Steel and Composite Structures, *Vol. 20, No. 4 (2016) 801-812* DOI: http://dx.doi.org/10.12989/scs.2016.20.4.801

The relationship between time-varying eccentricity of load with the corner lateral displacement response of steel structure during an earthquake

Kambiz Takin^{1a}, Behrokh H. Hashemi^{*2} and Masoud Nekooei^{1b}

¹ Department of Civil Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran ² Structural Engineering Research Center (SERC), International Institute of Earthquake Engineering & Seismology (IIEES), Tehran, Iran

(Received February 25, 2015, Revised December 12, 2015, Accepted December 10, 2015)

Abstract. In an actual design, none of the structures with shear behaviors will be designed for torsional moments. Any failure or damages to roofs, infills, shear walls, and braces caused by an earthquake, will inevitably result in relocation of center of mass and rigidity of the structure. With these changes, the dynamic characteristics of structure could be changed during an earthquake at any moment. The main objective of this paper is to obtain the relationship between time-varying eccentricity of load and corner lateral displacement. In this study, various methods have been used to determine the structural response for time-varying lateral corner displacement. As will be seen below, some of the structural calculation methods result in a significant deviation from the actual results, although these methods include the interaction effects of modes. Controlling the lateral displacement of structure can be performed in different ways such as, passive dampers, friction dampers, semi-active systems including the MR damper and active Systems. Selecting and locating these control systems is very important to bring the maximum safety with minimum cost into the structure. According to this study will be show the relation between the corner lateral displacements of structure and time-varying eccentricity by different kind of methods during an earthquake. This study will show that the response of the structure at the corners due to an earthquake can be very destructive and because of changing the eccentricity of load, calculating the maximum possible response of system can be carried out by this method. Finally, some kind of systems must be used for controlling these displacements. The results shows that, the CQC, DSC and exact methods is comply each other but the results of Vanmark method is not comfortable for these kind of buildings.

Keywords: torsional modes; shear modes; lateral displacement; eccentricity; earthquake; ABS, SRSS, CQC; vanmark; humar; Gupta and exact methods

1. Introduction

It is possible to assume the torsional response of structures due to earthquakes at each point of the structure's foundation is excited simultaneously. Therefore, if the center of mass and rigidity of the floors being along the same axis, the lateral forces of an earthquake will be translational

Copyright © 2016 Techno-Press, Ltd.

http://www.techno-press.org/?journal=scs&subpage=6

^{*}Corresponding author, Ph.D., Member of IEEA, E-mail: behrokh h h@yahoo.com

^a Ph.D. Candidate, E-mail: omran@engineer.com

^b Ph.D., E-mail: msnekooei@gmail.com

components. Thus, if the distance between the centers of mass and rigidity are considerable, the torsional moment in structure could be huge. (Kuo 1974). The torsional response may cause to increase the lateral displacement of structure at the corners of building and suffer severe damages to these members. The torsional response of the structure at the corners is much bigger than predicted. For calculate the amplification which produced by seismic excitation and accidental torsion in the elastic mode, the static eccentricities of loads are defined by codes with simple equations (De la Llera and Chopra 1994). Anastassiadis et al. (1998), perform a comprehensive study of the static eccentricities that are presented a set of equations for a one-storey building. A procedure to obtain the static torsional eccentricities for asymmetrical multi-storey buildings is presented by Tso and Moghadam (2000). They have used a method for determine the torsional radius for multi-storey buildings. Therefore, the inelastic torsional response is predictable, because the location of the center of rigidity varies at nonlinear static analysis. The presences of two seismic loads or the eccentricity in two orthogonal directions have importance in inelastic range (Fajfar et al. 2005). Therefore, calculate the eccentricities by static analysis, can be used in the elastic range. The static eccentricity is changed in each step of the nonlinear analysis. The seismic response of structures subjected to an earthquake may be significantly modified due to the torsional effects. Therefore, the floors of the building can rotate around a vertical axis. At First, the torsion occurs due to lack of symmetry in building due to a non-uniform stiffness or mass distribution in plan. This lack of symmetry may be accidental and unpredicted. The second is movement of the foundation due to an earthquake. Therefore, the torsional modes may also occur in symmetric structures. The buildings with different shaped plans which built as units; huge forces may occurs at the intersection of the arms at vibrational components and torsional mode (Tande and Patil 2013). The steel bracing is one of the efficient system which can be used to strengthen buildings. The Steel bracing reduce shear and torsional deformation of structure and the storey drifts (Kevadkar and Kodag 2013). Direct displacement design is a procedure, which allows one to distribute the earthquake forces to the levels of a multi-storey building to confine the interstorey drift (Bahmani et al. 2014).

Destruction of buildings during an earthquake captured the structural and earthquake engineers' attention for decades. Torsional moment in structures is one of the main unpredictable damages to buildings. There are two main factors, which causes to arise torsion moments in buildings. The first One is mismatching the center of mass and rigidity of each floor of the building and another is torsional moments generated by relocation of the center of mass and rigidity of floors due to local damages. Therefore, to understand the actual behavior of the structure during an earthquake, it's necessary to study and consider the torsional effects. The torsional phenomenon can be occurred in structures with shear behavior due to an earthquake or movement of heavy machineries in the building. The distance between the center of mass and rigidity of each floor may be varied during an earthquake, which leads to change shear and torsional modes of the structure. Torsional modes of the structure can be highly unpredictable and destructive and it must suitably consider in important structures, such as power plants, defensive structures, critical facilities, nuclear power plants, etc. Damage to structures, such as nuclear power plants, may lead to a humanitarian catastrophe. In addition, it imposes heavy cost and adverse effects for long years. Also, the design of structures against torsion that can be changed at any moment during an earthquake by using handheld computation and routine software is a very time consuming work and sometimes impossible. The aim of this study is reach to a solution for creating high reliability to resolve the concerns caused by torsion in the structure.

2. The mass and stiffness matrices

2.1 Reviewing the mass and stiffness matrices of a torsion-shear single-storey building by taking the degrees of freedom at the center of mass

The mass and stiffness matrices can be calculated based on the center of mass and stiffness. The easiest way is to consider the center of mass, therefore, the mass matrix remains diagonal and the stiffness matrix determines based on the distance between the center of mass and rigidity. A shear frame with a rigid roof has been considered, as shown in Fig. 1, and the center of mass and center of stiffness were identified as C.M. and C.R. The transitional and torsional degrees of freedom considered at the center of mass.

By referring to the definition of the mass and stiffness matrices, it is easy to define the elements of mass and stiffness matrices. By doing so, these matrices eventually can be obtained, as shown in Eqs. (1)-(2) (Hosseini and Legzian 2011).

$$[K] = \begin{bmatrix} K_{x} & 0 & -K_{x}e_{y} \\ 0 & K_{y} & K_{y}e_{x} \\ -K_{x}e_{y} & K_{y}e_{x} & K_{\theta} + K_{x}e_{y}^{2} + K_{y}e_{x}^{2} \end{bmatrix}$$
(1)

$$[M] = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_\theta \end{bmatrix}$$
(2)



Fig. 1 The center of mass and rigidity in one storey building



Fig. 2 The center of mass and rigidity in two floors building

2.2 Reviewing the mass and stiffness matrices of a torsion-shear two-floors building by taking the degrees of freedom at the center of mass

A shear frame with rigid roof is assumed, as shown in Fig. 2, and the center of mass and stiffness were identified as C.M. and C.R. The transitional and torsional degrees of freedom are considered at the center of mass.

The structural stiffness matrix elements are formed as Eq. (3).

$$[K] = \begin{bmatrix} K_{x1} + K_{x2} & 0 & -K_{x1}e_{y1} - K_{x2}e_{y12} & -K_{x2} & 0 & K_{x2}e_{y2} \\ 0 & K_{y1} + K_{y2} & K_{y1}e_{x1} + K_{y2}e_{12} & 0 & -K_{y2} & -K_{y2}e_{x2} \\ -K_{x1}e_{y1} & K_{y1}e_{x1} & +K_{y1}e_{y1}^{2} + K_{\theta22} & K_{x2}e_{y12} & -K_{y2}e_{x12} + K_{y2}e_{x2}^{2} + K_{x2}e_{y2}\Delta y \\ -K_{x2}e_{y12} & +K_{y2}e_{12} & +K_{y2}e_{x12}^{2} & K_{x2} & 0 & -K_{x2}e_{y2} \\ -K_{x2} & 0 & K_{x2}e_{y12} & K_{x2} & 0 & -K_{x2}e_{y2} \\ 0 & -K_{y2} & -K_{y2}e_{x12} & 0 & K_{y2} & K_{y2}e_{x2} \\ 0 & -K_{y2} & -K_{y2}e_{x12} & 0 & K_{y2} & K_{y2}e_{x2} \\ -K_{x2}e_{y2} & -K_{y2}e_{x2} & +K_{y2}e_{y2}^{2} + K_{x2}e_{y2}^{2} \\ -K_{x2}e_{y2} & -K_{y2}e_{x2} & +K_{y2}e_{x2}^{2} + K_{x2}e_{y2}Ay & -K_{x2}e_{y2} & K_{y2}e_{x2} \\ -K_{y2}e_{x2}Ax) \end{bmatrix}$$
(3)

Where, Δx and Δy are the displacement between two masses along x and y axis, e_{x1} and e_{x2} are the distance between the center of mass and rigidity in first and second floor along x axis, e_{y1} and e_{y2} are the distance between the center of mass and rigidity in first and second floor along y axis and e_{x12} and e_{y12} are the distance between the center of mass and rigidity of first and second floor along x and y axis. K_{x1} and K_{x2} are the transitional stiffness of first and second floor along x axis, K_{y1} and K_{y2} are the transitional stiffness of first and second floor along y axis, K_{y1} and K_{y2} are the transitional stiffness of first floor, $K_{\theta11}^*$ is the rotational stiffness of first floor around the center of rigidity of first floor, $K_{\theta11}^*$ is the rotational stiffness of first floor about the center of rigidity of second floor, $K_{\theta11}^*$ is the rotational stiffness of first floor about the center of mass of first floor, $K_{\theta22}^*$ is the rotational stiffness of second floor around the center of mass of second floor, $K_{\theta12}^*$ is the rotational stiffness of first floor around the center of mass of second floor, $K_{\theta21}^*$ is the rotational stiffness of first floor around the center of first floor. The mass matrix obtains from Eq. (4).

$$[M] = \begin{bmatrix} m_{x1} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{y1} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{\theta1} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x2} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{y2} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{\theta2} \end{bmatrix}$$
(4)

Where, m_{x1} and m_{x2} are the transitional mass in first and second floor along x axis, m_{y1} and m_{y2} are the transitional mass in first and second storey along y axis, $m_{\theta1}$ and $m_{\theta2}$ are the rotational mass in first and second floor (Hosseini and Legzian 2011). This approach can be used for n-floors buildings.

3. Different kinds of method for obtain the torsional response of structure

The Vanmark provided Eq. (5) to calculate the correlation coefficient between modes. In this equation, the modified damping is as following, $\overline{\zeta}_j = \zeta_j (1 - e^{-2\zeta_j \omega_j s})^{-1}$. While using this method in order to calculate the structural responses, some parts of the diagram have to be modified. In the lower eccentricities, the responses are reasonable, but in the higher values, the responses are not as expected to be. According to these results and the results that obtain in the following calculation, we must avoid from this method in high eccentricity values.

$$\rho_{jk} = \left[\frac{8r\bar{\zeta}_{j}(\bar{\zeta}_{j}+\bar{\zeta}_{k})[(1-r^{2})^{2}-4r(\bar{\zeta}_{j}-\bar{\zeta}_{k}r)(\bar{\zeta}_{k}-\bar{\zeta}_{j}r)]}{8r^{2}[(\bar{\zeta}_{j}^{2}+\bar{\zeta}_{k}^{2})(1-r^{2})^{2}-2(\bar{\zeta}_{k}^{2}-\bar{\zeta}_{j}^{2}r^{2})(\bar{\zeta}_{j}^{2}-\bar{\zeta}_{j}^{2}r^{2})]+(1-r^{2})^{4}}\right]^{-1}$$
(5)

Humar in 1984 used the Rosenblueth's equation with assumption $s = \infty$ (white noise) and $\zeta = \zeta_j$ = ζ_k , the correlation coefficient is calculated according to Eq. (6), which use in the New Zealand code. When the frequencies are very close to each other, the correlation coefficient in this method is bigger than CQC method, but by separating the frequencies from each other (correlation between modes reduced), its value decreases. Also, when the damping of the structural modes much vary together, the results of correlation coefficient has a higher error rate and amount, rather than calculated. It was more sensitive to high-frequency modes and more errors associated with them.

$$\rho_{jk} = 1 + \left[\frac{\sqrt{1-\zeta^2}}{\zeta} \left(\frac{\omega_j - \omega_k}{\omega_j + \omega_k}\right)\right]^2 \tag{6}$$

Gupta in 1990 offered the improved Rosenblueth's equation by using the results of various systems. Eq. (7), which is related to Gupta, where $C_{jk} = (0.16 - 0.5\zeta)(1.4 - |\omega_j^2 - \omega_k^2|) \ge 0.0$ and ζ is the mean of damping ratio.

$$\rho_{jk} = \frac{2\sqrt{\zeta_j \zeta_k}}{\zeta_j + \zeta_k} \left\{ 1 + \left(\frac{\sqrt{1 - \zeta^2} (\omega_j - \omega_k)}{\zeta_j \omega_j + \zeta_k \omega_k + C_{jk}} \right)^2 \right\}$$
(7)

One of the calculation methods in finding the exact solutions for structures with torsional behavior is mode-displacement method. To obtain the structural response in x, y and z directions with this method, the Eqs. (8)-(16) can be used. In these equations, S_{dxi} , S_{dyi} , S_{dzi} , S_{vxi} , S_{vyi} and S_{vzi} are the displacement, velocity and acceleration, $r = \omega_i / \omega_j$ and The unknown values of A_{ij} , B_{ij} , C_{ij} and D_{ij} are to calculate the interaction between two modes obtained from Eqs. (11)-(16).

$$R_{Dx}^{2} = \sum_{i=1}^{N} R_{i}^{2} + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \gamma_{i} \gamma_{j} \phi_{i} \phi_{j} \left(A_{ij} S_{dxi}^{2} + B_{ij} S_{vxi}^{2} + C_{ij} S_{dxj}^{2} + D_{ij} S_{vxj}^{2} \right)$$
(8)

$$R_{Dy}^{2} = \sum_{i=1}^{N} R_{i}^{2} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \gamma_{j} \gamma_{j} \phi_{i} \phi_{j} \left(A_{ij} S_{dyi}^{2} + B_{ij} S_{vyi}^{2} + C_{ij} S_{dyj}^{2} + D_{ij} S_{vyj}^{2} \right)$$
(9)

$$R_{Dz}^{2} = \sum_{i=1}^{N} R_{i}^{2} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \gamma_{j} \gamma_{j} \phi_{i} \phi_{j} \left(A_{ij} S_{dzi}^{2} + B_{ij} S_{vzi}^{2} + C_{ij} S_{dzj}^{2} + D_{ij} S_{vzj}^{2} \right)$$
(10)

$$u = -2(1 - 2\xi_j^2) \tag{11}$$

$$s = -2r^2(1 - 2\xi_j^2) \tag{12}$$

$$t = r^4 \tag{13}$$

$$\begin{bmatrix} 1 & u - s & 1 \\ u & 1 - t & s \\ 1 & 0 & t \end{bmatrix} \begin{bmatrix} A_{ij} \\ \overline{B}_{ij} \\ C_{ij} \end{bmatrix} = \begin{bmatrix} 1 \\ -(1 + r^2 - 4\xi_i \xi_j r) \\ r^2 \end{bmatrix}$$
(14)

$$B_{ij} = \overline{B}_{ij} / \omega_j^2 \tag{15}$$

$$D_{ij} = -B_{ij} \tag{16}$$

806

4. Numerical example

The dimension of considered a one story building is 40 by 40 meters and Coalinga earthquake record selected for this study. The transitional stiffness along x and y axis is 2950 t/m and rotational stiffness is 98300 t.m. The total mass of structure is 610 t and rotational mass is 18300 t.m². The mass and stiffness matrices by assuming centrality of the center of mass for the calculations are according to Eqs. (17)-(18).

$$[K] = \begin{bmatrix} K_x & 0 & -K_x e_y \\ 0 & K_y & -K_y e_x \\ -K_x e_y & -K_y e_x & K_\theta + K_x e_y^2 + K_y e_x^2 \end{bmatrix}$$

$$\Rightarrow [K] = \begin{bmatrix} 2950 & 0 & -2950e_y \\ 0 & 2950 & -2950e_x \\ -2950e_y & -2950e_x & 98300 + 2950e_y^2 + 2950e_x^2 \end{bmatrix}$$

$$[M] = \begin{bmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & J \end{bmatrix} = \begin{bmatrix} 610 & 0 & 0 \\ 0 & 610 & 0 \\ 0 & 0 & 18300 \end{bmatrix}$$
(17)
(17)

The approximate model of structure is formed by putting external braces at the corners, which can equipped by MR dampers or the other active or semi-active damping equipments, which has ability to control the lateral displacements caused by torsional modes. Fig. 3 shows an overview of structure with external bracing equipped by active of semi-active damping system.



Fig. 3 An overview of structure with external bracing equipped by active of semi-active damping system

5. The results

Fig. 4 shows the lateral displacement diagrams for different modes of structure for time-varying eccentricity. In the first mode, it initially shows an increment with increasing the eccentricity. Then, it decreases and increases again. In the second mode, it shows a decrease, and then, it increases. In the third mode, it shows a decreasing trend. According to this diagram, the results that obtained from the CQC, DSC and Humar methods show good agreement with the exact method so for buildings with torsional behavior, and these methods can be inferred that an acceptable level of accuracy in calculation of the structures with torsional behavior, despite showing some errors. The Gupta and Vanmark methods do not seem to be reliable methods in calculation of the structures with torsional behavior due to the sudden variations in the result and failure to comply with the exact methods. By using this method, some of the resulting values are much larger than the actual values. For example, in the first mode at eccentricities between 2.9 to 3.3 shows an instantaneous rise. This scenario occurs in the second mode too. The displacement diagram shows a sudden increment in an approximate eccentricity of 2.45, then decreases. The Gupta's results indicate that the displacement increases in the first mode and then experience a sharp decrease and then take incremental process. In second mode the displacement decreases, then increase, decrease and then increase again.

The analogy of the results in different methods has shown in the Figs. 4-6. With comparing all these results, it can be inferred that with increasing the eccentricity in the first mode, the results in all the methods show increasing with an initial decreasing. In ABS and SRSS methods, which the correlation between modes are not considered, show greater responses than the other methods. The results obtained from DSC, CQC, and Humar has shown a good level of accuracy in comparing with the exact method and can be utilized in the structures with torsional behaviors, despite showing some parts. However, the Gupta and Vanmark methods sound like they are not applicable



Fig. 4 The lateral displacement relative to structural eccentricity for all methods in the first mode



Fig. 5 The lateral displacement relative to structural eccentricity for all methods in the second mode



Fig. 6 The lateral displacement relative to structural eccentricity for all methods in the third mode

methods in calculation of the structures with torsional behaviors, because sudden variations have been shown in their results along with failure to comply with the exact solutions, but at lower values of eccentricities can be used. The displacement rate in the structure shows an increment in the first and the second modes, but in the third mode initially has increased then decreased. In the first and the second modes, the least values have shown in Gupta's method, but the greatest values have shown in Vanmark's method. In the first and the second modes, the values obtained from CQC, DSC, and Humar methods are higher than values in the exact method. In the third mode, the Vanmark's method initially shows the lowest values, then, increases with a large slope and gains the highest values comparing to the other methods. In the third mode, the values obtained by Humar and Gupta methods are close and the values obtained from CQC and DSC methods are almost the same. The lowest displacement rates in the third mode are shown in DSC and CQC methods. Regardless of the Vanmark's method, the highest values show up in Humar and Gupta methods. The exact method has the same rate and the exact values are somewhere between these two methods. In fact, this method shows sudden changes in the transitional modes. In the third mode, the Vanmark's method shows a different behavior in comparing with the other methods and it is not a reliable method in calculation of structures with torsional behavior whatsoever.

The curves in the Figs. 7-8 show the lateral displacement of the corners for different methods. The values obtained by the above methods, in calculation of structural responses, are in the same range, but the values obtained from the exact method show the lower values. With increasing the value of eccentricity in the closer corners to this, the lateral displacements of these corners increase. In the far corner, the displacement initially decreases, then increases, and eventually, slightly decreases. The explanation for the reason is that by increasing the eccentricity, the amount of torsional modes increase. These torsional modes are in rotation, which these rotational conditions cause to increase the lateral displacements in the near corners. These rotational conditions due to their reverse movement against the transitional displacement cause to see decreasing in the far corners. As can be seen in, the curves that related to Humar and Vanmark methods show the different trend and in Vanmark's method sudden increases in some areas are obvious. The lateral displacement values in the corners obtained from CQC and DSC methods are relatively very close to each other, or even in some points they are the same. The values obtained



Fig. 7 The curves of lateral displacement in corners which near to eccentricity



Fig. 8 The curves of lateral displacement in corners which far from eccentricity

from the exact method comparatively have shown higher values than the other methods, but the trend remains the same.

6. Conclusions

Torsion phenomenon more likely to happen during an earthquake in the structures with shear behavior and the torsional effects are not considered in the calculation of such structures. In order to find more accurate solutions for these torsional effects, it is required to use other methods such as, mode-acceleration, or mode-displacement methods, which are not employed in currently used commercial software. As calculations showed, the lateral displacement of the corners in torsional mode was significant and could threaten the health of structures during an earthquake. With development of the lateral displacement occurred in the structure, the generated plastic joints lead the structure towards failure. Therefore, it has to be considered in critical structures such as, power plants, governmental structures, and nuclear power plants, to prevent any colossal damages due to these torsional modes. Several approaches can be used in this field to be able to properly control the lateral displacement by using an active, semi-active, or inactive system when destructive torsional modes occur in the structure. As the results showed, the lateral displacements due to torsion at the corners of the structure can be several times bigger than calculated values in shear behavior. The results showed that the structural responses, obtained from some of the common methods, are not reasonable, although the correlation between vibrational modes is considered. Thus, using the methods like Vanmark, does not recommend for these types of structures. The results obtained by COC and DSC methods showed that not only their results are very close to each other, but also are valid for torsional structures. However, the responses that obtained from these methods are slightly less than the exact method. Therefore, a correction factor has to be

considered in these methods in order to obtain an accurate solution for the lateral displacement due to torsional modes.

References

- Anastassiadis, K., Athanatopoulos, A. and Makarios, T. (1998), "Equivalent static eccentricities in the simplified methods of seismic analysis of buildings", Earthq. Spectra, 14(1), 1-34.
- Bahmani, P., van de Lindt, J.W. and Dao, T.N. (2014), "Displacement-based design of buildings with torsion: Theory and verification", J. Struct. Eng. (ASCE), 140(6), 04014020.
- De la Llera, J.C. and Chopra, A.K. (1994), "Using accidental eccentricity in code-specified static and dynamic analysis of buildings", *Earthq. Eng. Struct. Dyn.*, **23**(7), 947-967. Dempsey, K.M. and Irvine, H.M. (1979), "Envelopes of maximum seismic response for a partially
- symmetric single storey building model", Earthq. Eng. Struct. Dyn., 7(2), 161-180.
- Fajfar, P., Marusic, D. and Perus, I. (2005), "Torsional effects in the pushover-based seismic analysis of buildings", J. Earthq. Eng., 9(6), 831-854.
- Hosseini, M. and Legzian, Gh. (2011), "Derivation of stiffness and mass matrices of torsional buildings in fully irregular state", 8(3), 125-136.
- Kevadkar, M.D. and Kodag, P.B. (2013), "Lateral load analysis of R.C.C. building", Int. J. Modern Eng Res. (IJMER), 3(3), 1428-1434.
- Kuo, P.T. (1974), "Torsional effects in structures subjected to dynamic excitations of the ground", Ph.D. Thesis; Rice University, Houston, TX, USA.
- Tande, S.M. and Patil, S.J. (2013), "Seismic response of asymmetric buildings", Int. J. Latest Trend. Eng. Technol. (IJLTET), 2(4), 365-369.
- Tso, W.K. and Moghadam, A.S. (1998), "Application of Eurocode 8 torsional provisions to multi-storey buildings", Proceedings of the 11th European Conference on Earthquake Engineering, Paris, France, September.