = 1, 2,...., 6.

For further reference, we shall use the following notations

$$\begin{gathered} \vec{Y}_1 = \left[\vec{Y}\right]_{R=-R_1},\quad \vec{Y}_2 = \left[\vec{Y}\right]_{R=-R_2},\quad \vec{Y}_3 = \left[\vec{Y}\right]_{R=-R_3}, \\ \vec{Y}_4 = \left[\vec{Y}\right]_{R=R_1},\quad \vec{Y}_5 = \left[\vec{Y}\right]_{R=R_2},\quad \vec{Y}_6 = \left[\vec{Y}\right]_{R=R_3}. \end{gathered} \tag{32}$$

Thus, the complementary solution of Eq. (25) has the following from

$$V_c = \sum_{j=1}^{3} A_j Y_j e^{-R_j x} \tag{33}$$

where the terms containing exponentials of growing nature in the space variable x have been discarded due to the regularity condition of the solution at infinity, A_1, A_2 and A_3 are constants to be determined from the boundary condition of the problem. From Eq. (25), there is the exponential function $e^{-\gamma^* x}$ in the inhomogeneous terms, which coincides with the exponential function in the homogeneous equation solution. Thus, the particular solution V_p should be sought in the form of a quasi-polynomial vector

$$V_p = h e^{-\gamma^* x} \tag{34}$$

where h is depend on y and t. Thus, the general solutions of the field variables can be written for x, y and t as

$$u(x,y,t) = \sum_{j=1}^{3} A_j u_j e^{-R_j x + \omega t + iqy} + \frac{f^* \gamma^* \left(b_{46} b_{52} - b_{45} b_{53} - b_{46} \gamma^{*2}\right) t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{35}$$

$$v(x,y,t) = \sum_{j=1}^{3} A_j v_j e^{-R_j x + \omega t + iqy} + \frac{f^* \left(b_{53} \gamma^{*2} - b_{41} b_{53} + b_{46} b_{54} \gamma^{*2}\right) t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{36}$$

$$T(x,y,t) = \sum_{j=1}^{3} A_j T_j e^{-R_j x + \omega t + iqy} + \frac{f^* \left(b_{41} b_{52} - b_{41} \gamma^{*2} - b_{52} \gamma^{*2} - b_{45} b_{54} \gamma^{*2} + \gamma^{*4}\right) t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{37}$$

$$\sigma_{xx}(x,y,t) = \sum_{j=1}^{3} \left(-R_j u_j + m_3 iq v_j - T_j\right) A_j e^{-R_j x + \omega t + iqy} + \frac{n_1 t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{38}$$

$$\sigma_{yy}(x,y,t) = \sum_{j=1}^{3} \left(-R_j v_j + m_3 iq u_j - T_j\right) A_j e^{-R_j x + \omega t + iqy} + \frac{n_2 t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{39}$$

$$\sigma_{xy}(x,y,t) = \sum_{j=1}^{3} \beta_1 \left(-R_j v_j + iq u_j\right) A_j e^{-R_j x + \omega t + iqy} + \frac{n_3 t}{n} e^{-\gamma^* x - \frac{y^2}{r^2} - \frac{t}{t_p}}, \tag{40}$$

where u_j, v_j and T_j are the component j of eigenvector for u, v and T respectively

$$\begin{aligned} n &= \gamma^{*6} - S_1 \gamma^{*4} + S_2 \gamma^{*2} + S_3, \\ n_1 &= -f^* \gamma^{*2} \left(b_{46} b_{52} - b_{45} b_{53} - b_{46} \gamma^{*2}\right) - \frac{y^2}{r^2} m_3 f^* \left(b_{53} \gamma^{*2} - b_{41} b_{53} + b_{46} b_{54} \gamma^{*2}\right) - \\ &\qquad f^* \left(b_{41} b_{52} - b_{41} \gamma^{*2} - b_{52} \gamma^{*2} - b_{45} b_{54} \gamma^{*2} + \gamma^{*4}\right), \end{aligned} \tag{41}$$

$$
\begin{aligned}
n_2 &= -\frac{y^2}{r^2} f^* \left(b_{53}\gamma^{*2} - b_{41}b_{53} + b_{46}b_{54}\gamma^{*2}\right) - m_3 f^* \gamma^{*2} \left(b_{46}b_{52} - b_{45}b_{53} - b_{46}\gamma^{*2}\right) - \\
&\qquad f^* \left(b_{41}b_{52} - b_{41}\gamma^{*2} - b_{52}\gamma^{*2} - b_{45}b_{54}\gamma^{*2} + \gamma^{*4}\right), \\
n_3 &= -m_1\gamma^* f^* \left(b_{53}\gamma^{*2} - b_{41}b_{53} + b_{46}b_{54}\gamma^{*2}\right) - \frac{y^2}{r^2} m_1 f^* \gamma^* \left(b_{46}b_{52} - b_{45}b_{53} - b_{46}\gamma^{*2}\right), \\
f^* &= -\frac{I_0\gamma^*}{2\pi r^2 t_p^2}.
\end{aligned} \tag{41}
$$

To complete the solution we have to know the constants A_1, A_2 and A_3, by using the boundary conditions Eq. (20) which give

$$
\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}^{-1} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} \frac{t}{n} e^{-\frac{y^2}{r^2} - \omega t - iqy - \frac{t}{t_p}}, \tag{42}
$$

where

$$
\begin{aligned}
& M_{11} = -R_1u_1 + m_3iqv_1 - T_1, \quad M_{12} = -R_2u_2 + m_3iqv_2 - T_2, \\
& M_{13} = -R_3u_3 + m_3iqv_3 - T_3, \\
& M_{21} = m_1\left(-R_1v_1 + iqu_1\right), \quad M_{22} = m_1\left(-R_2v_2 + iqu_2\right), \quad M_{23} = m_1\left(-R_3v_3 + iqu_3\right), \\
& M_{31} = T_1, \quad M_{32} = T_2, \quad M_{33} = T_3, \\
& M_1 = -S_1, \quad M_2 = -S_3, \quad M_3 = -f^*\left(b_{41}b_{52} - b_{41}\gamma^{*2} - b_{52}\gamma^{*2} - b_{45}b_{54}\gamma^{*2} + \gamma^{*4}\right).
\end{aligned} \tag{43}
$$

4. Numerical results and discussion

In order to illustrate the theoretical results obtained in the preceding section, we present some numerical values for the physical constants. For purposes of numerical evaluations, the copper material was chosen and the constants were taken by Abbas (2014c)

$$
\begin{aligned}
& \lambda = 7.76\times10^{10}\ \text{kg m}^{-1}\text{s}^{-2},\ \mu = 3.86\times10^{10}\ \text{kg m}^{-1}\text{s}^{-2}, \rho = 8954\,\text{kg m}^{-3}, \alpha_t = 1.78\times10^{-5}\ \text{k}^{-1}, I_0 = 10^5 \\
& K = 386\ \text{w m}^{-1}\text{k}^{-1},\ c_e = 383.1\ \text{J kg}^{-1}\text{k}^{-1},\ T_o = 293\ \text{k},\ \tau_o = 0.05,\ y = 0.5,\ \omega = 2 - 0.1i,\ q = 1.2.
\end{aligned}
$$

In the context of generalized thermoelastic theory with one relaxation time, the numerical technique outlined above was used to obtain the temperature, the components of displacement and the components of stress distributions. Here all the physical quantities and parameters are given in the non-dimensional forms and illustrated graphically as in figures 2-10. . It is notes that, in all figures, the solid line ( ) when (τ_o = 0.0) refer to the classical dynamical coupled theory (CT) while the dotted line ( ) when ($\tau_o \neq 0.0$) refer to the generalized thermoelstic theory with one relaxation time (LS).

Figs. 2-4 show the variation of temperature for two theories with respect to time t, the distance y and the distance x, respectively, as expect from Fig. 1 which validate the results when the surface of the half-space is taken to be traction free and thermally insulated.

Fig. 2 display the variation of temperature against time for two theories when ($x = 0.5$, $y = 0.5$) remains constants and it is observed that the temperature have a great effect for the thermal relaxation time. In both cases, the temperature attains maximum value after some time, and then continuously decreases to zero. Fig. 3 shows the variation of temperature along the distance y when ($t = 2.0$, $x = 0.5$) remains constants and it is noticed that the temperature has a maximum value at the origin then gradually decreases with increase the magnitude of y to close to zero. Fig. 4 indicates the variation of temperature along the distance x when ($t = 2.0$, $y = 0.5$) remains constants. It is clear that the temperature have maximum values at the surface and then continuously decreases with increase the distance x until it reaches to zero. The variation of horizontal displacement u with respect to the distance x when ($t = 2.0$, $y = 0.5$) remains constants are drown in Fig. 5. It indicates that when the surface of the half-space is taken to be traction free and thermally insulated, the displacement shows a negative value at the boundary of the half space and it attains a stationary maximum value after some distance and then it decreases to zero value. Fig. 6 display the variation of vertical displacement v with respect to x. It is always begin at the maximum value at the surface and then decreases with increases the distance x to close to zero.

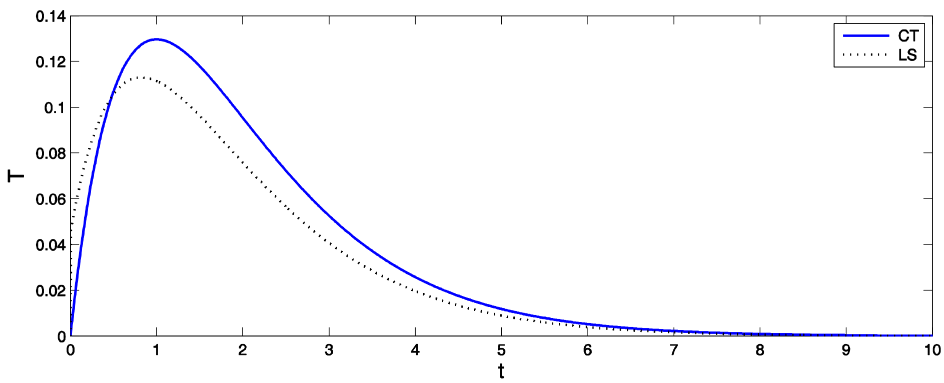

Fig. 2 Variation of temperature T with time t for two different theories

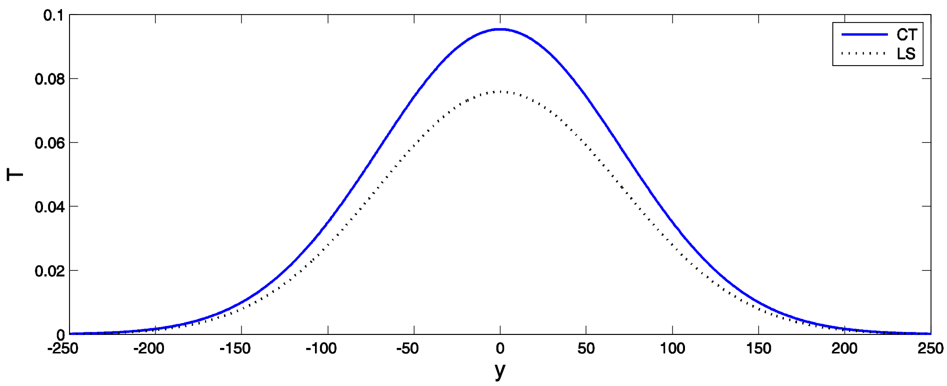

Fig. 3 Variation of temperature T with distance y for two different theories

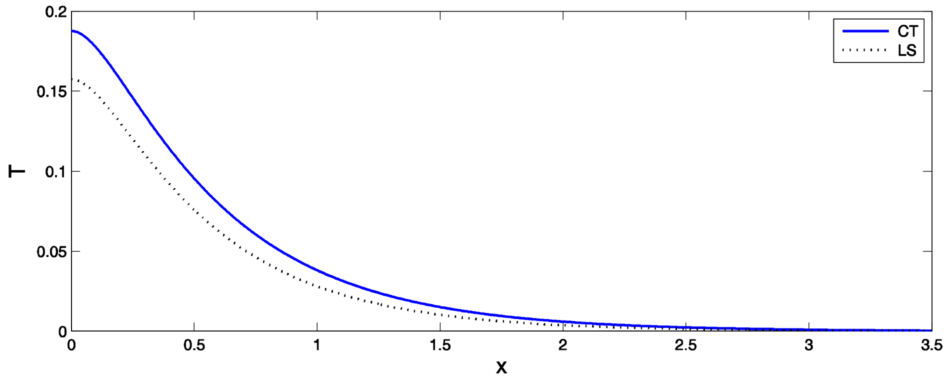

Fig. 4 Variation of temperature T with distance x for two different theories

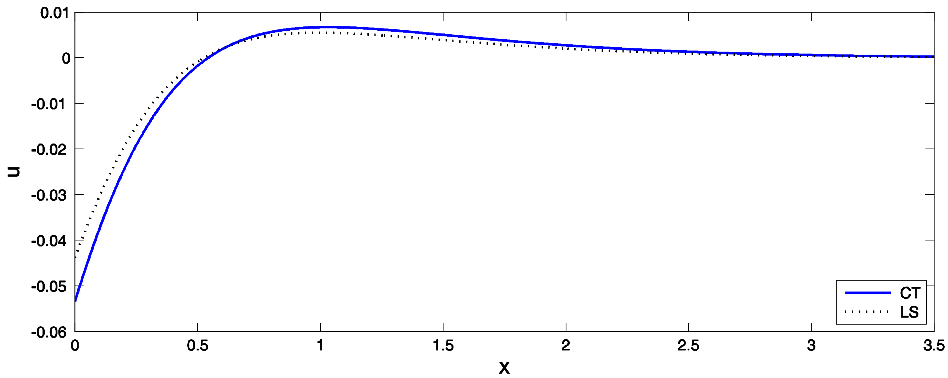

Fig. 5 Variation of horizontal displacement u with distance x for two different theories

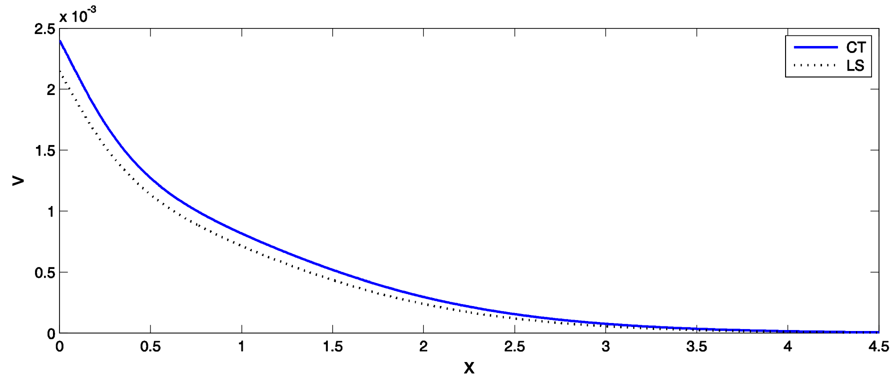

Fig. 6 Variation of vertical displacement v with distance x for two different theories

Figs. 7 and 8 present the variations of stress components σ_{xx} and σ_{xy} along the distance x respectively, in which we notes that, the stresses σ_{xx} and σ_{xy} are zero at x = 0 which agree with the boundary conditions of the problem. Fig. 9 shows the variation of stress σ_{yy} with respect to distance x in which we notice that, the magnitude of stress increases to maximum value then

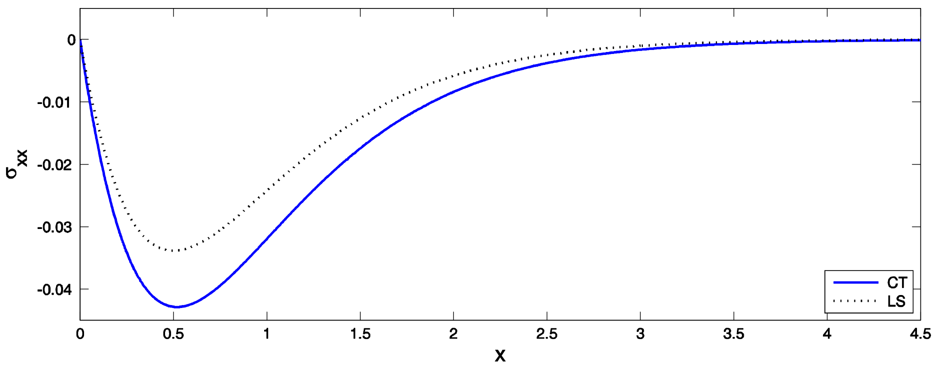

Fig. 7 The distribution of stress component σ_{xx} with distance x for two different theories

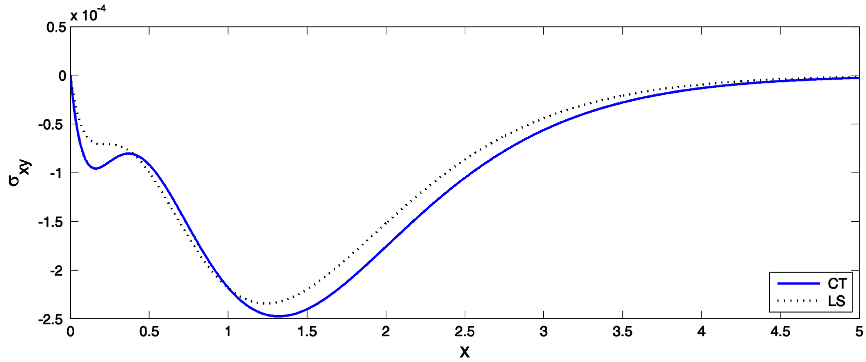

Fig. 8 The distribution of stress component σ_{xy} with distance x for two different theories

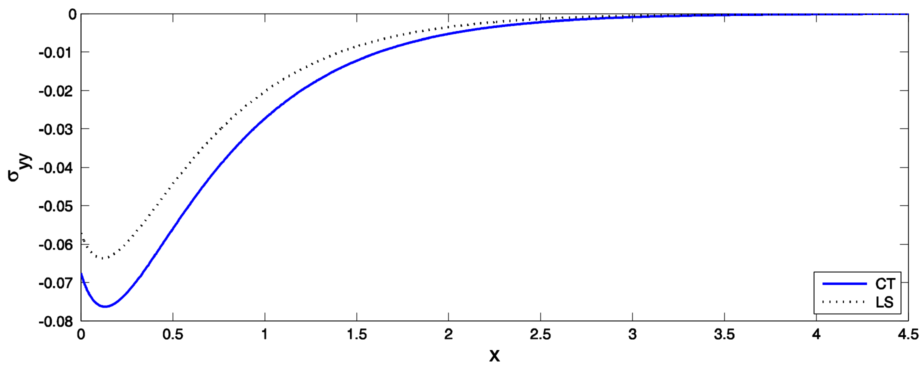

Fig. 9 The distribution of stress component σ_{yy} with distance x for two different theories

decrease to zero.

Finally, it easily to see that, the relaxation time has a great effect on the all physical quantities which supports the physical fact. The increasing of relaxation time act to reduce the magnitudes of the considered variables, which may be significant in some practical applications, can easily be taken under consideration and accurately assessed.

5. Conclusions

According to the above results, we can conclude that:

- The relaxation time parameter in the current model has significant effects on all the fields.
- The value of all physical quantities converges to zero with an increase in distance x and all functions are continuous.
- The comparison of different theories of thermoelasticity, *i.e.* LS theory and CT model are carried out.
- Analytical solutions based upon normal mode analysis for thermoelastic problem in solids have been developed and utilized.

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