Steel and Composite Structures, *Vol. 18, No. 5 (2015) 1197-1214*
DOI: http://dx.doi.org/10.12989/scs.2015.18.5.1197

Nonlinear time-varying analysis algorithms for modeling the behavior of complex rigid long-span steel structures during construction processes

Li-Min Tian* and Ji-Ping Hao

School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an 710055, P.R. China

(*Received January 25, 2014, Revised September 25, 2014, Accepted November 12, 2014*)

Abstract. There is a great difference in mechanical behavior between design model one-time loading and step-by-step construction process. This paper presents practical computational methods for simulating the structural behavior of long-span rigid steel structures during construction processes. It introduces the positioning principle of node rectification for installation which is especially suitable for rigid long-span steel structures. Novel improved nonlinear analytical methods, known as element birth and death of node rectification, are introduced based on several calculating methods, as well as a forward iteration of node rectification method. These methods proposed in this paper can solve the problem of element's 'floating' and can be easily incorporated in commercial finite element software. These proposed methods were eventually implemented in the computer simulation and analysis of the main stadium for the Universiade Sports Center during the construction process. The optimum construction scheme of the structure is determined by the improved algorithm and the computational results matched well with the measured values in the project, thus indicating that the novel nonlinear time-varying analysis approach is effective construction simulation of complex rigid long-span steel structures and provides useful reference for future design and construction.

Keywords: complex rigid long-span steel structures; construction mechanics; positioning principle; nonlinear time-varying analysis; node rectification; finite element analysis

1. Introduction

With long-span steel structures constantly emerging, several unique mechanical problems in construction process of structures are widely valued. It is broadly accepted that different construction procedures lead to different internal force and deformation distributions (Choi *et al.* 1992, Epaarachchi *et al.* 2002) and there is a great difference in mechanical behavior between design model one-time loading and step-by-step construction process (Cui *et al.* 2006, Wang *et al.* 2004, Mari 2000, Rosowsky *et al.* 1994). In order to meet the design requirement, it is necessary to analyze the structure during the construction process by using the method of construction mechanics.

The construction mechanics belongs to slow time-varying mechanics and combines structural mechanics and construction technology (Huang *et al.* 2007, Naumov 1994, Wang 2000). Its main

*Corresponding author, Ph.D., E-mail: tianlimin701@163.com

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

feature is that the research objects such as geometrical profiles, physical properties and boundary conditions change with the duration of construction (Case *et al*. 2014, Cruz *et al*. 1998). Several analysis methods for structural construction were proposed, and can be broadly classified into three categories: general finite element method, time-varying element method and topology method (Agkathidis and Brown 2013, Hajjar and AbouRizk 2002, Mohamed and AbouRizk 2005, Oakley and Knight 1995). Commercial software based on the finite element method is the most effective numerical procedures to solve the problem of construction mechanics. However, this results in inevitable singular error, since the long-span steel structures have some structural characteristics like long cantilever and more complex properties. There have been not enough researches on construction mechanics.

In this paper, an improved positioning principle for installation is developed, based on the theory of nonlinear beam-column elements and positioning for installation in bridge engineering. It is especially suitable for rigid long-span steel structures. Several computational methods for complex rigid steel structures are intensively studied, followed by the new practical calculating methods. All these methods were utilized in the computer simulation of main stadium for the Universiade Sports Center during the construction process and compared with the experimental results thereafter. It is proved that the proposed nonlinear time-varying analytical methods are accurate and reliable for modeling the steel structures during the construction process.

2. Theory of construction mechanics

It is necessary to consider the influence of nonlinearities in the computer simulation and analysis of complex structures during the construction process (Li 1994, Kanemitsu *et al*. 2001, Zijl *et al*. 2001). Generally, material is in the elastic state and the effect of elasto-plastic has been taken into account appropriately in the safety factor. For this reason, it is feasible to mainly consider the geometrical nonlinearity in the analysis of long-span steel structures during the construction process. Nonlinear problems are primarily solved by iterative and incremental method (Matthies and Strang 1979). Loads are usually divided into several steps in incremental method. In each step, the load increment ΔP corresponds to a displacement increment $\Delta\delta$. The final displacements are obtained via superposition and the numerical results of each load stage can also be obtained. It is proved that incremental method is an effective method for solving nonlinear problems during the construction process. Finite element equilibrium equations for geometric nonlinear problems are obtained from the principle of virtual work as

$$\psi(\delta) = \int [\overline{B}]^T \sigma dV - P = 0 \tag{1}$$

and

$$[\overline{B}] = [B_0] + [B_L] \tag{2}$$

in which,

$[\overline{B}]$ is the transformation matrix of displacement and strain, $[B_0]$ is the matrix of linear strain analysis and $[B_L]$ is the matrix caused by nonlinear deformation and it is related to δ.

In order to solve the Eq. (1) using Newton-Raphson iterative algorithm, the tangent stiffness matrix of element is obtained as follow

$$[K_T]^{NL} = [K_0]^L + [K_L]^{NL} + [K_\sigma]^{NL} \tag{3}$$

where

$[K_0]^L$ is the linear stiffness matrix, $[K_L]^{NL}$ is the matrix of initial displacement and $[K_\sigma]^{NL}$ is the geometric stiffness matrix.

Because the construction of long-span steel structures is a dynamic procedure, the influence of nonlinearity during the construction process is described by the method of motion. Consider the configuration of previous time step, the state of the next time step is obtained by Updated Lagrange Formulation (*U.L.*). Based on this formulation, the integration method of element stiffness matrix is updated after deformation and the nonlinearity of higher moment is ignored. The tangent stiffness matrix of element can be expressed as

$$[K_T]^{NL} = [K_0]^L + [K_\sigma]^{NL} \tag{4}$$

It is noted that the coupling effects between most of the commercial finite element software and geometrical nonlinearity of construction mechanics are available by using Updated Lagrange Formulation. The tangent stiffness matrix of the $i+1$ time step during the construction process can be described as follow

$$[K_{Ti+1}]^{NL} = [K_{Ti}]^{NL} + [K_\Delta]^{NL} \tag{5}$$

where $[K_\Delta]^{NL}$ represents the influence of newly increased element stiffness matrix. Therefore, finite element equilibrium equations in each time step during the construction process can be written as

$$[K_i]\delta_i = P_i \tag{6}$$

The internal forces and deformations of the structure during the construction process are obtained by superimposing the results for each construction stage.

3. Analysis algorithm of construction mechanics

3.1 Positioning principle

Xiang (2001) proposed three positioning principles for installation in bridge engineering, as shown in Fig. 1.

Since long-span steel structures have long cantilever and many other complex components in practical engineering application, the structure is unstable by using positioning principles for installation in bridge engineering directly. At critical condition, it cannot form a stable system and collapses. For this reason, an improved positioning principle named the positioning principle of node rectification for installation is developed, which is suitable for rigid long-span steel structures. In this approach, temporary supports are set up at the suitable position and newly increased nodes are updated according to the design coordinates. Then, the final deformations are obtained by removing temporary supports after the installation is completed, as shown in Fig. 2.

According to the topology of long-span steel structures, temporary supports should be set up at the terminal of dip and cantilever, as well as the position of top corner, as plotted in Fig. 3.

3.2 Nonlinear analysis algorithm

It is evident that different construction processes of structures lead to different internal forces and deformations distribution. There are several key mechanical problems in simulating construction process of structures, such as how to simulate time-variation structures and boundary conditions, and how to consider time-dependent materials. For this reason, construction process should be simulated by nonlinear time-varying algorithm. As previously mentioned, it is feasible to mainly consider the geometrical nonlinearity in the time-varying analysis of long-span steel

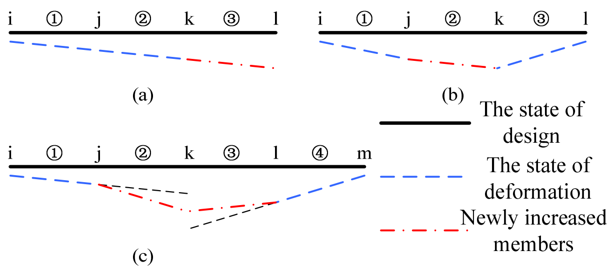

Fig. 1 Positioning principles of newly increased member

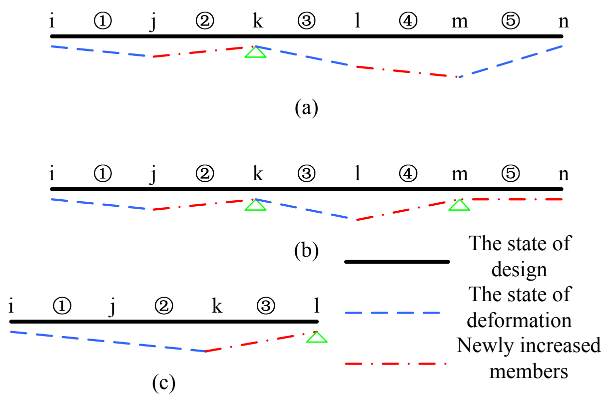

Fig. 2 Positioning principle of node rectification

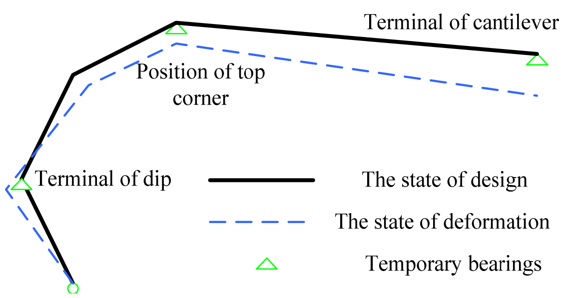

Fig. 3 Selection of location for temporary supports

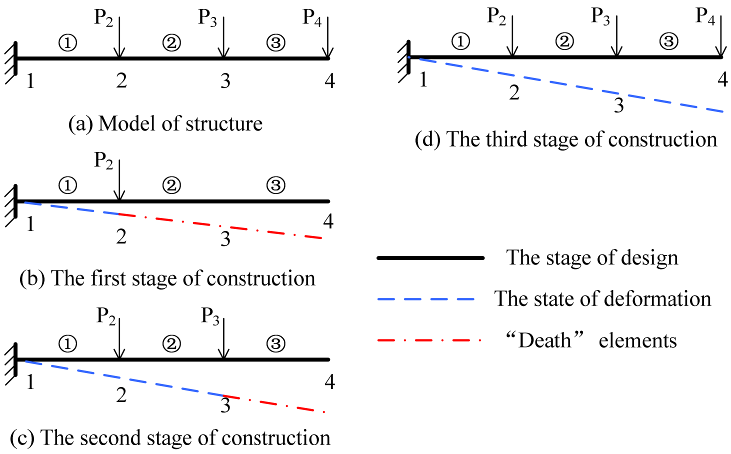

Fig. 4 Basic principle of the method of element birth and death

structures during the construction process. Therefore, nonlinear time-varying algorithm is the description of simulation method based on slow time-varying mechanics in which the nonlinearity is well taken into account during the construction process. Nowadays, the method of element birth and death, based on the theory of incremental finite element, is a common nonlinear analysis algorithm of construction mechanics (Guo and Liu 2008). The basic principle is illustrated in Fig. 4 through a three-segment cantilever beam.

The whole model of the beam is inputted based on the coordinates of design. Then the governing equation is obtained as follows after coordinate transformation for element stiffness matrix

$$\begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{22}^2 & K_{23}^2 & 0 \\ 0 & K_{32}^2 & K_{33}^2 + K_{33}^3 & K_{34}^3 \\ 0 & 0 & K_{43}^3 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{7}$$

in which K_{jm}^i is the submatrix of element stiffness matrix. Respectively, u_j and F_j is the displacement array of node and the load array.

At the beginning of the analysis, the stiffness matrix is multiplied by the coefficient of element birth and death ($\eta = 10^{-n}$, $n \geq 6$) and the load matrix is multiplied by zero. This process is defined as element's 'death'. The governing equation is updated as

$$\begin{bmatrix} \eta K_{11}^1 & \eta K_{12}^1 & 0 & 0 \\ \eta K_{21}^1 & \eta K_{22}^1 + \eta K_{22}^2 & \eta K_{23}^2 & 0 \\ 0 & \eta K_{32}^2 & \eta K_{33}^2 + \eta K_{33}^3 & \eta K_{34}^3 \\ 0 & 0 & \eta K_{43}^3 & \eta K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

The element ① is activated at the first stage of construction while the other elements remain in the state of 'death'. The governing equation is updated as follow

$$\begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1+\eta K_{22}^2 & \eta K_{23}^2 & 0 \\ 0 & \eta K_{32}^2 & \eta K_{33}^2+\eta K_{33}^3 & \eta K_{34}^3 \\ 0 & 0 & \eta K_{43}^3 & \eta K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

After diagonal element is transformed into 1, the loads (P_2) and boundary conditions ($u_1 = 0$) are introduced as

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & K_{22}^1+\eta K_{22}^2 & \eta K_{23}^2 & 0 \\ 0 & \eta K_{32}^2 & \eta K_{33}^2+\eta K_{33}^3 & \eta K_{34}^3 \\ 0 & 0 & \eta K_{43}^3 & \eta K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P_2 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Because the coefficient of element birth and death is a dinky number, the matrix of the Eq. (10) is an ill-conditioned matrix and it can be decomposed into two parts: ‘birth’ element and ‘death’ element. The equation of ‘birth’ element becomes

$$\begin{bmatrix} I & 0 \\ 0 & K_{22}^1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ P_2 \end{bmatrix} \tag{11}$$

The Eq. (11) can be solved as

$$K_{22}^1 u_2 = P_2 \tag{12}$$

On the other hand, the equation of ‘death’ element can be expressed as

$$\begin{bmatrix} K_{22}^1+\eta K_{22}^2 & \eta K_{23}^2 & 0 \\ \eta K_{32}^2 & \eta K_{33}^2+\eta K_{33}^3 & \eta K_{34}^3 \\ 0 & \eta K_{43}^3 & \eta K_{44}^3 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P_2 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

The Eq. (13) can be written as follow by dividing η on both sides

$$\begin{bmatrix} K_{22}^1/\eta+K_{22}^2 & K_{23}^2 & 0 \\ K_{32}^2 & K_{33}^2+K_{33}^3 & K_{34}^3 \\ 0 & K_{43}^3 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P_2/\eta \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

Substituting Eq. (12) into Eq. (14), gives

$$\begin{bmatrix} K_{22}^1/\eta+K_{22}^2 & K_{23}^2 & 0 \\ K_{32}^2 & K_{33}^2+K_{33}^3 & K_{34}^3 \\ 0 & K_{43}^3 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \left(K_{22}^1 u_2\right)/\eta \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

Because the coefficient of element birth and death is a dinky number, $\alpha = 1/\eta$ is a huge number. Then

$$\begin{bmatrix} \alpha K_{22}^1 & K_{23}^2 & 0 \\ K_{32}^2 & K_{33}^2+K_{33}^3 & K_{34}^3 \\ 0 & K_{43}^3 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \alpha K_{22}^1 u_2 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

That means

$$K_{22}^1 u_2 = P_2 \tag{17}$$

It is noted that 'birth' element and 'death' element are in the compatible state of deformation. u_3 and u_4 are 'floating' value caused by the deformation of 'birth' element while 'death' elements have not stress and strain. Surplus construction stages are analyzed by using the same method.

This approach requires newly increased members to install in a particular position after 'floating'. Moreover, the stiffness matrix may become singular because of the element floating problem. In order to solve the above-mentioned problems, the structure is modeled by using step-by-step modeling technique. In other word, the unerected elements do not appear in the stiffness matrix, which need manual intervention in the next step. For this reason, it is difficult to be realized in commercial finite element software. Therefore, these two techniques have distinct limitations. For some problems, these become very time consuming or in some cases these may suffer convergence problems and other difficulties, as described in the previous sections. Hence, an improved algorithm named the element birth and death of node rectification is presented. Based on the positioning principle of node rectification, the problem of element's 'floating' is solved by using temporary supports. Basic principle of the improved algorithm is illustrated in Fig. 5 through the three-segment cantilever beam as described in the previous sections.

As shown, the results of 'birth' elements by using the method known as element birth and death of node rectification are the same as those using the method of element birth and death at the first two stages of construction. However, because of the existence of temporary supports, displacement of node 4 is restrained and the problem of element's 'floating' is solved. Temporary supports still exists when the third stage of construction is analyzed. When beam ③ is activated, the governing equation which considers the coupling of geometrical nonlinearity can be expressed as

$$\begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1+K_{22}^2 & K_{23}^2 & 0 \\ 0 & K_{32}^2 & K_{33}^2+K_{33}^3 & K_{34}^3 \\ 0 & 0 & K_{43}^3 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{18}$$

After diagonal element is transformed into 1, the loads (P_4 which load application on temporary bearing directly has no effect on structure) and boundary conditions ($u_1 = 0$; $u_4 = 0$) are introduced as follow

$$\begin{bmatrix} I & 0 & 0 & 0 \\ K_{21}^1 & K_{22}^1+K_{22}^2 & K_{23}^2 & 0 \\ 0 & K_{32}^2 & K_{33}^2+K_{33}^3 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P_2 \\ P_3 \\ 0 \end{bmatrix} \tag{19}$$

At present, all of the 'death' element is activated and stiffness matrix is no longer singular. It is

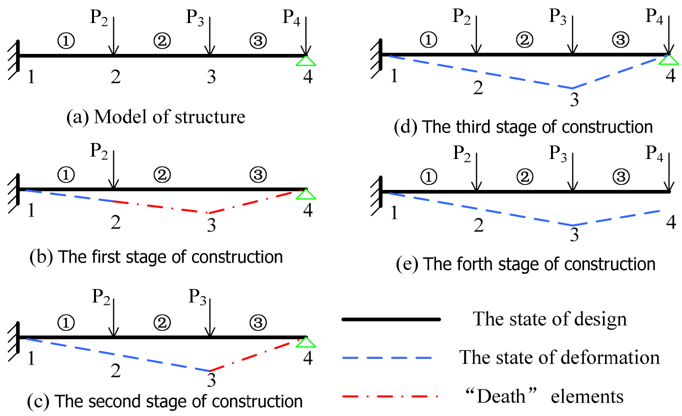

Fig. 5 Basic principle of the method named element birth and death of node rectification

feasible to obtain deformation on this stage of construction directly. Finally, the forth stage of construction (eliminate restriction of node 4) based on the first three stages of construction is analyzed. The governing equation with boundary conditions $u_1 = 0$ can be represented as

$$\begin{bmatrix} I & 0 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{22}^2 & K_{23}^2 & 0 \\ 0 & K_{32}^2 & K_{33}^2 + K_{33}^3 & K_{34}^3 \\ 0 & 0 & 0 & K_{44}^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \tag{20}$$

It seems that deformation of each node can be obtained when the installation is completed. And the governing Eq. (20) needs to be revised when integral lifting of grid structures is simulated. Rigid body displacements still exist because there are not sufficient restraints and may lead to convergence problem. This problem can be solved by increasing temporary boundary constraint when the *element birth and death of node rectification* method is used. Based on the studies mentioned above, the basic analysis procedure for the improved algorithm is proposed in Fig. 6.

4. Determination of pre-set deformation value

It is possible that deformation of accomplished structures disagrees with that of the design when the installation is completed if long-span steel structures with long cantilevers are directly installed according to the position of design. This detrimental effect can be eliminated by analysis of pre-camber value. The determination of pre-camber value is a dynamic procedure which iterates from analysis to pre-set.

Several calculating methods for determining the pre-set deformation value are implemented, such as general iteration method, forward iteration method, backward iteration method and stepped comprehensive iteration method (Xiang 2001). General iteration method doesn't consider the influence of construction process. Forward iteration method is an analysis method of going ahead based on the method of element birth and death. The stiffness matrix may still become singular because of the element floating problem, even though this method takes the influence of

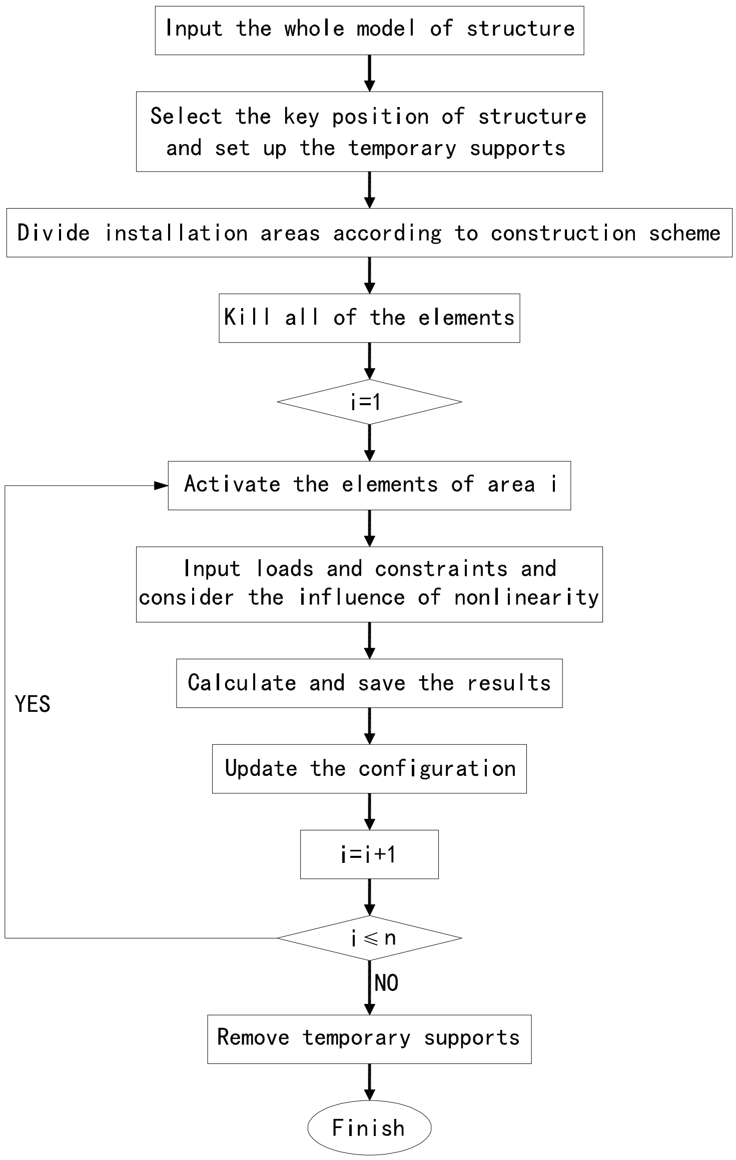

Fig. 6 Basic analysis procedure of the method named element birth and death of node rectification

construction process into account. Backward iteration method can solve the problem of 'floating' by analysis of reverse, but it needs to iterate in each step, therefore it is difficult to be realized in commercial finite element software. Stepped comprehensive iteration method which combines advantages between forward iteration method and backward iteration method also has distinct limitations. First of all, the problem of 'floating' is not solved. Furthermore, it is evident that different stage divisions will lead to different analysis results and inaccurate stage division will lead to large deviation. For this reason, an improved iteration algorithm named forward iteration of node rectification is developed in order to solve the tradeoff between the computational accuracy

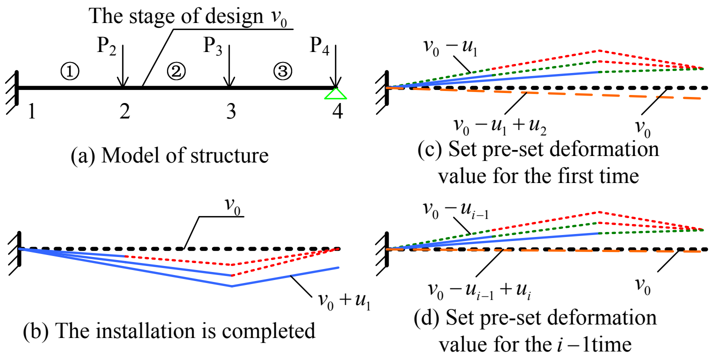

Fig. 7 Basic principle of the method named forward iteration of node rectification

and computation speed. Based on the positioning principle of node rectification, pre-set deformation value can be obtained by analyzing forward (use the *element birth and death of node rectification* method). The basic principle of the improved algorithm is illustrated in Fig. 7 through the three-section cantilever beam as described in the previous sections.

The process of iteration is as follow:

(1) Based on the state of design (v_0), deformation of accomplished structures (u_1) is obtained when the installation is completed by using the method named element birth and death of node rectification.

(2) Pre-set deformation state for the first time ($v_0 - u_1$) is given by exerting reversed deformation ($-u_1$) on the state of design.

(3) Based on the state of pre-set deformation ($v_0 - u_1$), the state of iteration for the second time ($v_0 - u_1 + u_2$) is obtained via the same method.

(4) If the absolute difference between v_0 and $v_0 - u_1 + u_2$ satisfy the relation: $|u_1 - u_2| \le [u]$ ($[u]$ is allowable error), the convergence criterion is satisfied and the iteration is over.

(5) If the absolute difference between v_0 and $v_0 - u_1 + u_2$ does not satisfy the relationship mentioned above, the iteration must continue until the convergence criterion is satisfied. Iterative algorithm can be expressed as

$$h_i = h_{i-1} \pm \delta \left| v_{i-1} - v_0 \right| \tag{21}$$

where h_i and h_{i-1} are the configurations of pre-set, v_{i-1}, which is calculated by h_{i-1}, is the configuration of accomplished structures. For the sake of simplicity, a parameter (δ) which can be described from 0.9 to 1.1 is introduced to incorporate the uncertainties arising from insufficient knowledge of the iteration coefficients. If v_{i-1} is greater than v_0, the Eq. (21) is plus. Conversely, the Eq. (21) is minus.

In the above studies, it is evident that cutting work based on the configuration of design will lead to deviation in factory. For this reason, the configuration of cutting work (h_p) should agree with the configuration of pre-set. And it can be obtained as follows by releasing internal force in the state of pre-set because the configuration of cutting work is the state of zero stress

$$h_p = h_i + [K]^{-1}(-\{F_i\}) \tag{22}$$

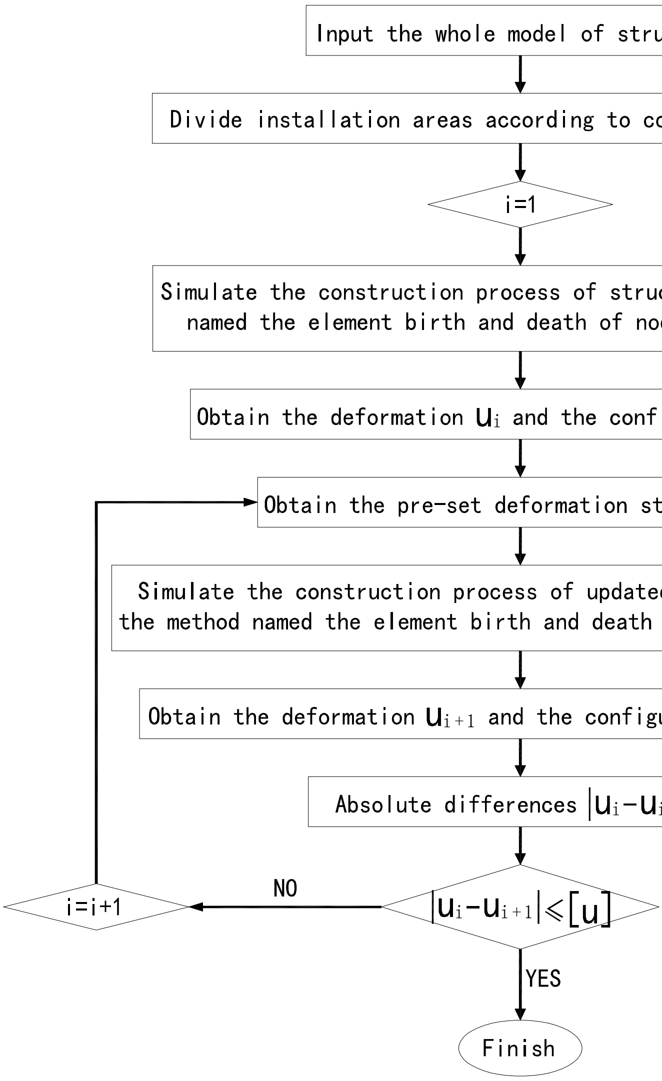

Fig. 8 Basic analysis procedure of the method named forward iteration of node rectification

in which [K] is the element stiffness matrix, {F_i} is the load vector which is equivalent to the internal force and the negative sign indicates the release of internal force.

In addition, maximum pre-set deformation value of structures should not exceed forward iteration deflection under loading condition in order to avoid overlarge horizontal thrust. Based on the above analysis, the basic procedure for the improved iteration algorithm is proposed here (Fig. 8).

5. Verification

In order to validate the two improved algorithms, a simple example is illustrated in this paper.

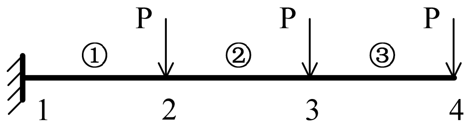

Fig. 9 Cantilever beam

Table 1 Results of analysis

Node numbering	The first stage of construction		The second stage of construction		The third stage of construction	
	Bending moment /kN·m	Deformation /mm	Bending moment /kN·m	Deformation /mm	Bending moment /kN·m	Deformation /mm
1	32.8 [32.8]	0 [0]	103 [103]	0 [0]	211 [211]	0 [0]
2	0 [0]	-1.7 [-1.7]	32.8 [32.8]	-6.0 [-6.0]	103 [103]	-13.1 [-13.1]
3	— [—]	— [—]	0 [0]	-17.6 [-17.6]	32.8 [32.8]	-41.9 [-41.8]
4	— [—]	— [—]	— [—]	— [—]	0 [0]	-76.3 [-52.7]

* Note: The brackets are results of the improved algorithm and '—' means the value does notexist

A three-section cantilever welded beam shown in Fig. 9 is analyzed. The section of beam is H400 × 250 × 10 × 14; the length of each segment is 3 m. The material of steel is Q235; the elastic modulus $E = 2.06 \times 10^5$ N/mm^2 and the density is 7.8×10^3 kg/m^3. Every node of connection but the fixed end has a vertical concentrated load $P = 10$ kN. Dead weight of the beam is considered.

5.1 Analysis of construction process

The internal force and deformation of the steel beam during the construction process is analyzed by using the conventional analysis algorithm (the method of element birth and death) and the improved scheme (the method named element birth and death of node rectification). The results for the above two algorithms are compared in Table 1.

Compared with the conventional method, it is noted that the maximum deformation of the steel beam obtained from the method named element birth and death of node rectification reduces from -76.3 mm to -52.7 mm by 31%. Consequently, the improved algorithm is verified to be effective through this example and the problem of element's 'floating' is solved well.

5.2 Analysis of pre-set deformation value

Based on the results for the proposed method, analysis of pre-set deformation value is obtained using the improved iteration algorithm named forward iteration of node rectification. Similarly,

Table 2 Iteration results of pre-deformation

The stage of calculation	The content of calculation	Vertical deformation of node/*mm*			
		1	2	3	4
1	u_1	0	-13.0	-41.9	-76.2
		[0]	[-13.1]	[-41.8]	[-52.7]
2	$-u_1$	0	13.0	41.9	76.2
		[0]	[13.1]	[41.8]	[52.7]
3	u_2	0	-12.9	-41.8	-75.9
		[0]	[-13.1]	[-41.7]	[-52.5]
4	$\|u_1 - u_2\|$	0 < 1	0.1 < 1	0.1 < 1	0.3 < 1
		[0 < 1]	[0 < 1]	[0.1 < 1]	[0.2 < 1]
Allowable error $[u]$ is satisfied and the iteration is over					

*Note: The brackets are results of the improved algorithm

the result of the conventional analysis algorithm (General iteration method) is obtained. Suppose the configuration of steel beam is controlled strictly, allowable error $[u]$ is 1 mm. The iteration results of pre-deformation for the above two algorithms are listed in Table 2. Thereinto, u_1 is the deformation of accomplished structures when the installation is completed by using the method named element birth and death of node rectification, u_2 is the deformation of accomplished structures via the same method based on the state of pre-set deformation.

From this table, it is seen that the tolerance is satisfied after only one iteration. Compared with the conventional method, it is noted that the allowable error obtained from the method named forward iteration of node rectification is more accurate. It is feasible to analyze pre-set deformation value of structures by using the method of forward iteration of node rectification. Moreover, the numbers of iteration are reduced from six to two when implementing the proposed method, rather than the conventional backward iteration method. Therefore, this improved method can be easily incorporated in commercial finite element software. Hence, this improved iteration algorithm can achieve the tradeoff between the computational accuracy and computation speed well.

6. Engineering application

6.1 Project profile

The roof of the main stadium for the Universiade Sports Center adopts the single-layer folded-plane latticed shell structure system. The structure is constituted by twenty units with similar shapes and the dimension of the architectural plane is 274 m × 289 m. The sections of main members are circular and the diameters are from 700 to 1,400 mm. On the other hand, secondary members are the box sections and the depths of sections are from 450 to 600 mm. Respectively, the material of main members is Q390 and Q420 and the material of secondary members is Q345. The main stadium during construction is shown in Fig. 10 and the overall rendering effect of the structure is plotted in Fig. 11. Because the structure has some structural characteristics such as

Fig. 10 The main stadium during construction

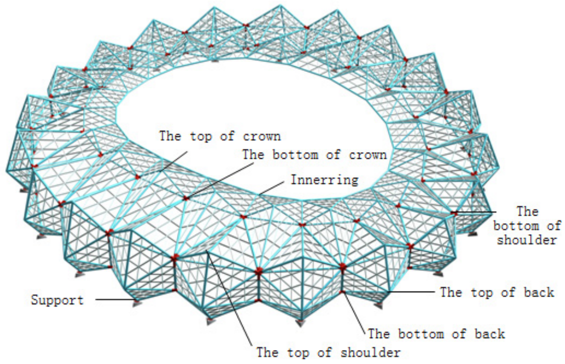

Fig. 11 Overall rendering effect of the structure

long cantilever and complex layout, it is necessary to analyze the internal force and deformation of the World University Games main stadium during the construction process by using the method of construction mechanics.

6.2 Construction scheme

Construction scheme of this project adopted the method of installation in situ which belongs to the positioning principle of node rectification. Therefore, it is feasible to analyze the construction process of the structure by using the improved algorithm named the element birth and death of node rectification. Three schemes were selected before construction. On the basis of the three schemes, the structure was divided into 22 construction areas as shown in Fig. 12. The construction areas of A and B were installed according to the clockwise direction at the same time. Then the temporary supports were dismantled in accordance with outside-in after the installation was completed. Respectively, every construction area was installed according to outside-in and in-outside in the scheme 1 and the scheme 2 (Fig. 13). For the scheme 3, the first thing was to

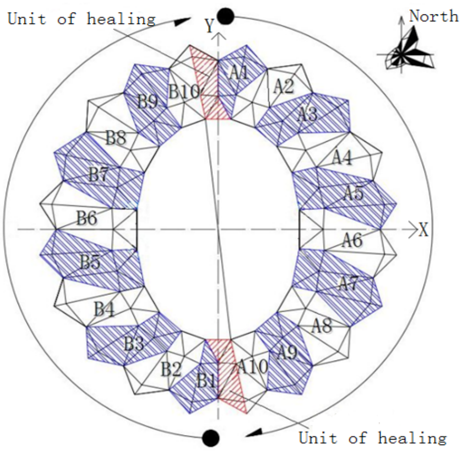

Fig. 12 Partition of structural unit

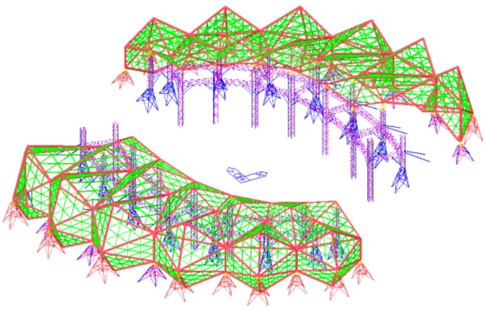

Fig. 13 Construction scheme 1 and 2

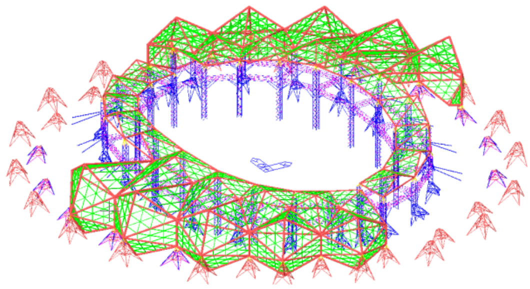

Fig. 14 Construction scheme 3

Table 3 Construction stages of the scheme 3

The stage of construction	The main structure	The temporary support
1	Install inner ring before healing area then install A1 and B1	The installation is finish
2	Install A2 and B2	—
3	Install A3 and B3	—
4	Install A4 and B4	—
5	Install A5 and B5	—
6	Install A6 and B6	—
7	Install A7 and B7	—
8	Install A8 and B8	—
9	Install A9 and B9	—
10	Install A10 and B10	—
11	Install healing members	—
12	—	dismantle the top of back
13	—	dismantle the bottom of back
14	—	dismantle the bottom of crown and inner ring circularly
15	—	dismantle is finish

Table 4 Contrasts between one-time loading and three kinds of schemes

Construction scheme	The maximum deformation of X /mm	The maximum deformation of Y /mm	The maximum deformation of Z /mm	The maximum stress of members /MPa
The scheme 1	75.86	50.93	-328.54	-280.12
The scheme 2	79.77	51.39	-347.51	-227.63
The scheme 3	80.18	51.01	-332.32	-218.56
One-time loading	73.45	28.38	-324.62	-114.29

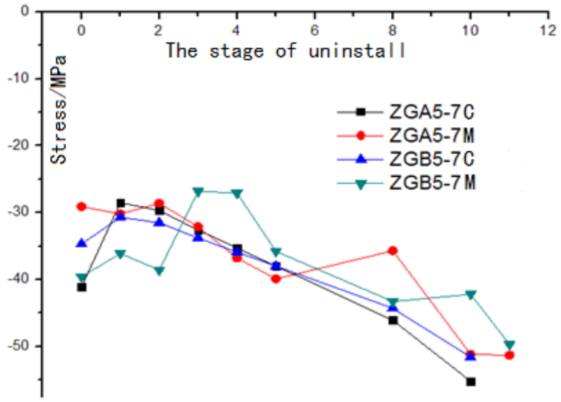

Fig. 15 Stress of crucial members in the unit 5

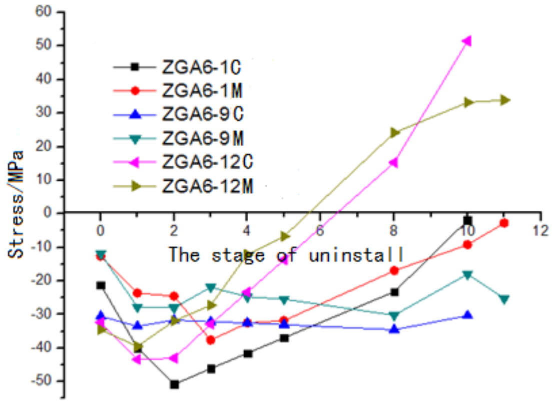

Fig. 16 Stress of crucial members in the unit A6

install all the temporary supports and install inner ring before healing areas. Secondly, the rest of construction areas were installed according to the clockwise direction at the same time (Fig. 14). The construction process of the scheme 3 was divided into 15 stages as listed in Table 3. In this paper, comparison of analysis results between one-time loading and three kinds of schemes by using the improved algorithm were listed in Table 4.

From this table, it is observed that the maximum stress of members by one-time loading was less than that in the step-by-step construction process obviously and the maximum stress of the scheme 3 was the smallest. Considering the site conditions (Tian 2013), the construction scheme 3 was the optimum construction scheme.

6.3 Comparison between calculation and measurement

As an application of the improved method, the mechanical behavior of the main stadium during the construction process in accordance with the optimum construction scheme had been estimated and the computational results were compared with the experimental data. In this paper, stress of crucial members from the west and the east during the dismantlement process were compared, as plotted in Figs. 15 and 16. In the figures, ZGA(B)i-jC represents the calculative results of the

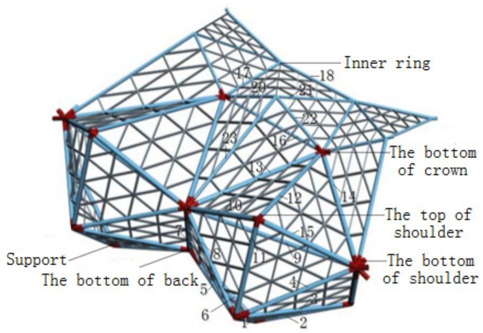

Fig. 17 Numberings of members

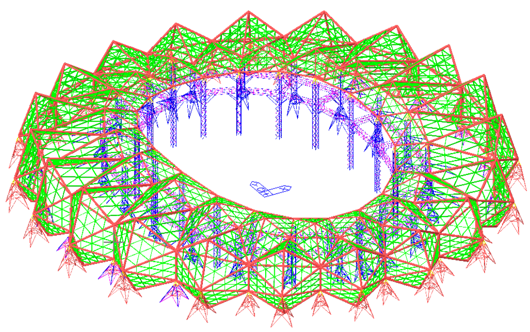
Fig. 18 FEA model of structure

member ‘j’ in the unit ‘i’ and ZGA(B)i-jM is the measured results of the member ‘j’ in the unit ‘i’. The numberings of members are shown in Fig. 17 and the FEA model of structure is shown in Fig. 18.

Following the results obtained herein, it is noted that the computational results by the improved methods matched well with the measured values in the project. Because the members and nodes of the FEA model are non-defective, the calculative results were greater than the measured results. Respectively, the maximum vertical deformation of the structure for the measured and simulation results were -316.6 mm and -332.3 mm after dismantlement and the difference was negligibly small. Moreover, it was close to the deformation of design (-328.5 mm). That indicates how important the accuracy of the algorithm should be, so as to obtain reliable results, and these results offer scientific reference for the selection of designation and construction scheme in the future.

7. Conclusions

Several calculating methods during construction process of complex rigid steel structures are intensively studied, followed by the new practical calculating methods. All these methods were utilized in the computer simulation of main stadium for the Universiade Sports Center during the construction process and compared with the experimental results thereafter. Based on the research work above, the following conclusions have been drawn:

- An improved positioning principle named the positioning principle of node rectification for installation, suitable for rigid long-span steel structures, is proposed, based on the theory of nonlinear beam-column elements and positioning for installation in bridge engineering. According to the topology of long-span steel structures, temporary supports should be set up at the terminal of dip, terminal of cantilever and position of top corner.
- An improved algorithm named the element birth and death of node rectification is developed. Based on the positioning principle of node rectification, the problem of element’s ‘floating’ is solved well by using temporary supports.
- An improved iteration algorithm named forward iteration of node rectification is also developed for determining the pre-set deformation value at the present. This improved iteration algorithm can be easily incorporated in commercial finite element software and achieve the tradeoff between the computational accuracy and computation speed.
- The optimum construction scheme of the main stadium for the Universiade Sports Center is determined by using the improved algorithm and the computational results matched well with the measured values in the project.
- The engineering application demonstrates that the improved nonlinear time-varying analytical methods are effective analytical methods of construction mechanics for complex rigid long-span steel structures.

Acknowledgments

The research described in this paper was financially supported by the Natural Science Foundation of China (Grant No. 51408623), the Research Foundation of Education Bureau of Shaanxi Province(Grant No. 14JK1400), the Opening Funds of State Key Laboratory of Building Safety and Built Environment (Grant No. BSBE2014-04), and Steel Structure Innovation Team of

Xi'an University of Architecture and Technology. Their financial support is highly appreciated.

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