

## Dynamic load concentration caused by a break in a Lamina with viscoelastic matrix

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**Abstract.** The effect of cutting off fibers on transient load in a polymeric matrix composite lamina was studied in this paper. The behavior of fibers was considered to be linear elastic and the matrix behavior was considered to be linear viscoelastic. To model the viscoelastic behavior of matrix, a three parameter solid model was employed. To conduct this research, finite difference method was used. The governing equations were obtained using Shear-lag theory and were solved using boundary and initial conditions before and after the development of break. Using finite difference method, the governing integro-differential equations were developed and normal stress in the fibers is obtained. Particular attention is paid the dynamic overshoot resulting when the fibers are suddenly broken. Results show that considering viscoelastic properties of matrix causes a decrease in dynamic load concentration factor and an increase in static load concentration factor. Also with increases the number of broken fibers, trend of increasing load concentration factor decreases gradually. Furthermore, the overshoot of load in fibers adjacent to the break in a polymeric matrix with high transient time is lower than a matrix with lower transient time, but the load concentration factor in the matrix with high transient time is lower.

**Keywords:** viscoelasticity; load concentration factor; transient stress; unidirectional lamina; finite difference method; break

### 1. Introduction

Today, due to the widespread use of composite materials in different industries, having enough and accurate knowledge of how stress distributes in these materials is vital. When in a composite material one or more of the fibers get ruptured or a crack develops, the amount of load that these fibers carried before rupture, is transferred to adjacent fibers and results in normal stress concentration which is quite temporary. On the other hand, polymeric materials have a widespread application in industries as a composite materials matrix. Polymeric materials possess a significant viscoelastic property which results in many changes in their mechanical properties under the passage of time or rate of loading (Shaw and MacKnight 2005).

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In the past, few researches were conducted on the analysis of transient stress due to break in fibers. One of the first studies in this field was conducted by Landis and McMeeking (1999). They analyzed stress in a composite material containing a broken fiber using a model close to shear-lag theory. In this analysis, the behavior of the matrix was considered complete elastoplastic. Beyerlein (2000) investigated the effect of pre-existing breaks in the fibers on the time dependent stress distribution and displacements in a unidirectional fiber composite material under constant in plane tension. He employed a Newtonian viscos model to calculate the shear deformation behavior of matrix and to calculate creep in it, meanwhile the fibers were considered to possess time-independent elastic properties. Balacó de Morais (2001) developed a model to predict the shear stress distribution along the broken fiber in a unidirectional fibrous composite material. In this model, it is assumed that the matrix behaves as elastic-perfectly plastic.

Some studies have been done in the field of dynamic stress of electro-elastic interaction and fracture behaviors of a cracked piezoelectric laminated structure. Wang and Noda (2001) studied the electro-elastic fracture problem for a laminate with two layers of piezoelectric material bonded to an elastic layer and investigated numerical values of the crack tip fields under transient electromechanical loading. In the other study, the finite element method has been used by Martines and Artemev (2009) to study the effect of fiber damage on the performance of active fibrous composite with piezoelectric fibers. Itou (2007) examined transient dynamic stresses around two rectangular cracks in a nonhomogeneous interfacial layer sandwiched between two dissimilar elastic half-spaces that included a ceramic half-space and a steel half-space. Su *et al.* (2007) studied the problem of interface cracks between dissimilar magneto-electro-elastic strips under out-of-plane mechanical and in-plane magneto-electrical impacts by using the integral transform and the Cauchy singular integral equation methods. The effects of the crack configuration and the main constitutive parameters of the magneto-electro-elastic materials on the dynamic response are examined. Aboudi (2013) presented a continuum model which is capable of generating the transient electroelastic field in piezoelectric composites of periodic microstructure, caused by the sudden appearance of localized defects. Several applications are presented for the sudden formation of cracks in homogeneous and layered piezoelectric materials which are subjected to various types of electromechanical loading, and for the sudden appearance of a cavity.

In the following, we can pointed to some works about transient stress. Rizk (2008) examined the transient thermal stress crack problem for two bonded dissimilar materials subjected to a convective cooling on the surface containing an edge crack perpendicular to the interface. Khalili *et al.* (2009) studied free and forced vibration of multilayer composite circular cylindrical shells under transverse impulse load as well as combined static axial loads and internal pressure. In the analysis of transient dynamic response, the impulse load was in the form of sine pulse, which is applied on a rectangular area and the effect of fiber orientation, axial load, internal pressure and some of the geometrical parameters on the time response of the shells has been investigated. Wünsche and Zhang (2010) developed a spatial symmetric time-domain boundary element method to study transient elastodynamic analysis of two-dimensional, piecewise homogeneous, anisotropic and linear elastic solids containing interior and interface cracks which subjected to impact loading. The spatial discretization is performed by a symmetric Galerkin-method. Chaudhuri (2011) employed an eigenfunction expansion technique to derive hitherto unavailable three-dimensional asymptotic stress field in the vicinity of the front of a semi-infinite through-thickness crack weakening an infinite transversely isotropic unidirectional fiber reinforced composite plate, of finite thickness and subjected to far-field mode I/II loadings. Souad *et al.* (2013) investigated the stress concentration factor in a fibre reinforced composite material with ceramic matrix. Using

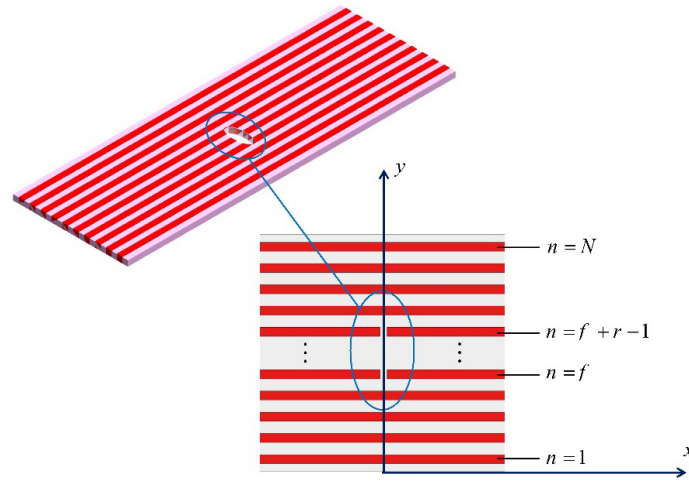


Fig. 1 Numbering of fibers in a lamina with a break

finite element method, they calculated the stress concentration factor for crack growth in the ceramic matrix and fiber-matrix interface.

Earlier studies in this field were conducted to investigate the stress concentration factors in steady state and few studies have been done about transient stress concentration factors. Furthermore, in the all of performed researches about polymeric matrix composite materials, the behavior of matrix has been assumed to be elastic. Whereas polymeric matrices possess featured viscoelastic behavior, therefore transient normal stress due to the sudden break of fibers in the polymeric composite materials with considering viscoelastic properties of the matrix has been chosen as the subject of this analysis.

## 2. Governing differential equations and assumptions

In this analysis, a filamentary lamina composite plate has been considered under tensile loading, as shown in Fig. 1, which one or more of fibers has been ruptured.

As shown in Fig. 1, the break is considered to be in the middle of the lamina, symmetrically. For further simplification, the cross sectional area of fibers was considered to be square. Considering the polymeric matrix of lamina and also low tensile modulus ratio of matrix rather than the fibers, shear-lag model seems to be reasonable. So, unidirectional fibers carry normal loads and imbed in a matrix which carries only shear. Lamina is considered to be under in plane tensile load. Moreover:

- Both matrix and fibers are considered homogenous.
- Uniform loading is assumed at infinity in the direction of fibers.
- Small deflection elasticity theory is used.
- A three-parameter solid model was used to model the viscoelastic behavior of matrix.
- A complete bond exists between the matrix and fiber.
- Fibers possess linear elastic behavior until rupture.

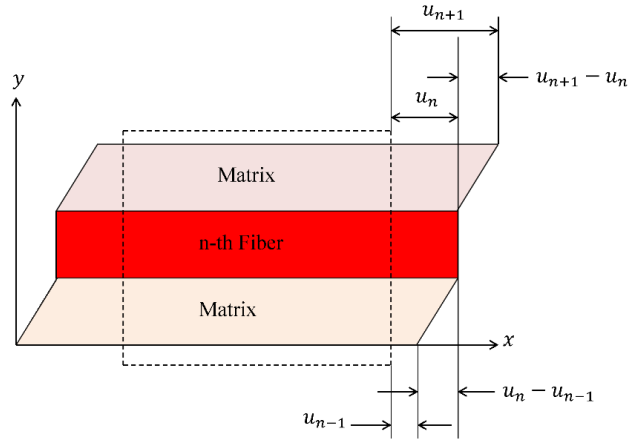


Fig. 2 Displacement in the fiber and surrounding matrix bays

Normal stresses and loads in each fiber ( $n$ -th fiber) is obtained from Eq. (1)

$$\sigma_n = E_f \varepsilon_n = E_f \frac{du_n}{dx} \rightarrow p_n = E_f A_f \frac{du_n}{dx} \quad (1)$$

Where  $u_n$  and  $p_n$  are displacement and tensile load in  $n$ -th fiber. Shear stress in the viscoelastic matrix (assuming that the lateral displacement is not function of  $x$ ) will be determined by Eq. (2)

$$\tau_{xy} = \int_0^t G(t-\zeta) \frac{\partial \gamma}{\partial \zeta} d\zeta \quad (2)$$

Where  $G$  is the relaxation function of matrix shear modulus (Brinson and Brinson 2008). By substituting  $\gamma = \frac{\partial u}{\partial y}$  and according to Fig. 2, the finite difference form of shear stress in the matrix bays between  $n$ -th fiber and  $(n-1)$ -th fiber in terms of fiber displacement and the space between them, i.e.,  $d$ , will be according to Eq. (3) and the shear stress between  $n$ -th and  $(n+1)$ -th fiber will be according to Eq. (4).

$$(\tau_{xy})_{n,n-1} = \frac{1}{d} \int_0^t G(t-\zeta) \left[ \frac{\partial}{\partial \zeta} \{u_n - u_{n-1}\} \right] d\zeta \quad (3)$$

$$(\tau_{xy})_{n+1,n} = \frac{1}{d} \int_0^t G(t-\zeta) \left[ \frac{\partial}{\partial \zeta} \{u_{n+1} - u_n\} \right] d\zeta \quad (4)$$

To model the viscoelastic properties of matrix, viscoelastic three-parameter solid model has been employed, as shown in Fig. 3.

Dimensionless relaxation modulus of this model is defined according to Eq. (5) (Brinson and Brinson 2008)

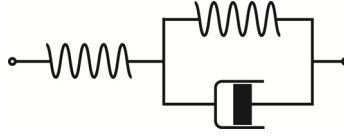
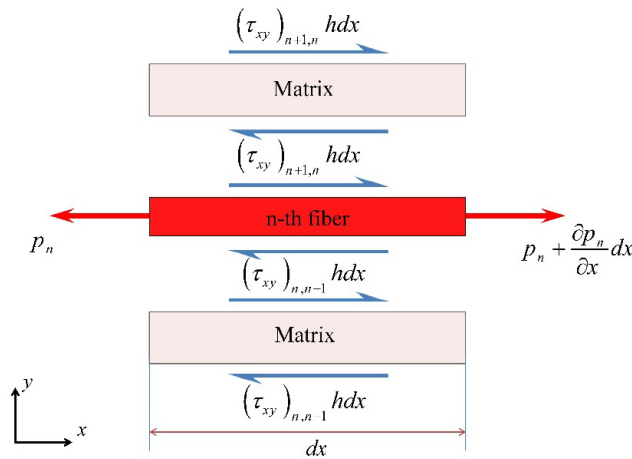


Fig. 3 Viscoelastic three-parameters solid model

Fig. 4 The applied forces on the  $n$ -th fiber

$$G(t) = \frac{G_1}{G_0} + \frac{G_2}{G_0} e^{-t/\zeta} \quad (5)$$

where

$$G_0 = G_1 + G_2 \quad \& \quad \zeta = \frac{\eta}{G_0} \quad (6)$$

According to Fig. 4 and by using shear-lag theory, equilibrium equations of  $n$ -th fiber in the moment of rupture were obtained according to Eq. (7).

$$\frac{\partial p_n}{\partial x} dx + \{(\tau_{xy})_{n+1,n} - (\tau_{xy})_{n,n-1}\} h dx = m dx \frac{\partial^2 u_n}{\partial t^2} \quad (7)$$

Where,  $h$  is thickness of lamina and  $m$  is mass per unit length of the fibers. It's important to know that the shear stresses in the free edges of lamina are equal to zero. Therefore equation of motion for fibers locating at the edges of the lamina is different. In the meantime, by substituting Eqs. (1), (3) and (4) in Eq. (7), the governing equations of fibers in terms of displacements are expressed according to Eq. (8). The governing equations contain an integral term which is due to the viscoelastic properties of the matrix.

$$E_f A_f \frac{\partial^2 u_n}{\partial t^2} + \frac{G_0 h}{d} \int_0^t G(t-\zeta) \times \left[ \frac{\partial}{\partial \zeta} \{u_{n+1} - 2u_n + u_{n-1}\} \right] d\zeta = m \frac{\partial^2 u_n}{\partial t^2} \quad (8)$$

To non-dimensionalize the equations of motion, dimensionless parameters were employed according to Eqs. (9).

$$P_n = \frac{p_n}{p}, \quad U_n = \sqrt{\frac{E_f A_f G_0 h}{p^2 d}} u_n, \quad \xi = \sqrt{\frac{G_0 h}{E_f A_f d}} x, \quad \tau = \sqrt{\frac{G_0 h}{md}} t, \quad S_{xy} = \sqrt{\frac{E_f A_f G_0 h}{p^2 d}} \tau_{xy}, \quad (9)$$

So, the equation of motion is written this way

$$\frac{\partial^2 U_n}{\partial \xi^2} + \int_0^\tau G(\tau - \zeta) \frac{\partial(U_{n+1} - 2U_n + U_{n-1})}{\partial \zeta} d\zeta = \frac{\partial^2 U_n}{\partial \tau^2} \quad (10)$$

Where the initial conditions are as follow

$$\frac{\partial U_n(\xi, 0)}{\partial \tau} = 0 \quad \& \quad P_n(\xi, 0) = \frac{\partial U_n(\xi, 0)}{\partial \xi} = 1 \quad (11)$$

$$U_n(\xi, 0) = 0 \quad n < f \quad \text{or} \quad n > f + r \quad (12)$$

$$P_n(0, \tau) = \frac{\partial U_n(0, \tau)}{\partial \xi} = 0 \quad f \leq n \leq f + r \quad (13)$$

$$P_n|_{\xi \rightarrow \infty} = 1 \quad (14)$$

### 3. Finite difference method

To solve the governing equation of motion, explicit finite difference method was used (Causon and Mingham 2010, Thomas 1995). The governing integro-differential equation is a second-order equation and has two variables of position and time. The fiber length is divided to equal divisions and the same divisions is done for time variable. Each fiber is divided to  $n_z$  equal divisions, that each division has a length of  $\Delta \xi$ . In a way that

$$L = n_z \Delta \xi, \quad \xi = (i-1) \Delta \xi \quad (15)$$

Where  $i$  is the number of each division. Each step of time after occurrence the break is demonstrated by  $j$  in a way that

$$T = n_t \Delta \tau, \quad \tau = (j-1) \Delta \tau \quad (16)$$

In the Eq. (16),  $T$  is the total amount of time after rupture, and  $n_t$  is the number of time divisions. Now, all terms of Eq. (10) are transformed to finite difference terms and substituted in the main equation. The dimensionless second-order differential of displacement in the form of central difference around point  $i$  is

$$\frac{\partial^2 U_n^{i,j}}{\partial \xi^2} = \frac{U_n^{i+1,j} - 2U_n^{i,j} + U_n^{i-1,j}}{\Delta \xi^2} \quad (17)$$

Where,  $U_n^{i,j}$  is representative of displacement in  $n$ -th fiber at a point  $\xi = (i-1)\Delta\xi$  far from the middle of the fiber at  $\tau = (j-1)\Delta\tau$  time after occurrence the break. The second-order differential of time as central difference about node  $j$  will be written according to Eq. (18) (Christensen 1982)

$$\frac{\partial^2 U_n^{i,j}}{\partial \tau^2} = \frac{U_n^{i,j+1} - 2U_n^{i,j} + U_n^{i,j-1}}{\Delta \tau^2} \quad (18)$$

The integral term in Eq. (10) will be expressed in finite difference form as Eq. (19). The first order derivative terms in this equation was written in form of backward difference

$$\Delta S_{xy} = \int_0^\tau G(\tau - \zeta) \left[ \frac{U_n^{i,j} - U_{n+1}^{i,j-1}}{\Delta \xi} - 2 \frac{U_n^{i,j} - U_n^{i,j-1}}{\Delta \xi} + \frac{U_{n-1}^{i,j} - U_{n-1}^{i,j-1}}{\Delta \xi} \right] d\zeta \quad (19)$$

The same time divisions has been used to transform the integral term, i.e.

$$\Delta \xi = \Delta \tau, \quad \tau = (j-1)\Delta \tau, \quad \zeta = k\Delta \tau \quad (20)$$

By numerical evaluating of the integral terms using rectangular method and by simplifying more, one can write

$$\begin{aligned} & \int_0^\tau G(\tau - \zeta) \frac{\partial(U_{n+1} - 2U_n + U_{n-1})}{\partial \xi} d\zeta \\ &= \sum_{k=0}^{j-1} \left\{ G((j-k)\Delta \tau) \times [(U_{n+1}^{i,k+1} - 2U_n^{i,k+1} + U_{n-1}^{i,k+1}) - (U_{n+1}^{i,k} - 2U_n^{i,k} + U_{n-1}^{i,k})] \right\} \end{aligned} \quad (21)$$

If  $M$  and  $E$  are considered to be as a unit diagonal matrix, the governing Eq. (10) can be written in index form according to Eq. (22) and by substituting Eqs. (17), (18) and (21).

$$\begin{aligned} & E_{n,n} \frac{U_n^{i+1,j} - 2U_n^{i,j} + U_n^{i-1,j}}{\Delta \xi^2} \\ &+ \sum_{k=0}^{j-1} \left\{ G((j-k)\Delta \tau) \times [(U_{n+1}^{i,k+1} - 2U_n^{i,k+1} + U_{n-1}^{i,k+1}) - (U_{n+1}^{i,k} - 2U_n^{i,k} + U_{n-1}^{i,k})] \right\} \\ &= M_{n,n} \frac{U_n^{i,j+1} - 2U_n^{i,j} + U_n^{i,j-1}}{\Delta \tau^2} \end{aligned} \quad (22)$$

By assuming  $s = \left( \frac{\Delta \tau}{\Delta \xi} \right)^2$  and after simplifying Eq. (22), we have

$$\begin{aligned} U_n^{i,j+1} &= \frac{\Delta \tau^2}{M_{n,n}} \sum_{k=0}^{j-1} G((j-k)\Delta \tau) \times [(U_{n+1}^{i,k+1} - 2U_n^{i,k+1} + U_{n-1}^{i,k+1}) - (U_{n+1}^{i,k} - 2U_n^{i,k} + U_{n-1}^{i,k})] \\ &+ 2 \left( 1 - \frac{sE_{n,n}}{M_{n,n}} \right) U_n^{i,j} + \frac{sE_{n,n}}{M_{n,n}} U_n^{i+1,j} + \frac{sE_{n,n}}{M_{n,n}} U_n^{i-1,j} - U_n^{i,j-1} \end{aligned} \quad (23)$$

Eq. (23) is the equation of displacement in the dimensionless distance  $\xi = i \Delta \xi$ , and after the dimensionless time step  $\tau = j \Delta \tau$ . Therefore to start solving and calculation of displacement  $U_n^{i,j+1}$  in each location of fibers and in a new time step, unknowns need to be calculated by using boundary and initial conditions. The displacement of fibers in the first two steps time is calculated using initial conditions. The first step is considered before the break occurrence of fibers. At this time, all fibers are intact and the exerted tensile stress is uniform. So, according to Eq. (11), one can write

$$P_n(\xi, 0) = \frac{\partial U_n(\xi, 0)}{\partial \xi} = \frac{U_n^{i+1,1} - U_n^{i,1}}{\Delta \xi} = 1 \quad (24)$$

With this assumption that lamina is symmetric and displacement of middle fibers is equal to zero, in the first time step  $i = 1$ ,  $U_n^{1,1} = 0$  and displacement of all points of fibers in this step is obtained using Eq. (25).

$$U_n^{2,1} = \Delta \xi, \quad U_n^{3,1} = 2\Delta \xi, \dots, \quad U_n^{N,1} = (n_z - 1)\Delta \xi, \quad \text{for } 1 \leq n \leq N \quad (25)$$

The second step ( $j=2$ ), is the moment of break occurrence in the fibers. Using initial conditions of relation (11) and expanding the velocity term in the form of forward difference, the displacement in various points can be expressed as Eq. (26).

$$\frac{\partial U_n(\xi, \Delta \tau)}{\partial \tau} = \frac{U_n^{i,2} - U_n^{i,1}}{\Delta \tau} = 0 \Rightarrow U_n^{i,2} = U_n^{i,1} \quad \text{for } 1 \leq n \leq N, \quad 1 \leq i \leq n_z \quad (26)$$

According to Eq. (12), the amount of displacement in intact fibers and in the middle of lamina (in  $\xi = 0$ ) is equal to zero. Therefore

$$U_n^{0,j} = 0 \quad n < f \quad \text{or} \quad n > f + r \quad (27)$$

To obtain displacement of broken fibers in the middle of the lamina, (in location of break), boundary condition of relation (13) is used. Using forward difference form around point  $i = 1$ , one can write

$$P_n(0, \tau) = \frac{\partial U_n(0, \tau)}{\partial \xi} = \frac{U_n^{2,j} - U_n^{1,j}}{\Delta \xi} = 0 \Rightarrow U_n^{1,j} = U_n^{2,j}, \quad f \leq n \leq f + r \quad (28)$$

In this way, by having Eq. (23) and boundary and initial conditions of Eqs. (25) to (28), one can obtain the displacements in every location of fibers and at various times (From  $j = 3$  and so on). Load concentration factor has been defined as ratio of tensile load in the intact fiber adjacent to broken fiber(s) at break location to load value in the same fiber which far away from break location.

$$K_r = \frac{P_{m,f+r+1} \Big|_{(0,\tau)}}{P_{m,f+r+1} \Big|_{(\infty,\tau)}} = P_{m,f+r+1} \Big|_{(0,\tau)} = \frac{\partial U_{m,f+r+1}}{\partial \xi}(0, \tau) \quad (29)$$



#### 4. Results and discussion

To investigate the effect of viscoelasticity of matrix on transient normal stress in the fibers which adjacent to the break, three types of materials with different viscoelastic properties were used as a composite material matrix.

Environmental conditions like temperature affect the viscoelastic properties of materials significantly and temperature increase or decrease could change and shift the transient time in polymeric materials. Fig. 5 shows typical relaxation modulus curves obtained from the creep experiments. For three typical temperatures in this study, the material exhibited time-temperature superposition with only horizontal shifting required to construct the master curves. The initial and the equilibrium modulus of these three cases were assumed to be equal. Therefore to investigate different states of matrix, as shown in Fig. 5, three type of materials with low, medium and high

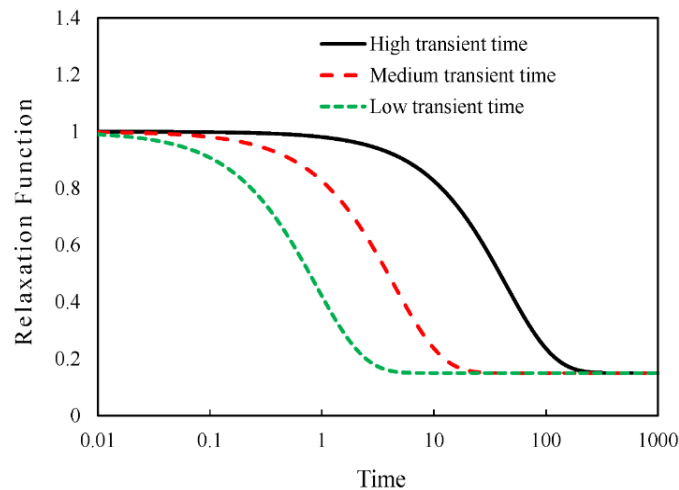


Fig. 5 Three typical state of matrix relaxation function

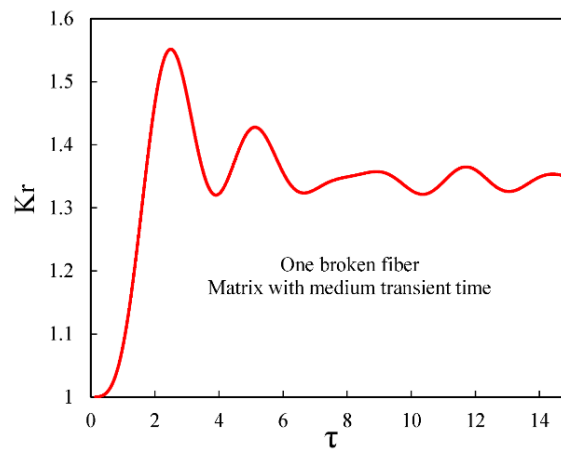


Fig. 6 Transient load concentration factor in a lamina with one broken fiber

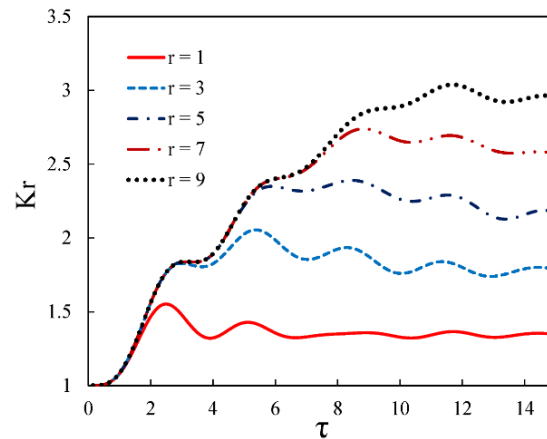


Fig. 7 The effect of broken fibers number on the load concentration factors for a matrix with high transient time

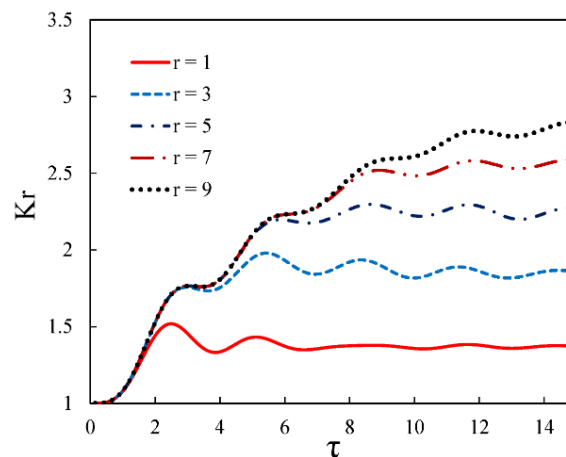


Fig. 8 The effect of broken fibers number on the load concentration factors for a matrix with medium transient time

transient time were used to consider the viscoelastic property of the matrix in different conditions on transient load of fibers adjacent to break.

To investigate this problem, static and dynamic load concentration factors due to one or more fibers being broken are determined. In Fig. 6, load concentration factor in fiber adjacent to the break in a lamina with viscoelastic matrix (with medium transient time) in which a break has occurred in one of its fibers, has been depicted. At first the load concentration factor overshoots and increases significantly. With the passage of time, load concentration factor decrease a little which is more than the state before the occurrence of break.

In this paper the amount of overshoot in the load concentration factor is called the dynamical load concentration factor and its amount at the steady state is called static load concentration factor. In Figs. 7-9, the effect of increase in number of broken fibers, i.e.,  $r$ , on the load concentration factors of first adjacent intact fiber near the break was shown. Fig. 7 shows the load concentration

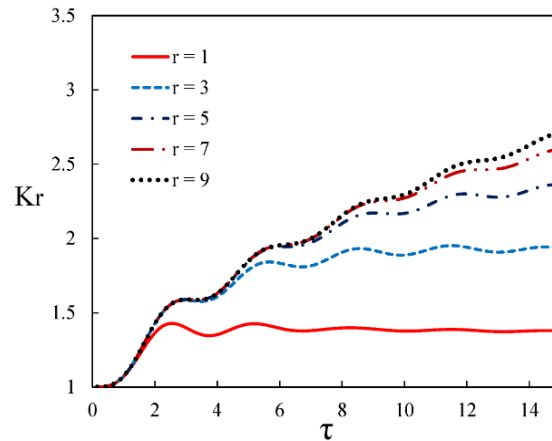


Fig. 9 The effect of broken fibers number on the load concentration factors for a matrix with low transient time

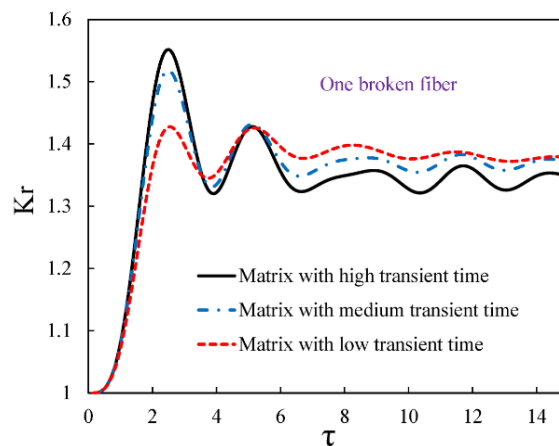


Fig. 10 The effect of transient time of matrix with one broken fiber on the load concentration factors

factors in a lamina with a viscoelastic matrix and high transient time. Same investigations have been shown for a viscoelastic matrix by medium transient time in Fig. 8 and low transient time in Fig. 9. By examining these three figures, one can conclude that by increasing the number of broken fibers in a lamina with viscoelastic matrix, the static and dynamic load concentration factors increase. What which is obviously seen in these three states is the fact that by increasing the number of broken fibers, the value of static load concentration factors approaches to the value of dynamic load concentration factor and the decrease in the load concentration factor after the first shock is lower, considerably.

In Fig. 10, the effect of transient time of viscoelastic matrix with one broken fiber has been examined on the load concentration factors. The most dynamic load concentration factor is observed in the viscoelastic matrix with high transient time. Meanwhile by using a viscoelastic matrix with low transient time, the value of dynamic load concentration factor gets less than other

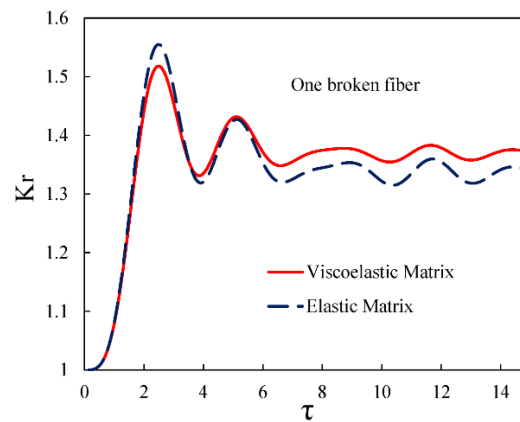


Fig. 11 Comparison between elastic and viscoelastic matrix with one broken fiber

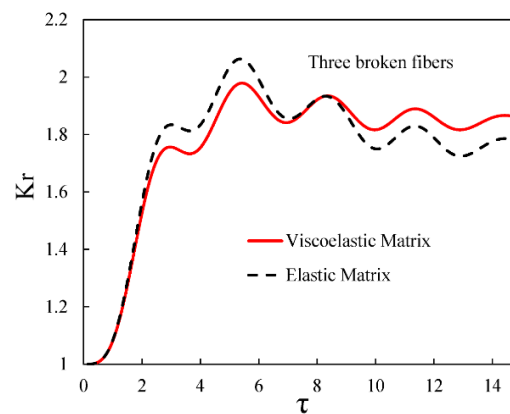


Fig. 12 Comparison between elastic and viscoelastic matrix with three broken fibers

states. This trend is quite contrary on the static load concentration factor. In a lamina which its matrix has a higher transient time, the static load concentration factor is less than other cases and for a lamina with lower transient time, this factor is higher.

In the following, the load concentration factor in terms of dimensionless time in a lamina with viscoelastic matrix has been evaluated with respect to a lamina with elastic matrix. This comparison is shown for a lamina with one, three and five broken fibers in Figs. 11, 12 and 13, respectively.

According to the depicted results, the dynamic load concentration factor resulted from viscoelastic property of the matrix is less than the elastic matrix and with increase the number of broken fibers, no change in this trend is seen.

In Table 1, the value of dynamic load concentration factors are shown for three different types of matrix (high, medium and low transient time) in terms of number of broken fibers. As can be seen, the dynamic load concentration factors in the viscoelastic matrix with low transient time is lower. Increase in the number of broken fibers increase this factor, although the rate of increase of this factor reduces with the number of broken fibers.

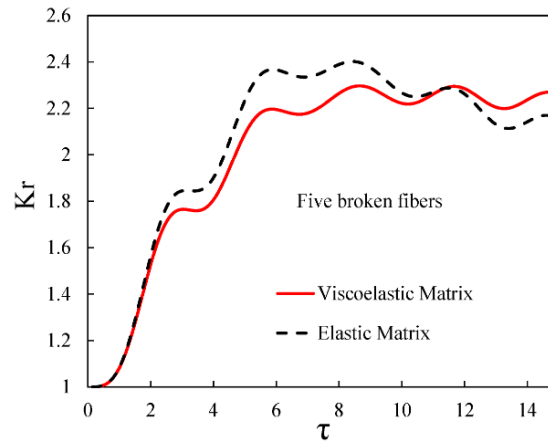


Fig. 13 Comparison between elastic and viscoelastic matrix with five broken fibers

Table 1 The effect of broken fibers number on dynamical load concentration factors

Number of broken fibers	Dynamical load concentration factor		
	High transient time	Medium transient time	Low transient time
1	1.5505	1.5172	1.4276
3	2.0543	1.9789	1.9515
5	2.3619	2.2968	2.2895
7	2.7395	2.5979	2.5898
9	3.0387	2.8337	2.7053

## 5. Conclusions

In this paper, the effect of viscoelasticity of polymeric matrix composite lamina on the load concentration factor due to a break in fibers has been investigated. The results showed that the load concentration factor overshoots greatly and increases, initially. With the passage of time the load concentration factor reduces a little bit with respect to the cases which were before the occurrence the break. The results also reveal that considering viscoelastic property of the matrix will result in reduction of dynamic load concentration factor and increase of static load concentration factor with respect to the cases that the matrix was considered elastic. With increase of broken fibers no change was seen in this trend. In addition, load mutation in the fibers adjacent to the break for the matrix with a high transient time was greater than the matrix with a low transient time, but steady-state load concentration factor for the matrix with a high transient time was comparatively less than the others.

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