Steel and Composite Structures, *Vol. 16, No. 6 (2014) 681-701*
DOI: http://dx.doi.org/10.12989/scs.2014.16.6.681

Topology optimization of nonlinear single layer domes by a new metaheuristic

Saeed Gholizadeh * and Hamed Barati [a]

Department of Civil Engineering, Urmia University, Urmia, West Azerbaijan Province, Iran

(*Received January 18, 2014, Revised March 13, 2014, Accepted March 21, 2014*)

Abstract. The main aim of this study is to propose an efficient meta-heuristic algorithm for topology optimization of geometrically nonlinear single layer domes by serially integration of computational advantages of firefly algorithm (FA) and particle swarm optimization (PSO). During the optimization process, the optimum number of rings, the optimum height of crown and tubular section of the member groups are determined considering geometric nonlinear behaviour of the domes. In the proposed algorithm, termed as FA-PSO, in the first stage an optimization process is accomplished using FA to explore the design space then, in the second stage, a local search is performed using PSO around the best solution found by FA. The optimum designs obtained by the proposed algorithm are compared with those reported in the literature and it is demonstrated that the FA-PSO converges to better solutions spending less computational cost emphasizing on the efficiency of the proposed algorithm.

Keywords: optimal design; meta-heuristic; firefly algorithm; lattice dome; geometrical nonlinearity

1. Introduction

One of the most challenging tasks in the field of structural engineering is to cover large spans, such as exhibition halls, stadium and concert halls, without intermediate columns. Space structures, especially domes, offer economical solutions to this problem. Domes provide a completely unobstructed inner space and economy in terms of materials. They are lighter compared with the more conventional forms of structures (Makowshi 1984). Although dome structures are economical forms of structural systems, structural optimization techniques can be effectively utilized to design these structures for optimum weight.

Structural optimization is an interesting activity that has received considerable attention in the last four decades. Usually, structural optimization problems involve searching for the minimum of the structural weight subject to various constraints. Topology optimization of structures is the most challenging research areas of the structural optimization field. In this class of optimization problems three types of design variables with different natures, including sizing, geometric and topological variables, are involved. The topology optimization problem has been identified as a more difficult but more important task than pure sizing and shape optimization, since potential

∗Corresponding author, Dr., Ph.D., E-mail: s.gholizadeh@urmia.ac.ir
[a] Ph.D. Student, E-mail: s57441@yahoo.com

Copyright © 2014 Techno-Press, Ltd.
http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

savings in material can be far better improved than by the sizing and shape optimization procedures.

The latticed domes are given special names depending on the form in which steel elements are connected to each other. Among the recent applications, the well known ones are lamella domes, network domes, and geodesic domes (Carbas and Saka 2012). Once diameter of a latticed dome is specified, its geometry can be defined by the total number of rings and height of crown. Therefore, in the process of topological design optimization of a lattice dome, these three parameters besides the cross-sectional areas of structural members must be considered as design variables.

Stochastic optimization algorithms are the general class of techniques which employ some degree of randomness to find optimal solutions to hard problems. In order to comprehensively explore a larger fraction of the design space, stochastic search techniques reveal their promising abilities in comparison with gradient-based optimization methods. Meta-heuristics are the most general of these kinds of algorithms, and are applied to a very wide range of problems. Meta-heuristics are simple to design and implement, and are very flexible. The past 20 years have witnessed the development of numerous meta-heuristics in various communities that sit at the intersection of several fields, including artificial intelligence, computational intelligence and soft computing. Most of the meta-heuristics mimic natural metaphors to solve complex optimization problems such as evolution of species, annealing process, ant colony, particle swarm, immune system, bee colony, and wasp swarm (Salajegheh and Gholizadeh 2012).

Despite the fact that the topological design optimization greatly improves the design, due to its complexity, this class of structural optimization problems has been investigated far less in comparison with pure sizing and shape optimization (Rozvany *et al*. 1995, Bendsoe and Sigmund 2003). During last years, many new meta-heuristic algorithms have been proposed for structural engineering applications (Gandomi *et al*. 2013a, b, Talatahari *et al*. 2012, 2013) and a number of researchers have employed meta-heuristics for topology optimization of lattice domes. Saka (2007) presented an algorithm to carry out the optimum topological design of single layer lattice geodesic domes taking into account the nonlinear response of the structure due to effect of axial forces on the flexural stiffness of members. He employed a coupled genetic algorithm to achieve optimization task. Carbas and Saka (2012) suggested an improved harmony search (HS) algorithm to determine the optimum number of rings, the optimum height of crown and tubular section designations for the member groups of lamella, network, and geodesic domes. Their proposed design algorithm also considered the geometric nonlinearity of these dome structures. Kaveh and Talatahari (2011) proposed a charged system search (CSS) based procedure for topology optimization of geodesic lattice domes considering geometric nonlinearity of the domes. Talaslioglu (2012) achieved size and topology optimization of geometrically nonlinear dome structures by minimizing both entire weight and joint displacements and maximizing load-carrying capacity using non-dominated sorting genetic algorithm II (NSGA II) as a multi-objective optimization tool.

In the present study, firefly algorithm (FA) (Yang 2009) and particle swarm optimization (PSO) (Eberhart and Kennedy 1995) are serially integrated in order to present a new meta-heuristic algorithm for topology optimization of lamella, network, and geodesic domes by taking into account the geometrically nonlinear response of the domes. PSO was inspired by the social behaviour of organisms such as bird flocking. As compared with other robust design optimization methods, PSO is more efficient, requiring fewer number of function evaluations while leading to better or the same quality of results (Hu *et al*. 2003, Hassan *et al*. 2005). However, PSO has some defects such as trapping into local optima in a complex search space. The FA is an optimization technique inspired by social behaviour of fireflies and the phenomenon of bioluminescent communication. The superiority of FA to PSO and GA was demonstrated using various test functions (Yang 2009, 2010). Gandomi *et al*. (2011) utilized the FA to solve benchmark mixed-variable and non-convex optimization problems. Gholizadeh and Barati (2013) compared the computational performance of PSO, HS, and FA in sizing and shape optimization of truss structures. Their results demonstrated the superiority of FA to HS and PSO.

In the novel algorithm proposed in this study, FA and PSO are serially integrated to provide a meta-heuristic algorithm with improved exploration and exploitation abilities compared with standard versions of FA and PSO meta-heuristics. This new algorithm is termed as FA-PSO meta-heuristic algorithm.

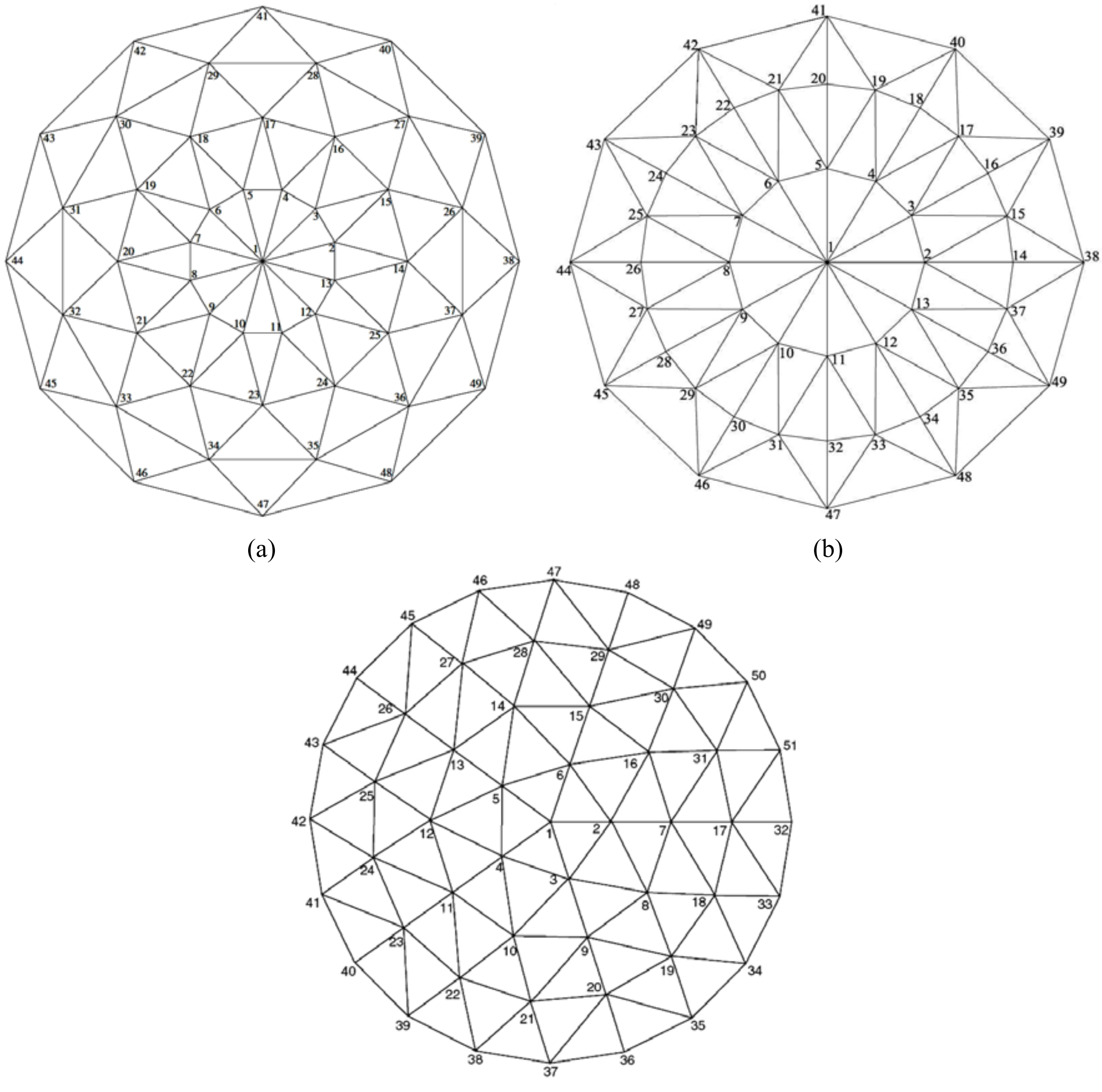
In the proposed algorithm, the number of rings, the height of crown and tubular section designations for the member groups of lamella, network, and geodesic domes are considered as design variables. The geometrically nonlinear behaviour of these dome structures is considered during the optimization process. In addition, the serviceability and strength requirements are considered in the design problems as specified in LRFD-AISC (1991). To perform structural analysis considering geometrical nonlinearity, ANSYS (2006) are employed. All of the required computer programs are coded in MATLAB (2006) platform. Furthermore, Optimization runs are performed on a standard PC equipped with a 3GHz Pentium IV CPU. The obtained numerical results demonstrate the efficiency of the proposed FA-PSO algorithm compared with other algorithms available in the literature in terms of optimal weight and computational cost.

2. Geometry generation of dome structures

The main contribution of the present study is to design single layer lamella, network and geodesic domes for optimal topology by a novel algorithm. The plan view of typical forms of the above mentioned domes are shown in Fig. 1. The geometry of these domes can be easily generated if the diameter of the dome (D), the total number of rings (n_r), and the height of the crown (h) are specified. In these domes, the distances between the rings on the meridian line are generally equal. Furthermore, the distances between all joints on the same ring are equal. The joint located at the crown is considered as the first joint. The procedures employed for geometry generation of lamella, network and geodesic domes are briefly explained in the next subsections. More details can be found in (Carbas and Saka 2012).

In lamella dome, each ring includes 12 joints. On the odd numbered rings, all of the first joints are located on the radius that makes angle of 15° with the x-axis. On the even numbered rings, the first joints are located on the intersection points of the x-axis and that ring. In the network dome, odd numbered rings contain 12 joints while on the even numbered rings there are 24 joints. The first joint of each ring is located on the intersection point of that ring and the x-axis. The other joints on each ring are numbered in a regular sequence. In geodesic dome, multiplying the ring number by 5 gives the total number of joints on that ring. In this type of dome, joints are numbered as well as the network dome.

Members are numbered in similar fashion. First member connects joints 1 and 2 for each dome type. In lamella and network domes the other 11 members connect first joint to the remaining joints located on the first ring. In geodesic dome the other four members connect joint 1 to joints 3 to 6. On the first ring of geodesic dome the members connect joints as 2-3, 3-4, 4-5, 5-6, 6-2. In lamella and network domes a similar procedure is followed. This process is repeated for each ring



(a) (b) (c)

Fig. 1 Plan view of (a) Lamella; (b) Network; and (c) Geodesic domes

and all members are numbered consequently. In order to compute x, y, and z coordinates of ith joint on these domes, the angle between the line that connects the joint to joint 1 (on plan) and the x-axis must be determined as shown in Fig. 2.

The angle α_i can be calculated for lamella, network and geodesic domes using the following equations (Carbas and Saka 2012).

$$\text{Lamella dome - the odd numbered rings: } \alpha_i = 15 + 30\left(i - j_r^1\right) \tag{1}$$

$$\begin{aligned}&\text{Lamella dome - the odd numbered rings: } \alpha_{\mathrm{i}} = 30\left(i - j_r^1\right)\\&\text{Network dome - the odd numbered rings: same as Eq. (2)}\end{aligned} \tag{2}$$

$$\text{Network dome - the even numbered rings: } \alpha_{\mathrm{i}} = 15\left(i - j_r^1\right) \tag{3}$$

$$\text{Geodesic dome: } \alpha_{\mathrm{i}} = 72\frac{\left(i - j_r^1\right)}{r} \tag{4}$$

In which i and j_r^1 are respectively the ith and first joints placed on the ring r.

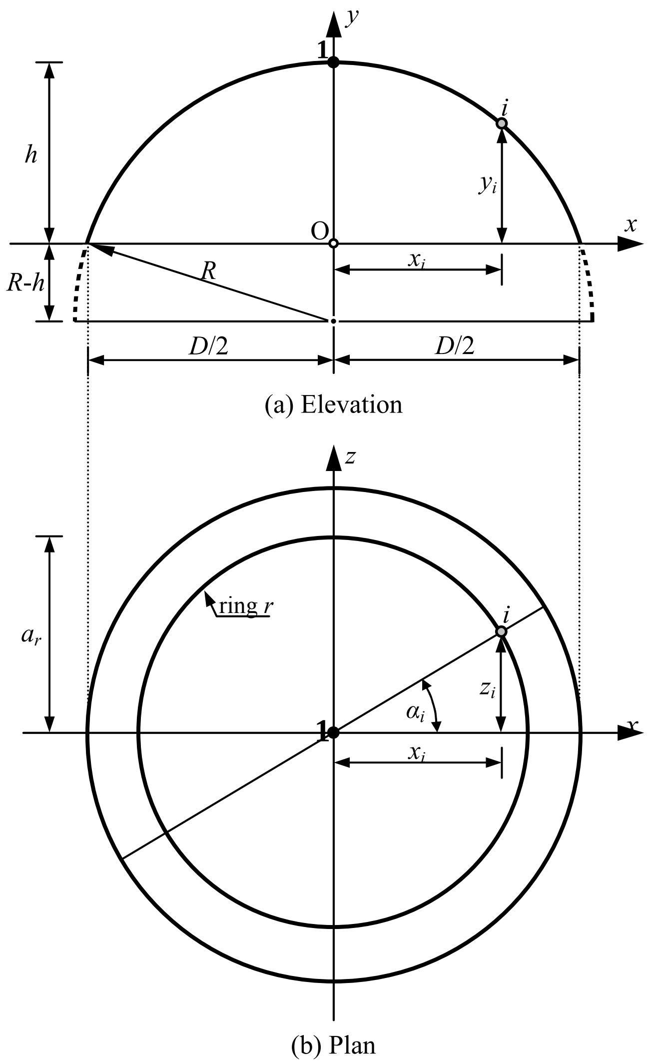


Fig. 2 Coordinates of *i*th joint on the *j*th ring of the domes

Considering Fig. 2, radius of dome (R) and x_i, z_i, y_i coordinates of the ith joint on the ring r can be computed as follows

$$R = \frac{D^2 + 4h^2}{8h} \tag{5}$$

$$x_i = a_r \cos(\alpha_i) \tag{6}$$

$$z_i = a_r \sin(\alpha_i) \tag{7}$$

$$y_i = \sqrt{R^2 - a_r^2} - (R - h) \tag{8}$$

where D and h are span and height of the dome, respectively. Also, a_r is the radius of the ring r.

3. Formulation of topology optimization problem of dome structures

In structural optimization problems the aim is usually to minimize an objective function, under some behavioural constraints (Gholizadeh 2013, Gholizadeh and Barzegar 2013). In the topology optimization problem of dome structures, the structural weight is considered as the objective function and the vector of design variables consists of number of rings, height of the crown and structural element groups' cross-sections. For a steel dome structure consisting of ne members that are collected in ng design groups, if the variables associated with each design group are selected from a list of steel pipe sections given by LRFD-AISC (1991), a discrete topology optimization problem can be formulated as follows

$$\text{Find} \quad X = n_r, h, [I_1, I_2, \ldots, I_k, \ldots, I_{ng}] \tag{9}$$

$$\text{To minimize} \quad f(X) = \sum_{i=1}^{ne} \rho\, A_i l_i \tag{10}$$

Subject to

$$g^s(X) = \det(\boldsymbol{K}(X)) > 0 \tag{11}$$

$$g_j^\delta(X) = \frac{\delta_j}{\delta_{all}} - 1 \le 0, \quad j = 1,2\ldots,nj \tag{12}$$

$$g_j^\delta(X) = \begin{cases} \text{for}\left(\dfrac{P_u}{\phi_c P_n}\right) < 0.2 \;\Rightarrow \left(\dfrac{P_u}{2\phi_c P_n} + \left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right)\right) - 1 \le 0 \\ \text{for}\left(\dfrac{P_u}{\phi_c P_n}\right) < 0.2 \;\Rightarrow \left(\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right)\right) - 1 \le 0 \end{cases}, \quad j = 1,2\ldots,ne \tag{13}$$

$$g_i^V(X) = \frac{V_u}{\phi_v P_n} - 1 \le 0, \quad j = 1,2\ldots,ne \tag{14}$$

where X is a vector of design variables; I_k is an integer value expressing the sequence numbers of steel sections assigned to kth group; h is the height of the crown; n_r is the total number of rings which is taken as 3, 4 or 5; ρ is material unit volume weight; A_i and l_i are cross-sectional area and length of the ith structural member, respectively; $g^s(X)$ is a stability constraint; $\boldsymbol{K}$ is the structural stiffness matrix; $g_j^\delta(X)$, δ_j and δ_{all} are the displacement constraint, displacement and allowable displacement of joint j, respectively; $g_i^\sigma(X)$ is the stress constraint of ith member; P_u is the required strength (tension or compression); P_n is the nominal axial strength (tension or compression); ϕ_c is the resistance factor; M_{ux} and M_{uy} are the required flexural strengths in the x and y directions; respectively; M_{nx} and M_{ny} are the nominal flexural strengths in the x and y directions; and ϕ_b is the flexural resistance reduction factor (ϕ_b = 0.9); V_u is the factored service load for shear; V_n is the nominal strength in shear and ϕ_v represents the resistance factor for shear given as 0.9.

In this study, the penalty function method (PFM) is employed to transform the constrained structural optimization problem into an unconstrained one as described below. The general approach of penalty function methods is to minimize the objective function as an unconstrained function but to provide some penalty to limit constraint violations (Vanderplaats 1984). Hence, the above constrained structural optimization problem is transformed into an unconstrained one. In this case, the pseudo unconstrained objective function can be represented as follows

$$\Phi\left(X,r_p\right)=f(X)\left(1+\xi\left(\begin{array}{l}\left(\max\left\{0,g^s\right\}\right)^2+\sum\limits_{j=1}^{nj}\left(\left(\max\left\{0,g_j^\delta\right\}\right)^2\right.\\ \left.+\sum\limits_{i=1}^{ne}\left(\left(\max\left\{0,g_i^\sigma\right\}\right)^2+\left(\left(\max\left\{0,g_i^V\right\}\right)^2\right.\right.\right.\end{array}\right)\right) \tag{15}$$

where Φ and ξ are the pseudo objective function and positive penalty parameter, respectively.

In the present study, an efficient meta-heuristic algorithm is proposed to minimize the above pseudo objective function. The next section describes the proposed algorithm.

4. Proposed meta-heuristic algorithm

In the present work, a new optimization algorithm is proposed by serially integration of FA and PSO meta-heuristics to tackle the mentioned topology optimization problem. It is demonstrated by Gandomi *et al*. (2011) that the PSO is a particular case of the FA if the randomization parameter of FA is set to zero. Hence, the basic concepts of the mentioned algorithms are briefly explained in the next sections.

4.1 Particle swarm optimization

The PSO has been proposed by Eberhart and Kennedy (1995) to simulate the motion of bird swarms. The particle swarm process is stochastic in nature; it uses a velocity vector to update the current position of each particle in the swarm. The velocity vector is updated based on the memory gained by each particle, conceptually resembling an autobiographical memory, as well as the knowledge gained by the swarm as a whole. Thus, the position of each particle in the swarm is updated based on the social behaviour of the swarm which adapts to its environment by returning to promising regions of the space previously discovered and searching for better positions over time. Numerically, the position of the ith particle, X_i, at iteration $t + 1$ is updated as follows

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{16}$$

where V_i^{t+1} is the corresponding updated velocity vector given as follows

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 \left(P_i^t - X_i^t\right) + c_2 r_2 \left(G_{\text{best}} - X_i^t\right) \tag{17}$$

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{k_{\max}} \cdot k \tag{18}$$

where V_i^t is the velocity vector at iteration t, r_1 and r_2 represents random numbers between 0 and 1; P_i^t represents the best ever particle position of particle i, and G_{best}^t corresponds to the global best position in the swarm up to iteration t. The remaining terms are problem dependent parameters; c_1 and c_2 are cognitive and social parameters, respectively; ω is the inertia weight which plays an important role in the PSO convergence behaviour; ω_{max} and ω_{min} are the maximum and minimum values of ω, respectively; k_{max}, and k are the number of maximum iterations and the number of present iteration, respectively.

Eqs. (17) and (18) indicate that for using PSO, internal parameters c_1, c_2, ω_{min} and ω_{max} must be determined.

4.2 Firefly algorithm

The FA is a new meta-heuristic optimization algorithm inspired by the flashing behaviour of fireflies. The FA is a population-based algorithm, which may share many similarities with PSO. Fireflies communicate, search for pray and find mates using bioluminescence with varied flashing patterns (Gandomi *et al*. 2011). In order to develop the firefly algorithm, natural flashing characteristics of fireflies have been idealized using the following three rules (Yang 2009)

(1) All of the fireflies are unisex; therefore, one firefly will be attracted to other fireflies regardless of their sex.
(2) Attractiveness of each firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright firefly will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
(3) The brightness of a firefly is determined according to the nature of the objective function.
(4) The attractiveness of a firefly is determined by its brightness or light intensity which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by (Yang 2010, Miguel *et al*. 2013).

$$\beta = \beta_0 e^{-\gamma . d^2} \tag{19}$$

where d is the distance of two fireflies, β_0 is the attractiveness at $d = 0$, and γ is the light absorption coefficient.

The distance between two fireflies i and j at X_i and X_j respectively, is determined as follows

$$d_{ij} = \left\| X_i - X_j \right\| = \sqrt{\sum \left(x_{i,k} - x_{j,k} \right)^2} \tag{20}$$

where $x_{i,k}$ is the k-th parameter of the spatial coordinate x_i of the i-th firefly.

In the FA, the movement of a firefly i towards a more attractive (brighter) firefly j is determined by the following equation (Yang 2010)

$$X_i^{t+1} = X_i^t + \beta_0 e^{-\gamma . d_{ij}{}^2} \left(X_i^t - X_j^t \right) + \lambda (r - 0.5) \tag{21}$$

where the second term is related to the attraction, while the third term is randomization with λ being the randomization parameter between 0 and 1; r is a random number generator uniformly distributed in [0, 1].

4.3 FA-PSO meta-heuristic

The main drawback of standard version of the PSO is that its exploration and exploitation abilities are not balanced (Angeline 1998). This means that PSO may converge to local optima in complex problems, such as the mentioned topology optimization problem. In the standard version of FA, it has been also observed by Yang (2009) that: the solutions are still changing as the optima are approaching. This implies that, as well as the PSO, FA may not able to provide appropriate convergence rate in the complex optimization problems. Yang (2010) illustrated that the convergence behaviour of the FA can be improved by reducing the randomness gradually. As a slight modification, in the present work the following simple equation has been used to gradually decrease the randomization parameter λ as the optima are approaching.

$$\lambda = \lambda_{\max} - \frac{\lambda_{\max} - \lambda_{\min}}{t_{\max}} \cdot t \tag{22}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum values of the randomization parameter; $t_{\max}$ and t are the numbers of maximum iterations and present iteration, respectively.

Hence, the internal parameters β_0, γ, $\lambda_{\min}$ and $\lambda_{\max}$ must be determined before using of the FA.

In the present study, FA and PSO are serially integrated to propose an efficient optimization algorithm having improved computational performance. In fact, the proposed algorithm includes two stages. As the superiority of the FA to the PSO has been demonstrated in the previous works (Yang 2009, 2010), in the first stage of the proposed algorithm FA is utilized to perform a global search through the design space. The best solution found in this stage is termed as X_{best}^{FA}. In the second stage, PSO is employed to implement another optimization process by utilizing the information derived in the first stage. To improve the solution quality, a specific initial swarm is generated for PSO. In this case, X_{best}^{FA} is directly transformed to the initial swarm and the remaining ones are selected from the neighbourhood of the X_{best}^{FA} in the design space as follows

$$X = \boldsymbol{N}\left(X_{\text{best}}^{FA}, \eta X_{\text{best}}^{FA} \right) \tag{23}$$

where $\boldsymbol{N}\left(X_{\text{best}}^{FA}, \eta X_{\text{best}}^{FA} \right)$ represents a vector of random normally distributed numbers with the mean of X_{best}^{FA} and the standard deviation of $\eta X_{\text{best}}^{FA}$.

As the parameter η can seriously affect the convergence behaviour of the proposed algorithm, a sensitivity analysis should be performed to determine its best value.

This meta-heuristic algorithm is termed as FA-PSO and its flowchart is shown in Fig. 3. The proposed FA-PSO algorithm includes some internal parameters affecting its computational performance at different extents. In this study, a sensitivity study is carried out on the effect of these parameters in the convergence behaviour of the algorithm and the results are presented in the numerical results section.

5. Application of FA-PSO for topology optimization

The design variables of the topology optimization problem of the latticed domes are defined by Eq. (9). It is clear that once the values of n_r and h are specified, the cross-sections of the element groups should be selected from an available section list in such a way that the domes satisfies the

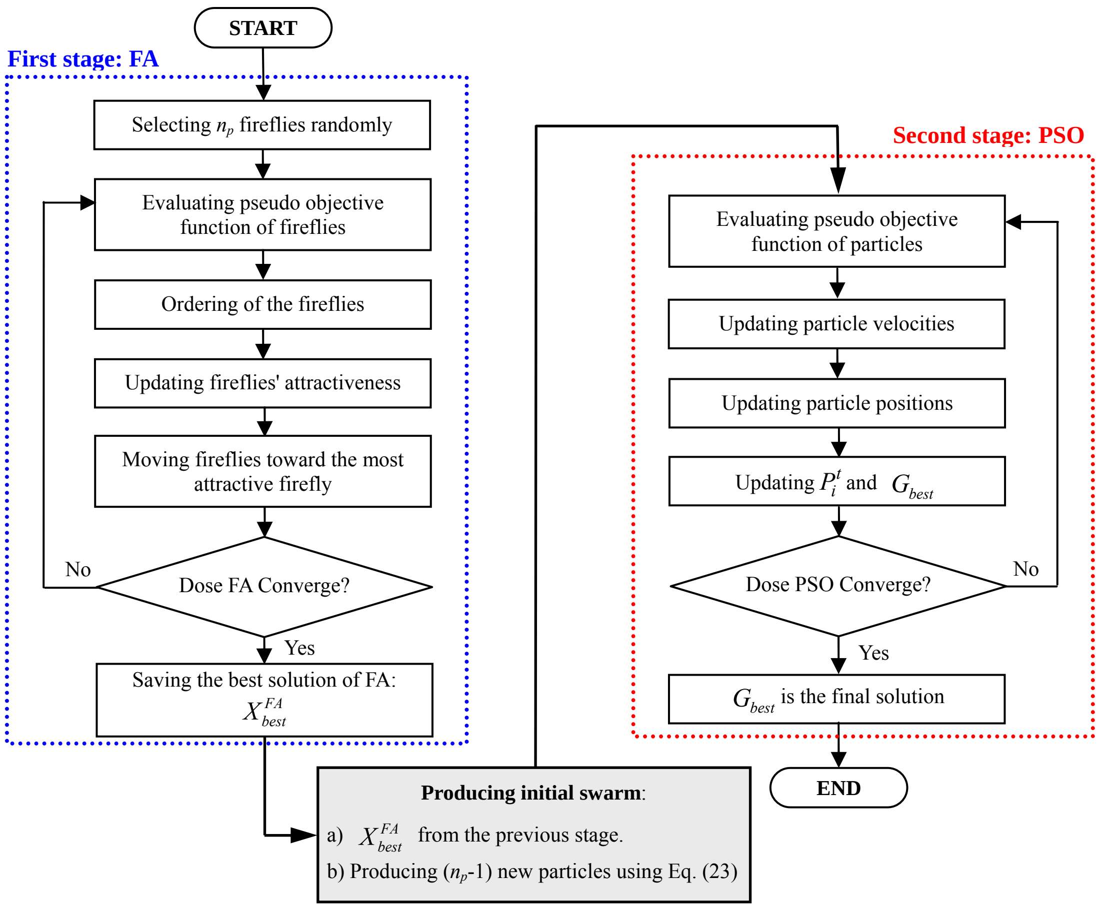


Fig. 3 Flowchart of FA-PSO meta-heuristic

design requirements specified by the code of practice. The algorithm proposed in this study randomly selects cross sections of the element groups from the sections list including 37 steel pipe sections given by LRFD-AISC (1991). Using the selected sections, ANSYS is utilized to perform geometrically nonlinear structural analysis. If the latticed dome losses its overall stability during the nonlinear analysis, the selected set of cross sections are rejected and a new set is selected. In case that the dome satisfies the stability constraint, the displacement and stress constraints will be checked and the optimization process will be continued until a termination criterion is met. Reaching the maximum number of generations is taken into account as the termination criterion of the present study.

As already explained, FA-PSO includes two stages. During the first stage, FA explores the design space and finds the best solution X_{best}^{FA}.

$$X_{\text{best}}^{FA} = n_{r,\text{best}}^{FA}, h_{\text{best}}^{FA}, [I_1, I_2, \ldots, I_k, \ldots, I_{ng}]_{\text{best}}^{FA} \tag{24}$$

In the second stage, PSO is used to perform a local search in the neighbourhood of the best solution X_{best}^{FA} found by FA and the initial swarm of PSO is generated using Eq. (23). In this case, to reduce the diversity of the process for better exploiting inside the design space, the number of rings is fixed and therefore it is removed from the list of the design variables during the second stage. Thus, in the second stage, all of the generated domes have $n_{r,\text{best}}$ rings while their cross sections and height are variable.

6. Numerical examples

In order to design the single layer lamella, network, and geodesic domes (shown in Figs. 1a, 1b, and 1c) for optimum topology, PSO, FA and FA-PSO meta-heuristics are employed in this study. For all the domes, the cross sections of element groups are selected from a list of steel tubular sections given in LRFD-AISC (1991). The crown height is changed from 1.00 to 8.75 m with the increment of 0.25 m. The diameters of all domes are taken as 20 m. The total number of rings can be 3, 4, or 5. The modulus of elasticity and the yield strength of the steel are taken as 205 kN/mm^2 and 250 MPa, respectively. For all the domes, displacement of joints 1 to 3 is limited to 28 mm in z-direction. Furthermore, the allowable displacement of joints 2 to 3 in both x- and y-directions is 33 mm. Two load cases are considered for design of the domes. The firs one includes an equipment loading of 500 kN at the crown while the second one is unsymmetrical loading of 15 kN concentrated loads applied on each joint of the one half of the domes. In the presented design examples, the population size for PSO, FA and FA-PSO meta-heuristics is chosen to be 20 and the maximum number of generations is limited to 500. In the case of FA-PSO, the maximum number of generations for each of the algorithms is limited to 250. Therefore, for all of the algorithms maximum number of structural analyses is equal to 10000.

In this paper, in order to find the best setting of the internal parameters of PSO (i.e., c_1, c_2, $\omega_{\min}$ and $\omega_{\max}$) a sensitivity analysis is carried out. For this reason combinations of three sets (1.0, 3.0), (2.0, 2.0) and (3.0, 1.0) for (c_1, c_2), and three sets (0.0, 0.5), (0.2, 0.7), (0.4, 0.9) for ($\omega_{\min}$, $\omega_{\max}$) are selected and PSO is used to obtain the optimum results for the combinations of these parameter values. For each combination of the parameters 25 independent optimization runs are performed and based on the obtained results, the best setting is determined. The results demonstrate that, for all of the presented examples, the best setting of parameters yielding the least weight is $c_1 = 1.0$, c_2

= 3.0, ω_{min} = 0.4, and ω_{max} = 0.9.

For determining the best combination of FA internal parameters (i.e., β_0, γ, λ_{min} and λ_{max}) another sensitivity analysis is carried out. As well as Miguel *et al*. (2013) in this study the value of β_0 is also taken to be 1.0 (i.e., β_0=1.0). Four values 0.1, 1.0, 10.0 and 100 are considered for γ and three sets (0.0, 0.5), (0.2, 0.7), (0.4, 0.9) are selected for (λ_{min}, λ_{max}) and FA is employed to perform 25 optimization runs in the case of each combination of these parameters to determine the best setting of them. The results indicate that, for all of the presented examples, the optimal combination of parameters is β_0 = 1.0, γ = 1.0, λ_{min} = 0.4, and λ_{max} = 0.9.

In the proposed FA-PSO meta-heuristic, the above mentioned best setting of parameters of PSO and FA are used and four values 0.05, 0.10, 0.15 and 0.20 are selected for η. In each case 25 independent optimization runs are performed and the results demonstrate that in the case of η = 0.10 the least weight can be obtained. Therefore, the best setting of parameters of FA-PSO meta-heuristic is β_0 = 1.0, γ = 1.0, λ_{min} = 0.4, λ_{max} = 0.9, η = 0.10, c_1 = 1.0, c_2 = 3.0, ω_{min} = 0.4, and ω_{max} = 0.9.

6.1 Lamella dome

Table 1 presents the results (in terms of best, worst and average structural weights and corresponding standard deviation) of 25 independent optimization runs of lamella dome subject to

Table 1 Results of 25 independent optimization runs performed for lamella dome subject to load case 1

Weight (kg)	PSO	FA	FA-PSO
Best	3174.4	3147.4	2998.2
Worst	3343.4	3316.4	3098.2
Average	3256.2	3212.3	3031.4
Standard deviation	74.5	49.6	31.1

Table 2 Results of optimization of lamella dome subject to load case 1

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	3	3	3	3
h (m)	5.75	5.75	5.50	5.75
I_1	PIPST 127	PIPST 127	PIPST 127	PIPST 102
I_2	PIPDEST 51	PIPST 76	PIPEST 64	PIPDEST 51
I_3	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_4	PIPST 32	PIPEST 19	PIPST 19	PIPST 25
I_5	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_6	PIPST 13	PIPST 13	PIPST 13	PIPST 13
Weight(kg)	3443.9	3174.4	3147.4	2998.2
Max. displacement (mm)	-4.97	-23.60	-23.82	-24.14
Max. strength ratio	1.00	0.99	0.99	1.00
Required structural analyses	25000	10000	10000	10000

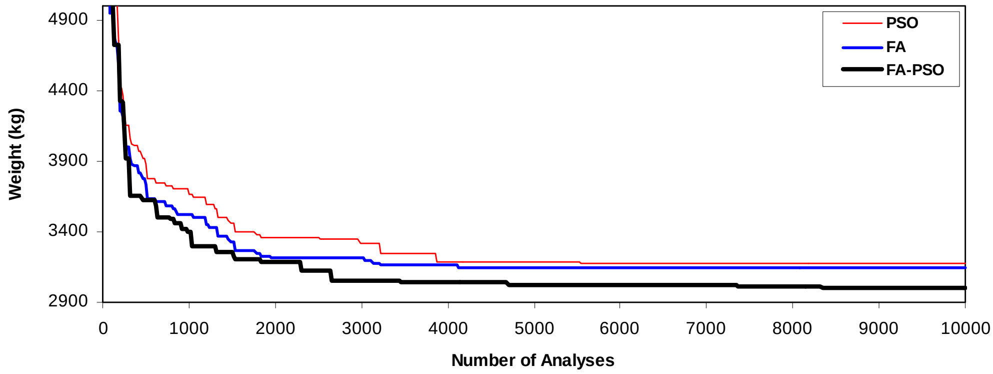


Fig. 4 Convergence histories for the lamella dome subject to load case 1 using PSO, FA and FA-PSO

Table 3 Results of 25 independent optimization runs performed for lamella dome subject to load case 2

Weight (kg)	PSO	FA	FA-PSO
Best	2235.2	2221.9	2185.7
Worst	2354.3	2294.1	2249.8
Average	2311.3	2279.1	2208.3
Standard deviation	59.1	37.2	11.6

load case 1, associated to the above mentioned best settings of the internal parameters of various algorithms.

This results indicate that the best weights obtained by FA-PSO is 2998.2 kg which is 5.55% and 4.74% lighter than the best weights of PSO (3174.4 kg) and FA (3147.4 kg), respectively. The results demonstrate that FA-PSO outperforms the PSO and FA meta-heuristics. As expected the computational performance of the FA is better than that of the PSO.

The best results obtained in the present work are compared with those obtained by Carbas and Saka (2012) in Table 2.

The convergence histories of the PSO, FA and FA-PSO meta-heuristics are shown in Fig. 4. It can be observed that FA-PSO possesses better convergence rate in comparison with PSO and FA.

These results indicate that the best weight obtained by FA-PSO is 12.94% lighter than the weight reported by Carbas and Saka (2012). In addition, the FA-PSO requires 10000 structural analyses while the algorithm proposed by Carbas and Saka (2012) requires 25000 ones. These results demonstrate that the FA-PSO not only found the best design overall but required also much less structural analyses compared with the other optimization algorithm.

In the case of lamella dome subject to unsymmetrical loading, the results of 25 independent optimization runs associated to the best settings of the internal parameters are given in Table 3.

The results presented in Table 3 imply again that the computational performance of FA-PSO is better than those of the PSO and FA. The best weight found by FA-PSO is 2.21% and 1.63% lighter than those of the PSO and FA, respectively. Table 4 compares the best results obtained in the present work with those reported by Carbas and Saka (2012).

Table 4 Results of optimization of lamella dome subject to load case 2

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	3	3	3	3
h (m)	4.75	5.00	4.75	5.75
I_1	PIPST 38	PIPST 38	PIPST 38	PIPST 38
I_2	PIPST 51	PIPEST 38	PIPST 51	PIPEST 38
I_3	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_4	PIPST 19	PIPEST 19	PIPST 25	PIPEST 13
I_5	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_6	PIPST 13	PIPST 13	PIPST 13	PIPST 13
Weight (kg)	2212.1	2235.2	2221.9	2185.7
Max. displacement (mm)	+1.232	-7.09	-7.35	-7.66
Max. strength ratio	0.95	0.96	0.97	0.99
Required structural analyses	25000	10000	10000	10000

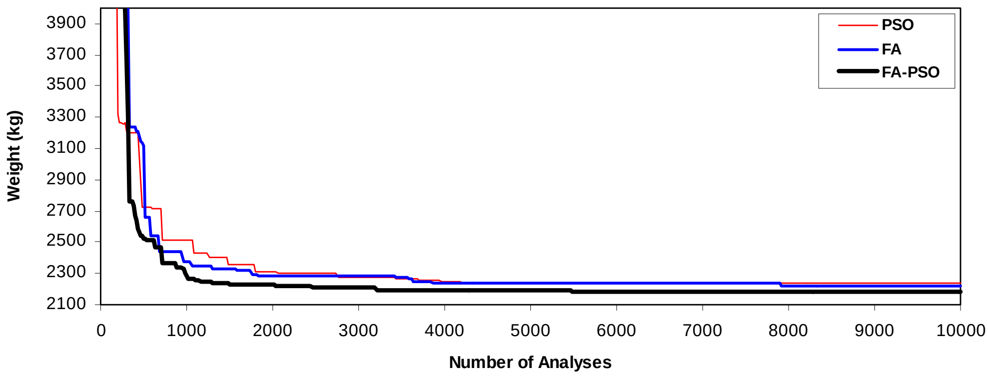


Fig. 5 Convergence histories for the lamella dome subject to load case 2 using PSO, FA and FA-PSO

Fig. 5 compares the convergence histories of PSO, FA and FA-PSO indicating the better convergence rate of the FA-PSO with respect to PSO and FA.

The best weight and required computational demand of FA-PSO meta-heuristic algorithm are 1.19% and 60% less than those of the algorithm reported by Carbas and Saka (2012). These results demonstrate the efficiency of the FA-PSO meta-heuristic in comparison with other algorithms.

6.2 Network dome

The results of 25 independent optimization runs performed, using the best settings of the internal parameters, for optimization of network dome subject to load case 1 are summarized in Table 5.

Table 5 Results of 25 independent optimization runs performed for network dome subject to load case 1

Weight (kg)	PSO	FA	FA-PSO
Best	3975.4	3944.7	3920.4
Worst	4245.1	4198.3	3974.1
Average	4176.3	4113.9	3942.1
Standard deviation	87.2	66.8	25.1

Table 6 compares the best results of the present work with those of the reported by Carbas and Saka (2012)

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	5	3	5	5
h (m)	3.75	5.00	3.75	3.75
I_1	PIPST 203	PIPST 127	PIPST 203	PIPST 203
I_2	PIPST 13	PIPEST 64	PIPST 13	PIPST 13
I_3	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_4	PIPEST 51	PIPEST 25	PIPEST 64	PIPEST 51
I_5	PIPST 51	PIPST 64	PIPST 51	PIPST 51
I_6	PIPEST 25	PIPST 13	PIPEST 32	PIPEST 25
I_7	PIPST 51	N.A.	PIPST 51	PIPST 51
I_8	PIPEST 19	N.A.	PIPST 25	PIPEST 19
I_9	PIPST 51	N.A.	PIPST 51	PIPST 51
I_{10}	PIPST 13	N.A.	PIPST 13	PIPST 13
Weight(kg)	3920.4	3975.4	3944.7	3920.4
Max. displacement (mm)	-25.47	-26.62	-27.38	-25.47
Max. strength ratio	1.00	0.96	1.00	1.00
Required structural analyses	25000	10000	10000	10000

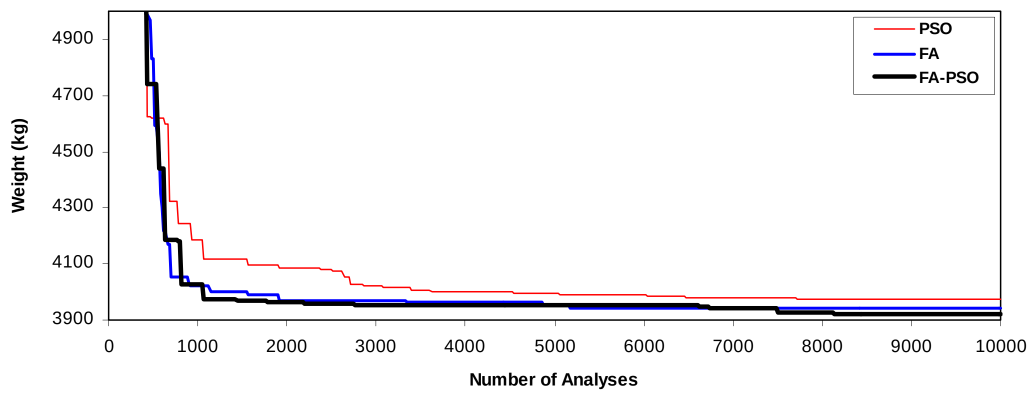


Fig. 6 Convergence histories for the network dome subject to load case 1 using PSO, FA and FA-PSO

These results demonstrate the efficiency of FA-PSO in comparison with PSO and FA. The best weight obtained by FA-PSO is 1.38% and 0.62% lighter than the best weights of PSO and FA, respectively.

Table 6 compares the best results of the present work with those of the reported by Carbas and Saka (2012).

Fig. 6 shows the convergence histories of the algorithms used to optimize network dome for load case 1. It is clear that the convergence rate of the FA-PSO is better than those of the PSO and FA.

The best design obtained in this paper and that reported by Carbas and Saka (2012) are identical. However, FA-PSO meta-heuristic requires much less number of structural analyses in comparison with the proposed algorithm by Carbas and Saka (2012). The results of 25 independent optimization runs for design of network dome subject to load case 2 are given in Table 7.

These results demonstrate the efficiency of FA-PSO in comparison with PSO and FA. The best weight of FA-PSO is 1.27% and 1.21% lighter than the best weights of PSO and FA, respectively.

Table 8 compares the best results of the present work with those of reported by Carbas and Saka (2012). Comparison of convergence histories of PSO, FA and FA-PSO for the network dome subject to load case 2, shown in Fig. 7, implies the superiority of FA-PSO to the PSO and FA.

Table 7 Results of 25 independent optimization runs performed for network dome subject to load case 2

Weight (kg)	PSO	FA	FA-PSO
Best	3200.7	3198.8	3159.9
Worst	3304.8	3287.1	3272.6
Average	3286.2	3265.6	3201.3
Standard deviation	67.1	43.4	27.6

Table 8 Results of optimization of network dome subject to load case 2

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	3	3	3	3
h (m)	4.75	4.5	4.75	4.5
I_1	PIPEST 38	PIPST 51	PIPST 51	PIPST 51
I_2	PIPST 51	PIPST 64	PIPEST 51	PIPEST 51
I_3	PIPST 64	PIPST 64	PIPST 64	PIPST 64
I_4	PIPST 38	PIPEST 32	PIPST 38	PIPEST 32
I_5	PIPST 64	PIPST 64	PIPST 64	PIPST 38
I_6	PIPST 13	PIPST 13	PIPST 13	PIPST 13
Weight (kg)	3168.5	3200.7	3198.8	3159.9
Max. displacement (mm)	+1.934	-5.68	-5.38	-6.01
Max. strength ratio	0.98	1.00	1.00	1.00
Required structural analyses	25000	10000	10000	10000

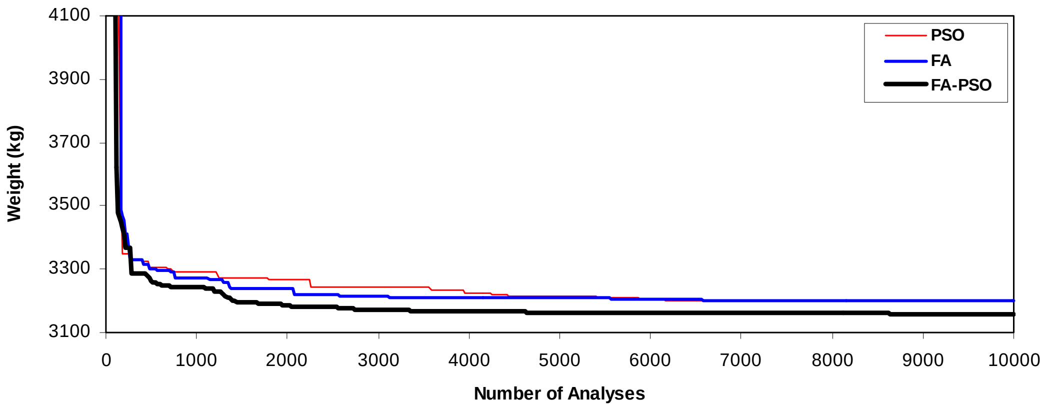


Fig. 7 Convergence histories for the network dome subject to load case 2 using PSO, FA and FA-PSO

Table 9 Results of 25 independent optimization runs performed for geodesic dome subject to load case 1

Weight (kg)	PSO	FA	FA-PSO
Best	3627.5	3615.2	3523.3
Worst	3790.5	3730.7	3623.1
Average	3740.3	3694.4	3564.3
Standard deviation	98.62	58.13	42.14

Table 10 Results of 25 independent optimization runs performed for geodesic dome subject to load case 2

Weight (kg)	PSO	FA	FA-PSO
Best	2108.1	2068.9	1986.5
Worst	2180.8	2127.4	2023.1
Average	2169.8	2098.2	2006.0
Standard deviation	26.0	19.2	7.9

The best weight obtained by FA-PSO is slightly better than that of reported by Carbas and Saka (2012). However, the computational performance of FA-PSO meta-heuristic algorithm is considerably better compared with the proposed algorithm by Carbas and Saka (2012) in terms of required structural analyses during the optimization process.

6.3 Geodesic dome

The results of 25 independent optimization runs performed, using the best settings of the internal parameters, for optimization of geodesic dome subjected to load cases 1 and 2 are summarized in Tables 9 and 10, respectively.

The results show that, in load case 1, the best weight obtained by FA-PSO is 2.87% and 2.54% lighter than the best weights of PSO and FA, respectively. While, for load case 2, the best weight obtained by FA-PSO is 5.76% and 3.98% lighter than the best weights of PSO and FA,

Table 11 Results of optimization of geodesic dome subject to load case 1

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	5	5	3	3
h (m)	4.25	4.25	5.75	5.75
I_1	PIPST 305	PIPST 305	PIPST 203	PIPST 203
I_2	PIPST 13	PIPST 13	PIPEST 64	PIPST 76
I_3	PIPST 89	PIPST 89	PIPST 89	PIPST 89
I_4	PIPEST 51	PIPST 64	PIPEST 38	PIPEST 25
I_5	PIPST 64	PIPST 64	PIPST 76	PIPST 76
I_6	PIPEST 25	PIPEST 32	PIPST 13	PIPST 13
I_7	PIPST 51	PIPST 51	N.A.	N.A.
I_8	PIPEST 13	PIPST 25	N.A.	N.A.
I_9	PIPST 51	PIPST 51	N.A.	N.A.
I_{10}	PIPST 13	PIPST 13	N.A.	N.A.
Weight(kg)	3631.4	3627.5	3615.2	3523.3
Max. displacement(mm)	-27.77	-26.12	-22.32	-23.96
Max. strength ratio	1.00	0.99	0.98	0.98
Required structural analyses	25000	10000	10000	10000

Table 12 Results of optimization of geodesic dome subject to load case 2

Design variables	Carbas and Saka (2012)	Present work		
		PSO	FA	FA-PSO
n_r	3	3	3	3
h (m)	5.00	5.25	5.25	5.25
I_1	PIPST 32	PIPST 32	PIPST 32	PIPST 32
I_2	PIPST 64	PIPEST 51	PIPST 51	PIPST 51
I_3	PIPST 64	PIPEST 51	PIPEST 51	PIPEST 51
I_4	PIPST 51	PIPEST 51	PIPEST 51	PIPST 51
I_5	PIPEST 51	PIPST 64	PIPST 64	PIPST 64
I_6	PIPST 13	PIPST 13	PIPST 13	PIPST 13
Weight(kg)	2034.2	2108.1	2068.9	1986.5
Max. displacement (mm)	+0.675	-2.24	-2.24	-2.45
Max. strength ratio	0.99	0.98	0.98	0.99
Required structural analyses	25000	10000	10000	10000

respectively. These results demonstrate the efficiency of FA-PSO in comparison with PSO and FA.

The best results obtained in optimization of geodesic dome subject to load cases 1 and 2 are compared with those of reported by Carbas and Saka (2012) in Tables 11 and 12, respectively.

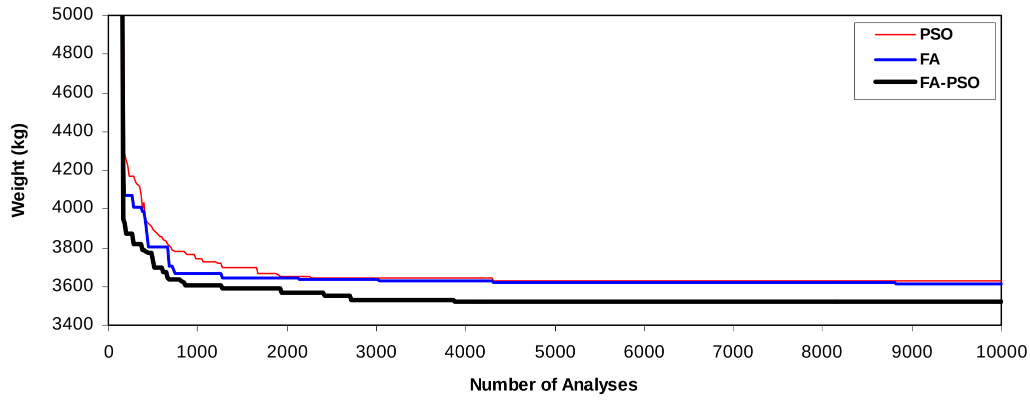


Fig. 8 Convergence histories for the geodesic dome subject to load case 1 using PSO, FA and FA-PSO

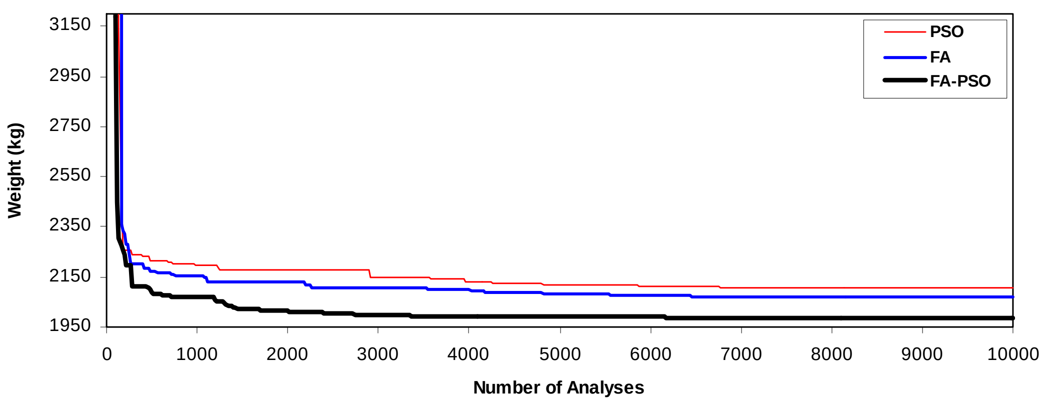


Fig. 9 Convergence histories for the geodesic dome subject to load case 2 using PSO, FA and FA-PSO

Convergence histories of PSO, FA and FA-PSO for the geodesic dome subject to load cases 1 and 2 are shown in Figs. 8 and 9, respectively. It can be easily observed that the convergence rateof FA-PSO is better than those of the PSO and FA.

The results show that the best weight obtained by FA-PSO in load cases 1 and 2 are respectively 2.98% and 2.34% lighter than the best weights reported by Carbas and Saka (2012) at considerably lower computational cost.

7. Conclusions

An efficient meta-heuristic algorithm is proposed for topology optimization of geometrically nonlinear single layer dome structures. To achieve this purpose, two popular meta-heuristic algorithms, FA and PSO, are serially integrated and the resulted algorithm is denoted as FA-PSO. In the proposed algorithm, exploration and exploitation tasks are carried out by FA and PSO, respectively. Lamella, network and geodesic single layer lattice domes are designed for optimal topology subject to two load cases using PSO, FA and FA-PSO meta-heuristics. Design variables of the optimization problem are the number of rings, the height of crown and tubular section of the member groups. Furthermore, the constraints of the optimum topology design problem are handled according to LRFD-AISC code. In the case of each design example, a sensitivity study is carried out to find the best setting of internal parameters of the algorithms. Numerical results demonstrate that, in all the design examples, the proposed FA-PSO algorithm outperforms both the standard PSO and FA meta-heuristics. In addition, it is observed that the computational performance of FA is better than that of the PSO. The results, obtained using the proposed FA-PSO meta-heuristic, are compared with those of reported in the literature. It is demonstrated that FA-PSO not only converges to lightweight designs but it also requires much less number of structural analyses compared with other algorithms. Therefore, FA-PSO algorithm can be efficiently and reliably employed to find optimum topology of geometrically nonlinear latticed domes spending low computational cost.

References

American Institute of Steel Construction (AISC) (1991), *Manual of Steel Construction-Load Resistance Factor Design*, (3rd Edition), AISC, Chicago, IL, USA.

Angeline, P. (1998) "Evolutionary optimization versus particle swarm optimization: Philosophy and performance difference", *Proceeding of the 7th International Conference on Evolutionary Programming Conference*, San Diego, CA, USA, March.

ANSYS Incorporated (2006), ANSYS Release 10.0.

Bendsoe, M.P. and Sigmund, O. (2003), *Topology Optimization*: *Theory*, *Methods and Applications*, Springer, Berlin, Germany.

Carbas, S. and Saka, M.P. (2012), "Optimum topology design of various geometrically nonlinear latticed domes using improved harmony search method", *Struct*. *Multidis*. *Optim*., **45**(3), 377-399.

Eberhart, R.C. and Kennedy, J. (1995), "A new optimizer using particle swarm theory", *Proceedings of the 6th International Symposium on Micro Machine and Human Science*, Nagoya, Japan, IEEE Press, pp. 39-43.

Gandomi, A.H., Yang, X.S. and Alavi, A.H. (2011), "Mixed variable structural optimization using firefly algorithm", *Comput*. *Struct*., **89**(23-24), 2325-2336.

Gandomi, A.H., Alavi, A.H. and Talatahari, S. (2013a), *Swarm Intelligence and Bio-Inspired Computation*: *Theory and Applications*, Chapter 15: "Structural optimization using krill herd algorithm", Elsevier, (Editors: XS Yang *et al*.), 335-349.

Gandomi, A.H., Yang, X.S. and Alavi, A.H. (2013b), "Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems", *Eng*. *Comput*., **29**(1), 17-35.

Gholizadeh, S. (2013), "Layout optimization of truss structures by hybridizing cellular automata and particle swarm optimization", *Comput*. *Struct*., **125**, 86-99.

Gholizadeh, S. and Barati, H. (2011), "Optimum design of structures using an improved firefly algorithm", *Int*. *J*. *Optim*. *Civil Eng*., **2**, 327-340.

Gholizadeh, S. and Barzegar, A. (2013), "Shape optimization of structures for frequency constraints by sequential harmony search algorithm", *Eng*. *Optim*., **45**(6), 627-646.

Hassan, R., Cohanim, B., de Weck, O. and Venter, G. (2005), "A comparison of particle swarm optimization and the genetic algorithm", *The 1st AIAA Multidisciplinary Design Optimization Specialist Conference*, No. AIAA-2005-1897, Austin, TX, USA.

Hu, X., Eberhart, R. and Shi, Y. (2003), "Engineering optimization with particle swarm", *IEEE Swarm Intelligence Symposium* (*SIS 2003*), Indianapolis, IN, USA, April, pp. 53-57..

Kaveh, A. and Talatahari, S. (2011), "Geometry and topology optimization of geodesic domes using charged

system search", *Struct. Multidis. Optim.*, **43**(2), 215-229.
Makowshi, Z.S. (1984), *Analysis, Design and Construction of Braced Domes*, Granada Publishing Ltd., London, UK.
MATLAB (2006), The language of technical computing [software], The MathWorks.
Miguel, L.F.F., Lopez, R.H. and Miguel, L.F.F. (2013), "Multimodal size, shape, and topology optimization of truss structures using the Firefly algorithm", *Adv. Eng. Softw.*, **56**, 23-37.
Rozvany, G.I.N., Bendsoe, M.P. and Kirsch, U. (1995), "Layout optimization of structures", *App. Mech. Rev.*, **48**(2), 41-119.
Saka, M.P. (2007), "Optimum topological design of geometrically nonlinear single layer latticed domes using coupled genetic algorithm", *Comput. Struct.*, **85**(21-22), 1635-1646.
Salajegheh, E. and Gholizadeh, S. (2012), "Structural seismic optimization using meta-heuristics and neural networks: A review", *Comput. Tech. Rev.*, **5**, 109-137.
Talaslioglu, T. (2012) "Multiobjective size and topolgy optimization of dome structures", *Struct. Eng. Mech., Int. J.*, **43**(6), 795-821.
Talatahari, S., Gandomi, A.H. and Yun, G.J. (2012), "Optimum design of tower structures by firefly algorithm", *Struct. Des. Tall. Spec. Buil.*, **23**(5), 350-361.
Talatahari, S., Kheirollahi, M., Farahmandpour, C. and Gandomi, A.H. (2013), "A multi-stage particle swarm for optimum design of truss structures", *Neural. Comput. Appl.*, **23**(5), 1297-1309.
Vanderplaats, G.N. (1984), *Numerical Optimization Techniques for Engineering Design: With Application*, (2nd Ed.), McGraw-Hill, NewYork, NY, USA.
Yang, X.S. (2009), "Firefly algorithms for multimodal optimization", *Stochastic Algorithms: Foundations and Applications* (Eds O. Watanabe and T. Zeugmann), SAGA 2009, Lecture Notes in Computer Science, 5792, Springer-Verlag, Berlin, pp. 169-178.
Yang, X.S. (2010), "Firefly algorithm, Levy flights and global optimization", *Research and Development in Intelligent Systems* XXVI, (Eds M. Bramer, R. Ellis, M. Petridis), Springer London, UK, pp. 9-18.

CC