Steel and Composite Structures, *Vol. 21*, *No. 1* (*2016*) *217-247*
DOI: http://dx.doi.org/10.12989/scs.2016.21.1.217

Parametric modeling and shape optimization design of five extended cylindrical reticulated shells

J. Wu [1,2a], X.Y. Lu [*2], S.C. Li [1b], Z.H. Xu [1c], Z.D. Wang [3d], L.P. Li [1e] and Y.G. Xue [1f]

[1] *Geotechnical and Structural Engineering Research Center*, *Shandong University*, *Ji'nan 250061*, *Shandong*, *China*
[2] *Institute of Engineering Mechanics*, *Shandong Jianzhu University*, *Ji'nan 250101*, *Shandong*, *China*
[3] *Shantui Construction Machinery co.*, *Ltd*, *Ji'ning 272073*, *Shandong*, *China*

(*Received March 21*, *2015*, *Revised March 08*, *2016*, *Accepted March 13*, *2016*)

Abstract. Five extended cylindrical reticulated shells are proposed by changing distribution rule of diagonal rods based on three fundamental types. Modeling programs for fundamental types and extended types of cylindrical reticulated shell are compiled by using the ANSYS Parametric Design Language (APDL). On this basis, conditional formulas are derived when the grid shape of cylindrical reticulated shells is equilateral triangle. Internal force analysis of cylindrical reticulated shells is carried out. The variation and distribution regularities of maximum displacement and stress are studied. A shape optimization program is proposed by adopting the sequence two-stage algorithm (RDQA) in FORTRAN environment based on the characteristics of cylindrical reticulated shells and the ideas of discrete variable optimization design. Shape optimization is achieved by considering the objective function of the minimum total steel consumption, global and locality constraints. The shape optimization for three fundamental types and five extended types is calculated with the span of 30 m~80 m and rise-span ratio of 1/7~1/3. The variations of the total steel consumption along with the span and rise-span ratio are analyzed with contrast to the results of shape optimization. The optimal combination of main design parameters for five extended cylindrical reticulated shells is investigated. The total steel consumption affected by distribution rule of diagonal rods is discussed. The results show that: (1) Parametric modeling method is simple, efficient and practical, which can quickly generate different types of cylindrical reticulated shells. (2) The mechanical properties of five extended cylindrical reticulated shells are better than their fundamental types. (3) The total steel consumption of cylindrical reticulated shells is optimized to be the least when rise-span ratio is 1/6. (4) The extended type of three-way grid cylindrical reticulated shell should be preferentially adopted in practical engineering. (5) The grid shape of reticulated shells should be designed to equilateral triangle as much as possible because of its reasonable stress and the lowest total steel consumption.

Keywords: extended cylindrical reticulated shell; APDL; parametric modeling; shape optimization

*Corresponding author, Ph.D., Professor, E-mail: luxy5504@163.com
[a] Ph.D. Student, E-mail: wujing9516@163.com
[b] Ph.D., Professor, E-mail: lishucai@sdu.edu.cn
[c] Ph.D., Lecturer, E-mail: zhenhao_xu@sdu.edu.cn
[d] M Eng., E-mail: 15705479068@163.com
[e] Ph.D., Associate Professor, E-mail: yuliyangfan@163.com
[f] Ph.D., Professor, E-mail: xieagle@sdu.edu.cn

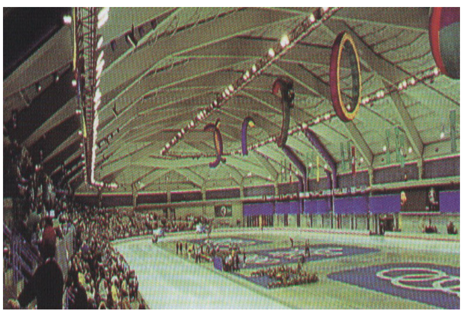
http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

1. Introduction

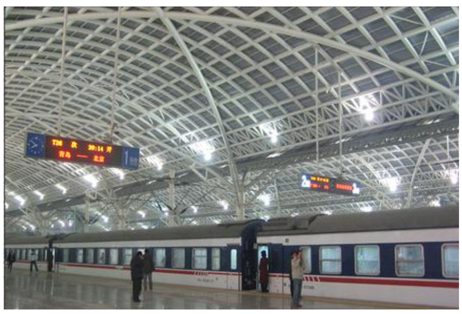
In recent decades, spatial structure develops rapidly in the world. The technology of spatial structure is known as the symbol of national building level. Reticulated shell is a both old and young structure type among the many spatial structures. It belongs to spatial skeletal structures, and its rod-system is generated by connecting the nodes according to certain rules. Owing to its saving material, reasonable stress, large stiffness, single rods, easy fabrication and installation, etc. (Levy *et al*. 1994, Shen and Chen 1996, Deng and Dong 1999), it has been widely used in modern large-span building structures (Rajan 1995, Dong and Yao 2003). With the continuous improvement of industrial technology and computational analysis technology, reticulated shell increasingly shows its rich and colorful external forms and artistic creative potentials, which cannot be matched by a planar structure. And it also reflects the beauty and mystery of nature (Saka 1991, Luo *et al*. 2012, Lu *et al*. 2012).

The types of reticulated shell have cylindrical reticulated shell, spherical reticulated shell, hyperbolic paraboloid reticulated shell, folded plate reticulated shell, etc. (Shen and Chen 1996, Lu *et al*. 2013). Because the Gaussian curvature of cylindrical reticulated shell is zero, and its grid division and rods layout are relatively simple, thus, cylindrical reticulated shell is one of the most commonly used types. Its grid shape is generally triangle. It can also be rectangle, square, diamond and so on. Cylindrical reticulated shell is usually adopted when the building plane is square or rectangle, especially narrow plane. Typical engineering examples have Speed Skating Museum of Calgary in Canada, Qingdao Railway Station in China (Fig. 1), etc.

According to different grid types, there are five typical cylindrical reticulated shells, i.e., single diagonal rod - cylindrical reticulated shell, double slash rod - cylindrical reticulated shell, herringbone - cylindrical reticulated shell, lamella - cylindrical reticulated shell, three-way grid - cylindrical reticulated shell. Five extended cylindrical reticulated shells are obtained by changing distribution rule of diagonal rods based on the first three cylindrical reticulated shells. The mechanical properties of extended types are better than the corresponding fundamental types. Therefore, the extended cylindrical reticulated shells should have a wide range of applications in modern architectures. However, the number of nodes and rod elements of reticulated shells is too many and the variation of span, rise, grid size, type and other parameters can cause structural internal force reallocation. So the workload of re-modeling is very large. What is more, it is quite difficult to carry out high efficient internal force analysis and shape optimization design. Conventional modeling of these structures often relies on hand-modeling rather than on parametric



(a) Speed Skating Museum of Calgary



(b) Qingdao Railway Station

Fig. 1 Engineering examples of cylindrical reticulated shell modeling in domestic and foreign studies. Relevant research is seldom related to the specific work of parametric modeling. In addition, the total steel consumption of reticulated shells is very large, and shape optimization design can be got obvious economic benefits.

At present, the research on optimization design of reticulated shells is not much. There are three reasons: Firstly, spatial beam element is adopted at the time of conducting finite element analysis. Each node has six degrees of freedom, and constraint condition is complex. Secondly, the effects of geometric nonlinearity must be considered when carrying out optimization design, which makes the problem more complicated. Thirdly, design variables and constraints of reticulated shells are too many, so that it is difficult to converge to the optimal solution when optimizing. According to the properties of design variables, structural optimization design can be divided into continuous variable optimization design and discrete variable optimization design. According to different difficulty levels, it also can be divided into section optimization, shape optimization, topology optimization, layout optimization and lectotype optimization. The variables for optimal design are usually discrete in practical engineering. Section optimization and shape optimization of reticulated shells have become research priorities of domestic and international scholars for a long time.

Nowadays, the section optimization has already possessed perfect theoretical system and abundant practical experience. Moreover, shape optimization has also got some progress. The present research achievements are mainly as follows: Suzuki and Kikuchi (1991) discussed shape and topology optimization of a linearly elastic structure by using a modification of the homogenization method together with various examples which may justify validity and strength of the present approach for plane structures. Eschenauer *et al*. (1994) addressed a novel method of topology and shape optimization. The basic idea was the iterative positioning of new holes (so-called "bubbles") into the present structure of the component. This concept was therefore called the "bubble method". Jenkins studied structural optimization with the genetic algorithm (Jenkins 1991a, b) and natural algorithm (Jenkins 1997). Saka and Kameshki (1998) investigated optimum design of nonlinear elastic framed domes, i.e., an algorithm was presented for the optimum design of three-dimensional rigidly jointed frames which took into account the nonlinear response due to the effect of axial forces in members. He *et al*. (2001) proposed that chaos optimization algorithm could be used in the optimization design of double-layer cylindrical reticulated shell, which realized the synchronous optimization of the rise to span ratio, grid size and vault thickness as continuous variables. Allaire *et al*. (2002) studied a level-set method for numerical shape optimization of elastic structures. Their approach combined the level-set algorithm of Osher and Sethian with the classical shape gradient. Although this method was not specifically designed for topology optimization, it could easily handle topology changes for a very large class of objective functions. Zhang and Dong (2003) presented a structural optimization algorithm. In the process of optimum design, both the stress constraints and displacement constraints were considered. The geometrical nonlinearity was taken into account during the computation of stresses and displacements. A computer program was developed, and the design example verified the effectiveness of the proposed method. Xu *et al*. (2006) investigated an optimal method, and this optimum design was performed by the combination of the direct searching method and the criterion. Wang and Tang (2006) proposed an optimum method based on the optimality criteria, which could be used in optimization design of single-layer reticulated shells. The constraints of displacement, stress, member stability and structural stability were considered. Vyzantiadou *et al*. (2007) proposed structural systems. The proposed computational method produced algorithms using fractal mathematics, and could generate forms applicable to shells. Yas *et al*. (2007)

proposed the stacking sequence optimization of a laminated cylindrical shell for obtaining maximum natural frequency and buckling stress, simultaneously. Rahami *et al*. (2008) introduced a combination of energy and force method, and genetic algorithm was employed as an optimization tool for minimizing the weight of the truss structures. Oudjene *et al*. (2009) presented a response surface methodology (RSM) for shape optimization problem, which was based on Moving Least-Square (MLS) approximation and adaptive moving region of interest. To avoid a local optimum and to obtain an accurate solution at low cost, an efficient strategy which allowed for improving the RSM accuracy in the vicinity of the global optimum was presented. Wu *et al*. (2010) investigated a new design concept of MAS, and a shape optimization method with finite element analysis was applied on two-dimensional (2D) stent models. Qian (2010) presented an approach for analytically computing the full sensitivities of both the positions and weights of NURBS control points in structural shape optimization. Such analytical formulation allowed accurate calculation of sensitivity and had been successfully used in gradient-based shape optimization. Dietl and Garcia (2010) proposed a new approach to change the shape of the beam to concentrate the strain in sections of the beam where it could contribute the most to transduction. A vibration model of beams with non-uniform width was developed and validated with shaker table tests. Durgun and Yildiz (2012) introduced a new optimization algorithm, called the Cuckoo Search Algorithm, for solving structural design optimization problems. Xia *et al*. (2012) presented a level set solution to the stress-based structural shape and topology optimization. First, a novel global measure of stress was proposed, and the optimization problem was formulated to minimize the global measure of stress subject to a constraint of material volume. Kaveh and Khayatazad (2013) employed ray optimization for size and shape optimization of truss structures. The goal function of the present optimization method was the minimization of truss weight under the required constraints. Yildiz (2013) investigated a comparison of evolutionary-based optimization techniques for structural design optimization problems. Furthermore, a hybrid optimization technique based on differential evolution algorithm was introduced for structural design optimization problems. Emmanuel *et al*. (2014) used ANN and GA for buckling optimization of laminated composite plate with elliptical cutout. Schulz (2014) provided a novel framework for analyzing shape Newton optimization methods by exploiting a Riemannian perspective. A Riemannian shape Hessian was defined possessing often sought properties like symmetry and quadratic convergence for Newton optimization methods. In addition, the publications (He *et al*. 2002, Wang 2012, Kaveh and Ahmadi 2014, Kaveh and Zolghadr 2014, Thrall *et al*. 2014) also considered the structural optimization design.

In the present study, five extended cylindrical reticulated shells are proposed by changing distribution rule of diagonal rods based on three fundamental types. According to geometric feature of cylindrical reticulated shells, the generation methods of nodes and rod elements under cylindrical coordinate system are investigated. Macro programs are compiled by using the ANSYS Parametric Design Language (APDL). Users can easily get the required models only by inputting five parameters, i.e., the shell span (D), rise (F), length (L), transverse copies (M) and longitudinal copies (N). On this basis, conditional formulas are derived when the grid shape of cylindrical reticulated shells is equilateral triangle. Internal force analysis of cylindrical reticulated shells is carried out. The variation and distribution regularities of maximum displacement and stress are studied. A shape optimization program is proposed by adopting the sequence two-stage algorithm (RDQA) in FORTRAN environment based on the characteristics of cylindrical reticulated shells and the ideas of discrete variable optimization design. Shape optimization is achieved by considering the objective function of the minimum total steel consumption, global and locality constraints. The shape optimization for three fundamental types and five extended types is calculated with the span of 30 m~80 m and rise-span ratio of 1/7~1/3. The variations of the total steel consumption along with the span and rise-span ratio are analyzed with contrast to the results of shape optimization. The optimal combination of main design parameters for five extended cylindrical reticulated shells is investigated. The total steel consumption affected by distribution rule of diagonal rods is discussed. The research results provide reference for actual cylindrical reticulated shells.

2. Structural features of five extended cylindrical reticulated shells

A straight line moves parallelly along two curves, and the two curves have the same radius of curvature, then a cylindrical reticulated shell is formed, which is shown in Fig. 2.

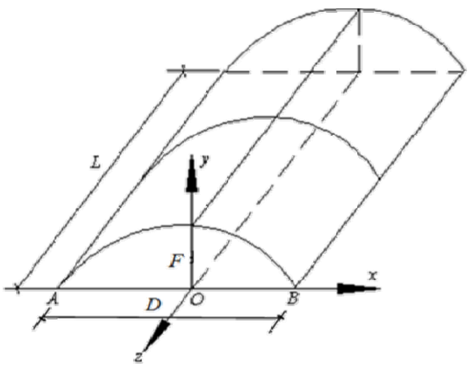
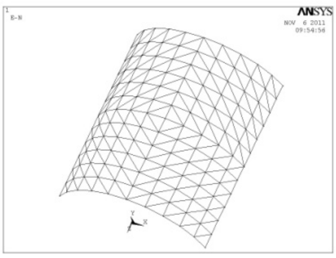
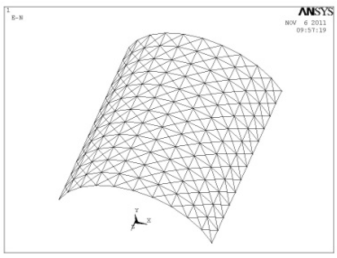


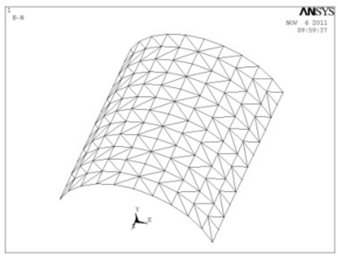
Fig. 2 Structure diagram of cylindrical reticulated shell



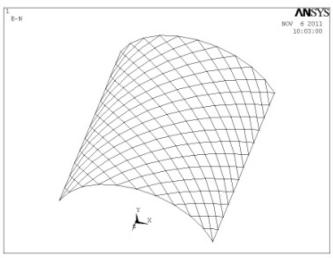
(a) Single diagonal rod type



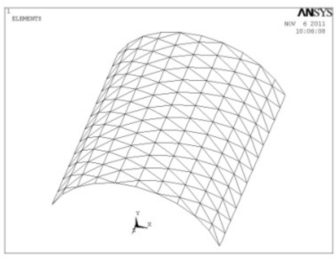
(b) Double slash rod type



(c) Herringbone type



(d) Lamella type



(e) Three-way grid type

Fig. 3 Five fundamental types of cylindrical reticulated shell

In the above coordinates, curved surface equation of cylindrical surface can be expressed as Eq. (1)

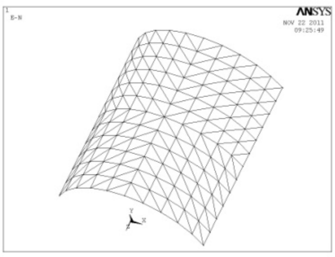
$$x^2 + (y + R - F)^2 = R^2 \tag{1}$$

Where R is radius of curvature, F is the rise of cylindrical reticulated shell. The main geometric parameters of describing a cylindrical reticulated shell have: span (D), rise (F), length (L), transverse copies (M) and longitudinal copies (N).

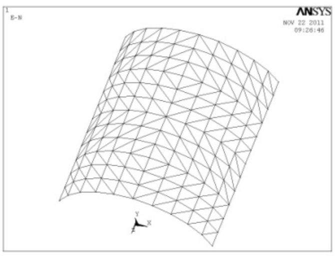
According to different grid types, cylindrical reticulated shells have five fundamental types (Fig. 3), i.e., single diagonal rod type, double slash rod type, herringbone type, lamella type, three-way grid type.

The cylindrical reticulated shells developed by five fundamental types are called extended cylindrical reticulated shells. They are formed by changing distribution rule of diagonal rods based on fundamental types of cylindrical reticulated shell. The extended cylindrical reticulated shell is a new-type structure, and its structure type can be varied as required.

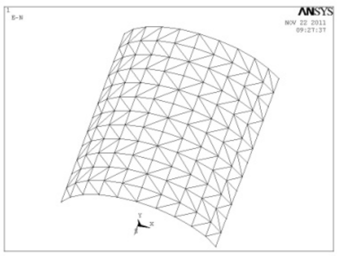
(1) Extended type 1 of single diagonal rod - cylindrical reticulated shell, as shown in Fig. 4(a). All diagonal rods are reversed direction based on fundamental type of single diagonal rod.
(2) Extended type 2 of single diagonal rod - cylindrical reticulated shell, as shown in Fig. 4(b). The arrangement of diagonal rods is the same as fundamental type of single diagonal rod in the former 1/2 length. The diagonal rods cycle again in the back 1/2 length.
(3) Extended type 1 of herringbone - cylindrical reticulated shell, as shown in Fig. 4(c). All diagonal rods are reversed direction based on fundamental type of herringbone.
(4) Extended type 2 of herringbone - cylindrical reticulated shell, as shown in Fig. 4(d). The span is divided into m groups, and diagonal rods of even-number groups are reversed direction, so the diagonal rods are arranged in an X shape.



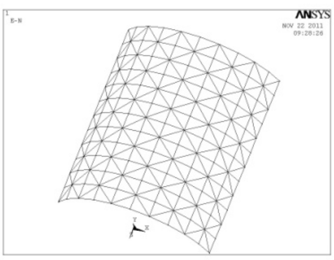
(a) Extended type 1 of single diagonal rod



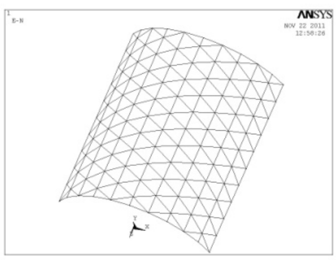
(b) Extended type 2 of single diagonal rod



(c) Extended type 1 of herringbone



(d) Extended type 2 of herringbone



(e) Extended type of three-way grid

Fig. 4 Five extended types of cylindrical reticulated shell

(5) Extended type of three-way grid - cylindrical reticulated shell, as shown in Fig. 4(e). The grid shape of fundamental type of three-way grid is changed, which makes the triangle become equilateral triangle, i.e., all rod elements are equal in length and all angles are 60 degrees.

3. Parametric modeling

ANSYS Parameter Design Language is called APDL for short, which is a secondary development tool provided by ANSYS. It is an interpreted language like FORTRAN. Script programs compiled by using APDL can automatically complete most of the GUI operation tasks. They can be even used to address some GUI unattainable functions, such as solver control, parametric modeling, etc. Meanwhile, APDL also has some other features; such as repeatedly executing a command, macro language, if-then-else branch statements, do-loop circulation, and so on (Chen and Liu 2009, Gong and Xie 2010, Wu *et al*. 2015a, b). Moreover, it is the basis of some advanced operations, such as optimization design and adaptive meshing, etc.

The parametric modeling idea of using APDL is as follows. First, the nodes are generated by node coordinates, then, the rod elements are generated by connecting nodes according to different grid types. As for different cylindrical reticulated shells, their difference is the connection of transverse rod elements, longitudinal rod elements and diagonal rod elements. Below are parametric modeling methods of extended cylindrical reticulated shells.

3.1 The inputting of geometric parameters

The inputting of geometric parameters can be achieved by ANSYS graphical user interface (GUI), which adopts multi-parameter input dialog box. The procedure is as follows.

```
MULTIPRO, 'start',6
*cset,1,3,D, 'Span (m)',40
*cset,4,6,F, 'Rise (m)',10
*cset,7,9,L, 'Length (m)',60
*cset,10,12,M, ' Transverse copies',18
*cset,13,15,N, 'Longitudinal copies',20
*cset,16,18,z, 'the structure type',1
1 - Extended type 1 of single diagonal rod - cylindrical reticulated shell
2 - Extended type 2 of single diagonal rod - cylindrical reticulated shell
3 - Extended type 1 of herringbone - cylindrical reticulated shell
4 - Extended type 2 of herringbone - cylindrical reticulated shell
5 - Extended type of three-way grid - cylindrical reticulated shell
*cset,61,62, 'Please input geometry parameters'
MULTIPRO, 'end'
```

3.2 The definition for rod element types, material properties and real constants

For the spatial structure, its rod elements not only sustain axial tension and pressure, but also bear bending moment, thus, beam4 element is chosen. Then the material properties including elastic modulus, Poisson's ratio and density of steel are defined, i.e., $E = 2.06 \times 10^{11}$ N/m^2, $\mu = 0.3$, $\rho = 7850$ kg/m^3. The real constants are mainly cross-sectional area of rod elements, which can be got from the allowable discrete set of cross-sectional area.

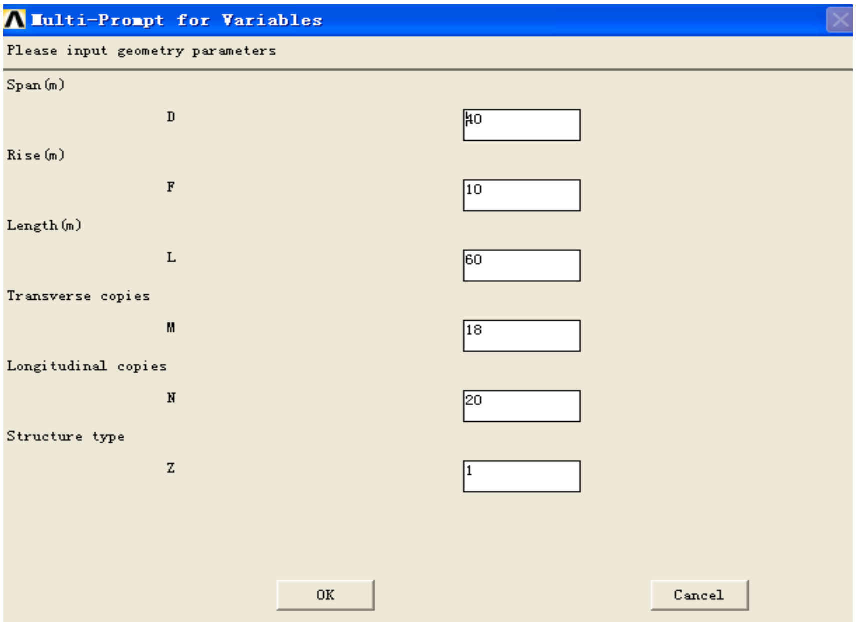


Fig. 5 The input window of geometrical parameters

3.3 The calculation of node coordinates

As for the coordinate system shown in Fig. 2, the centre of a circle is not at the origin of coordinates. To facilitate this analysis, the origin of *OXYZ* coordinates is moved to the center point of cylindrical curved surface, thereby establishing a new coordinate system. The *xy*-direction views of new coordinate system are shown in Fig. 6. Suppose there are $m + 1$ nodes in the span direction and $n + 1$ nodes in the length direction. Any two adjacent nodes $(i, i+1)$ are selected. The central angle formed by i, $i+1$ and O is expressed as $\beta_i = 2\beta / m$. The coordinates of each node can be determined by its geometrical parameters. The origin of *OXYZ* coordinates is translated to the node O' for easy analyzing in ANSYS. The OO' can be obtained, i.e., $OO' = R - F$.

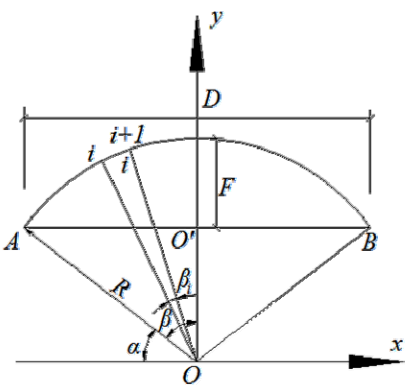


Fig. 6 The *xy*-direction views of cylindrical reticulated shell

In conclusion, the j-th node coordinate at the i-th row is expressed as follows

$$\begin{cases} x = -R\cos[\alpha + (i-1)\beta_i] \\ y = R\sin[\alpha + (i-1)\beta_i] - R + F \\ z = -(j-1)L/n \end{cases} \tag{2}$$

Wherein the value range of variables (i, j) is as follows, $i = 1, 2, ..., m+1$; $j = 1, 2, ..., n+1$.

The program's commands of generating nodes are *N*, *NODE*, *X*, *Y*, *Z* in ANSYS. Wherein the *X*, *Y*, *Z* are the coordinates of nodes.

3.4 Setting up rod elements

The program's commands of generating rod elements are *E*, P_1, P_2 in ANSYS. Wherein the P_1, P_2 are the node numbers on both ends of rod element.

As shown in Fig. 7, the six nodes *A*, *B*, *C*, *D*, *E*, *F* in rows i, i+1, columns j, j+1, j+2 are selected. According to arrangement rules of nodes, the numbers of six nodes are as follows.

$$A:(j-1)(m+1)+i \qquad B:j(m+1)+i \qquad C:(j+1)(m+1)+i$$
$$D:(j-1)(m+1)+i+1 \quad E:j(m+1)+i+1 \quad F:(j+1)(m+1)+i+1$$

The rod elements can be connected based on the above numbers. Transverse rod elements and longitudinal rod elements should be first connected. Then generating diagonal rod elements. The diagonal rod elements are first connected in rows, then in columns. When connecting rod elements on the same row or the same column, only adjacent nodes can be connected, and non-adjacent nodes can not be connected.

3.5 The programming

Taking the extended type of three-way grid - cylindrical reticulated shell as an example. Its method of parametric modeling is as follows:

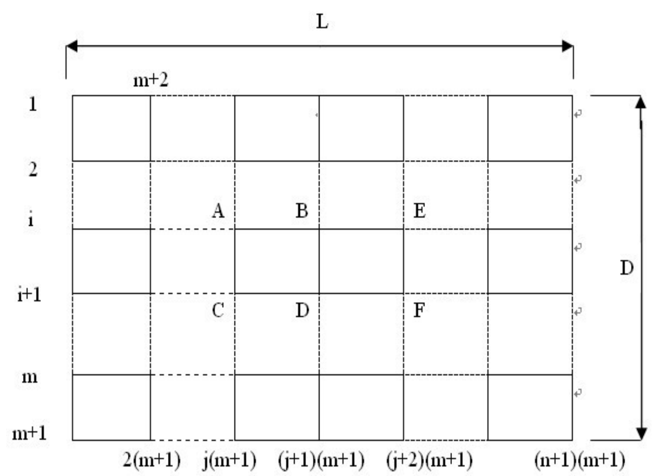


Fig. 7 The nodes distribution of cylindrical reticulated shells

In the first place, the node coordinates are generated by using N, NODE, X, Y, Z commands. The m+1 nodes are generated in the span direction, meanwhile, the n+1 nodes are generated in the odd rows of length direction, and the 2n+1 nodes are generated in the even rows of length direction. Then the rod elements are generated by using E, P_1, P_2 commonds. The double *DO *END DO loop control statements are adopted in the program, and control variables are i and j. The rod elements in the span direction are first connected, and only the rod elements of j = 1 and j = n+1 in columns are connected. Then the rod elements in the length direction are connected, and the rod elements of odd rows and even rows are connected respectively. The connection of diagonals rod elements can be referred to Fig. 7. The nodes A and D, D and E, B and C, B and F are connected respectively, thus, a diagonals rod element of X-type is formed. Then the X-type circulates along the span direction and the length direction. The loop statement are *DO, i, 2, m, 2 and *DO, j, 2, n, 2. The ranges of i and j are $0 \le i \le m$, $0 \le j \le n$.

3.6 The derivation of conditional formulas

On the premise of a known span (D), rise (F), length (L), the grid shape of cylindrical reticulated shells can be equilateral triangle when transverse copies (M) is proportional to longitudinal copies (N). The derivation process is as follows.

It can be obtained from Fig. 6, the curve is divided into M portions. The radius of curvature is calculated according to geometrical relationship.

$$R = \frac{4F^2 + D^2}{8f} \tag{3}$$

$$\angle AOO' = \beta = \arcsin\frac{D}{2R} \tag{4}$$

Suppose the side length of equilateral triangle is x. It can be obtained from the length direction and span direction.

$$Nx = L \tag{5}$$

$$\sin(\pi/3)Mx = 2R\beta \tag{6}$$

It can be obtained from Eqs. (3), (4), (5), (6).

$$\frac{\mathrm{M}}{N} = \frac{4F^2 + D^2}{2\sqrt{3}FL}\arcsin\left(\frac{4DF}{4F^2 + D^2}\right) \tag{7}$$

Suppose rise-span ratio is $k = F/D$, long-span ratio is $s = L/D$. Then the formula (7) can be expressed with k and s.

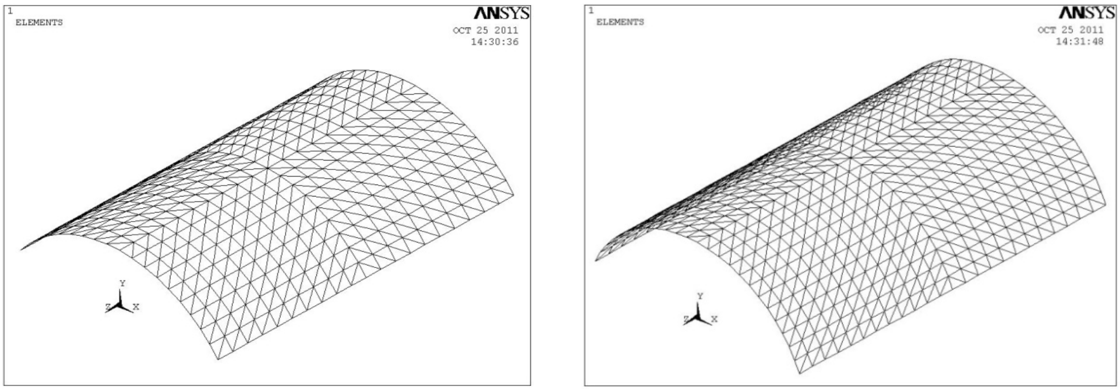
$$\frac{M}{N} = \frac{4k^2 + 1}{2\sqrt{3}ks}\arcsin\left(\frac{4k}{4k^2 + 1}\right) \tag{8}$$

Therefore, if the k and s are konwn, and M, N satisfy the formula (8), then the grid shape of cylindrical reticulated shells is equilateral triangle, i.e., the length of all rod elements is equal, and all angles are 60 degrees.

The side length of equilateral triangle is $x = L/N$. According to technical specification for reticulated shells (JGJ7 2010), the length of rod elements can adopt 1.5 m~3.0 m when the span is less than 50 m. And the length of rod elements can adopt 2.5 m~3.5 m when the span is 50 m~100 m. In addition, according to geometric feature of cylindrical reticulated shells, cylindrical reticulated shells are axisymmetric structures in the span direction, so the transverse copies (M) is even number. The M and N can be determined based on the above points. For example, if $D = 40$ m, $F = 10$ m, $L = 48$ m, substituting it into formula (7), then $M/N = 11/10$. Taking $M = 22$, $N = 20$, the length of rod elements can meet requirements.

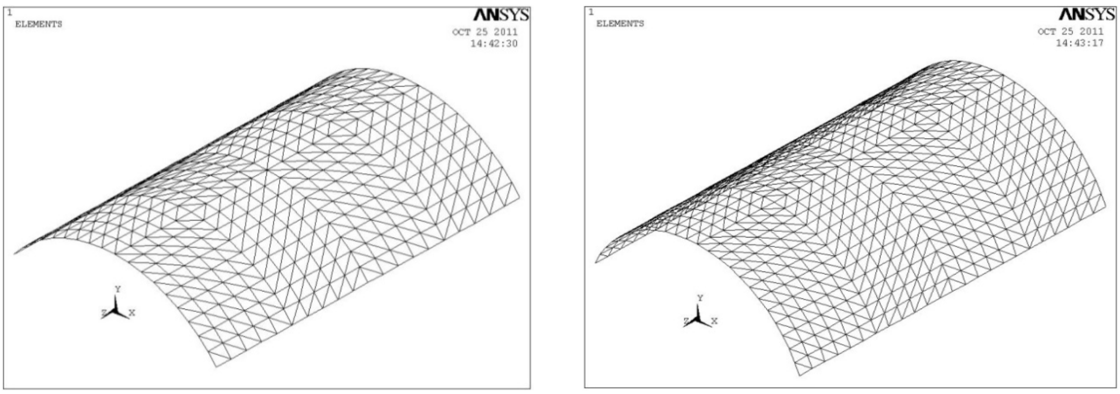
3.7 Parametric modeling examples

The types of cylindrical reticulated shells (1, 2, 3, 4, 5), span (D), rise (F), length (L), transverse copies (M) and longitudinal copies (N) are input in dialog box (Fig. 5), then the parametric modeling can be achieved. Parametric modeling examples are shown in Figs. 8-12.



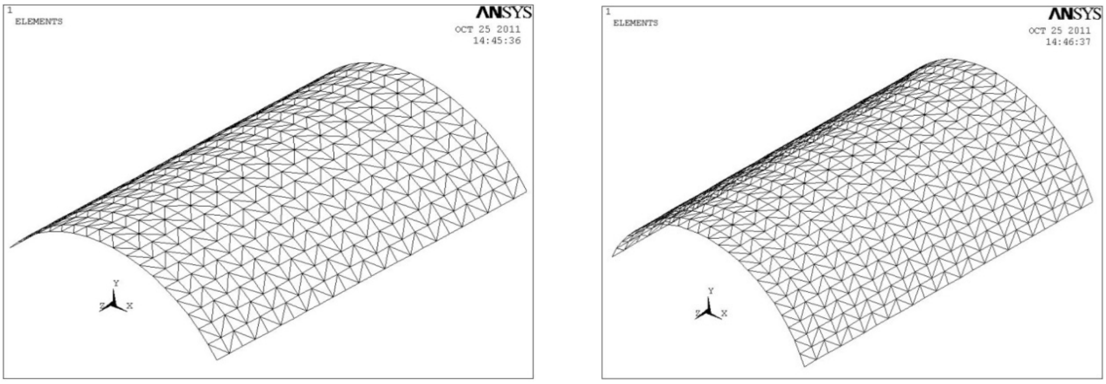
(a) $D = 40$ m, $L = 60$ m, $F = 10$ m, $M = 18$, $N = 20$ (b) $D = 60$ m, $L = 90$ m, $F = 20$ m, $M = 24$, $N = 24$

Fig. 8 Extended type 1 of single diagonal rod



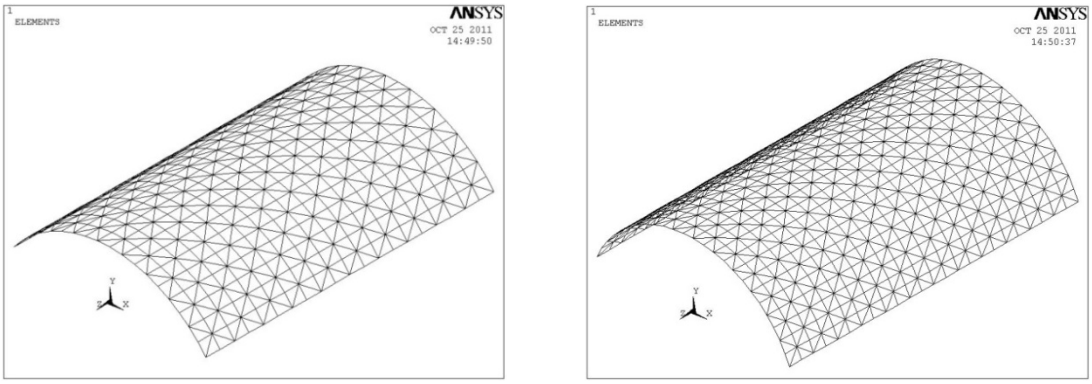
(a) $D = 40$ m, $L = 60$ m, $F = 10$ m, $M = 18$, $N = 20$ (b) $D = 60$ m, $L = 90$ m, $F = 20$ m, $M = 24$, $N = 24$

Fig. 9 Extended type 2 of single diagonal rod



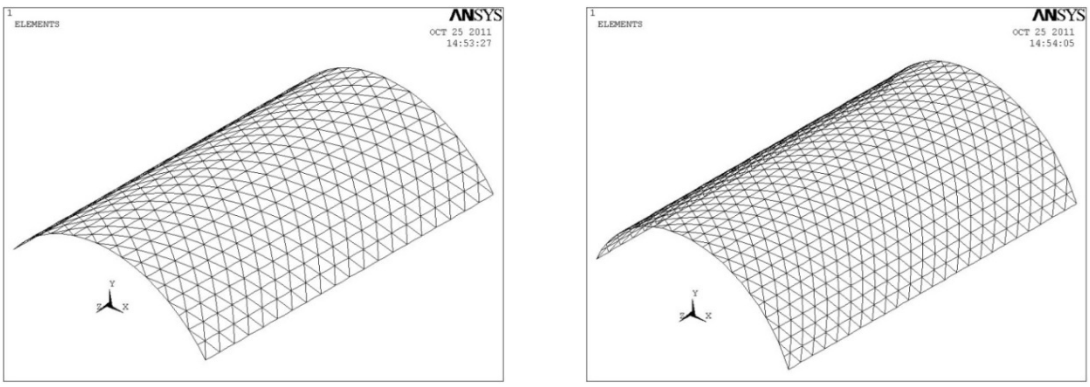
(a) D = 40 m, L = 60 m, F = 10 m, M = 18, N = 20 (b) D = 60 m, L = 90 m, F = 20 m, M = 24, N = 24

Fig. 10 Extended type 1 of herringbone



(a) D = 40 m, L = 60 m, F = 10 m, M = 18, N = 20 (b) D = 60 m, L = 90 m, F = 20 m, M = 24, N = 24

Fig. 11 Extended type 2 of herringbone



(a) D = 40 m, L = 60 m, F = 10 m, M = 18, N = 20 (b) D = 60 m, L = 90 m, F = 20 m, M = 24, N = 24

Fig. 12 Extended type of three-way grid

Table 1 The model parameters of internal force analysis for cylindrical reticulated shells

Structure type	Single diagonal rod, extended type 1 of single diagonal rod, extended type 2 of single diagonal rod, herringbone, extended type 1 of herringbone, extended type 2 of herringbone, three-way grid, extended type of three-way grid.		
Span (m)	30, 40, 50, 60, 70, 80		
Length-span ratio	1.2	Rise-span ratio	1/3
Uniform load (KN/m^2)	2.35	Steel density (kg/m^3)	7850
Elastic modulus (N/m^2)	2.06×10^{11}	Poisson ratio	0.3

4. The internal force analysis

As for cylindrical reticulated shells, the basal principle and methods of internal force analysis can be summarized as two categories (Wu *et al*. 2015a, b). The first category is imitative shell method based on continuity assumption, and the second category is finite element method of truss structures based on discretization assumption. For imitative shell method, the structures are analyzed and studied according to the basic theory of elastic thin shells. Its purpose is to obtain displacement and stress of the structures and then convert into internal force of cylindrical reticulated shells. The basal principle of finite element method for truss structures is that the grids constituted by rod elements originally can disperse into individual element. And a rod can be usually considered as a basic element when conducting internal force analysis.

The finite element method of truss structures is usually adopted when making internal force analysis for reticulated shells. Space beam elements are adopted as rod elements, and the nodes of reticulated shells are assumed to be ideal rigid joints. This method can be applied in static, dynamic and buckling analysis of all types of reticulated shells (Shang and Qiu 2005).

The internal force analysis for fundamental types and extended types of cylindrical reticulated shells is carried out by using ANSYS. The selected model parameters are shown in Table 1.

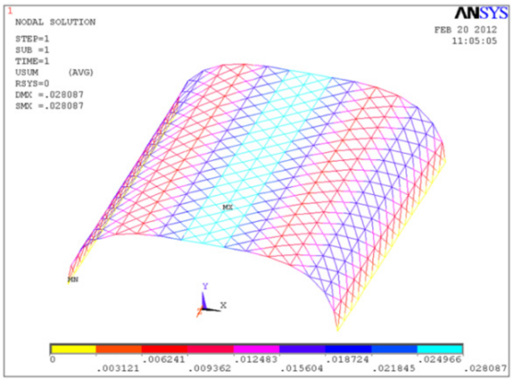
4.1 The loading programming

In order to analyze the internal force of cylindrical reticulated shells more easily and quickly, the loading programm is compiled with the APDL after parametric modeling.

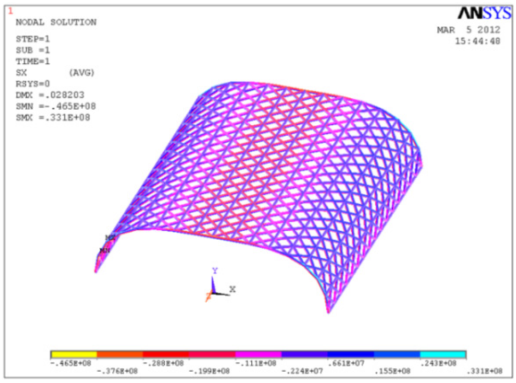
Since this is a three-dimensional spatial structural problem, the rod elements not only subject to axial tension and pressure, but also subject to bending moment. Beam4 space beam element is selected in the present study. Beam4 element is a tensile and compressive, torsional and bending element in the axial direction, meanwhile, each node has six degrees of freedom, which can translate along the *X*, *Y*, *Z* directions and rotate around the *X*, *Y*, *Z* axis in the node coordinate system. Constraint condition of the boundary points of cylindrical reticulated shells is simply supported, which cannot translate along the X, Y, Z directions but can rotate around the *X*, *Y*, *Z* axis. The program's commands are *D*, *P51X*, , , , , ,*UX*,*UY*,*UZ*, , ,. The uniform loads vertically downward effect on the nodes of cylindrical reticulated shells. The internal force of each node is calculated by the formula (9).

$$P_i = \frac{Q \times S}{n} \tag{9}$$

Wherein, Q is the uniform load, S is the projected area, n is the total number of nodes.

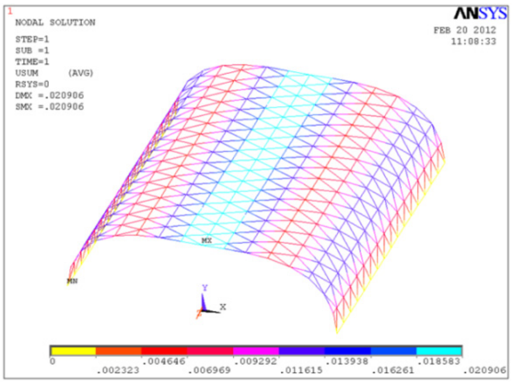


(a) Displacement contour

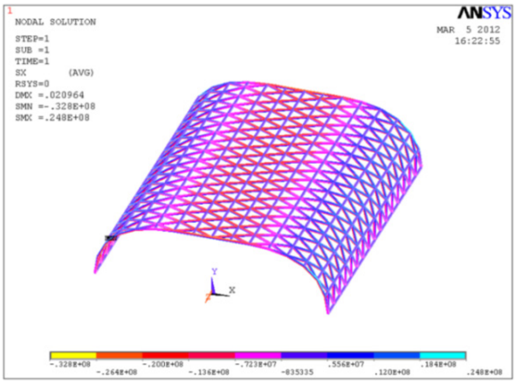


(b) Stress contour

Fig. 13 Three-way grid - cylindrical reticulated shell (D = 30 m)



(a) Displacement contour



(b) Stress contour

Fig. 14 Extended type of three-way grid - cylindrical reticulated shell (D = 30 m)

Table 2 The results of internal force analysis for three-way grid - cylindrical reticulated shell

Span (m)	Length (m)	Rise-span ratio	Transverse grid number	Longitudinal grid number	The maximum displacement(m)	The maximum stress (MPa)
30	36	1/3	16	16	0.028087	33.1
40	48	1/3	20	20	0.096489	63.3
50	60	1/3	20	20	0.292328	125
60	72	1/3	24	24	0.618201	189
70	84	1/3	28	28	1.161	257
80	96	1/3	32	32	2.002	345

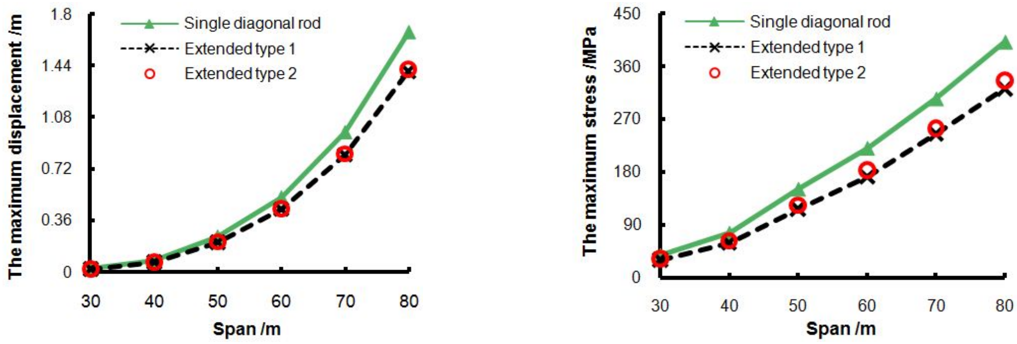
The corresponding program's commands are *F*,*P51X*,*FY*,(-1)*(2350*D*L)/(n). Wherein D is the span, L is the length.

Table 3 The results of internal force analysis for extended type of three-way grid - cylindrical reticulated shell

Span (m)	Length (m)	Rise-span ratio	Transverse grid number	Longitudinal grid number	The maximum displacement (m)	The maximum stress (MPa)
30	36	1/3	16	16	0.020906	24.8
40	48	1/3	20	20	0.072234	46.9
50	60	1/3	20	20	0.219176	93
60	72	1/3	24	24	0.463932	134
70	84	1/3	28	28	0.872232	194
80	96	1/3	32	32	1.504	260

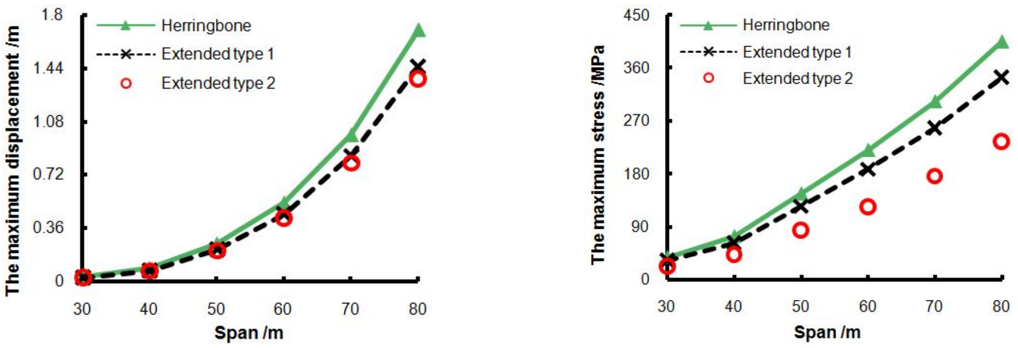
4.2 The analysis results

The loading programm is carried out in ANSYS. Then the internal force analysis for fundamental types and extended types of cylindrical reticulated shell is achieved. Take the three-way grid - cylindrical reticulated shell for example. Displacement contour and stress contour are shown in Figs. 13-14. Analysis results are shown in Tables 2-3.



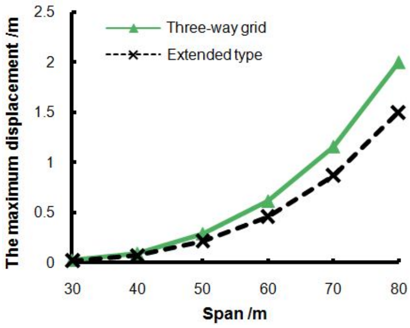
(a) The maximum displacement of nodes (b) The maximum stress of rod elements

Fig. 15 The results of internal force analysis for single diagonal rod and its extended types

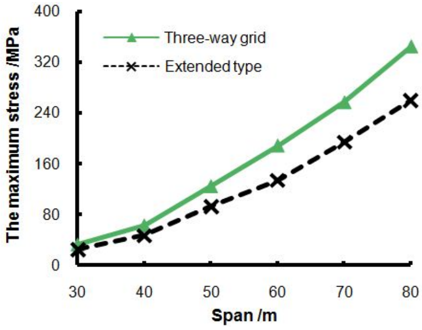


(a) The maximum displacement of nodes (b) The maximum stress of rod elements

Fig. 16 The results of internal force analysis for herringbone and its extended types



(a) The maximum displacement of nodes



(b) The maximum stress of rod elements

Fig. 17 The results of internal force analysis for three-way grid and its extended type

According to the analysis results, the maximum displacement and the maximum stress are obtained in the displacement contour and stress contour. The maximum displacement of cylindrical reticulated shells all occurs at the position of 1/2 span. To make the results comparable, the relevant parameters are unified, and fetch values from the Tables 2-3. The maximum displacement and stress of single diagonal rod type, herringbone type, three-way grid type under the different span are shown in Figs. 15-17.

The following conclusions can be obtained from Figs. 15-17.

(1) As for fundamental type and extended types of single diagonal rod - cylindrical reticulated shell, it can be obtained from Fig. 15. The maximum displacement and the maximum stress increase with the span. The maximum displacement and the maximum stress of extended types are less than its fundamental type. In Fig. 15(a), the maximum displacements of extended type 1 and extended type 2 are basically the same. In Fig. 15(b), the maximum stress of extended type 1 is less than extended type 2. Therefore, the mechanical performance of extended type 1 of single diagonal rod is better.

(2) As for fundamental type and extended types of herringbone - cylindrical reticulated shell, it can be obtained from Fig. 16. The maximum displacement and the maximum stress increase with the span. The maximum displacement and the maximum stress of extended types are less than its fundamental type. In Fig. 16(a), the difference of the maximum displacement between extended type 1 and extended type 2 is smaller, but as the span increases, the maximum displacement of extended type 2 is gradually less than extended type 1. In Fig. 16(b), the maximum stress of extended type 2 is far less than extended type 1, and as the span increases, the difference gets bigger and bigger. Therefore, the mechanical performance of extended type 2 of herringbone is better.

(3) As for fundamental type and extended type of three-way grid - cylindrical reticulated shell, it can be obtained from Fig. 17. The maximum displacement and the maximum stress increase with the span. The maximum displacement and the maximum stress of extended type are far less than its fundamental type. And as the span increases, the difference between fundamental type and extended type gets bigger and bigger. Therefore, the mechanical performance of extended type of three-way grid is better.

In order to compare the overall mechanical properties of the eight cylindrical reticulated shells, the change rules of the maximum displacement and the maximum stress along with span are shown in Figs. 18-19.

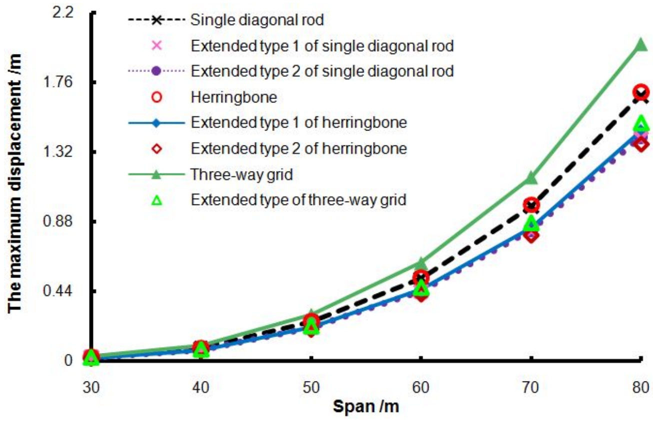


Fig. 18 The maximum displacement of eight cylindrical reticulated shells under different span

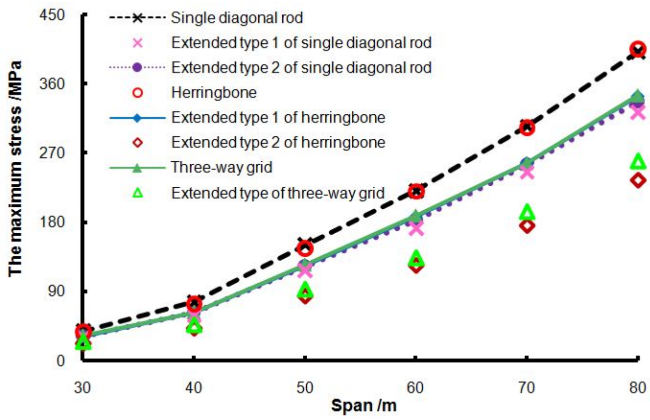


Fig. 19 The maximum stress of eight cylindrical reticulated shells under different span

(1) It can be clearly seen from Fig. 18, the maximum displacement of three-way grid - cylindrical reticulated shell is maximal, and the extended type 2 of herringbone - cylindrical reticulated shell is minimal. Overall, the maximum displacements of eight cylindrical reticulated shells are basically the same when the span is less than 50m. As for extended types of single diagonal rod, extended types of herringbone and extended type of three-way grid, the difference of the maximum displacement is small when the span is from 30m to 80m.

(2) It can be clearly seen from Fig. 19, the maximum stresses of single diagonal rod and herringbone - cylindrical reticulated shells are maximal, and the extended type 2 of herringbone and extended type of three-way grid - cylindrical reticulated shells are minimal.

The following conclusion can be obtained according to the above analysis results.

Mechanical properties of cylindrical reticulated shell are related to its arrangement of diagonal rod. The extended types of single diagonal rod are got by changing the direction of diagonal rod. The mechanical properties are improved after changing the direction of diagonal rod. In the two extended types of herringbone, X-type arrangement of diagonal rod of extended type 2 is better than its fundamental herringbone type, i.e., the forces of extended type 2 is more reasonable. As for the three-way grid - cylindrical reticulated shell, equilateral triangle grid of extended type is better than isosceles triangle grid of its fundamental type, i.e., the forces of extended type is more reasonable.

5. The shape optimization

The nodes distribution of cylindrical reticulated shell has a regularity, which is determined by macroscopic surface shape and geometric parameters (*D*, *F*, *L*, *M*, *N*) of the structures. With regard to this kind of structures, cross-section optimization and shape optimization are carried out in order when conducting optimization design. Cross-section optimization adopts relative difference quotient algorithm (RDQA) based on discrete variables. Optimal cross-section size is sought by presetting macroscopic surface parameters. Then, on the basis of cross-section optimization, the optimal solution is got by changing macroscopic surface parameters with the goal of minimizing the total steel consumption, i.e., shape optimization.

5.1 Mathematical models of shape optimization

The Mathematical model of the shape optimization mainly includes design variables, objective function and constraint conditions. Detailed information can be referred in Wu *et al.*'s previous study (Wu *et al.* 2015a, b).

5.2 The two-stage optimization method

(1) The first-stage (cross-section) optimization:

The sequence two-stage optimization algorithm based on discrete variables is adopted. The first stage makes use of a one-dimensional search algorithm to process local constraints, such as stress constraints, stability constraints, slenderness ratio constraints, etc. The second stage takes advantage of relative difference quotient algorithm (RDQA) to handle whole constraints (Deng and Dong 1999).

Mathematical models (Lu *et al.* 2013, Sun *et al.* 2002) of cross-section optimization are as follows

$$\begin{aligned} & P_1 \quad Seeking\ A \\ & \min W = \sum_{i=1}^{m} \rho_i l_i(S) Ai + \sum_{j=1}^{n} \rho_j V_j \\ & s.t. \quad \sigma_{\mathrm{w}i} \le [\sigma] \\ & \qquad \lambda_i \le [\lambda] \\ & \qquad x_i \in S_i \end{aligned} \tag{10}$$

(2) The second-stage (shape) optimization:

The aim is to seek optimal node locations along declining direction of the total weight, which can improve mechanical properties of the structures and provide an improved structural style for next round of cross-section optimization.

Mathematical models (Lu *et al*. 2013, Sun *et al*. 2002) of shape optimization are as follows

$$
\begin{aligned}
&P_2 \quad Seeking\, M, N \\
&\min W = \sum_{i=1}^{m} \rho_i l_i (M, N) A_i + \sum_{j=1}^{n} \rho_j V_j \\
&s.t. \quad \delta_{\max} \le [\delta]
\end{aligned}
\tag{11}
$$

Given the range of M and N, optimal combination of M and N is searched with the goal of minimizing the total steel consumption of cylindrical reticulated shells.

5.3 The design concept of shape optimization

As for shape optimization, the cross-section optimization should be carried out firstly, which includes many methods, e.g., the combination of the direct searching method and the criterion method (Xu *et al*. 2006), the combination of the one-dimensional search method and the relative difference quotient method (Lu *et al*. 2012, Wu *et al*. 2015a, b) and so on. However, there is a mistake in our previous study (Wu *et al*. 2015a, b) where the latter method is applied in our program instead of the former. Thus, the flow chart and the corresponding description is not right and is corrected in the present study.

For cylindrical reticulated shells, the number of rod elements and nodes is the main factors affecting the total weight of structures. This study takes the total steel consumption of cylindrical reticulated shells (including the weight of rod elements and nodes) as objective function. Meanwhile, M and N are taken as design variables (Lu *et al*. 2012, Wu *et al*. 2015a, b). A shape optimization program is compiled in FORTRAN environment. The optimizer can use ANSYS to model, resolve and optimize in the background, so that the parameterization can be achieved.

According to the design concept of two-stage optimization (Sun *et al*. 2002, Chen 1989, Zhang and Hou 1998), the span (D) and rise-span ration (F/D) are determine firstly, providing the range of transverse copies (M) and longitudinal copies (N), then one-dimensional search method and relative difference quotient method are adopted for cross-section optimization. At this stage, the cross-sectional area of rod elements and the volume of nodes are used as design variables. Secondly, in order to make the total steel consumption least, the optimal M and N are selected at this span and rise-span ration based on the cross-section optimization results, and shape optimization of this rise-span ration is achieved. Thirdly, the cross-section and shape optimization of another rise-span ration is carried out, and the corresponding optimal solution is also obtained. Finally, combining the above optimal solutions under different rise-span ratios, the optimal rise-span ratio is selected by considering the objective function of the minimum total steel consumption. The shape optimization of this span is achieved, and the optimal F/D, M, N and the minimum total steel consumption are obtained.

The range of M and N is defined beforehand in optimization program based on theoretical and practical problems. Under the certain rise to span ratio, from one side, if the M and N are too large, the grid of the shells will be too thick, thus the construction will be hard. From the other side, if the M and N are too small, the grid of the shells will be too thin, and this will lead to unreasonable stress problems. Therefore, as for cylindrical reticulated shells of different span and height, the range of M and N is selected appropriately based on theoretical and practical experiences in optimization program.

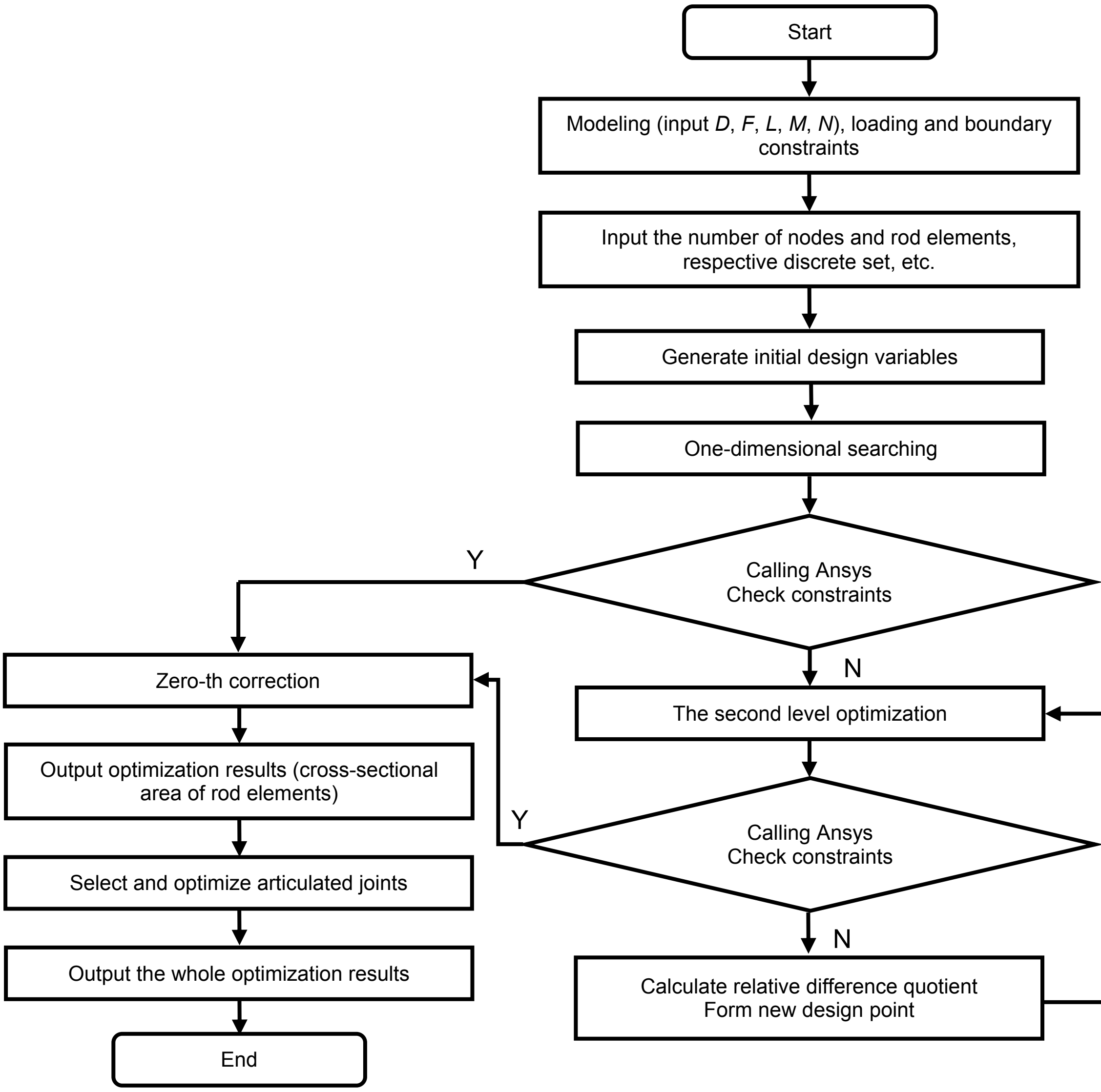


Fig. 20 Cross-section optimization flowchart of cylindrical reticulated shells

In order to present the process of cross-section optimization intuitively, Fig. 20 shows the cross-section optimization flowchart of cylindrical reticulated shells.

5.4 The programming of shape optimization

An optimum design program is written by using standard FORTRAN language, and it can be run after connecting with ANSYS. The program is compiled in FORTRAN environment, and the shape optimization is carried out by calling pre-processing and post-processing results of ANSYS directly. According to the optimized results, the real constants of rod elements in each group are modified, and they are read from lsjhao_result.txt. The new real constants are sent to the ANSYS for finite element analysis, then, the results are passed again to FORTRAN for optimization. The cycle is kept going until it satisfies the constraints.

ANSYS software has a secondary development function. APDL language and macro commands can be used for modeling of cylindrical reticulated shells, defining material properties, grouping of rod elements, defining boundary constraints and imposing nodal loads. Then the finite element model can be analyzed and solved by using ANSYS. Pre-treatment information extraction is mainly extracting the number of nodes and rod elements. Post-processing information extraction includes the extraction of nodal displacement, nodal numbers, and length of rod elements, axial force and bending moments of beam elements.

The commands of calling ANSYS for finite element analysis in FORTRAN are as follows.

Result systemqq('"D:\Program Files\Ansys Inc\v100\ANSYS\bin\intel\ansys100" -b -p ane3fl -i e:\ modle.txt-o e:\ shuchu.txt')

The commands of reading FORTRAN optimization results in ANSYS are as follows.

*DIM, rr,, m1,2

*VREAD, rr (1,1),e:\ lsjhao_zhizhen,txt,,JIK,2,m1,,1

Wherein m1 is the number of rod elements groups.

5.5 The results of shape optimization and analysis

The shape optimization for fundamental types and extended types of cylindrical reticulated shell is carried out. Its structural parameters are shown in Table 4.

Table 4 The model parameters of shape optimization for cylindrical reticulated shells of fundamental type and extended type

Structure type	Single diagonal rod, extended type 1 of single diagonal rod, extended type 2 of single diagonal rod, herringbone, extended type 1 of herringbone, extended type 2 of herringbone, three-way grid, extended type of three-way grid.		
Span (m)	30, 40, 50, 60, 70, 80		
Rise-span ratio	1/7, 1/6, 1/5, 1/4, 1/3		
Length-span ratio	1.2	Allowable stress (N/mm^2)	215
Uniform load (KN/m^2)	2.35	Steel density (kg/m^3)	7850
Elastic modulus (N/m^2)	2.06×10^{11}	Poisson ratio	0.3

Table 5 The optimal results of extended type 1 of single diagonal rod- cylindrical reticulated shell

Type	Span (m)	The optimal steel consumption (kg)	The amount of steel per unit area (kg/m^2)	The optimal rise - span ratio (F/D)	The number of optimal grids	
					M	N
Extended type 1 of single diagonal rod	30	12028.28	11.14	1/6	12	20
	40	29118.79	15.16	1/6	14	22
	50	56228.05	18.74	1/6	16	24
	60	98788.86	22.87	1/6	18	28
	70	159562.9	27.14	1/6	22	32
	80	236958.9	30.85	1/6	24	34

Table 6 The optimal results of extended type 2 of single diagonal rod- cylindrical reticulated shell

Type	Span (m)	The optimal steel consumption (kg)	The amount of steel per unit area (kg/m^2)	The optimal rise - span ratio (F/D)	The number of optimal grids	
					M	N
Extended type 2 of single diagonal rod	30	12585.59	11.65	1/6	12	20
	40	29697.05	15.47	1/6	14	22
	50	57333.92	19.11	1/6	18	26
	60	102211.3	23.66	1/6	18	28
	70	163398.7	27.79	1/6	22	32
	80	240044.0	31.25	1/6	24	36

Table 7 The optimal results of extended type 1 of herringbone- cylindrical reticulated shell

Type	Span (m)	The optimal steel consumption (kg)	The amount of steel per unit area (kg/m^2)	The optimal rise - span ratio (F/D)	The number of optimal grids	
					M	N
Extended type 1 of herringbone	30	11995.55	11.11	1/6	12	20
	40	29804.21	15.52	1/6	14	20
	50	57064.59	19.02	1/6	18	24
	60	102835.0	23.57	1/6	18	28
	70	165272.3	28.11	1/6	22	32
	80	243252.5	31.67	1/6	24	36

Table 8 The optimal results of extended type 2 of herringbone- cylindrical reticulated shell

Type	Span (m)	The optimal steel consumption (kg)	The amount of steel per unit area (kg/m^2)	The optimal rise - span ratio (F/D)	The number of optimal grids	
					M	N
Extended type 2 of herringbone	30	10545.40	9.76	1/6	12	22
	40	25494.69	13.28	1/6	14	22
	50	48424.04	16.14	1/6	18	26
	60	85193.75	19.72	1/6	18	28
	70	139075.6	23.65	1/6	22	32
	80	211941.5	27.59	1/6	24	34

According to technical specification for reticulated shells (JGJ7 2010), the length of rod elements can adopt 1.5 m~3.0 m when the span is less than 50 m. And the length of rod elements can adopt 2.5 m~3.5 m when the span is 50 m~100 m. In addition, the rod elements of cylindrical reticulated shells adopt hot-rolling seamless pipe (calculated by YB 231-70). The cross-sectional area of rod elements is discrete. Constraints of cylindrical reticulated shells are simply supported. The optimal results of five extended cylindrical reticulated shells are shown in Tables 5-9.

Table 9 The optimal results of extended type of three-way grid - cylindrical reticulated shell

Type	Span (m)	The optimal steel consumption (kg)	The amount of steel per unit area (kg/m^2)	The optimal rise - span ratio (F/D)	The number of optimal grids	
					M	N
Extended type of three-way grid	30	9520.692	8.82	1/6	12	18
	40	22644.45	11.79	1/6	14	20
	50	42973.02	14.32	1/6	18	24
	60	75472.35	17.47	1/6	18	26
	70	120983.2	20.57	1/6	22	30
	80	178469.4	23.24	1/6	24	32

The following conclusions are reached from Tables 5-9:

(1) The optimal steel consumption of five extended cylindrical reticulated shells increases with the span. As for cylindrical reticulated shells of different span and rise-span ratio, the number of optimal grids is existed after optimization, and at this moment, the total steel consumption is the least.
(2) The density of grid division has obvious implications for structural total steel consumption. The total steel consumption will increase rapidly whether the density is too dense or too sparse. M and N will increase as the span increases for the same type of cylindrical reticulated shell.
(3) When the span is between 30 m to 80 m, the optimal rise-span ratio unchanged at 1/6.

5.6 Discussion

In order to compare change rule of total steel consumption for extended types and fundamental types of cylindrical reticulated shell after shape optimization, the total steel consumption with the span of 30 m, 50 m, 60 m and 80 m are shown by using curves.

According to the optimization results for fundamental type and extended types of single diagonal rod- cylindrical reticulated shell, the structural total steel consumption under the same span and different rise-span ratio are shown in Fig. 21.

The following conclusions are reached from Fig. 21:

(1) When the span is between 30 m~80 m, the total steel consumption of extended types is almost the same as fundamental type of single diagonal rod - cylindrical reticulated shell. The total steel consumption of extended type 1 is slightly smaller than the fundamental type, and the total steel consumption of extended type 2 is slightly larger than the fundamental type.
(2) As for fundamental type and extended types of single diagonal rod - cylindrical reticulated shell, the total steel consumption firstly decreases and then increases with rise-span ratio increases under the certain span. The total steel consumption is the least when the rise-span ratio is 1/6.
(3) When the rise-span ratio is between 1/7 ~ 1/5, the total steel consumption of these three cylindrical reticulated shells changes more gently. When the rise-span ratio is greater than 1/5, it increases rapidly. Therefore, the rise-span ratio of single diagonal rod - cylindrical reticulated shells should try not to exceed 1/5 in practical engineering.

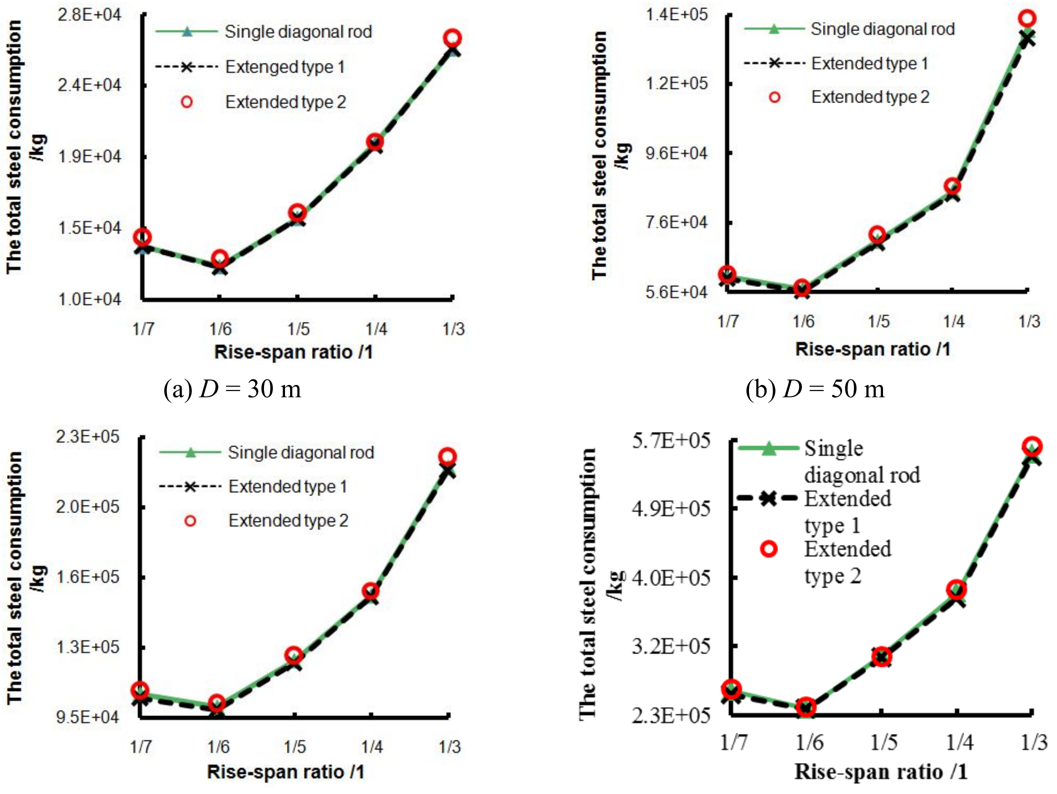
According to the optimization results for fundamental type and extended types of herringbone - cylindrical reticulated shell, the structural total steel consumption under the same span and different rise-span ratio are shown in Fig. 22.

The following conclusions are reached from Fig. 22:

(1) When the span is between 30 m~80 m, the total steel consumption of fundamental type is almost the same as extended type 1 of herringbone - cylindrical reticulated shell, and the total steel consumption of extended type 1 is slightly larger than the fundamental type. The total steel consumption of extended type 2 is obviously smaller than the fundamental type, so the extended type 2 is the lightest in these three cylindrical reticulated shells.

(2) As for fundamental type and extended types of herringbone - cylindrical reticulated shell, the total steel consumption firstly decreases and then increases with rise-span ratio increases under the certain span. The total steel consumption is the least when the rise-span ratio is 1/6.

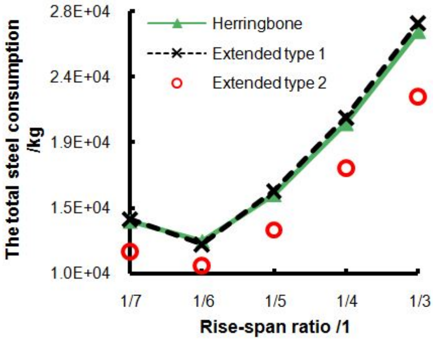
(3) When the rise-span ratio is between 1/7~1/5, the total steel consumption of these three cylindrical reticulated shells changes more gently. When the rise-span ratio is greater than 1/5, it increases rapidly. Therefore, the rise-span ratio of herringbone - cylindrical reticulated shells should try not to exceed 1/5 in practical engineering.



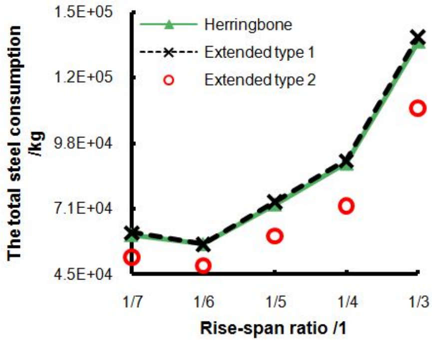
(a) $D = 30$ m (b) $D = 50$ m

(c) $D = 60$ m (d) $D = 80$ m

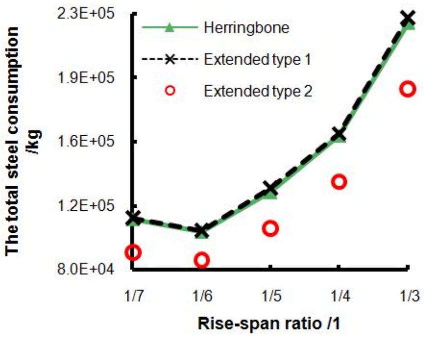
Fig. 21 The total steel consumption for fundamental type and extended types of single diagonal rod - cylindrical reticulated shell



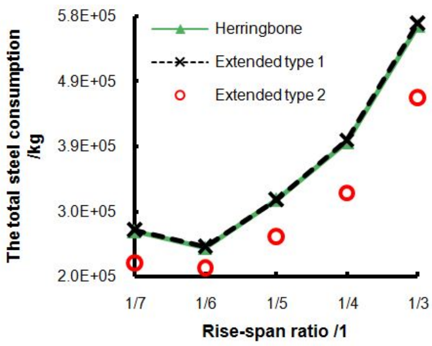
(a) $D = 30$ m



(b) $D = 50$ m



(c) $D = 60$ m



(d) $D = 80$ m

Fig. 22 The total steel consumption for fundamental type and extended types of herringbone - cylindrical reticulated shell

According to the optimization results for fundamental type and extended types of three-way grid - cylindrical reticulated shell, the structural total steel consumption under the same span and different rise-span ratio are shown in Fig. 23.

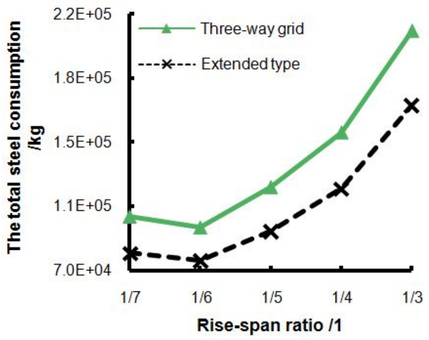
The following conclusions are reached from Fig. 23:

(1) When the span is between 30m~80m, the total steel consumption of extended type is obviously smaller than the fundamental type of three-way grid - cylindrical reticulated shell.

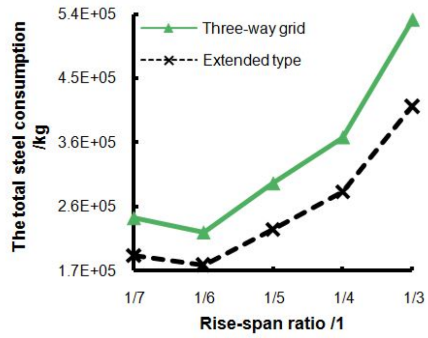
(2) As for fundamental type and extended type of three-way grid - cylindrical reticulated shell, the total steel consumption firstly decreases and then increases with rise-span ratio increases under the certain span. The total steel consumption is the least when the rise-span ratio is 1/6.

(3) When the rise-span ratio is between 1/7~1/5, the total steel consumption of these two cylindrical reticulated shells changes more gently. When the rise-span ratio is greater than 1/5, it increases rapidly. Therefore, the rise-span ratio of three-way grid - cylindrical reticulated shells should try not to exceed 1/5 in practical engineering.

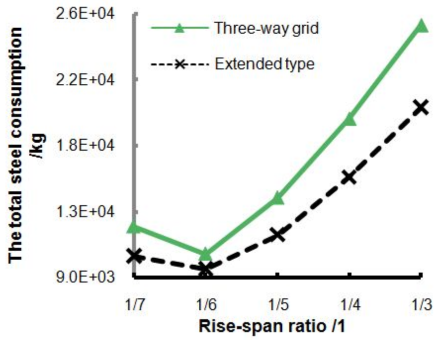
In order to compare the total steel consumption of these eight cylindrical reticulated shells after optimization, the total steel consumption of different cylindrical reticulated shells under the same span and different rise-span ratio are shown in Figs. 24-27.



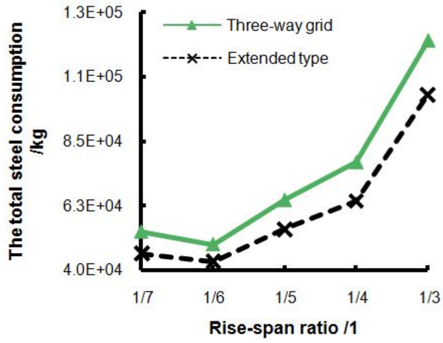
(a) D = 30 m



(b) D = 50 m



(c) D = 60 m



(d) D = 80 m

Fig. 23 The total steel consumption for fundamental type and extended type of three-way grid - cylindrical reticulated shell

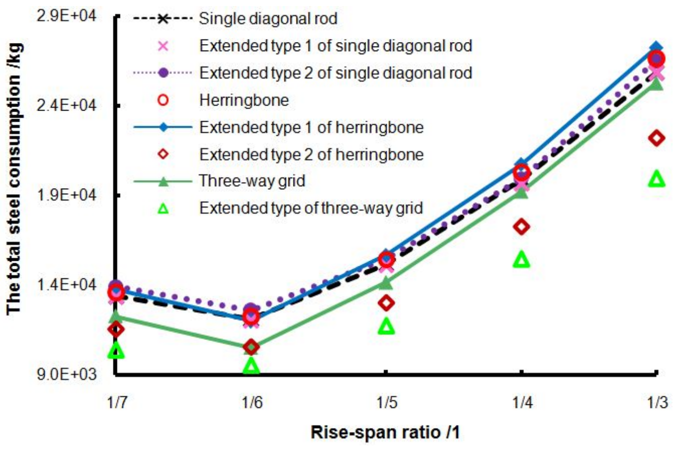


Fig. 24 The total steel consumption of eight cylindrical reticulated shells when span is 30m

The following conclusions can be obtained from Figs. 24-27:

(1) Under the certain span, the total steel consumption of eight cylindrical reticulated shells firstly decreases and then increases with rise-span ratio increases. And the total steel

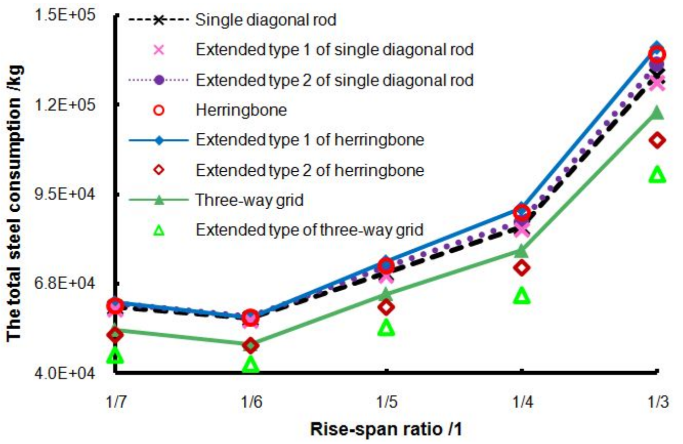


Fig. 25 The total steel consumption of eight cylindrical reticulated shells when span is 50m

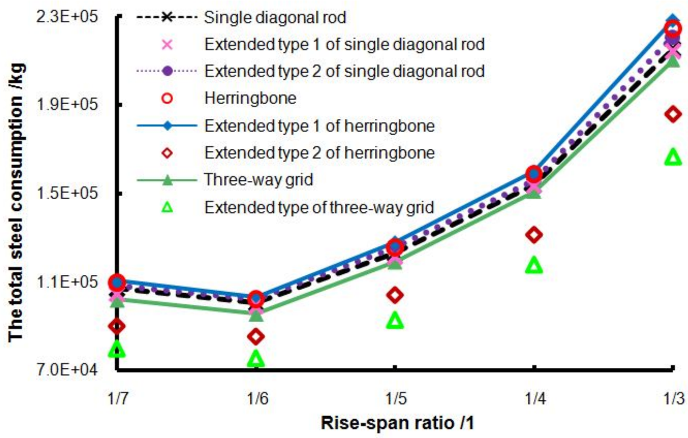


Fig. 26 The total steel consumption of eight cylindrical reticulated shells when span is 60m

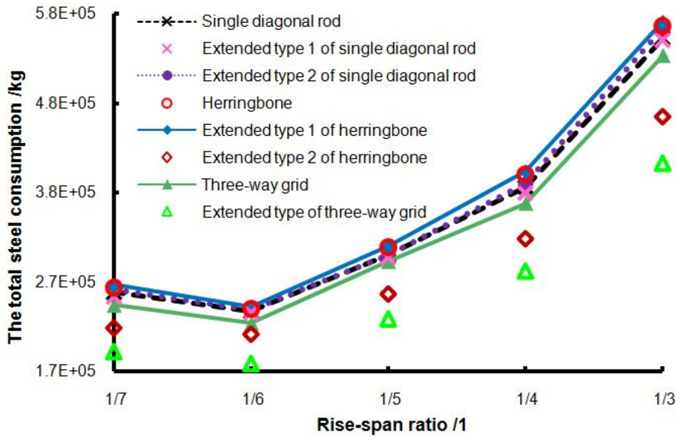


Fig. 27 The total steel consumption of eight cylindrical reticulated shells when span is 80m

consumption is the least when the rise-span ratio is 1/6.

(2) Among the eight cylindrical reticulated shells, the total steel consumption of extended type of three-way grid - cylindrical reticulated shell is the minimum after shape optimization, followed by extended type 2 of herringbone - cylindrical reticulated shell, and the extended type 1 of herringbone - cylindrical reticulated shell is the maximum.

(3) The total steel consumption of cylindrical reticulated shells is connected with their grid shape on the premise that the span, length, rise and the grid number are the same. The grid shape of single diagonal rod type and herringbone type is right triangle. The grid shape of fundamental type of three-way grid is isosceles triangle, and the extended type of three-way grid is equilateral triangle. Under the same model parameters, compared to the total steel consumption of different grid shape after shape optimization, the total steel consumption of equilateral triangular grid (extended type of three-way grid) is always minimal.

6. Conclusions

In the present study, as for five extended cylindrical reticulated shells, an efficient parametric modeling method and a shape optimization method are proposed and compiled in APDL and FORTRAN language. The maximum displacement and stress of cylindrical reticulated shells are analyzed. Shape optimization is carried out based on the objective function of minimizing total steel consumption and the restriction condition of strength, stiffness, slenderness ratio, stability. The variations of total steel consumption along with the span and rise-span ratio are analysed with contrast to the results of shape optimization. The optimal combination of main design parameters for five extended cylindrical reticulated shells is investigated. The total steel consumption affected by distribution rule of diagonal rods is discussed. The results show that:

- Parametric modeling method is simple, efficient and practical, which can provide the possibility for quickly generating different types of cylindrical reticulated shells.
- The maximum displacement and stress of five extended cylindrical reticulated shells are smaller than their fundamental types. Wherein the extended type 1 of single diagonal rod, extended type 2 of herringbone and extended type of three-way grid have better mechanical properties.
- Under the certain span, the total steel consumption of the eight cylindrical reticulated shells firstly decreases and then increases with rise-span ratio increases. And the total steel consumption is the least when the rise-span ratio is 1/6.
- The total steel consumption of extended type of three-way grid - cylindrical reticulated shell is the minimum after shape optimization, followed by extended type 2 of herringbone - cylindrical reticulated shell, and the extended type 1 of herringbone - cylindrical reticulated shell is the maximum.
- From the viewpoint of internal force analysis and shape optimization, the extended type of three-way grid - cylindrical reticulated shell is preferable. Thus, this structure type should be the first choice in in practical cylindrical reticulated shell projects.
- As for the grid of equilateral triangle, its length of all rod elements is equal. The distribution of rod elements is homogeneous, and mechanical properties of the material can be fully utilized and played. Therefore, in practical engineering, the grid shape of reticulated shells should be designed to equilateral triangle as much as possible because of its reasonable stress and the lowest total steel consumption.

Acknowledgments

We would like to acknowledge the financial support from the National Basic Research Program of China (973 Program, No.: 2013CB036000), the National Natural Science Foundation of China (Grant No. 51479106, 51509147), the promotive research fund for excellent young and middle-aged scientists of Shandong Province (Grant No. 2014GN028) and the China Postdoctoral Science Foundation (Grant No. 2014M551908).

References

Allaire, G., Jouve, F. and Toader, A.M. (2002), "A level-set method for shape optimization", *Comptes Rendus Mathematique*, **334**(12), 1125-1130.

Chen, L.Z. (1989), *The Optimal Method of Discrete Variable-Principle and Application*, China Machine Press, Beijing, China.

Chen, Z.H. and Liu, H.B. (2009), *APDL Parametric Calculation and Analysis*, China Water Power Press, Beijing, China.

Deng, H. and Dong, S.L. (1999), "Shape optimization of spatial reticulated shell structures", *J. Zhejiang Univ.* (*Engineering Science*), **33**(4), 371-375.

Dietl, J.M. and Garcia, E. (2010), "Beam shape optimization for power harvesting", *J. Intell. Mater. Syst. Struct.*

Dong, S.L. and Yao, J. (2003), "Future and prospects of reticulated shells", *Spatial structure*, **9**(1), 31-34.

Durgun, İ., and Yildiz, A.R. (2012), "Structural design optimization of vehicle components using cuckoo search algorithm", *Mater. Test.*, **54**(3), 185-188.

Emmanuel, N.P., Padmanaban, K.P. and Vasudevan, D. (2014), "Buckling optimization of laminated composite plate with elliptical cutout using ANN and GA", *Struct. Eng. Mech., Int. J.*, **52**(4), 815-827.

Eschenauer, H.A., Kobelev, V.V. and Schumacher, A. (1994), "Bubble method for topology and shape optimization of structures", *Struct. Optimiz.*, **8**(1), 42-51.

Gong, S.G. and Xie, G.L. (2010), *ANSYS Parametric Programming and Command Manual*, China Machine Press, Beijing, China.

He, Y.J., Qi, D.L. and Dong, S.L. (2001), "Application of chaos optimization algorithm in the optimization of double-layer cylindrical latticed shell", *J. China Coal Soc.*, **26** (6), 663-666.

He, Y.J., Qi, D.L. and Dong, S.L. (2002), "Application of genetic algorithm in the optimization of double-layer cylindrical latticed shell", *J. China Coal Soc.*, **26**(6), 663-666.

Jenkins, W.M. (1991b), "Towards structural optimization via the genetic algorithm", *Comput. Struct.*, **40**(5), 1321-1327.

Jenkins, W.M. (1991a), "Structural optimization with the genetic algorithm", *The Struct. Eng.*, **69**(24), 418-422.

Jenkins, W.M. (1997), "On the application of natural algorithms to structural design optimization", *Eng. Struct.*, **19**(4), 302-308.

JGJ7 (2010), *Technology Procedures of Space Grid Structures*, China Building Industry Press, Beijing, China.

Kaveh, A. and Ahmadi, B. (2014), "Sizing, geometry and topology optimization of trusses using force method and supervised charged system search", *Struct. Eng. Mech., Int. J.*, **50**(3), 365-382.

Kaveh, A. and Khayatazad, M. (2013), "Ray optimization for size and shape optimization of truss structures", *Comput. Struct.*, **117**, 82-94.

Kaveh, A. and Zolghadr, A. (2014), "A new PSRO algorithm for frequency constraint truss shape and size optimization", *Struct. Eng. Mech., Int. J.*, **52**(3), 445-468.

Levy, R., Hanaor, A. and Rizzuto, N. (1994), "Experimental investigation of prestressing in double-layer grids", *Int. J. Space Struct.*, **9**(1), 21-26.

Lu, X.Y., Zhao, X.W. and Huang, L.L. (2012), "Shape optimizing design of kiewiti spherical reticulated shell", *Adv. Mater. Res.*, **424**, 324-329.

Lu, X.Y., Zhao, X.W. and Chen, S.Y. (2013), *The Optimization of Reticulated Shell Structures Based On Discrete Variables*, Building Industry Press, Beijing, China.

Luo, Z., Zhang, N., Gao, W. and Ma, H. (2012), "Structural shape and topology optimization using a meshless Galerkin level set method", *Int. J. Numer. Method. Eng.*, **90**(3), 369-389.

Oudjene, M., Ben-Ayed, L., Delameziere, A. and Batoz, J.L. (2009), "Shape optimization of clinching tools using the response surface methodology with Moving Least-Square approximation", *J. Mater. Process. Technol.*, **209**(1), 289-296.

Qian, X. (2010), "Full analytical sensitivities in NURBS based isogeometric shape optimization", *Comput. Method. Appl. Mech. Eng.*, **199**(29), 2059-2071.

Rajan, S.D. (1995), "Sizing, shape, and topology design optimization of trusses using genetic algorithm", *J. Struct. Eng.*, **121**(10), 1480-1487.

Rahami, H., Kaveh, A. and Gholipour, Y. (2008), "Sizing, geometry and topology optimization of trusses via force method and genetic algorithm", *Eng. Struct.*, **30**(9), 2360-2369.

Saka, M.P. (1991), "Optimum design of steel frames with stability constraints", *Comput. Struct.*, **41**(6), 1365-1377.

Saka, M.P. and Kameshki, E.S. (1998), "Optimum design of nonlinear elastic framed domes", *Adv. Eng. Software*, **29**(7), 519-528.

Schulz, V.H. (2014), "A riemannian view on shape optimization", *Found. Comput. Math.*, **14**(3), 483-501.

Shang, X.J. and Qiu, F. (2005), *ANSYS Structural Finite Element Senior Analysis Method and Sample Applications* ,China Water Power Press, Beijing, China.

Shen, Z.Y. and Chen, Y.J. (1996), *Grid and Lattice Shell*, Tongji University Press, Shanghai, China.

Sun, H.C., Chai, S. and Wang, Y.F. (2002), *Structural Optimization with Discrete Variables*, Dalian University of Technology Press, Dalian, China.

Suzuki, K. and Kikuchi, N. (1991), "A homogenization method for shape and topology optimization", *Comput. Appl. Mech. Eng.*, **93**(3), 291-318.

Thrall, A.P., Zhu, M., Guest, J.K., Paya-Zaforteza, I. and Adriaenssens, S. (2014), "Structural optimization of deploying structures composed of linkages", *J. Comput. Civil Eng.*, **28**(3), 04014010.

Vyzantiadou, M.A., Avdelas, A.V. and Zafiropoulos, S. (2007), "The application of fractal geometry to the design of grid or reticulated shell structures", *Comput.-Aid. Des.*, **39**(1), 51-59.

Wang, Z.D. (2012), "Shape optimization of five developed cylinder latticed shell", Mater Dissertation; Shandong Jianzhu University, Jinan, China.

Wang, C.W. and Tang, G. (2006), "Sectional optimum design of single-layer lattice shells considering structural stability", *Spatial Structure*, **12**(3), 31-34.

Wu, W., Petrini, L., Gastaldi, D., Villa, T., Vedani, M., Lesma, E. and Migliavacca, F. (2010), "Finite element shape optimization for biodegradable magnesium alloy stents", *Annals Bomed. Eng.*, **38**(9), 2829-2840.

Wu, J., Lu, X.Y., Li, S.C., Xu, Z.H., Li, L.P., Zhang, D.L. and Xue, Y.G. (2015a), "Parametric modeling and shape optimization of four typical Schwedler spherical reticulated shells", *Struct. Eng. Mech.*, *Int. J.*, **56**(5), 813-833.

Wu, J., Lu, X.Y., Li, S.C., Zhang, D.L., Xu, Z.H., Li, L.P. and Xue, Y.G. (2015b), "Shape optimization for partial double-layer spherical reticulated shells of pyramidal system", *Struct. Eng. Mech.*, *Int. J.*, **55**(3), 555-581.

Xia, Q., Shi, T., Liu, S. and Wang, M.Y. (2012), "A level set solution to the stress-based structural shape and topology optimization", *Comput. Struct.*, **90**, 55-64.

Xu, J., Yang, S.S. and Diao, Y.S. (2006), "Optimized design of single - layer reticulated shell", *Spatial Structure*, **12**(3), 35-37.

Yas, M.H., Shakeri, M. and Ghasemi-Gol, M. (2007), "Two-objective stacking sequence optimization of a cylindrical shell using genetic algorithm", *Scientia Iranica*, **14**(5), 499-506.

Yildiz, A.R. (2013), "Comparison of evolutionary-based optimization algorithms for structural design

optimization", *Eng*. *Appl*. *Artif*. *Intell*., **26**(1), 327-333.
Zhang, N.W. and Dong, S.L. (2003), "Optimum design of single-layer lattice shells considering the effect of geometrical nonlinearity", *Spatial Structure*, **9**(1), 31-34.
Zhang, B.H. and Hou, C. (1998), *Optimization Design of Civil Structures*, Tongji University Press, Shanghai, China.

CC