Steel and Composite Structures, Vol. 2, No. 3 (2002) 209-222
DOI: http://dx.doi.org/10.12989/scs.2002.2.3.209

Theoretical and experimental studies of unbraced tubular trusses allowing for torsional stiffness

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(Received February 14, 2001, Accepted January 30, 2002)

Abstracts. This paper describes the buckling phenomenon of a tubular truss with unsupported length through a full-scale test and presents a practical computational method for the design of the trusses allowing for the contribution of torsional stiffness against buckling, of which the effect has never been considered previously by others. The current practice for the design of a planar truss has largely been based on the linear elastic approach which cannot allow for the contribution of torsional stiffness and tension members in a structural system against buckling. The over-simplified analytical technique is unable to provide a realistic and an economical design to a structure. In this paper the stability theory is applied to the second-order analysis and design of the structural form, with detailed allowance for the instability and second-order effects in compliance with design code requirements. Finally, the paper demonstrates the application of the proposed method to the stability design of a commonly adopted truss system used in support of glass panels in which lateral bracing members are highly undesirable for economical and aesthetic reasons.

Keywords: buckling; tubular sections; torsional stiffness; advanced analysis; nonlinear integrated design and analysis.

1. Introduction

Trusses have been used extensively as load transfer systems for large span structures in facades, roofs and bridges. Apart from the two dimensional type of truss, large span space trusses have become more popular than pre-stressed or reinforced concrete structures as coliseum roofs, due to their high capacity to weight ratio. In reality, structural members of a welded truss are normally subjected to axial forces

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Fig. 1 Bow trusses in support of glazing

and moments. The information on the buckling strength of the structure is essential for production of an economical and safe design. Fig. 1 shows a common form of truss system with unsupported compressive chord during wind suction. Note that glass panels fixed to the truss by flexible sealant are not generally assumed as effective lateral restraining members in Hong Kong.

Current practice for strength checking uses the ultimate limit state design in conjunction with the first-plastic-hinge concept. In this philosophy, the design capacity of a structure is taken as the load at which the first plastic hinge is formed. Although methods for large deflection and inelastic analysis are available and developed (Liew, White and Chen 1992, Chan and Chui 1997 and Clarke 1994), many important advantages of using the second-order analysis such as contribution of tension members under high torsional moment against overall frame instability were not demonstrated. On the other hand, the design method used in practice to-date is based on linear analysis with individual member check carried out using formulae and charts in the design codes.

One major difficulty in using design codes and linear analysis is the problem of accurate assessment of the effective length of a structural member. BS5950 (1990) provides some guidelines for the estimation of effective length, but, like many other national codes such as the LFRD (1993), many important considerations are ignored. For instance, the effectiveness of restraint, the variation of member stiffness under axial force and the buckling effect of variable axial forces in a column are not considered. This ignorance is understandable since charts and design formulae can hardly cover all possible cases. In summary, the structural system is idealised as a group of isolated members which cannot reflect the actual behaviour of the frame and, consequently, large errors, either over- or under-design of the structures, may result.

For simple reference to a national code, the BS5950 (1990) is selected for comparison, but the concept is equally applicable to other national codes.

2. An illustrative experiment about torsional effect on buckling

Shown in Fig. 2 is the simply supported truss of 4.8m span and under an increasing downward point load at mid-span, making the top chord in compression and eventually buckle out of the plane of the truss. This simple truss carries a similar mechanics as a larger span truss in practice. In conventional design method, the effective length for this out-of-plane buckling is taken as the distance between the supports whilst the in-plane effective length is assumed as the distance between connections. The true design load will then be based on the smaller failure load calculated from the in-plane and out-of-plane buckling modes and, in the present study, on the more critical out-of-plane buckling mode.

When using the distance between supports as effective length, it is then taken as the span of the truss. From this effective length, the buckling capacity of the top chord is calculated and the applied load is determined as 7.8 kN. The tested load was obtained as 34 kN, which is about 4 times of the design load. This shows an uneconomical result from our conventional design method.

On the other hand, the proposed method based on the performance-based design concept of simulating the actual response of the truss under an increasing load calculates the design load as 32 kN which is equivalent to the conventional method of using half the span of the truss as the effective length. Whilst details of the theory are given in this paper with this test further elaborated in Example 1, it can be seen that the complete design process does not require the assumption of effective length. The checking of adequate strength of restraining members and variation of member stiffness under tension or compression loads are also included in the proposed method.

Fig. 2 Set-up of the 4.8 m steel truss under point load at mid-span of bottom chord

3. Comments on the methods of buckling analysis and design

Geometrical nonlinearity and member instability can be considered by the P-Δ and P-δ effects shown in Fig. 3. The P-Δ effect accounts for the gross change in structural geometry and it is included in BS5950 (1990) through the amplification factor in clause 5.6.3 for sway frame only or the limited frame method in Appendix E for both sway and no-sway frames. In computer analysis, this effect can be included by adding the displacements to nodal co-ordinates at every iteration for equilibrium. The P-δ effect is referred to as the additional moment of axial force and deflection along an element. Both the element stiffness and stress will be varied by this P-δ effect. The use of the buckling strength curve in design codes allows for the P-δ effect for some simplified and idealised conditions.

This elastic critical load factor (λ_{cr}) is frequently used by various codes as a parameter for design against instability. The eigenvalue method seeks for the condition of vanishing of the determinant of the tangent stiffness for determination of λ_{cr} as follows.

$$|K_L + \lambda_{cr} K_G| = 0 \tag{1}$$

in which K_L and K_G are respectively the linear and the geometric stiffness matrices, λ_{cr} is the load factor causing the determinant to vanish. Simplified methods for determination of λ_{cr} for regular frames are available, but computer analysis is needed for more general and irregular framework. The recent book by Task Committee of Effective Length of ASCE (1997) provides the most widely used linear and geometric stiffness matrices of a beam-column element.

The current checking against instability is mostly based on the eigenvalue type of bifurcation analysis like clause 5.6 for elastic and plastic design of rigid frames. The bifurcation or eigenvalue type of analysis is noted to be simple for program developers, but deficient in design of most practical skeletal structures. These deficiencies are highlighted as follows.

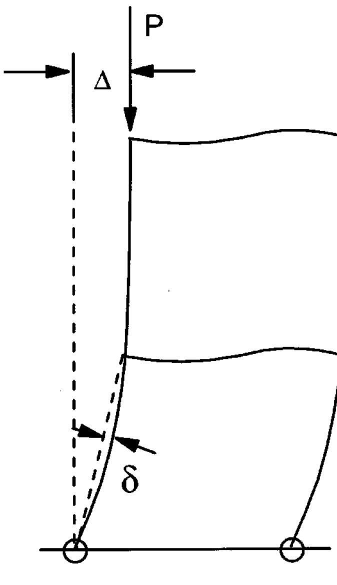

Fig. 3 The P-δ and P-Δ effects

1. The eigenvalue method is insensitive to member initial imperfection, which is unrealistic and not consistent with member design in codes which specify a mandatory value of initial imperfection of, for example, 0.001 of member length of hot-rolled tubular sections.

2. The result is an upper bound solution and the required factor of safety is unknown. Its application to special structures like snap-through buckling in domes and stability check of scaffolding requires additional consideration. The margin of over-estimation is problem-dependent and cannot be generalised. Merchant (1954) proposed the well-known interactive formula in Eq. (2) for computation of collapse load capacity, which does not consider initial imperfection and gives an empirical solution.

$$\frac{1}{\lambda_{ult}} = \frac{1}{\lambda_{cr}} + \frac{1}{\lambda_p} \tag{2}$$

in which $\lambda_{ult,}$ λ_{cr} and λ_p are the ultimate, elastic buckling and rigid plastic collapse load factors respectively.

3. Deflections cannot be computed in the method. A separated check for deflection is tedious and inconsistent.

As the bifurcation type of eigenvalue analysis does not update the element stiffness according to the deformed geometry, or mathematically, it ignores the large deflection matrix (see Equation 19.9, p.503, Zienkiewics 1997), it cannot handle any structure exhibiting large deflection behaviour leading to a considerable secondary moment in the analysis part of a design procedure.

In this paper, a performance-based second-order analysis and design procedure, which can be utilised with a typical second-order analysis computer program, with the developed software, is described. The method is compared against analytical result of columns with different boundary conditions and test results of the unbraced truss discussed previously described in this paper. The application of the method to the design of a 13.2 m span truss with supports at two ends is further used to illustrate the practicality of the method. The proposed technique allows for various complex and subjective checking required in a design code which includes assumption of effective length, snap-through instability, change of element stiffness due to axial force, required strength of restraining members etc. Most interestingly, the contribution and interactive behaviour of tension members and compressive members against system buckling through the torsional stiffness of the constitutive members is considered rigorously. This point appears to have been ignored in the design codes, but represents an important structural phenomenon for design of steel frames and trusses, as indicated in the example for testing of the 4.8 m span truss. This torsional stiffening effect in members leads to a substantial saving in design, which, to the authors best knowledge, is not covered in the design codes. Recognising this important structural behaviour, the engineer can design and scheme structures more efficiently and economically.

4. Interactive member behaviour in trusses

When loaded, a force distribution process is activated in a truss system. To predict the structural response accurately, it is necessary to establish the interactive behaviour of the structural system. For instability, the response becomes complex since the contributory member stiffness will be combined as a system and instability is normally in the form of a system instability since a member is seldom completely isolated.

A typical load versus stress curve is shown in Fig. 4. At the early loading stage where buckling is

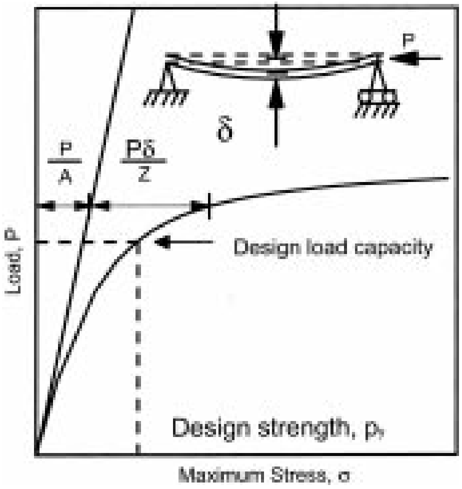

Fig. 4 The load vs. stress plot

remote, the stress is proportional to load. However, after a certain deflection, the P-δ effect becomes significant and the curve becomes non-linear so that the same load increment does not produce the same increase in stress when first loaded.

To allow for the effect of member instability and second-order stress, the permissible axial force in each structural member must be checked against axial force and deflection along member, generally termed as the P-δ effect. The well-known Perry Robertson formula is employed for determination of the design strength of an imperfect strut under the action of compression (see, for example, Appendix C, BS5950, 1990). The rationale behind this formula is the coincidence of design strength with the first yield load of the strut. The Perry's constant was further modified to allow for the effect of residual stress and to better fit experimental results.

In addition to the second-order moment described above, the gross change of geometry for the complete structure will generate an additional moment which is catered for in a design code like the BS5950 (1990) by a moment amplification formula for sway frames. Alternatively, this effect can be considered by using the modified effective length for sway and non-sway frames. These formulae are only applicable to simple structures with regular layout and therefore, for more complex structures, resort must be made from other methods.

In many occasions, the load factor causing instability in a truss needs to be calculated and the strength of individual members has to be checked as well. The assessment of the effective length for a member is essential in the design procedure. This important process, however, is normally carried out by manual judgement which is subjective and controversial, leading to argument between checking and the design engineers. Furthermore, checking against snap-through buckling of the whole structure may be complicated by the conventional method.

This paper uses the same strength criterion of a design code such as the BS5950 (1990) for the design of struts under compression. Therefore, the first yield load will be taken as the load capacity calculated from the Perry formula specified in BS5950 (1990). Consequently, when the effective length is obvious, such as the case of a simple column, the present method gives the same result as in the design code. However, for cases where the manual assessment of effective length is uncertain, the present method can still compute the design strength accurately by tracing the second-order non-linear equilibrium path which automatically allows for the effect of effective length. Using this concept, the important system buckling check in place of the member check is considered in the design process.

5. Finite element for beam-columns

An accurate element is essential in a second-order analysis since divergence or incorrect answer may be resulted when an inferior element is used. This section is devoted to the investigation of the performance of the most widely used cubic Hermite and the newly proposed pointwise equilibrium polynomial (PEP) element in a buckling analysis. The study here assists the practicing engineers in selection of a suitable member for his buckling check. The deficiency of the widely used cubic element has been reported by a numerical study by Liew *et al.* (1992), and will be confirmed analytically here.

In the finite element context using the most widely adopted cubic Hermite element, the geometric stiffness is used to allow for the effect of axial on the element stiffness (see, for examples, Appendix A in ASCE 1977). The corresponding rotation stiffness matrix for the simple column with both ends pinned is given by the following expression.

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} 4 + \dfrac{2PL^2}{15EI} & 2 - \dfrac{PL^2}{30EI} \\ 2 - \dfrac{PL^2}{30EI} & 4 + \dfrac{2PL^2}{15EI} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{3}$$

in which M_1 and M_2 are moments about the two ends, E is the Young's modulus of elasticity, I is the second moment of area, L is the member length, P is the axial force and θ_1 and θ_2 are the nodal rotations.

Similar to Eq. (1), the eigenvalue equation will then be obtained by setting the determinant of the above stiffness equation to zero as,

$$\begin{vmatrix} 4 + \dfrac{2PL^2}{15EI} & 2 - \dfrac{PL^2}{30EI} \\ 2 - \dfrac{PL^2}{30EI} & 4 + \dfrac{2PL^2}{15EI} \end{vmatrix} = 0 \tag{4}$$

Solving for the axial force P, we obtain the buckling load P_{cr} as,

$$P_{cr} = \frac{12EI}{L^2} \tag{5}$$

This buckling load is larger than the Euler's buckling load ($P_{cr} = \dfrac{\pi^2 EI}{L^2}$) by 21.6%.

Using an improved finite element stiffness, named as the pointwise equilibrium polynomial (PEP) element in Chan and Zhou (1995), and ignoring initial imperfection for simplicity, the stiffness equation is given by,

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} c_1 & c_2 \\ c_2 & c_1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{6}$$

in which,

$$c_1 = \frac{3(80)^2 + 10(80)q + \left(\frac{61}{7}\right)q^2 + \left(\frac{23}{1,260}\right)q^3}{H_1^2}$$

$$c_2 = \frac{48^2 + 6(48)q + \left(\frac{29}{5}\right)q^2 + \left(\frac{11}{420}\right)q^3}{H_2^2}$$

$$H_1 = 80 + q;\ H_2 = 48 + q;\ q = \frac{PL^2}{EI}$$

Solving for q under the condition of vanishing determinant in Eq. (6), we have the buckling load P_{cr} as which is the same as the Euler's buckling load. Although the finite element in Eq. (6) appears to be more complicated, the computational effort in computer in minimal and we just need to input the value of P and the accurate stiffness equation can then be determined.

From this simple comparison, the limitation of the widely used cubic Hermite element for elastic buckling analysis is demonstrated. The selected PEP element is more accurate than the conventional element. Note that, for an effective second-order analysis, the coupling and bowing effects must also be determined rigorously and the uncoupled stiffness equation in most analyses using either finite element or stability function cannot be assumed. The final axial force and moments relationship with the axial shortening and rotations in the PEP element can be summarised as follows.

$$P = \frac{\partial U}{\partial e} + \frac{\partial U}{\partial q}\frac{\partial q}{\partial e}$$

$$= EA\left[\frac{e}{L} + b_1(\theta_1 + \theta_2)^2 + b_2(\theta_1 - \theta_2)^2 + b_{vs}\frac{v_{mo}}{L}(\theta_1 - \theta_2) + b_{vv}\left(\frac{v_{mo}}{L}\right)^2\right] \tag{7}$$

$$M_{1,2} = \frac{\partial U}{\partial \theta_{1,2}} + \frac{\partial U}{\partial q}\frac{\partial q}{\partial \theta_{1,2}}$$

$$= \frac{EI}{L}\left[c_1(\theta_1 + \theta_2) \pm c_2(\theta_1 - \theta_2) \pm c_o\left(\frac{v_{mo}}{L}\right)\right] \tag{8}$$

in which e is the shortening between the two end nodes, v_{mo} is the amplitude of member initial imperfection and U is the strain energy function. Other coefficients (c_0, b_1, b_2, b_{vs} and b_{vv}) are given by Chan and Zhou (1995). The last two terms in Eq. (7) and the last term in Eq. (8) are to account for the initial curvature expressed in terms of member imperfection. The stiffness can be expanded to 3-dimensional problems by repeating the procedure to the other principal axis (Chan and Zhou 1995). Note that the axial force in equation does not only depend on the axial shortening, but also on the end rotations due to the bowing effect. Similarly, moment along a member depends on the axial force as well as the end rotations due to the P-δ effect. The reason for the accurate expression for the element stiffness in Eq. (6) is due to the satisfaction of equilibrium of shear and moment at the mid-span of the element of which the condition is not satisfied by the widely used cubic Hermite element under axial force.

With the expressions in Eqs. (7) and (8), we can then formulate the tangent and secant stiffness equations and employ the Newton-Raphson method for an incremental-iterative analysis. The book by Livesley (1964) provides a precise introduction of the Newton-Raphson analysis procedure. For conventional design, the stress, deflection or the response of a structure are required to be determined at a particular fixed load level. The conventional or the modified Newton-Raphson method for iteration at a fixed load level is used in this case. When the complete equilibrium path is sought in both the pre- and the post-buckling path, the arc-length with minimum residual displacement method (Chan and Chui 2000) is utilised in order to avoid divergence near limit load.

6. Design strength determination

A criterion must be checked for design strength determination. The section capacity check is utilized in many design codes for strength checking. This criterion is adopted here for consistency with the current design requirements so that the results can be directly used for practical design. Therefore, the design load capacity of the complete system is achieved when any section in a member of a structural framework satisfies the following condition.

$$\frac{P}{p_y A} + \frac{M}{p_y S} = 1 \tag{9}$$

in which P is the applied axial force, M is the resultant moment allowing for both P-Δ and P-δ effects ($M = \sqrt{M_x + M_y}$) at a section, p_y is the design strength and S is the elastic or plastic sectional modulus, depending on whether an elastic or the first-plastic hinge approach is adopted.

Unlike the conventional second-order analysis (see Chapter 1 in ASCE 1997), the second-order analysis software is not used to determine the effective length of individual members. Instead, the complete design process does not require computation or assumption of the effective length for determination of buckling load of members and the load capacity is assessed by the sectional capacity check in Eq. (9).

7. Numerical examples

To illustrate the application of the method for design and buckling analysis of tubular trusses, three examples are selected and solved. The first problem involves the checking of load capacity of a simple truss against the tested result. In the second problem, a 13.2 m truss commonly used as a lateral supporting truss in glazing systems is analyzed and designed.

In the analysis of frame structures, the direction as well as the magnitude of initial imperfection affects the buckling strength, determined as the load causing the satisfaction of maximum sectional capacity in Eq. (9). Most design codes like the BS5950 (1990) assumes a geometric initial imperfection of 0.001 of the column and this value is further modified here in order to generate the same design curve as the design code so that any sectional type and fabrication group such as cold-formed, hot-rolled and welded sections can be handled. In addition, the direction of initial imperfection is assumed to be the same as the deflected shape of the structures composed of perfectly straight members.

In computer analysis, the initial imperfection of 0.1% of the member length is assumed for all elements. For a member with several elements, the nodal co-ordinates between these elements are assigned an appropriate value using a half-sine function. Based on this modeling scheme, a member, whether it is modeled by one or several elements, will be described as a curved member based on the half sine function with mid-span imperfection of 0.1% of its length. For cases where the member is not pin-supported at two ends, this initial shape before load application is still assumed and it appears to be reasonable since the initial and imperfect shape of the structure is not dependent on the boundary conditions.

7.1. Experiment of a simple truss of 4.8 m span

The truss of nominal size 4.8 m wide ×1 m deep shown in Figs. 2 and 5 was tested. One end of the truss is allowed to slide freely along the longitudinal *x*-axis and to rotate about all axes by simply placing the member onto the supporting platform. The other supporting end is welded to a flat plate fixed onto the support so that torsional twist and displacements in all directions are prevented. All members of the truss are made of 48.3×3.2 Circular Hollow Section (CHS) and grade 43 steel of design strength 275 N/mm^2.

A point load at the mid-span bottom of the truss was applied to the truss until buckling, which was indicated by an excessive deflection of the top chord. Deflections at several nodal locations were measured against the load. This loading arrangement made the top chord in compression and buckled laterally.

In the design of the truss, a simple question will be raised. What is the effective length of the top chord against buckling in out-of-plane direction ? A simple widely used assumption for this effective length determination is the distance between chord for in-plane buckling and the distance between support for buckling out-of-plane.

When using this conventional approach of assuming the distance between supports as effective length, it is then taken as 4.798 m and the slenderness ration (L_e/r) for the tubular sections of 48.3×3.2 CHS of grade 43 steel is 299.9. From BS5950(1990), the permissible stress is 21 N/mm^2 and the permissible load in top chord is equal to $p_y A$ or 9.513 kN. The applied load generating this compressive load is then calculated as 7.8 kN.

In the experiment, the tested buckling load of about 34 kN is much higher than the design load

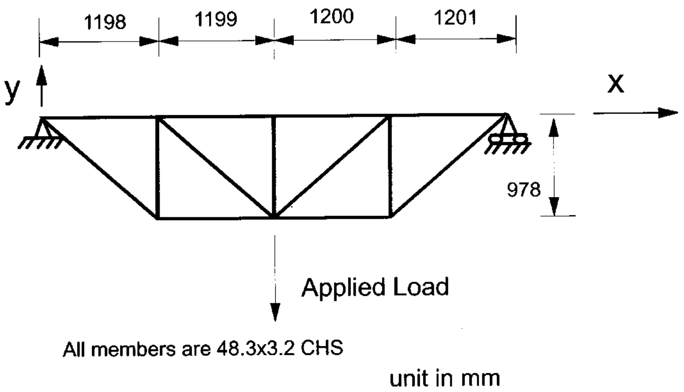

Fig. 5 Geometry of the tested truss

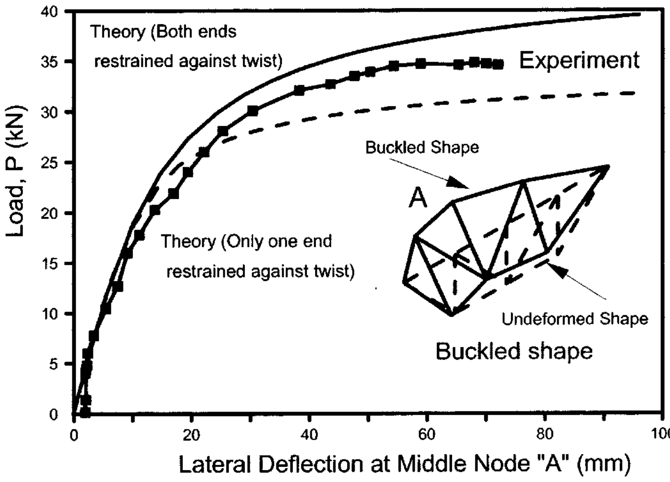

Fig. 6 Load versus deflection of simple truss

calculated from the conventional method of 7.8 kN by 4.4 times. This shows the uneconomical output by the conventional design method following strictly to the design code.

The experimental load versus deflection plot for the lateral deflection at mid-span node is also shown in Fig. 6, together with the computational results. In the theoretical analysis, the nodal co-ordinates are taken from Fig. 5, with allowance of initial imperfection specified in Eq. (9). For the first case, one end was assumed free to rotate longitudinally and the second case assumed this twist is restrained about the longitudinal x-axis. A 0.5% notional force is further applied in order to fulfil the code requirement. Nevertheless, it was noted that the notional force is unimportant for buckling analysis when the member initial imperfection was considered since both of them are disturbances to activate buckling. The objective of this notional force is to simulate the imperfection like the out-of-plumbness in a frame.

It can be seen in Fig. 6 that the theory simulating the actual condition is close to the tested results. The computed applied force, P, satisfying Eq. (9) is 32 kN at a lateral displacement of 107 mm. The calculated buckling strength results is less than the tested load of 34.2 kN. It was difficult to determine precisely the elastic buckling load of the truss since the elastic load capacity increases exponentially with displacement. This uncertainty is eliminated when using the proposed Eq. (9).

The buckled shape of the truss is plotted in Fig. 6. It can be seen that the bottom tension member deflects whilst the top compression member with the complete truss twists, demonstrating the system buckles simultaneously. This contribution by the torsional stiffness of the tension member stiffens the compression member against buckling significantly and its consideration will, therefore, make the design more economical.

When we assume the truss is restrained against twist, the design buckling strength is 39.5 kN. It can be seen in Fig. 6 that the deviation between the two sets of computational results increases when the deflection entered the non-linear range, demonstrating the stiffening-tension member effect activated when the structure behaved non-linearly. Linear analysis cannot therefore reveal this phenomenon for a planar truss.

When we use the concept of effective length, we encounter a problem of varying axial force in the buckling chord. This effect is not considered in most national codes which consider only the geometrical and boundary conditions. By conventional analysis using the maximum load in top chord, we can obtain the same result as our buckling analysis if the effective length is assumed as 2.311 m or the effective length factor is taken as 0.482. In this case, the buckling stress from Table 27a (BS5950, 1990) is then equal to 86 N/mm^2 and the permissible buckling load is then 39 kN, which can be produced by an applied point load of 32 kN.

Following the conservative assumption of using the distance between support equal to 4.7985 m as the effective length, the buckling stress from BS5950 (1990) is 21 N/mm^2 and the buckling applied load is only 7.8 kN. It differs from our computer and test result by about 4 times !

This example demonstrates the versatility and accuracy of the computer method in predicting the buckling load of a tubular truss against out-of-plane buckling. It further illustrates the significance of the torsional effect in buckling and the stiffening-tension member effect.

7.2. Design of a 13.2 m span glazing truss with support at top and at bottom

The simply supported truss of 13.5 m span and 1.2 m depth was constructed as a glazing supporting system. The sections are made of 114.3×6.3 CHS of grade 50 steel with design strength 355 MPa according to BS5950 (1990), since its thickness is less than 16 mm and the section is plastic. All members are welded with the supports free to rotation in all directions except in the rotation about the longitudinal axis of the truss. All displacements are restrained with an exception along the longitudinal axis of the truss in order to allow building and thermal movement. In the analysis, all member connections are assumed rigid.

The difficulty in the design of the truss is on the assessment of effective length of the back and compressive chord during wind suction. The architect requires the member size to be the smallest possible and the task of this example is to determine the maximum design wind load. Only the ultimate limit state is considered for demonstration purposes.

The equilibrium path of the structure against increasing uniformly distributed load, *w*, is plotted in Fig. 7. The condition stipulated in Eq. (9) is met at a design load of 7.52 kN/m. The analysis and design is completed once the computer analysis is finished, which consumes only a few seconds of computer time. If we take the maximum load in this compressive chord for back-analysis, the effective length is

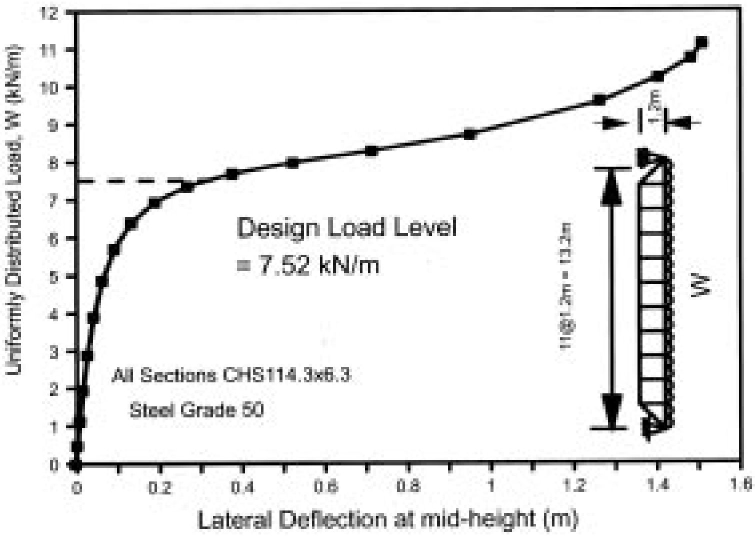

Fig. 7 Analysis and design of 13.2 m span truss

determined as 6.8 m. This implies that we will obtain the same result if we assume the effective length ratio of 0.515 for the compressive chord and use the BS5950 (1990) for determination of load capacity of the chord member. Undoubtedly, the value can hardly be assessed by inspection and manual judgement. In fact, owing to the variation of geometrical properties, the effective length ratio may change from one structure to another (Chan, Koon and Sun 1999).

It is interesting to note that the truss is considerably strengthened by the use of hollow sections stiff in torsion. If we replace the CHS sections by an open section of same cross-sectional properties but with a torsional moment of area of 1% of the CHS (an approximate order of reduction for an open and a closed section of the same weight per metre), the truss buckles at a load of only 1.5 kN/m. The reason for the large difference between the buckling resistance of closed and open sections is due to the effectiveness of torsional restraint against movement of the compression chord. When this torsional stiffness is large, the chord is restrained from lateral movement during buckling whilst a small torsional stiffness allows this movement to occur, in a way similar to the buckling of a column with roller supports at ends. The judgement of effective length for members with this type of semi-movable supports is complicated and unreliable. This contribution of member torsional stiffness is seldom considered in conventional design and the computer method demonstrates its importance in full. The tremendous increase in buckling resistance is also due to the restraint provided by tension member through the torsional stiffness. The buckling of the compressive member forces the tension member to deform simultaneously as a unique system through the transfer of moment by torsion, which significantly improves the buckling resistance of the structural system.

8. Conclusions

A computer method for determining the buckling load of a structural system with allowance for the stiffening-tension member effect is presented. With the use of a second-order analysis software, the buckling load of an unbraced steel truss composed of circular hollow section is computed and compared with the experimental result. The computer method is further applied to the design of a 13.2 m truss used as a glazing supporting truss under wind pressure. Unlike the conventional second-order analysis assisting the engineer to assess the effective length of individual members, the effective length is not needed to be assumed in the present method and, therefore, the inconvenience or uncertainty in assuming an effective length is eliminated. Also, design is complete simultaneously with analysis, leading to an efficient practical design procedure.

When instability occurs in a structural system, both the compression and the tension members will deform significantly as a unique single system. This process will utilise the flexural and torsional stiffness of the tension member against the system buckling and therefore significant saving in material can be resulted. However, this interesting effect is seldom considered in practical design. This paper studies this effect by a computer procedure which was further verified by a test of a simple truss.

Acknowledgements

The first author acknowledges the support by the RGC Grant Council of the Hong Kong SAR Government for the project “Static and dynamic analysis for steel structures [Q193]”. The valuable comments of the reviewer is also greatly acknowledged.

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