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Prediction of Poisson's ratio degradation in hygrothermal aged and cracked $[\theta_m/90_n]_s$ composite laminates

M. Khodjet-Kesba [*1], E.A. Adda bedia [2a], A. Benkhedda [1b] and B. Boukert [1]

[1] *Laboratoire des sciences aéronautiques, Institut d'aéronautique et des études spatiales, Université de Blida1, Blida, Algérie*
[2] *Laboratoire des Matériaux et Hydrologie, Université de Sidi Bel Abbes, Sidi Bel Abbes, Algérie*

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Abstract. The Poisson ratio reduction of symmetric hygrothermal aged $[\theta_m/90_n]_s$ composite laminates containing a transverse cracking in mid-layer is predicted by using a modified shear-lag model. Good agreement is obtained by comparing the prediction models and experimental data published by Joffe *et al*. (2001). The material properties of the composite are affected by the variation of temperature and transient moisture concentration distribution in desorption case, and are based on a micro-mechanical model of laminates. The transient and non-uniform moisture concentration distribution give rise to the transient Poisson ratio reduction. The obtained results represent well the dependence of the Poisson ratio degradation on the cracks density, fibre orientation angle of the outer layers and transient environmental conditions. Through the presented study, we hope to contribute to the understanding of the hygrothermal behaviour of cracked composite laminate.

Keywords: transverse cracking; poisson ratio; hygrothermal effect; Tsai model; desorption

1. Introduction

For multidirectional laminates, the first damage which appears under mechanical loading is often matrix cracking in the off-axis plies. After transverse matrix cracking in $[\theta_m/90_n]_s$ composite laminates, there is a high interlaminar stress concentration at the crack tip. This stress often induces local delamination at the interface between laminas (Zhang and Minnetyan 2006, Hallett *et al*. 2008, Farrokhabadi *et al*. 2011, Jalalvand *et al*. 2013, Van der Meer and Sluys 2013, Zubillaga *et al*. 2015).

Matrix cracking and their effects on the material properties degradation have gained much attention both experimentally, numerically and analytically. Such as, the shear-lag method (Steif 1984, Berthelot *et al*. 1996, Selvarathinam and Weitsman 1999, Kashtalyan and Soutis 2000, Nairn and Mendels 2001, Yokozeki and Aoki 2005, Tounsi *et al*. 2006, Adda bedia *et al*. 2008) and

*Corresponding author, Ph.D. Student, E-mail: mkhojet@hotmail.fr
[a] Professor, E-mail: addabed@yahoo.com
[b] Ph.D., E-mail: benkhedda90@hotmail.com

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variational approach (Hashin 1985, 1986, Li and Hafeez 2009, Vingradov and Hashin 2010, Barbero and Cosso 2014, Hajikazemi and Sadr 2014, Huang *et al*. 2014, Katerelos *et al*. 2015) are among the most analytical methods used to study the evolution of the transverse cracking in composite laminates.

The finite element model was used by Tay and Lim (1996), Joffe and Krasnikovs (2001), Singh and Talreja (2009), Akula and Garnich (2012), to analyse the dependence of the COD (crack opening displacement) on the crack density and for different angle-ply composite laminates. The finite element model has shown quite interesting results in comparison with the experimental data. Li *et al*. (2009) presented a technique for modelling multidirectional laminates in the presence of regular crack arrays in multiple layers with no more than two different fibres orientations. The main disadvantage of this method is that it allows modelling no more than two crack directions.

Crack-Faces-Displacement (CFD) models are based on the micromechanical theorem, which states that the global laminate strains can be computed by averaging the strains over the volume of each layer. By means of the divergence theorem, it turns out that this process is equivalent to averaging the displacement components over the surfaces of each layer. Lundmark and Varna (2011) have developed calculation scheme to predict stiffness reduction in the entire crack density region. The degradation of thermo-elastic properties of a laminate strongly depends on the intralaminar crack surface opening and sliding during loading. Convergence has been proved to occur after a very small number of iterations, but the final result was not in satisfactory agreement with FE models.

A generalised plane strain model for the evaluation of the stress fields in $[0/90]_s$ laminates loaded in tension with a regular crack array in the 90° ply was proposed by McCartney (1992). The in-plane stresses were considered constant through the thickness of each layer and they were written as the sum of their nominal value, computed with a stress perturbation, function of the transverse coordinate of the 90° ply and the Classical Lamination Theory (CLT). A fourth order differential equation was obtained for the stress perturbation function, by solving the 2D equilibrium equations averaged relation between the transverse strain and displacement over the ply thickness, and making use of the averaged. The result was coincident to that of Hashin (1985) apart from the fact that a generalised plane strain hypothesis was assumed by McCartney.

The hygrothermal effect in angle-ply composite laminates was studied by Adda Beddia *et al*. (2008) using a complete parabolic shear-lag model to predict the effect of transverse cracks on the stiffness degradation. A General expression for longitudinal modulus reduction versus transverse crack was obtained by introducing the stress perturbation function (Bouazza *et al*. 2007). The results illustrate well the dependence of the degradation of elastic properties on the cracks density, fibre orientation angle and hygrothermal conditions.

In this paper, a modified shear lag model (parabolic analysis and progressive shear) is used to predict the effect of transverse cracks on the Poisson ratio degradation of transient hygrothermal aged composite laminates. Good agreement is obtained by comparing prediction with experimental published by Joffe and Krasnikovs (Joffe *et al*. 2001). Then, the cracked $[\theta_m/90_n]_s$ composite laminates are initially exposed to the hygrothermal ageing and submitted to transient and non-uniform moisture concentration distribution for desorption case. For that, the model which will enable us to introduce ageing and to see its development on the fibre and matrix scales is the transient Tsai model. This model allows us to predict the most representation of hygrothermal effects on cracked composite laminate, compared to other works already done (Amara *et al*. 2014, Adda bedia *et al*. 2008, Tounsi *et al*. 2006).

2. Theoretical analysis

In the absence of a unified theory for the mechanical characterization of the composites materials with long fibres, many formulations were proposed. In the bibliography, we can quote, the rule of mixtures method, the contiguity method who is based on the fibres arrangement (Staab 1999, Maurice 2001), the semi-empirical method based of Halpin-Tsai (Haplin and Tsai 1968) and the additional technique method based on the fibres emplacement (Chamis 1984).

In this paper, we used the rule of mixtures method applied to the composites with anisotropic fibres, which was modified by Hahn as is described in Ref (Tsai 1988). Therefore, the longitudinal Young modulus for unidirectional composite is

$$E_x = E_m \cdot V_m + E_{fx} \cdot V_f \tag{1}$$

The modified micromechanics method for the transverse modulus for graphite/epoxy (T300/ 5208) composite is

$$E_y = \frac{1 + 0.516\left(V_m/V_f\right)}{\dfrac{1}{E_{fy}} + \dfrac{0.516\left(V_m/V_f\right)}{E_m}} \tag{2}$$

In the same manner, we can obtain the shear modulus

$$G_{xy} = \frac{1 + 0.316\left(V_m/V_f\right)}{\dfrac{1}{G_{fx}} + \dfrac{0.316\left(V_m/V_f\right)}{G_m}} \tag{3}$$

$$\nu_{xy} = V_f \upsilon_f + V_m \upsilon_m \tag{4}$$

In the same manner, we can obtain the shear modulus

$$V_f + V_m = 1 \tag{5}$$

E_f, G_f and υ_f are the Young's modulus, shear modulus and Poisson's ratio, respectively, of the fibre and E_m, G_m and υ_m are the corresponding properties for the matrix.

2.1 Poisson's ratio reduction model in the [*θ*/*90*]$_s$ *laminates*

We consider a symmetric [θ/90]$_s$ laminate subjected to uniaxial loads. It is assumed that the 90° ply has developed continuous intralaminar cracks in fibre direction which extend from edge to edge in the *z* direction. The spacing between two cracks is 2.l_0 (Fig. 1).

Loading is applied only in x-direction and the far field applied stress is defined by $\sigma_c = (1/2\mathrm{h})N_x$, where N_x is applied load. The following analysis will be performed assuming generalised plane strain condition

$$\varepsilon_y^{\theta} = \varepsilon_y^{90} = \varepsilon_y = const \tag{6}$$

The symbol $(\bar{\ })$ over stress and train components denotes volume average. They are calculated using the following expressions:

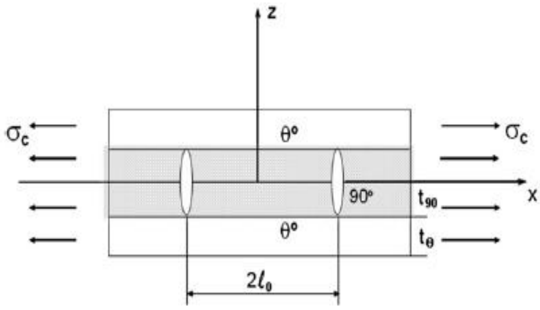

Fig. 1 Transverse cracked $[\theta/90]_s$ composite laminate and geometric model

- In the $\theta°$ layer.

$$\overline{f^{\theta}} = \frac{1}{2l_0}\frac{1}{t_\theta}\int_{-l_0}^{+l_0}\int_{t_{90}}^{h} f^{\theta}\,dx\,dz = \frac{1}{2\overline{l_0}}\frac{1}{\alpha}\int_{-\overline{l_0}}^{+\overline{l_0}}\int_{1}^{\bar{h}} f^{\theta}(\bar{x},\bar{z})d\bar{x}\,d\bar{z} \tag{7}$$

- In the $90°$ layer.

$$\overline{f^{90}} = \frac{1}{2l_0}\frac{1}{t_{90}}\int_{-l_0}^{+l_0}\int_{0}^{t_{90}} f^{90}dx\,dz = \frac{1}{2\overline{l_0}}\frac{1}{\alpha}\int_{-\overline{l_0}}^{+\overline{l_0}}\int_{0}^{1} f^{90}(\bar{x},\bar{z})d\bar{x}\,d\bar{z} \tag{8}$$

By using the strains in the $\theta°$ layer (which is not damaged and, hence, strains are equal to laminate strains, $\varepsilon_x = \bar{\varepsilon}_x^{\theta}$, etc.) and assuming that the residual stresses are zero, the Poisson's ratio of the laminate with cracks may be defined by the following expression

$$\nu_{xy} = -\frac{\varepsilon_y}{\bar{\varepsilon}_x^{\theta}} \tag{9}$$

2.2 Averaged stress-strain relationships for laminates with cracks

Constitutive equations that give the relationship between strain and stresses are:

- In the $90°$ layer.

$$\begin{Bmatrix} \varepsilon_x^{90} \\ \varepsilon_y^{90} \\ \varepsilon_z^{90} \end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} & S_{23} \\ S_{12} & S_{11} & S_{12} \\ S_{32} & S_{12} & S_{22} \end{bmatrix} \begin{Bmatrix} \sigma_x^{90} \\ \sigma_y^{90} \\ \sigma_z^{90} \end{Bmatrix} \tag{10}$$

- In the $\theta°$ layer.

$$\begin{Bmatrix} \varepsilon_x^{\theta} \\ \varepsilon_y^{\theta} \\ \varepsilon_z^{\theta} \end{Bmatrix} = \begin{bmatrix} S_{xx}^{\theta} & S_{xy}^{\theta} & S_{xz}^{\theta} \\ S_{xy}^{\theta} & S_{yy}^{\theta} & S_{yz}^{\theta} \\ S_{xz}^{\theta} & S_{yz}^{\theta} & S_{zz}^{\theta} \end{bmatrix} \begin{Bmatrix} \sigma_x^{\theta} \\ \sigma_y^{\theta} \\ \sigma_z^{\theta} \end{Bmatrix} \tag{11}$$

Where S_{ij} is the compliance matrix for unidirectional composite with $\theta°$ fibre orientation. In order to calculate the laminate elastic property we need $\bar{\varepsilon}_x^{\theta}$. By averaging Eqs. (8) and (9), we obtain averaged constitutive equations of the 90° and θ° layer. In averaged relationships we have $\bar{\sigma}_z^{90} = \bar{\sigma}_z^{\theta} = 0$ which follows from the force equilibrium in *z* direction

$$\int_{-l_0}^{+l_0} \sigma_z^i \; dx = 0, \quad i = 90, \theta \tag{12}$$

Averaged constitutive equations corresponding to in-plane normal stress and strain components are

$$\begin{Bmatrix} \bar{\varepsilon}_x^{\theta} \\ \varepsilon_y \end{Bmatrix} = \begin{bmatrix} S_{xx}^{\theta} & S_{xy}^{\theta} \\ S_{xy}^{\theta} & S_{yy}^{\theta} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x^{\theta} \\ \bar{\sigma}_y^{\theta} \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} \bar{\varepsilon}_x^{90} \\ \varepsilon_y \end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} \\ S_{12} & S_{11} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x^{90} \\ \bar{\sigma}_y^{90} \end{Bmatrix} \tag{14}$$

Eqs. (13) and (14) are obtained from the 3D strain-stress relationships but because of Eq. (12) the result is similar as in classical thin-laminate theory (CLT). In fact, for an undamaged laminate, the averaged stresses and strains are equal to the laminate theory stresses and strains and Eqs. (13) and (14) are still applicable.

Force equilibrium equations for a damaged (or undamaged) laminate are:

- In *x*-direction:

$$N_x = \int_0^{t_{90}} \sigma_x^{90} dz + \int_{t_{90}}^{h} \sigma_x^{\theta} dz = \sigma_c (t_{90} + t_{\theta}) \tag{15}$$

Leading to

$$\bar{\sigma}_x^{90} t_{90} + \bar{\sigma}_x^{\theta} t_{\theta} = \sigma_c (t_{90} + t_{\theta}) \tag{16}$$

- In *y*-direction:

$$N_y = 0 \implies \int_0^{t_{90}} \sigma_y^{90} dz + \int_{t_{90}}^{h} \sigma_y^{\theta} dz = 0 \tag{17}$$

From which follows

$$\bar{\sigma}_y^{90} t_{90} + \bar{\sigma}_y^{\theta} t_{\theta} = 0 \tag{18}$$

Eqs. (13), (14), (16) and (18) contain seven unknowns: four stress components and three strain components ($\bar{\varepsilon}_x^{90}$, $\bar{\varepsilon}_x^{\theta}$ and ε_y). The total number of equations is six. Hence, one of the unknowns may be considered as independent, and the rest of them can be expressed as linear functions of it. Choosing the stress $\bar{\sigma}_x^{90}$ as independent, and solving with respect to it the system of Eqs. (13), (14), (16) and (18) we obtain

$$\varepsilon_y = g_1 \bar{\sigma}_x^{90} + f_1 \sigma_c, \quad \bar{\varepsilon}_x^{90} = g_2 \bar{\sigma}_x^{90} + f_2 \sigma_c, \quad \bar{\varepsilon}_x^{\theta} = g_3 \bar{\sigma}_x^{90} + f_3 \sigma_c \tag{19}$$

Expressions for g_i, f_i, *i* = 1, 2, 3 through laminate geometry and properties of constituents are given as follows

$$g_1 = t_{90}\frac{S_{12}S_{yy}^{\theta} - S_{11}S_{xy}^{\theta}}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}}, \qquad f_1 = \frac{S_{11}S_{xy}^{\theta}(t_{\theta} + t_{90})}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}} \tag{20}$$

$$g_2 = S_{22} - \frac{S_{12}(S_{12}t_{\theta} + S_{xy}^{\theta}t_{90})}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}}, \qquad f_2 = \frac{S_{12}S_{xy}^{\theta}(t_{\theta} + t_{90})}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}} \tag{21}$$

$$g_3 = \frac{t_{90}}{t_{\theta}}\left(S_{xy}^{\theta}\frac{(S_{12}t_{\theta} + S_{xy}^{\theta}t_{90})}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}} - S_{xx}^{\theta}\right), \quad f_3 = \frac{t_{\theta} + t_{90}}{t_{\theta}}\left(S_{xx}^{\theta} - \frac{(S_{xy}^{\theta})^2 t_{90}}{S_{11}t_{\theta} + S_{yy}^{\theta}t_{90}}\right) \tag{22}$$

2.3 Stress and strain perturbation caused by cracks

In order to obtain an expression for average stress $\bar{\sigma}_x^{90}$ in the repeatable unit, we consider the axial stress perturbation caused by the presence of two cracks. Without losing generality the axial stress distribution may be written in the following form

$$\sigma_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} f_1(\bar{x},\bar{z}) \tag{23}$$

$$\sigma_x^{\theta} = \sigma_{x0}^{\theta} + \sigma_{x0}^{90} f_2(\bar{x},\bar{z}) \tag{24}$$

Where σ_{x0}^{90} is the CLT stress in 90° layer and σ_{x0}^{θ} is CLT stress in the θ° layer (laminate theory routine), $-\sigma_{x0}^{90} f_1(\bar{x},\bar{z})$ and $\sigma_{x0}^{90} f_2(\bar{x},\bar{z})$ are stress perturbation caused by the presence of crack. Normalising factors in form of far field stresses in perturbation functions are used for convenience. Averaging Eqs. (23) and (24) using the integral force equilibrium in the *x*-direction (Eq. (15)), we obtain

$$\bar{\sigma}_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90}\frac{1}{2\bar{l}_0}R(\bar{l}_0) \tag{25}$$

$$\bar{\sigma}_x^{\theta} = \sigma_{x0}^{\theta} + \sigma_{x0}^{90}\frac{1}{2\alpha\bar{l}_0}R(\bar{l}_0) \tag{26}$$

In the following function

$$R(\bar{l}_0) = \int_{-\bar{l}_0}^{+\bar{l}_0}\int_0^1 f_1(\bar{x},\bar{z})\,d\bar{z}\,d\bar{x} \tag{27}$$

$R(\bar{l}_0)$is called the stress perturbation function. It's related to axial stress perturbation in the 90° layer and depends on the crack density.

The average stress $\bar{\sigma}_x^{90}$ involved in Eq. (19) is now expressed through the stress perturbation function (Eq. (27)). Conditions used to obtain expressions (Eq. (19)) are the same as used in CLT. Hence, substituting $\bar{\sigma}_x^{90} = \sigma_{x0}^{90}$ where σ_{x0}^{90} is the x-axis stress in the 90° layer according to CLT, we obtain CLT solution: $\bar{\varepsilon}_x^{90} = \varepsilon_{x0}^{90} = \varepsilon_{x0}$, $\bar{\varepsilon}_x^{\theta} = \varepsilon_{x0}^{\theta} = \varepsilon_{x0}$ and $\varepsilon_y = \varepsilon_{y0}$.

Substituting Eq. (25), which contains two terms, in Eq. (19) the result has two terms. The first term according to the discussion above is equal to CLT strain but the second one is a new term related to the stress perturbation function $R(\bar{l}_0)$

$$\varepsilon_y = \varepsilon_{y0} - \sigma_{x0}^{90} g_1 \frac{1}{2\overline{l_0}} R(\overline{l_0}) \tag{28}$$

$$\bar{\varepsilon}_x^{90} = \varepsilon_{x0} - \sigma_{x0}^{90} g_2 \frac{1}{2\overline{l_0}} R(\overline{l_0}) \tag{29}$$

$$\bar{\varepsilon}_x^{\theta} = \varepsilon_{x0} - \sigma_{x0}^{90} g_3 \frac{1}{2\overline{l_0}} R(\overline{l_0}) \tag{30}$$

The stress σ_{x0}^{90} in the 90° layer of an undamaged laminate under mechanical loading may be calculated using CLT

$$\sigma_{x0}^{90} = Q_{22}\varepsilon_{x0}(1 - \upsilon_{12}\upsilon_{xy}^{0}) \tag{31}$$

Here, υ_{xy}^{0} is the Poisson's ratio of the undamaged laminate.

2.4 Expression for Poisson's ratio coefficient

For ideally distributed crack spacing in 90° layers of symmetric and balanced laminates, the Poisson's ratio reduction model (Joffe *et al*. 2001, Amara *et al*. 2006) was used to show that the crack spacing $2l_0$ reduces the extensional Poisson's ratio. By substituting Eqs. (28) and (30) in Eq. (9) it follows that

$$\frac{\upsilon_{xy}}{\upsilon_{xy}^{0}} = \frac{1 - b\bar{\rho}R(\overline{l_0})}{1 + a\bar{\rho}R(\overline{l_0})} \tag{32}$$

Where $\bar{\rho} = \frac{1}{2\overline{l_0}}$, $\overline{l_0} = \frac{l_0}{t_{90}}$ is the noramalized crack density and *a*, *b* are known functions, dependent on elastic properties and geometry of the θ° and 90° layer

$$a = \frac{E_{90}t_{90}}{E_x^{\theta}t_{\theta}}\left(\frac{1 - \upsilon_{12}\upsilon_{xy}^{0}}{1 - \upsilon_{12}\upsilon_{21}}\right)\left(1 + \upsilon_{xy}^{\theta}\,\frac{S_{xy}^{\theta}t_{90} + S_{12}t_{\theta}}{S_{yy}^{\theta}t_{\theta} + S_{11}t_{\theta}}\right) \tag{33}$$

$$b = \frac{E_{90}t_{90}}{\upsilon_{xy}^{0}}\left(\frac{1 - \upsilon_{12}\upsilon_{xy}^{0}}{1 - \upsilon_{12}\upsilon_{21}}\right)\left(\frac{S_{xy}^{\theta}S_{11} - S_{12}S_{yy}^{\theta}}{S_{yy}^{\theta}t_{90} + S_{11}t_{\theta}}\right) \tag{34}$$

E_x^{θ} and E_{90} are the Young's moduli of θ° and 90° layers respectively. From Eq. (32) it's clear that the function $R(\overline{l_0})$ is the only one unknown. Hence, reduction of Poisson's ratio modulus depends on the form of this function of crack density. Solution for this function can be found by using different analytical models such as shear-lag models.

2.5 Computation of the stress perturbation function using shear lag model

In this work, we have used two models developed by Berthelot *et al*. (1996). This latter is modified by introducing the stress perturbation function. The stress perturbation function $R(\overline{l_0})$ is found as

$$R(a) = \int_{-\bar{l_0}}^{+\bar{l_0}} \frac{\cosh(\xi \bar{x})}{\cosh(\xi \bar{l_0})} d\bar{x} = \frac{2}{\xi} \tanh(\xi \bar{l_0}) \tag{35}$$

Where, ξ is the shear-lag parameter

$$\xi^2 = \bar{G} \frac{t_{90}(t_{90}E_{90} + t_\theta E_\theta)}{t_\theta E_{90} E_\theta} \tag{36}$$

The coefficient $\bar{G}$ depends on used assumptions about the longitudinal displacement and shears stress distribution:

- The first case assumes that the longitudinal displacement is parabolic in the thickness of 90° layer

$$u_{90}(x,z) = \overline{u_{90}}(x) + \left(z^2 - \frac{t_{90}^2}{3}\right) A_{90}(x) \tag{37}$$

  The variation of the longitudinal displacement is to be determined by the thickness of θ° layers

$$u_\theta(x,z) = \overline{u_\theta}(x) + f(z) A_\theta(x) \tag{38}$$

- The second case assumes that the shear stresses, similar in θ° and 90° layers, can be obtained by assuming that the transverse displacement is independent of the longitudinal coordinate

$$\sigma_{xz}^i = G_{xz}^i \gamma_{xz}^i \tag{39}$$

$$\gamma_{xz}^i = \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x} \approx \frac{\partial u_i}{\partial z} \qquad i = \theta°, 90° \tag{40}$$

The coefficient $\bar{G}$ is done by

$$\bar{G} = \frac{3G}{t_{90}} \tag{41}$$

The shear modulus G of the elementary cell

$$G = \frac{G_{xz}^{90}}{1 - 3 \frac{G_{xz}^{90}}{G_{xz}^{\theta}} \frac{f(t_{90})}{t_{90} f'(t_{90})}} \tag{42}$$

Two different analytical functions of the variation function have been considered (Berthelot *et al*. 1996)

- In the case of the assumption of a parabolic variation of longitudinal displacement in both θ° and 90° layers,

  By replacing the function $f(z) = z^2 - 2(t_\theta + t_{90})z + \frac{2}{3} t_\theta^2 + 2 t_\theta t_{90} + t_{90}^2$ in the Eq. (42), the shear modulus for parabolic assumption will be in the form

$$G = \frac{G_{xz}^{90}}{1 + \alpha \frac{G_{xz}^{90}}{G_{xz}^{\theta}}} \tag{43}$$

- In the case when the variation of the longitudinal displacement is supposed progressive in $\theta°$ - layer:

We use the function $f(z) = \frac{\sinh \alpha\eta_t}{\alpha\eta_t} - \cosh \alpha\eta_t \left(1 + \alpha - \frac{z}{t_{90}}\right)$ in the Eq. (42), the shear modulus for progressive shear assumption will be in the form

$$G = \frac{G_{xz}^{90}}{1 + 3\alpha \frac{\alpha\eta_t (\tanh \alpha\eta_t)^{-1} - 1}{\alpha{\eta_t}^2} \frac{G_{xz}^{90}}{G_{xz}^{\theta}}} \tag{44}$$

3. Results and discussion

A computer code based on the preceding equations was written to compute the Poisson ratio loss.

3.1 Comparison of predictions with experimental data

In this section, we will validate the results of the present programme without taking into account the hygrothermal effect on the material properties. The results are compared with experimental data for glass/epoxy laminates with elastic properties in Table 1. (Joffe *et al*. 2001).

In Figs. 2 to 4 the degradation of the normalised effective Poisson's ratio are shown against crack density and compared with experimental data published by Joffe *et al*. (2001). The results show good agreement between the experimental results and those predicted using the analytical

Table 1 Material properties of glass/epoxy laminate used in calculations (Joffe *et al*. 2001)

Proprieties material	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	$G_{TT'}$ (GPa)	υ_{LT}	$\upsilon_{TT'}$	t_{90} (mm)
Glass/epoxy	44.73	12.76	5.8	4.49	0.297	0.42	0.144

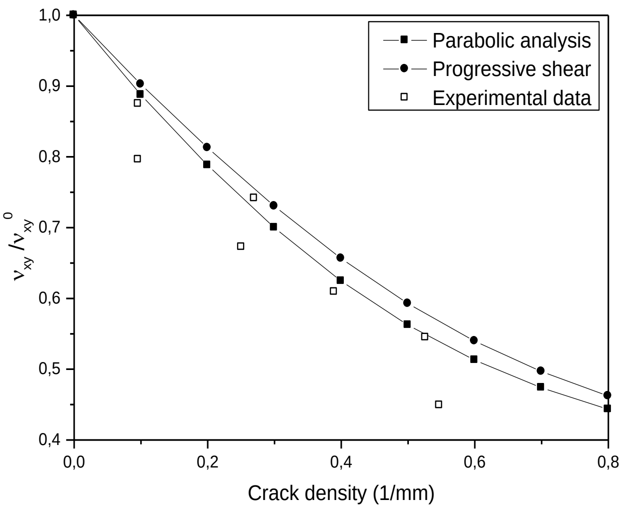

Fig. 2 Poisson's ratio degradation due to transverse cracks in $[0_2/90_4]_s$ GF/EP laminate

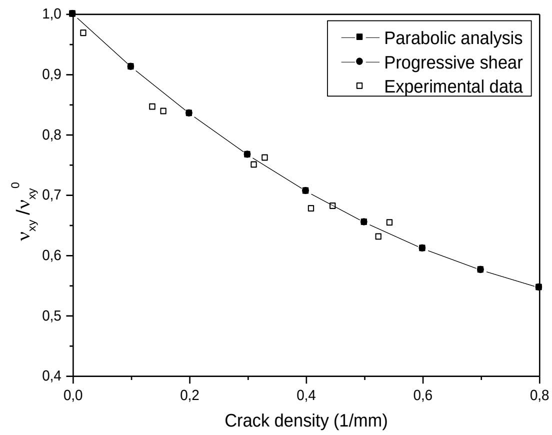

Fig. 3 Poisson's ratio degradation due to transverse cracks in $[\pm15/90_4]_s$ GF/EP laminate

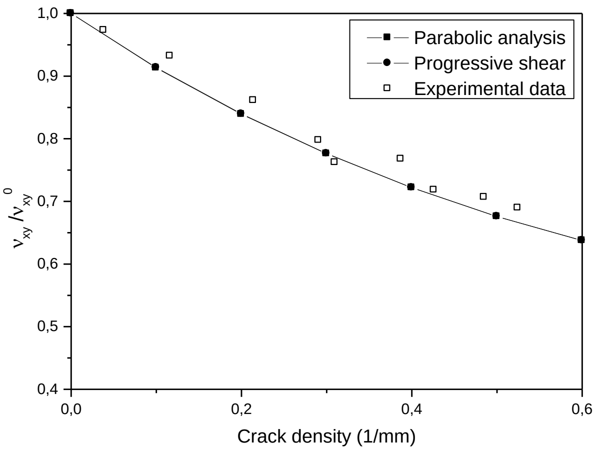

Fig. 4 Poisson's ratio degradation due to transverse cracks in $[\pm30/90_4]_s$ GF/EP laminate

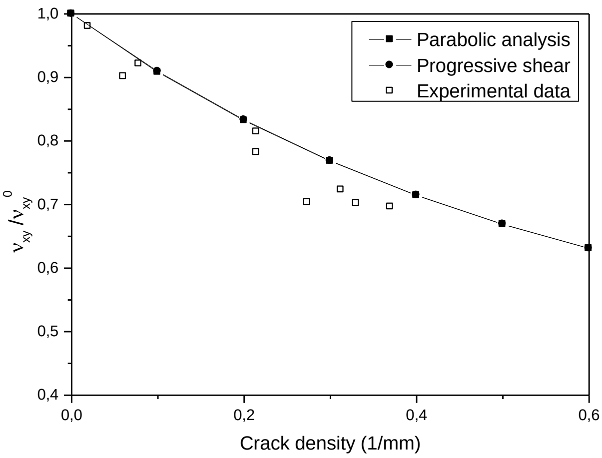

Fig. 5 Poisson's ratio degradation due to transverse cracks in $[\pm40/90_4]_s$ GF/EP laminate

models. Fig. 5 show another comparison of results obtained by the present method with the experimental data for $[\pm 40/90_4]_s$ GF/EP laminate. The present models give less accurate predictions of Poisson's ratio degradation as a function of crack density.

3.2 Influence of transient hygrothermal conditions on the reduced Poisson's ratio

The study here has been focussed on the Poisson's ratio reduction due to transverse cracking in $[\theta_m/90_n]_s$ composite laminate when this latter is initially exposed to the hygrothermal conditions. The model which will enable us to introduce ageing and to see its development on the fibre and matrix scales is Tsai model (1988). The principle of this simplified method is to take the real distribution of moisture concentration through each ply using its serial expansion (Vergnaud 1992) and this to determine the exact expression of the Poisson's ratio under hygrothermal effect.

Tsai (1988) proposes the adimensional temperature T^*, which is the essential parameter for evaluation of the hygrothermal effect in stress distribution

$$T^* = \frac{T_g - T_{opr}}{T_g - T_{rm}} \tag{45}$$

Where T_g is the glass transition temperature, T_{opr} is the operating temperature and T_{rm} is the room temperature. We further assume that moisture suppresses the glass transition temperature T_g^0 by simple temperature shift.

$$T_g = T_g^0 - g \cdot c \tag{46}$$

Where T_g^0 is the glass transition temperature at dry state, g is the temperature shift per unit moisture absorbed and c is the moisture absorbed.We can use the exponents of T^* to empirically fit the matrix of the mechanical properties as function of moisture and temperature.

$$\frac{E_m}{E_m^0} = \frac{G_m}{G_m^0} = \frac{\nu_m}{\nu_m^0} = (T^*)^a \tag{47}$$

E_m^0, G_m^0 and ν_m^0 are the Young's modulus, shear modulus and Poisson's ratio, respectively, of the matrix at room temperature and a is the empirical constant. The same exponents of T^* is used to empirically fit the fibre of the mechanical properties as function of moisture and temperature.

$$\frac{E_{fx}}{E_{fx}^0} = \frac{E_{fy}}{E_{fy}^0} = \frac{G_{fx}}{G_{fx}^0} = \frac{\nu_{fx}}{\nu_{fx}^0} = (T^*)^f \tag{48}$$

E_{fx}^0, E_{fy}^0, G_{fx}^0 and ν_{fx}^0 are the longitudinal and transversal Young's modulus, shear modulus and Poisson's ratio, respectively, of the fibre at room temperature and f is the empirical constant.

It is assumed that E_m, G_m, ν_m, E_{fx}^0, E_{fy}^0, G_{fx}^0 and ν_{fx}^0 are function of temperature and moisture (as Eqs. (47) and (48)), then E_x, E_y and G_{xy} (as Eqs. (1), (2) and (3)) will be also function of temperature and moisture. In Tables 2 and 3, we found the data parameters which influence the mechanical characteristics of graphite/epoxy material.

Let us consider a laminated plate of thickness h made of polymer matrix composite, submitted on it two sides to the same dry environment. The plate is considered to be infinite in both x and y

Table 2 Fibre and matrix characteristics of graphite/epoxy material (T300/5208) (Tsai 1988) (T = 22℃ and C = 0.5%)

E_{fx}^{0} (Gpa)	E_{fx}^{0} (Gpa)	E_{m}^{0} (Gpa)	ν_{fx}^{0}	ν_{m}^{0}	G_{fx}^{0} (Gpa)	G_{m}^{0} (Gpa)	V_f
259	18.69	3.4	0.25	0.35	19.69	1.26	0.7

Table 3 Parameters of temperature and moisture characteristics variation (Tsai 1988)

T_g^0 (℃)	T_{room} (℃)	g (℃/c)	a	f
160	22	2000	0.35	0.04

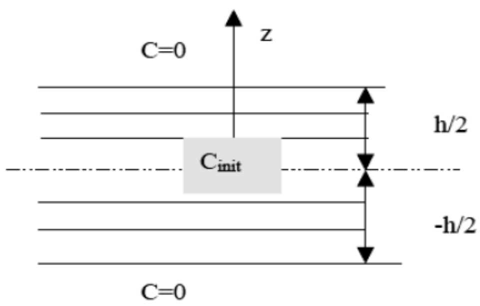

Fig. 6 Desorption phase

directions and the moisture vary only in the z direction. The initial moisture concentration C_{init} is uniform at t = 0. Both sides of the plate are suddenly exposed to a zero moist environment (Fig. 6). The moisture concentration inside the plate is described by Fick equation (Shen and Springer 1981, Benkhedda *et al*. 2008, Tounsi *et al*. 2005) with diffusivity D_z.

$$\frac{\partial C}{\partial t} = D_z \frac{\partial^2 C}{\partial z^2} \tag{49}$$

The moisture diffusion characteristics are given in Table 4.

With the initial conditions

$$C = C_{init} \quad \text{for} \quad -h/2 \le z \le h/2 \quad \text{and} \quad t = 0 \tag{50}$$

$$C = 0 \quad \text{for} \quad z = -h/2; \quad z = h/2 \quad \text{and} \quad t > 0 \tag{51}$$

The initial conditions being uniform and the boundary conditions are constants, the unidemen-

Table 4 Moisture diffusion characteristics (Tsai 1988)

Diffusivity, mm^2/s	$D_z = 0.57\exp(-4993/T)$ T: Temperature (K)
Moisture concentration at the surface as a function of relative humidity, %	$C_{ini} = 0.015 \cdot H$ H: Relative humidity (%)

sional solution of Fick equation can be expressed as (Khodjet Kesba *et al*. 2015, Rezoug *et al*. 2011, Benkhedda *et al*. 2008)

$$C(z_t, t) = \left[\frac{4C_{ini}}{\Pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos\left(\frac{(2n+1)\Pi z_k}{h}\right)\exp\left(\frac{-D_z(2n+1)^2\Pi^2 t}{h^2}\right)\right] \tag{52}$$

The concentration distribution through the thickness (h = 1 mm) with the initial temperature and moisture are T_{op} = 22°C and C_{ini} = 0.5% is presented in Fig. 7.

Time t being given, the first step is to compute the on-axis free expansions E_x, E_y, G_{xy} and υ_{xy}. These expansions are computed at each point z_k of the thickness.Finally, the Poisson's ratio degradation in $[\theta/90_3]_s$ composite laminate as of crack density is evaluated compared to the initial Poisson's ratio of the same uncracked laminate and for the same environmental case. We note that this initial Poisson's ratio of the uncracked laminate is a function of temperature and moisture distribution. Consequently, Eq. (32) becomes

$$\frac{\upsilon_{xy\,(i)}}{\upsilon^0_{xy\,(i)}} = \frac{1 - b_{(i)}\bar{\rho}R_{(i)}(\overline{l_0})}{1 + a_{(i)}\bar{\rho}R_{(i)}(\overline{l_0})} \tag{53}$$

The index (i) represents the considered case of the environmental conditions

The Poisson's ratio degradation is represented in $[\theta/90_3]_s$ cracked laminate exposed to hygrothermal conditions with a parabolic variation of longitudinal displacement in both θ° and 90° layers. Transient and non-uniform moisture concentration have been selected to represent the effect of temperature and moisture in the cracked laminates for desorption case. Three sets of environmental conditions are considered. For environmental case 1, Top = 22°C and C = 0%. For environmental case 2, Top = 60°C and C = 0.5%. For environmental case 3, Top = 120°C and C = 1%. The time chosen for simulation is taken equal to *tsat* = 4222 h with *tsat* is the moisture saturation for T300/5208.

In Fig. 8 and 9 the normalised Poisson's ratio, respectively are plotted as a function of crack density with different fibre angles θ°, and as a function of fibre angle-ply θ° with four different

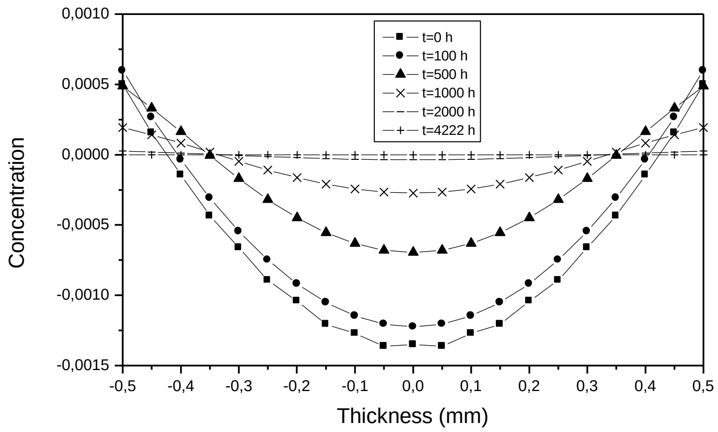

Fig. 7 Concentration distribution through the thickness at a different time of use with T_{op} = 22°C and C_{ini} = 0.5% for graphite/epoxy (T300/5208)

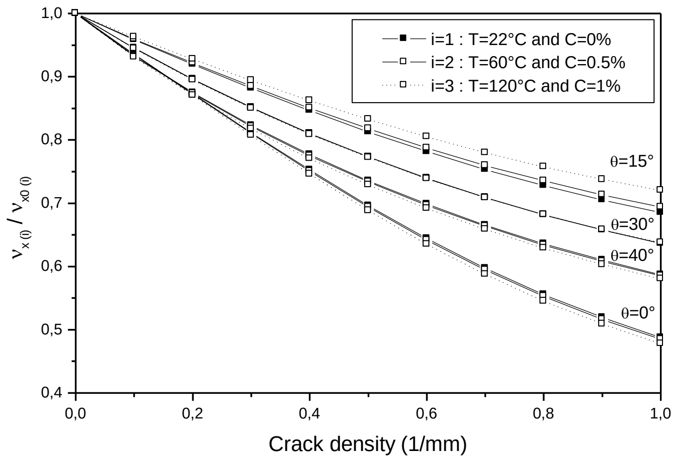

Fig. 8 Hygrothermal effect on the Poisson's ratio degradation due to transverse cracks in a $[\theta/90_3]_s$ graphite/epoxy (T300/5208)

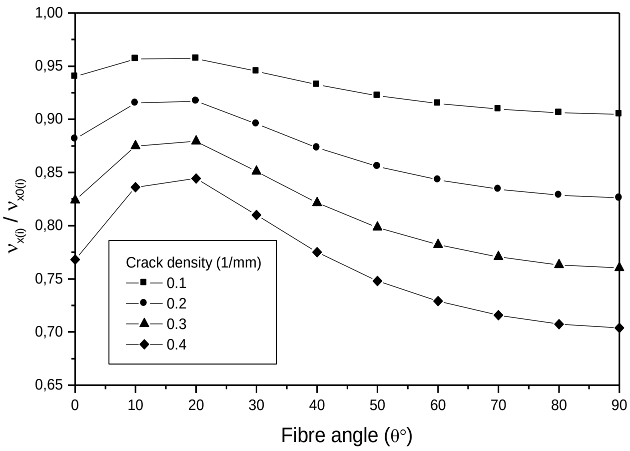

Fig. 9 Crack density effect on the Poisson's ratio degradation due to fibre angle-ply $\theta°$ in a $[\theta/90_3]_s$ graphite/epoxy (T300/5208) with Top = 120℃ and C = 0.1%

values of crack densities. It has been observed that the Poisson's ratio reduces monotonically with increasing fibre angle $\theta°$ and also at high crack density, except for fibre angle $\theta = 0°$ which gives a more accurate reduction of Poisson's ratio as a function of crack density. We note also that the transient hygrothermal effect has a significant impact when the fibre angle $\theta°$ is small and at high crack density.

4. Conclusions

The Poisson's ratio reduction was predicted using simple analytical models on the $[\theta_m/90_n]_s$ composite laminate including the effect of transverse cracks and under different environmental conditions by the variation of temperature and transient moisture concentration distribution in desorption case. The results show good agreement between prediction models and experimental data. On the other hand, when the cracked composite laminate is subjected to hygrothermal conditions, the Poisson's ratio depends largely on the crack density, fibre orientation angle $\theta°$ of the outer layers and transient hygrothermal conditions. Through this theoretical study, we hope that our prediction will be a support for future experimental research.

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