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Minimum-weight design of stiffened shell under hydrostatic pressure by genetic algorithm

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Abstract. In this paper, optimization of cylindrical shells under external pressure to minimize its weight has been studied. Buckling equations are based on standard of ABS underwater vehicles. Dimension and type of circumferential stiffeners, and its distance from each other are assumed as variables of optimization problem. Considering the extent of these variables, genetic algorithms have been used for optimization. To study the effect of hydrostatic pressure on the shell and its fabrication according to the existing standards, geometrical and construction as well as stress and buckling constraints have been used in optimization algorithm and also penalty functions are applied to eliminate weak model. Finally, the best model which has the minimum weight considering the applied pressure has been presented.

Keywords: hydrostatic pressure; genetic algorithms; stiffener ring; buckling; cylindrical shell

1. Introduction

Cylindrical shells are widely used in different industries. In marine industries, these shells are used for construction of underwater vessels. Cylindrical shells are under hydrostatic pressure when placed under deep water. For shells under hydrostatic pressure, buckling is very important because sometimes it occurs sooner than the final strength. To increase tolerable limit of buckling pressure in cylindrical shells under external hydrostatic pressure, stiffener rings are used in definite distances. These rings sometimes have been used inside the shell and sometimes out of it.

Extensive studies have been conducted on the buckling of cylindrical shells under hydrostatic pressure. Ross (2001) mentioned that if long cylindrical or conical shells had no stiffener rings, their buckling resistance under external hydrostatic pressure reduced considerably. If stiffener rings are not strong enough or shell is very long, the entire shell can be destroyed. This kind of destruction was recognized as general instability by Kendrick (1965).

In fact, it is well known that an appropriate design for the circumferential stiffeners and shell skin may permit the structure to carry loads several times higher than the first buckling one. This design has shown consequential potential for further weight decrease. On the other hand, the use of very effective aluminum alloy structures is currently made possible by validated design procedures, analysis methods and by the accessibility of large amount of data.

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The optimum buckling design of composite and aluminum stiffened plate with several stiffeners and loading conditions has been studied by some investigator using different methods. Nagendra *et al*. (1996) recommended an improved genetic algorithm (GA) to find the best stacking sequence of the skin and stiffeners laminate, and the stiffener size for minimum weight of a composite stiffened panel under buckling constraint. They presented an optimized design with weight reduction by about 4%. Bedair (1997) studied the effect of stiffener position on the permanence of stiffened plates in compression and plane bending. The results have shown that the optimum position for the stiffener depends on the relative proportions of the plate and the stiffener. Walker (2002) investigated the effect of stiffener arrangement and boundary conditions on the optimal ply angles and the buckling load.

Todoroki and Sekishiro (2008) presented a stacking sequence optimization to maximize the buckling load of blade-stiffened panels with strength constraint using the iterative fractal branch and bound method. In this procedure, the strength constraint was executed by a response surface method. The results showed that an optimal stacking sequence of the stiffened panel can be achieved by this procedure at a low computational cost. Alinia (2005) studied the optimization of plate stiffeners subjected to shear load. The study has shown that the optimum geometric parameters of the stiffeners are related to the point when the plate buckling shape changes from a global mode to local mode. Kang and Kim (2005) investigated the weight optimization of composite structures by considering the post-buckling behavior. In order to estimate the post-buckling behavior, a nonlinear finite element analysis code was used and the modified genetic algorithm was applied as the optimization procedure.

Hu and Yang (2007) studied the effect of end conditions, curvatures, aspect ratios, thicknesses and cutouts on the optimal fiber orientations and optimal buckling loads of composite cylindrical. Wang *et al*. (2010) concentrated on the optimization of a composite plate and a T-section stringer stiffened panel for maximizing the buckling load under a given weight. Abdi *et al*. (2011) studied the buckling behavior of optimum laminated composite cylindrical shells subjected to axial compression and external pressure. They used GA as optimization algorithm. The number and fiber orientation of layers were selected as optimization design variables. Their study had no constrain which must be considered.

Lee *et al*. (2013) have studied design load optimization of composite sandwich cylinders under external hydrostatic pressure and determined that both the buckling and the static material failure should be considered in the design of the composite sandwich cylinder. Ghasemi and Hajmohammad (2013) used response surface method (RSM) and GA to optimize stacking sequences of laminated composite materials and increase buckling capacity load. Walker and Tabakov (2013) have proposed and demonstrated the optimal design of engineering structures with manufacturing tolerances as design variables and used a GA to obtain the optimize results.

All these previous studies have contributed to useful guidelines, which help the design of stiffened shells under some loading conditions. However, there is no research focused on the optimum design for the minimum weight of cylindrical stiffened shell under buckling and strength criteria or geometrically constraint such as margin of displacement between stiffeners and some of continuous and discrete variables such as thickness of shell, displacement between stiffeners, size of stiffener and also kind of stiffeners like I, H, L, T and S- section profiles.

Among different optimization designs, the method of consecutive linear programming has been successfully implemented for many large scale structural problems (Zienkiewicz and Campbell 1973, Vanderplaats 1984). One of the discrete and continuous optimization techniques, genetic algorithm was submitted for the optimization of composite structures, and many researchers represented that GA was a suitable solution to complex optimization problems such as composite structures (Nagendra *et al*.1996, Perry *et al*. 1997). Above all, weight-minimization is the final goal for many researchers because the major inducement for the use of optimization concept is weight reduction.

Optimization with stability constraints has been investigated widely in the past. The researchers (Khot *et al*. 1976, Khot 1983) represented an optimality criterion method for determining the minimum weight design of linear space truss structures subjected to stability constraints. Muc and Muc-Wierzgoń (2012) have presented an evolution strategy to optimize stacking sequences of structures. They deal only thickness and stacking sequence optimization problems for circular cylindrical shells subjected to various dynamic and static constraints, respectively.

In this research, optimization of cylindrical shells under external pressure has been studied to minimize its weight. Variables of optimization problem are dimensions, the type of circumferential stiffeners and its distance from each other. Therefore, according to the extent of these variables, genetic algorithms have been used for optimization. To study the effect of hydrostatic pressure on the shell and its fabrication according to the existing standards, geometrical and construction as well as stress and buckling constraints have been used in optimization algorithm. Finally, the best model which has the minimum weight on the basis of the applied pressure has been presented.

2. Mathematical modeling of optimization problem

2.1 Objective function

In this section, mathematical modeling of optimization problem has been obtained. For an optimization problem, we search to find the best answer which maximizes or minimizes a function called objective function. Problem solving space is limited by constraints that mainly govern physics of the problem. Although all answers which are included inside permissible solving space are acceptable for designer, designer searches to find the answer which is preferred over other answers in terms of the problem type. In the first section of this research, objective function of the problem is studied and in the second section, constraints governing of the problem and their applications are studied. In this research, the objective function of the problem based on different components weight of the shell has been considered. The formed sheet of shell has been reinforced by stiffener ring with the same material to increase resistance against external pressure and the resulting inductive stresses. In this study, length and external diameter of the shell are constant and the optimization is performed on the thickness of shell, type of ring, geometrical dimensions and the number of rings. In this section, attempt has been made to obtain objective function of the problem after mathematical modeling of reinforcing rings and shell weight functions.

2.1.1 Shell weight function

Based on hypotheses of the problem, the desired problem is a cylindrical shell which has been made by aluminum with fixed external diameter and length. The properties of aluminum are presented in Table 1.

According to the thickness of the shell as the only variable of this equation, shell weight function is obtained through Eq. (1)

$$W_{shell} = \pi D_m t_s L_s \rho_s \tag{1}$$

Table 1 Properties of material, Mechanics of materials (1992)

Density (Kg/m^3)	Elastic modulus (GPa)	Poisson's ratio	Yield strength (MPa)	Ultimate strength (MPa)
2710	72.7	0.28	215	305

where D_m, t_s, L_s and ρ_s are medium diameter, thickness, length and density of shell, respectively. Medium diameter of the shell is substituted by Eq. (2) and in terms of fixed external diameter and variable value of the shell thickness

$$D_m = \frac{(D_{out} + D_{in})}{2} = \frac{(D_{out} + D_{in} - 2t_s)}{2} = D_{out} - t_s \tag{2}$$

For performing optimization process, a case study needs to optimize its variables which can effect on the weight of specimen. Also this case will be used in future works.

By substituting the known values of the specimen which include length of shell, 9 m, external diameter of shell, 75 cm, density of shell, 2710 kg/cubic meter , shell weight objective function is obtained as Eq. (3) in weight equation

$$W_{shell} = 24390\pi(0.75t_s - t_s^2) \tag{3}$$

2.1.2 Stiffener rings weight function

Stiffener rings mean a metal section which is connected to a part or the whole of shell under pressure as inscribed in circle or circumscribed about circle or continuously or discontinuously and locally reinforces power of resistance against inductive stresses. In this study, it is assumed that stiffener rings are inserted locally or as circumscribed about the circumference of the shell. In the cross section used in rings, they are selected based on the related standards such as Stahl etc. Since optimization of the type and geometry of the cross sections has been raised and considering aluminum material of the stiffener rings and necessity of formation instead of being purchased from the market, all used dimensions in definition of a cross section have been considered as variables of the problem. Tables available in Stahl standard have been used to define geometrical constraints which control dimensions of the rings.

To define cross sections of stiffener rings, standards ABS (2012) and ASME sec VIII (2010) have been referred. In these references, the following five sections have been recommended for reinforcing shell under pressure. As seen in the following figures, four cross sections can be defined using five parameters t_{b1}, t_{b2}, t_w, h, b.

By applying this technique, different cross sections with fixed parameters can be defined. This work helps us deal with weight functions and constraints and also reduce equations relating to the figure effectively. The fifth section is equal to definition of the second section in terms of parametric definition and the only difference is definition of moments of inertia. This section is shown in Fig. 5.

Moments of inertia of Sections 1 to 4 can be expressed only by one parametric equation (due to symmetry) in which doesn't hold true for the fifth section. For this reason, this ring will be optimized separately by applying equations relating to equations of its moments in constraints subprogram and the obtained result will be compared with the most optimal state of other four rings.

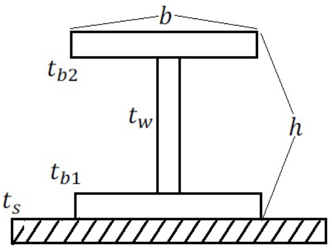

Fig. 1 Design variables of I-section

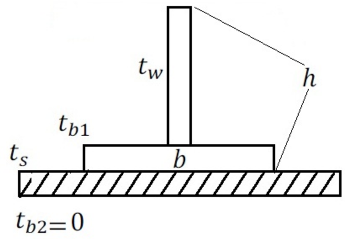

Fig. 2 Design variables of inverse T-section

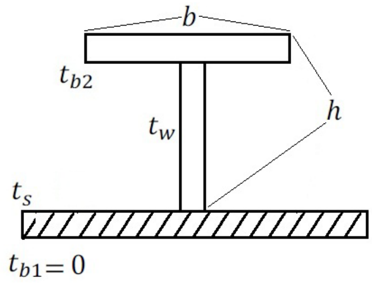

Fig. 3 Design variables of T-section

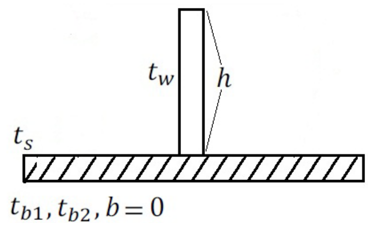

Fig. 4 Design variables of flat bar

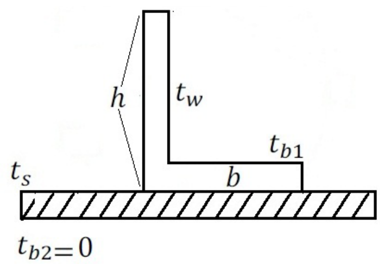

Fig. 5 Design variables of L- section

Cross section of all these rings is written in terms of the five parameters and weight function of each stiffener ring can be obtained as Eq. (4) by assuming that stiffener ring has circumscribed about the desired shell from inside ($\theta = 2\pi$).

$$W_{ring} = S_{ring} \times l_{ring} \times \rho_{ring} \tag{4-1}$$

$$S_{ring} = (h - t_{b1} - t_{b2})t_w + b(t_{b1} + t_{b2}) \tag{4-2}$$

$$l_{ring} = R_i \times \theta = \pi(D_0 - 2t_s) \tag{4-3}$$

$$W_{ring} = 2710\pi(0.75 - 2t_s)(h - t_{b1} - t_{b2})t_w + b(t_{b1} + t_{b2}) \tag{4-4}$$

2.1.3 Final objective function

Final shell and stiffener rings objective function are obtained from sum of the shell weight and ring number functions. In this section, the number of stiffener rings is also included in the optimization problem and algorithm selects it. Finally, weight function is obtained as Eq. (5)

$$W_{total} = 24390\pi(0.75 - t_s^2) + \sum_{i=1}^{n} 2710\pi(0.75 - 2t_s)(h - t_{b1} - t_{b2})t_w + b(t_{b1} + t_{b2}) \tag{5}$$

2.2 Constraints governing the problem

Constraints governing of this problem majorly result from the limitations which ABS standard (2012) has applied on the stresses inducted due to external pressure in shell and stiffeners. Another class of the constraints is geometrical constraints which designer includes for proper and rapid direction of algorithm based on standards on limits and dimensions of variables. The following figures show the schematic diagram of cylindrical shell and detail of reinforcement rings.

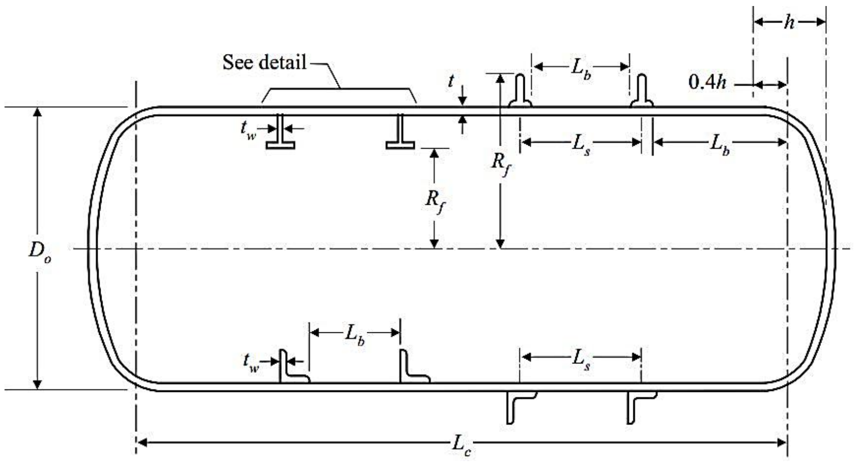

Fig. 6 Schematic diagram of cylindrical shell, ABS (2012)

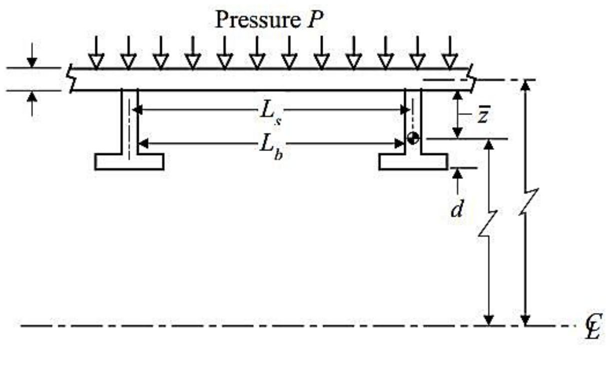

Fig. 7 Detail of reinforcement rings, ABS (2012)

2.2.1 Inter-stiffener strength

2.2.1.1 von mises buckling pressure for a cylinder

To calculate inter- stiffener strength, von Mises buckling pressure and yield stress are calculated under designed midline. The von Mises buckling pressure can be obtained according to Eq. (6)

$$P_m = \frac{2.42E\left(\frac{t_s}{2R}\right)^{\frac{5}{2}}}{(1-\upsilon^2)^{\frac{3}{4}}\left[\frac{L}{2R} - 0.45\left(\frac{t_s}{2R}\right)^{\frac{1}{2}}\right]} \tag{6}$$

where P_m, t_s, R, E, υ are von Mises buckling pressure, thickness of shell, average radius of shell, elastic modulus and Poisson's ratio, respectively. In addition L is larger number between l_s and l_b, that l_b is distance among stiffeners and l_s is length of compartments.

Yield stress in midline can be obtained according to Eq. (7)

$$P_y = \frac{\sigma_y \frac{t_s}{R}}{1-F} \tag{7}$$

where P_y is yield pressure in midline and σ_y is the minimum yield stress, which is specified for the shell. Further, F will be obtained by the appendix equations.

Inter-stiffener strength (P_c) is obtained according to the following rules and in terms of pressures calculated according to Eq. (8)

$$P_c = \begin{cases} P_m & \text{if} \quad \frac{P_m}{P_y} \le 1 \\ P_y\left(1 - \frac{P_y}{2P_m}\right) & \text{if} \quad 1 < \frac{P_m}{P_y} \le 3 \\ \frac{5}{6}P_y & \text{if} \quad \frac{P_m}{P_y} > 3 \end{cases} \tag{8}$$

Finally, the highest permissible working pressure is calculated based on inter-stiffener strengths according to Eq. (9)

$$P_{a1} = P_c \times \beta, \quad \beta = 0.8 \tag{9}$$

Output constraint of this limiting pressure is obtained based on Eq. (10)

$$Cons(1) = P - P_{a1} \tag{10}$$

2.2.1.2 Longitudinal stresses at stiffeners

One of the limiting factors in stresses is longitudinal stress at stiffeners which reaches critical stress. This stress is defined according to Eq. (11)

$$P_L = \frac{2\sigma_y . t_s}{R}\left[1 + \left(\frac{12}{1-\vartheta^2}\right)^{\frac{1}{2}} \gamma H\right]^{-1} \tag{11}$$

Permissible working pressure is defined as Eq. (12) for reinforced shell based on inductive longitudinal stress at stiffeners

$$P_{a2} = P_L \times \beta, \quad \beta = 0.67 \tag{12}$$

Finally, output constraint of this section can be expressed according to Eq. (13)

$$Cons(2) = P - P_{a2} \tag{13}$$

2.2.2 Overall buckling strength

Limited external pressure is obtained from Eq. (14) according to the overall buckling mode between the heavy supporting and reinforcing elements

$$P_n = \left(\frac{Et_s}{R}\right) A_1 + \frac{EIA_2}{LR^3} \tag{14}$$

where I is moment of inertia of combined section consisting of stiffener with an effective length of shell L_e about the centroidal axis of the combined section parallel to the axis of the cylinder. Its factors can be calculated according to Eq. (15).

$$\lambda = \frac{\pi R}{L_C} \tag{15-1}$$

$$A_1 = \frac{\lambda^4}{\left[A_2 + \left(\frac{\lambda^2}{2}\right)\right]\left[n_l^2 + \lambda^2\right]^2} \tag{15-2}$$

$$A_2 = n_l^2 - 1 \tag{15-3}$$

where L_C is the maximum distance between two heavy stiffeners or one stiffener and Quasi-spherical end or the entire length of a shell. In case knobs are quasi-spherical, this length also includes 40% of the depth of each knob. Since in this problem a cylindrical shell without knob and without using heavy stiffeners is considered, this length is total length of the shell and it is equal to 9 m. The parameter n_l indicates the number of buckling modes which is an integer of larger than or equal to 2 as recommended by standard. Considering application of this parameter in the above equation, it can be seen that the smaller the number becomes, the more critical pressure increases, therefore, number 2 is selected for this model in the next analyses. Then, permissible working pressure is obtained based on general buckling strength according to Eq. (16).

$$P_{a3} = P_n \times \beta, \quad \beta = 0.5 \tag{16}$$

Finally, output constraint of this section can be expressed according to Eq. (17)

$$Cons(3) = P - P_{a3} \tag{17}$$

2.2.3 Stiffener rings strength

All stiffeners should be connected to shell using the continuous welds. Each stiffeners ring which is connected to the cylindrical shell should satisfy all of the following strength formulas based on maximum stress in stiffener, stiffener tripping, local buckling of webs and flanges and stiffener flexural inertia. These formulations are applied to stiffeners that outer flanges (where fitted) are symmetric about the web. But by special consideration, these can be used in other geometries. Based on definition available in ABS standard (2012), all stiffeners are among the light stiffeners and these stiffeners don't reduce compartment length of shell.

2.2.4 Limitations of the stress

Yield pressure (P_t) which includes circumferential (hoop) stress and the bending stress arising from possible out-of-roundness are calculated by satisfying Eq. (18)

$$\sigma_y = \frac{P_t.\sigma_y}{P_{yf}} + \frac{EC\delta(n^2-1)P_t}{(P_n - P_t)R^2} \tag{18}$$

As seen above, yield stress function has been repeated among this equation complicatedly and the equation cannot be simply ordered. To solve the equation, the rules relating to solution of the equations in Matlab software have been used. In this equation, the distance of the stiffener flange from the neutral axis of the combined stiffener and effective shell section L_e is denoted "C" and δ is allowable out-of-roundness which is equal to 0.5 percent of average radius or $0.005R$.

P_{yf} is calculated from Eq. (19)

$$P_{yf} = \frac{\sigma_y.t_s.R_f}{R^2\left[1-\left(\frac{\vartheta}{2}\right)-\gamma\right]} \tag{19}$$

where R_f is radius of the shell to vertex of the stiffener ring. Also, the maximum allowable working pressure based on stiffener stresses is given by

$$P_{a4} = P_t \times \beta, \quad \beta = 0.5 \tag{20}$$

Finally, output constraint of this section can be expressed according to Eq. (21)

$$Cons(4) = P - P_{a4} \tag{21}$$

2.2.5 Stiffener tripping

Circumferential tripping stress for flanged stiffeners attached to the shell is obtained as follows

$$\sigma_T = \frac{EI_z}{A_s R\overline{\overline{Z}}} \tag{22}$$

where I_z is moment of inertia around its radial axis which passes web of ring, $\overline{Z}$ is the distance between stiffener cross section center and the nearest surface of shell and A_s is the area of stiffener cross section alone. Tripping stress calculated in this equation should be larger than the yield stress applied in design. Therefore, output constraint of this stress is expressed based on Eq. (23)

$$Cons(5) = \sigma_y - \sigma_T \tag{23}$$

2.2.6 Local buckling

To address the possibility of local buckling of the flanges and webs of a stiffener cross section welded to the shell, the following slenderness limits are to be met:
Flat bars, other outstands: Width/Thickness $\geq 0.3\sqrt{E/\sigma_y}$
Web of flanged stiffener: Depth/Thickness $\geq 0.9\sqrt{E/\sigma_y}$
Therefore, according to Eq. (24)

$$Cons(6) = \begin{cases} h/t_w \leq 0.3\sqrt{E/\sigma_y} \\ \text{or} \\ (h - t_{b1} - t_{b2}) \leq 0.9\sqrt{E/\sigma_y} \end{cases} \tag{24}$$

2.2.7 Inertia requirements

The moment of inertia for the combined section consisting of a stiffener welded to the shell and the effective shell length L_e has not to be less than I obtained from the following

$$I = \frac{PD_o L_s R_s^2}{(6E\eta)}, \quad \eta = 0.50 \tag{25}$$

where R_s is radius of the shell to central point of cross section of stiffener. Finally, output constraint of this standard requirement is calculated based on Eq. (26)

$$Cons(7) = I - I_o \tag{26}$$

2.2.8 Geometrical constraints governing variables

Unlike the problem variables which are geometrical parameters, it is inevitable the constraints which control ratios among these geometrical dimensions (if available) are needed. In the studied constraints, the lack of constraint which can find and control the constraint was felt. Constraints resulting from stress approaches majorly control permissible limits of variables. To proper and rapid direction of optimization algorithms and by referring to Stahl cross section standard, standard ratios among dimensions like depth, width, thickness of web and flange of the stiffener rings were extracted. The information governs output ratios as constraints of Eq. (27).

$$Cons(8) = 3(t_{b1} + t_{b2}) - h \tag{27-1}$$

$$Cons(9) = h - 11(t_{b1} + t_{b2}) \tag{27-2}$$

$$Cons(10) = 2.5(t_{b1} + t_{b2}) - b \tag{27-3}$$

$$Cons(11) = b - 9(t_{b1} + t_{b2}) \tag{27-4}$$

3. Problem optimization using genetic algorithm

3.1 Genetic algorithm

Genetic Algorithm (GA) is the optimization method inspired by living nature (living creatures) which can be considered in classifications as a direct and random numerical method. This algorithm is an iteration algorithm and its primary principles have been adopted from genetics as mentioned before.

Main factors of biological planes in living creatures are chromosomes and genes and their function is such that better and stronger genes and chromosomes remain and weaker genes are destroyed. In other words, result of mutual operation of genes and chromosomes is survival of better creatures.

The GA has several main differences from conventional search methods due to its imitation of nature:

(1) The GA works with bit strings each showing total set of variables while most methods independently face special variables.

(2) The GA performs random selection for searching guidance and there is no need for information of derivative.
In the GA, search methods act based on natural genetic and selection mechanism. These algorithms select the most suitable strings among the organized random information. In each generation, a new group of strings is created using the best parts of previous sequences and new random part for reaching a proper answer. Although algorithms are random, they are not regarded as simple random algorithms. They efficiently explore the past information in search space to move toward the best answer in a new search point with better answers.

(3) The GA follows potential laws not definite rules.

(4) The GA considers several points of search space in each iteration. Therefore, there is low chance of converging to a local maximum. In most conventional search methods (gradient method), governing decision rule acts such that it moves from one point to another point. These methods can have many hazardous maximum because they may converge to a local maximum. However, the GA produces full populations of strings (points). It makes each point and form as a new population which includes improved points by combining their contents. Aside from a search, concurrent observation of some points in the GA makes their adaptation with parallel machines possible.

3.1.1 Genetic algorithm mechanism

The GA as an optimization computational algorithm effectively searches for different zones of answer space according to a set of answer space points in any computational iteration. Although value of objective functions is not calculated in search mechanism, the calculated value of objective function for each point is involved in statistical averaging of objective function in all subspaces on which point is dependent and these subspaces are statistically averaged in parallel in terms of objective function. This mechanism is called implicit parallelism. This trend causes space search to lead to the zones in which statistical mean of objective function is high and there are more absolute optimal points. Because answer space is multilaterally searched in this method unlike single-path methods, there is low probability of convergence to a local optimal point.

In any iteration, each one of the strings available in population of strings is decoded and value of objective function is obtained for it. Based on the obtained values of objective function in population of strings, each string is assigned a fitness value. This fitness value will determine probability of selection for each string. Based on this selection probability, a set of strings is selected and new strings replace strings of the primary population by applying genetic functions on them so that the number of strings population can be fixed in computational iterations. Random mechanisms which act on selection and deletion of string are such that the strings with larger fitness value have more probability of combining and producing new strings and are more resistant than other strings in replacement stage. In this regard, population of sequences is completed in a competition based on objective function in different generations and average value of objective function increases in population of strings.

The GAs execute main search in answer space. These algorithms start with reproduction, which are responsible for creation of set of primary search points called primary population and are determined selectively or randomly. Since GAs use statistical methods to guide search operations to optimal point, the existing population is selected based on fitness of individuals for the next generation. Then, genetic operators include selection, combination, mutation and other potential operators are applied and new population is created. After that, new population replaces the previous population and this cycle continues. New population usually has more fitness that is population improves from one generation to another generation. When we reach the possible maximum generation or convergence is achieved or stop criteria have been fulfilled, search will be successful.

3.2 Problem optimization

Although the GAs follow a specified path and pattern for achieving optimal result, each of them has its unique procedure based on method of program coding. The algorithm used in this project utilizes three instruments of elitism, penalty function and control function for achieving the global optimum rapidly and safely. Elitism means the process that some members which have obtained the best ultimate functions in each generation are transferred directly to the next generation after participating in genetic operators and are not replaced by their children like other members of population. In this regard, the best genes of each generation remain in that generation which is not easily lost and accelerates convergence by increasing reproduction.

Penalty and control functions have been designed to control and prevent answers by entering genetic population from unauthorized solution space. Their procedure is such that each member of population which doesn't satisfy one of the constraints is penalized heavily and it does not qualified for competition to enter the elite groups. Furthermore, each member which violates the defined limits is rapidly excluded from population and is replaced by random members. In the program used for all variables, upper and lower limits have been defined and value of variable cannot violate these limits. These limits have been placed for the variables by referring to Stahl standard and considering physics of the problems.

Two important parameters which play significant role in progress and convergence of the GA are the number of generations and iterations of algorithm and the number of members of one generation. Selection of small values of these two parameters can lead to immature algorithm and convergence to a local optimum. Since the number of large solution space is large but mathematical computations are limited and light in this problem and requirement for achieving the best possible answer may be preferred over cost considerations, relatively values of 1000 and

10000 have been considered for these two parameters.

Other three parameters which play main role in each generation and generally in promotion and convergence of algorithm are coefficient of elitism, combination and mutation. These three parameters which represent the number of population members which have been recognized as elite and are protected against genetic operators enter reproduction operation and form the next generation and in each generation, some of its genes randomly change. According to the physics and conditions governing problem and referring to the GA references, values of elitism, combination and mutation were selected as 0.01, 0.95 and 0.2, respectively.

4. Result and discussion

4.1 Optimization diagrams

Power of the GA to solve optimization problems can be shown by diagrams. These diagrams usually show power of algorithm convergence and its dependency on main parameters of algorithm. Review of the main parameters of the GA which affect its computational loads effectively are the number of generations and the number of each generation members. Fig. 8 shows variations of the objective function in terms of the number of algorithm generations and iterations. To achieve this algorithm, another main algorithm or the number of each generation members should be constant. In this case, the number of each generation was considered as 50. The following diagram is obtained by different executions of the algorithm. In this diagram can be found that the algorithm rapidly has entered authorized solution space. The higher the number of generations increases, the more the optimal results become. Based on the diagram and for this problem, the required number of generations is about 200 generations for closing to optimal result reasonably while optimal result has been obtained in 600 generations.

To study dependency of the objective function value on the number of generations' members, different executions have been investigated in fixed number of 600 generations. These variations are shown in Fig. 9. It can be clearly shown that the algorithm could be closed to limits of optimal

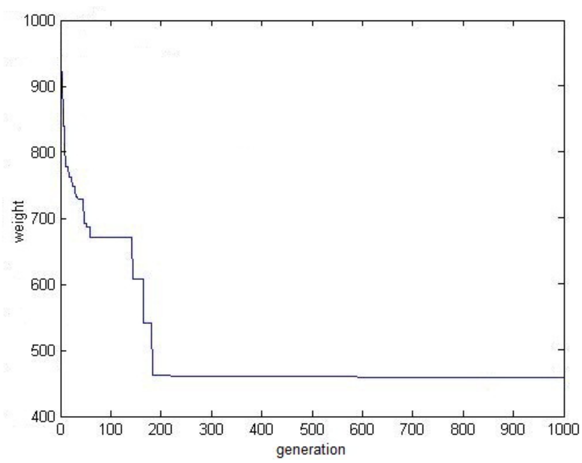

Fig. 8 Variations of weight in terms of the number of generations and iterations of algorithm

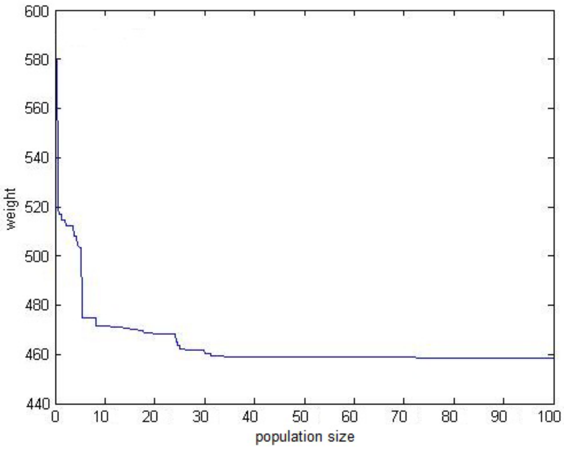

Fig. 9 Dependency of objective function value on the number of generations' members

result reasonably only by population of 35 chromosomes while it has achieved optimal result by population of 75 chromosomes.

4.2 Optimal dimensions

After different executions of the program and changes which have been made in some parts of program, final optimal dimensions of the program were obtained as follows

Table 2 shows that stiffener ring program number 1 (I- section stiffener) has been selected for this problem and algorithm could freely select all types of the defined rings and has selected this model for all rings. Even output dimensions of the algorithm are close to the dimensional ratio of wide beams which presented in Stahl standard. Although this ring has larger cross section than other rings and consequently it should be heavier, because of parametric advantage of this stiffener by reducing dimensions of stiffener and shell, it can achieved lighter design while it should be safe.

Optimal weights of different cross sections are given in Table 3.

Although only these common and known models of the stiffener rings were studied, the program is able to study all of the available profiles only by including some geometrical constraints. Selection power of 1 to 100 the stiffener rings was given to the program and the program finally selected 15 rings for reinforcing shell. It was found when the program was required to select at most 10 rings or at least 20 rings, final weight of the set was much higher

Table 2 Results of optimizing stiffened shell

Variables	Optimum weight (kg)	Shell thickness (m)	Ring quantity	Ring height (m)	t_w (m)	t_{b1} (m)	t_{b2} (m)	b (m)
	W_{total}	t_s	n	h				
Optimum value	482.00	0.008	15	0.0489	0.003	0.0024	0.0024	0.0325

Table 3 Optimal weights of different cross sections

Stiffener type	I-section	Inverse T-section	T-section	Flat bar	L-section
Optimum weight (kg)	482.00	487.09	518.22	594.24	496.84
Ring quantity	15	14	11	19	15

Table 4 Results of optimizing *L*-section stiffener

Variables	Optimum weight (kg)	Shell thickness (m)	Ring quantity	Ring height (m)	t_w (m)	t_{b1} (m)	t_{b2} (m)	b (m)
	W_{total}	t_s	n	h				
Optimum value	496.84	0.008	15	0.047	0.0087	0.0043	0.00	0.022

than the obtained optimal value. In other word, the program likes to reduce thickness of the shell by increasing the number of rings until thickness decrease of the shell leads to critical constraints of stresses. This point is the optimal one for the stiffener rings, but it is different for flat bar ring. This ring shows weak performance to reinforce shell due to weak properties of moment and cross section. In this case, the algorithm not only increases the number of rings, but also increases thickness of the shell a little to satisfy some constraint resulting in considerable increase of final weight.

4.3 Designing optimum reinforcement ring with L cross section

Since *L* cross section ring is a preferred reinforcement in manufacturing process and inertia moments of *L* section has not the same parametric equation as in other sections, this ring separately was studied. In this optimization process, thickness of the shell has been regarded desirable and constant of 8 mm and other dimensions of reinforcement ring have been considered as variable parameters.

Dimensions presented by algorithm show that the algorithm tries to promote inertia moment of the ring. The algorithm has reached this goal by increasing height of the ring and thickness of the ring web. Although this ring is not among the best optimization rings based on weight due to improper conditions of its moments of inertia, it can be regarded as the best rings due to simple production if multi-objective optimization is performed considering cost functions. These dimensions have been presented in Table 4.

5. Conclusions

In this paper, optimization of cylindrical shells under external pressure has been studied to minimize its weight. Variables of optimization problem are dimensions, the type of circumferential stiffeners and its distance from each other. So, according to the extent of these variables, genetic algorithms have been used for optimization.

To study the effect of hydrostatic pressure on shed and its fabrication considering the existing standards , geometrical and construction as well as stress and buckling constraints have been used in optimization algorithm. Finally, the best model which has the minimum weight according to the applied pressure has been presented. It is found that:

(1) I-section stiffener ring has been obtained for this problem as a best selection, although algorithm could freely select all types of the defined rings. The weight of specimen is sum of weight of the shell and stiffeners. So in the optimization process, the algorithm reached I-section stiffener ring by changing all of the variables. Although this ring has larger cross section than the other ones and consequently it should be heavier, due to parametric advantage of the stiffener, by reducing dimensions of I-section stiffener ring and shell, it can achieve lighter design while it should be safe.
(2) It was found that when the program was required to select at most 10 rings or at least 20 rings, final weight of the set was much higher than the obtained optimal value.
(3) Flat bar stiffener shows weak performance to reinforce shell due to weak properties of moment and cross section.
(4) In this case, the algorithm not only increases the number of rings, but also increases thickness of the shell a little to satisfy some constraint resulting in considerable increase of final weight.
(5) Optimization results showed that diverse in design variables can decrease weight about 10 percent.

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References

Abdi, B., Mozafari, H., Ayob, A., Kohandel, R. and Alibeigloo, A. (2012), “Buckling behavior of optimal laminated composite cylindrical shells subjected to axial compression and external pressure”, *Appl. Mech. Mater.*, **121**, 48-54.

ABS, Section 6 (2012), Rules for building and classing underwater vehicles, systems and hyperbaric facilities, metallic pressure boundary components, American Bureau of Shipping, Houston, TX, USA.

Alinia, M.M. (2005), “A study into optimization of stiffeners in plates subjected to shear loading”, *Thin-Wall. Struct.*, **43**(5), 845-860.

An International Code (2010), ASME boiler and pressure vessel code, Section VIII rules for construction of pressure vessels - division 1.

Bedair, O.K. (1997), “Influence of stiffener location on the stability of stiffened plates under compression and in-plane bending”, *Int. J. Mech. Sci.*, **39**(1), 33-49.

Beer, F.P., Johnston, E.R. and DeWolf, J.T. (1992), *Mechanics of Materials*, McGraw-Hill, New York, NY, USA.

Ghasemi, A.R. and Hajmohammad, M.H. (2013), “Optimization of stacking sequence for buckling load using the response surface method and genetic algoritms in laminated composite materials”, *J. Computat. Method. Eng.*, **31**(2), 131-140.

Hu, H.T. and Yang, J.S. (2007), “Buckling optimization of laminated cylindrical panels subjected to axial compressive load”, *Compos. Struct.*, **81**(3), 374-385.

Lee, G.C., Kweon, J.H. and Choi, J.H. (2012), "Optimization of composite sandwich cylinders for underwater vehicle application", *Compos. Struct.*, **96**, 691-697.

Muc, A. and Muc-Wierzgoń, M. (2012), "An evolution strategy in structural optimization problems for plates and shells", *Compos. Struct.*, **94**(4), 1461-1470.

Kang, J.H. and Kim, C.G. (2005), "Minimum-weight design of compressively loaded composite plates and stiffened panels for postbuckling strength by genetic algorithm", *Compos. Struct.*, **69**(2), 239-246.

Khot, N.S. (1983), "Nonlinear analysis of optimized structure with constraints on system stability", *AIAA J.*, **21**(8), 1181-1186.

Khot, N.S., Venkayya, V. and Berke, L. (1976), "Optimum structural design with stability constraints", *Int. J. Numer. Method. Eng.*, **10**(5), 1097-1114.

Nagendra, S., Jestin, D., Gürdal, Z., Haftka, R.T. and Watson, L.T. (1996), "Improved genetic algorithm for the design of stiffened composite panels", *Comput. Struct.*, **58**(3), 543-555.

Perry, C.A., Gurdal, Z. and Starnes, J.H. (1997), "Minimum-weight design of compressively loaded stiffened panels for postbuckling response", *Eng. Optimiz.*, **28**(3), 175-197.

Ross, C.T.F. (2001), *Pressure Vessels under External Pressure*, Saxe-Coburg Publications, pp. 357-386.

Kendrick, S. (1965), "The buckling under internal pressure of ring-stiffened circular cylinders", *Trans. RINA*, **107**(1), 135-156.

Todoroki, A. and Sekishiro, M. (2008), "Stacking sequence optimization to maximize the buckling load of blade stiffened panels with strength constraints using the iterative fractal branch and bound method", *Compos. Part B: Eng.*, **39**(5), 842-850.

Vanderplaats, G.N. (1984), *Numerical Optimization Techniques for Engineering Design: With Applications*, McGraw-Hill, New York, NY, USA.

Walker, M. (2002), "The effect of stiffeners on the optimal ply orientation and buckling load of rectangular laminated plates", *Comput. Struct.*, **80**(27), 2229-2239.

Walker, M. and Tabakov, P.Y. (2013), "Design optimization of anisotropic pressure vessels with manufacturing uncertainties accounted", *Int. J. Pres. Ves. Pip.*, **104**, 96-104.

Wang, W., Guo, S., Chang, N. and Yang, W. (2010), "Optimum buckling design of composite stiffened panels using ant colony algorithm", *Compos. Struct.*, **92**(3), 712-719.

Zienkiewicz, O.C. and Campbell, J.S. (1973), "Shape optimization and sequential linear programming", *Optim. Struct. Des.*, 109-126.

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Appendix

$$F = \frac{A\left[1 - \frac{\vartheta}{2}\right]G}{A + t_w t_s + \frac{2NLt_s}{\theta}}$$

$$G = \frac{2(\sinh Q \cos Q + \cosh Q \sin Q)}{\sinh(2Q) + \sin(2Q)}$$

$$N = \frac{\cosh(2Q) - \cos(2Q)}{\sinh(2Q) + \sin(2Q)}$$

$$Q = \frac{\theta}{2}$$

$$\theta = \left[3(1 - \vartheta^2)\right]^{\frac{1}{4}}.M$$

$$M = \frac{L}{\sqrt{Rt_S}}$$

$$A = A_S\left(\frac{R}{R_S}\right)$$