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Relative static and dynamic performances of composite conoidal shell roofs

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Abstract. Conoidal shells are doubly curved stiff surfaces which are easy to cast and fabricate due to their singly ruled property. Application of laminated composites in fabrication of conoidal shells reduces gravity forces and mass induced forces compared to the isotropic constructions due to the high strength to weight ratio of the material. These light weight shells are preferred in the industry to cover large column free open spaces. To ensure design reliability under service conditions, detailed knowledge about different behavioral aspects of conoidal shell is necessary. Hence, in this paper, static bending, free and forced vibration responses of composite conoidal shells are studied. Lagrange's equation of motion is used in conjunction with Hamilton's principle to derive governing equations of the shell. A finite element code using eight noded curved quadratic isoparametric elements is developed to get the solutions. Uniformly distributed load for static bending analysis and three different load time histories for solution of forced vibration problems are considered. Eight different stacking sequences of graphite-epoxy composite and two different boundary conditions are taken up in the present study. The study shows that relative performances of different shell combinations in terms of static behaviour cannot provide an idea about how they will relatively behave under dynamic loads and also the fact that the points of occurrence of maximum static and dynamic displacement may not be same on a shell surface.

Keywords: conoidal shell; composite material; finite element method; forced vibration; Newmark's method

1. Introduction

A shell structure is capable of covering large unsupported areas of stadiums, airports and shopping malls with small material consumption. They utilize their inplane stiffness in addition to bending stiffness due to the presence of curvature in their geometry. A doubly curved shell is aesthetically appealing and more rigid than singly curved shells. Use of a singly ruled surface is advantageous from execution point of view as it can be generated by placing straight shuttering between two curved boundaries. Conoidal shell (Fig. 1) is very popular in the industry as it is doubly curved and singly ruled at the same time. Moreover conoidal shell allows entry of daylight and natural air which is preferred in food processing and medicine units.

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A closed form solution cannot be obtained for dynamic analysis of shells with complex geometry and boundary condition. Such problems can be solved efficiently by finite element method. Application of finite element method to analyze shell configurations by singly curved finite element was first reported by Ergatoudis *et al*. (1968). Later it was modified by Greene *et al*. (1968) by introducing doubly curved finite element to solve dynamic problems of isotropic shells. The use of laminated composites to fabricate shells became preferred to civil engineers from second half of the last century. The reasons were high strength/stiffness to weight ratio, low cost of fabrication and better durability of the composites. Moreover, the stiffness of laminated composites can be altered by varying the fiber orientations and lamina thicknesses which gives design flexibility. Moreover the mass induced seismic forces and hence foundation costs for composites are less. Naturally study of composite shells has emerged as an active area of research. Reddy (1984) reported exact displacements and fundamental frequencies of laminated cylindrical and spherical shells. Fundamental frequencies of cylindrical, conoidal, elliptic paraboloidal and hyperbolic paraboloidal shells were reported by Chakravorty *et al*. (1995a, 1995b, 1998) for varying boundary conditions and laminations. Nayak and Bandyopadhyay (2005, 2006) worked on free and forced vibrations of conoidal shells using the finite element method. Bending and free vibration characteristics of laminated conoidal shells with and without stiffeners were studied by Das and Chakravorty (2007, 2008, 2009, 2010, 2011). Kumari and Chakravorty (2010, 2011) reported bending characteristics of delaminated conoidal shells. Pradyumna and Bandyopadhyay (2008, 2011) used higher order shear deformation theory to study vibration and dynamic instability behaviors of laminated conoidal shells.

A shell surface may subjected to short time dynamic forces in its service life by internal wind suction, snow loading in low temperature areas and seismic waves. A detail dynamic study including free and forced vibration studies is thus required to ensure long and uninterrupted service life of the laminated shells. Feeling this necessity a number of researchers like Chakravorty *et al*. (1998), Nayak and Bandyopadhyay (2006), Reddy and Chandrashekhara (1985), Lee and Han (2006), Nanda and Bandyopadhyay (2008, 2009) and Ribeiro (2008) worked on forced vibration of laminated shells. Reddy and Chandrashekhara (1985) reported linear and nonlinear transient responses of laminated spherical shells. Forced vibration responses of laminated cylindrical and spherical shells were reported by Lee and Han (2006). Nanda and Bandyopadhyay (2008, 2009) worked on nonlinear dynamic responses of laminated cylindrical and spherical

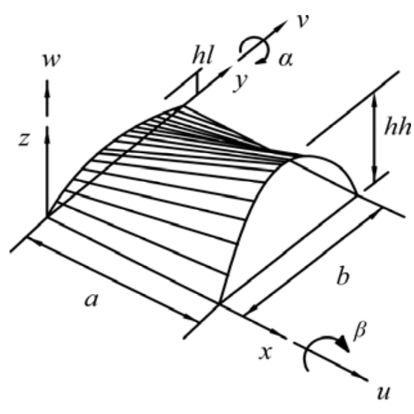


Fig. 1 Conoidal shell with degrees of freedom

shells subjected to transient loads. Similar studies on laminated conoidal shells are really scanty. Chakravorty *et al*. (1998) and Nayak and Bandyopadhyay (2006) were the only authors who reported transient responses of laminated conoidal shells. Nayak and Bandyopadhyay (2006) reported forced vibration of stiffened isotropic conoidal shells and Chakravorty *et al*. (1998) carried out limited study on transient responses of composite conoids. Hence, the present paper aims to study forced vibration characteristics of laminated composite conoidal shells subjected to three different load time histories. Complicated boundary conditions are chosen which the shell may have in industrial conditions.

2. Mathematical formulation

A composite conoidal shell of uniform thickness (h) and radii of curvatures R_y and R_{xy} is considered in the present study. The thickness of the shell may consist of any numbers of thin laminae in each of which the fibers are orientated at an angle θ with respect to the x axis of the shell (Fig. 2). The constitutive relationship of laminated composites is stated below

$$\{F\} = [D]\{\varepsilon\} \tag{1}$$

where

$$\{F\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x dz \\ \sigma_y dz \\ \tau_{xy} dz \\ \sigma_x zdz \\ \sigma_y zdz \\ \tau_{xy} zdz \\ \tau_{xz} dz \\ \tau_{yz} dz \end{Bmatrix} \qquad \text{(refer to Fig 3)}$$

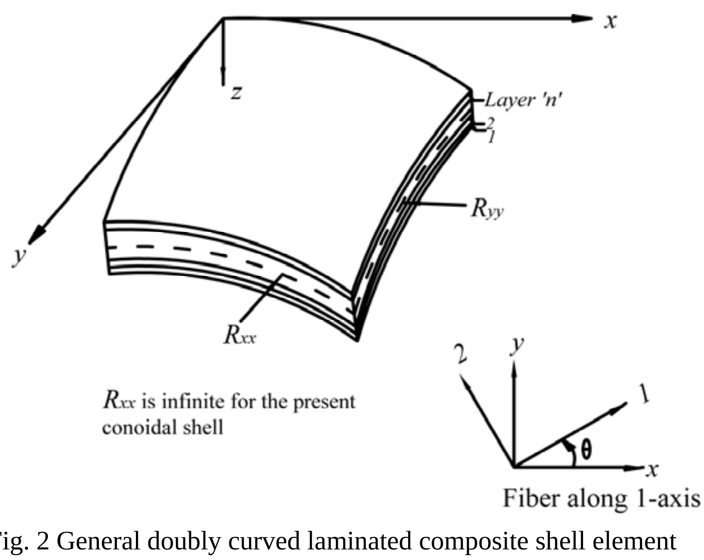


Fig. 2 General doubly curved laminated composite shell element

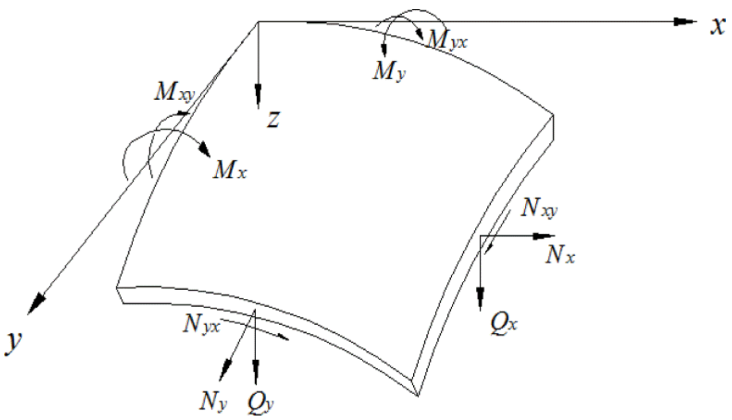


Fig. 3 Shell stress resultants

The laminate stiffness matrix [D] is given by

$$[D]=\begin{bmatrix}[A] & [B] & [0]\\ [B] & [D] & [0]\\ [0] & [0] & [S]\end{bmatrix} \tag{2}$$

$$\{\varepsilon\}=\{\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0 \quad k_x \quad k_y \quad k_{xy} \quad \gamma_{xz}^0 \quad \gamma_{yz}^0\}^T \tag{3}$$

The stiffness coefficients are

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{np}(Q_{ij})_k(z_k - z_{k-1}),\\
B_{ij} &= \frac{1}{2}\sum_{k=1}^{np}(Q_{ij})_k(z_k^2 - z_{k-1}^2), \qquad\qquad i,j=1,2,6\\
D_{ij} &= \frac{1}{3}\sum_{k=1}^{np}(Q_{ij})_k(z_k^3 - z_{k-1}^3)\\
S_{ij} &= \sum_{k=1}^{np}F_iF_j(G_{ij})_k(z_k - z_{k-1}) \qquad\qquad i,j=1,2
\end{aligned} \tag{4}$$

$(Q_{ij})_k$ are elements of the of axis elastic constant matrix which is expressed below

$$[Q_{ij}]^k = [T]^T[Q_{ij}]_{on}[T] \tag{5}$$

where

$$[Q_{ij}]_{on} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix} \tag{6}$$

$Q_{11} = (1-\nu_{12}\nu_{21})^{-1}E_{11}$, $Q_{22} = (1-\nu_{12}\nu_{21})^{-1}E_{22}$, $Q_{12} = (1-\nu_{12}\nu_{21})^{-1}E_{11}\nu_{21}$ and $Q_{66} = G_{12}$

The transformation matrix

$$[T] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \tag{7}$$

where $m = \cos\theta$ and $n = \sin\theta$.

The element strain vector is given by

$$\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy},\ \gamma_{xz},\ \gamma_{yz}\}^T = \{\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0,\ \gamma_{xz}^0,\ \gamma_{yz}^0\}^T + z\{k_x,\ k_y,\ k_{xy},\ k_{xz},\ k_{yz}\}^T \tag{8}$$

where the first vector contains the mid-surface strains for a conoidal shell and the second vector contains the change of curvatures due to loading. The strain displacement matrix adopted in the present study is same as reported by Das and Chakravorty (2007).

2.1 Finite element formulation

An eight noded curved quadratic isoparametric element is used here which consists of five degree of freedoms (u, v, w, α and β) at each node. The following shape functions are used to relate the element degree of freedoms with their nodal values.

$$\begin{aligned} N_i &= \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1) && i, j = 1, 2, 3, 4 \\ N_i &= \frac{1}{2}(1+\xi\xi_i)(1+\eta^2) && i = 5, 7 \\ N_i &= \frac{1}{2}(1+\eta\eta_i)(1+\xi^2) && i = 6, 8 \end{aligned} \tag{9}$$

2.2 Governing differential equation

Lagrange's equation of motion is derived using Hamilton's principle to express the governing differential equation of motion for an undamped forced vibration condition of an elastic system undergoing small displacement.

$$\frac{d}{dt}\left\{\frac{\partial U_2}{\partial \dot{d}_e}\right\} - \left\{\frac{\partial U_2}{\partial d_e}\right\} + \left\{\frac{\partial}{\partial d_e}(U_1 + U_3)\right\} = 0 \tag{10}$$

where U_1 = work done by conservative forces in shell element and is given by

$$U_1 = \frac{1}{2}\iint_A \{d_e\}^T [B]^T [D][B]\{d_e\} dx\, dy \tag{11}$$

U_2 = kinetic energy due to vibratory motion of an element and is expressed as

$$U_2 = \frac{1}{2}\iint_A \{\dot{d}_e\}^T [N]^T [m][N]\{\dot{d}_e\} dx\, dy \tag{12}$$

where dot (.) represents derivative with respect to time and $[m]$ is element mass matrix

$[m]=\begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$; $m=\rho h$ and $I=\dfrac{\rho h^3}{12}$. Where, ρ is mass density of shell.

The work done U_3 by surface tractions in an element is given by

$$U_3=-\iint_A \{d_e\}^T [N]^T [q]\, dx\, dy \tag{13}$$

where $[q]=\begin{bmatrix}0 & 0 & q & 0 & 0\end{bmatrix}^T$.

In the present analysis the conoidal shell is subjected to transverse loads only and the load is represented by the scalar notation 'q' hereafter.

Using Eqs. (11)- (12) - (13) into Eq. (10), we can get the following relationship

$$\left\{\iint_A [N]^T [m][N]\, dx\, dy\right\}\{\ddot{d}_e\}+\left\{\iint_A [B]^T [D][B]\, dx\, dy\right\}\{d_e\}=\iint_A [N]^T [q]\; dx\, dy \tag{14}$$

The coefficient of the element acceleration vector $\{\ddot{d}_e\}$ of Eq. (14) represents the consistent element mass matrix $[M_e]$ and that of the element displacement vector $\{d_e\}$ represents the element stiffness matrix $[K_e]$ and the term on the right hand side represents the consistent element nodal load vector $\{Q_e\}$.

Thus, Eq. (14) results in

$$[M_e]\{\ddot{d}_e\}+[K_e]\{d_e\}=\{Q_e\} \tag{15}$$

where

$$[M_e]=\iint_A [N]^T [m][N] dx\, dy,\quad [K_e]=\iint_A [B]^T [D][B] dx\, dy,\quad \{Q_e\}=\iint_A [N]^T \{q\} dx\, dy \tag{16}$$

The element mass matrix, stiffness matrix and load vector are transformed to isoparametric coordinates ξ and η for numerical integration by Gauss quadrature rule. The global stiffness $[K]$ and mass $[M]$ matrices and global load vector $\{Q\}$ of the shell are obtained as

$$[K]=\sum_{i=1}^{n}[K_e],\quad [M]=\sum_{i=1}^{n}[M_e],\quad [Q]=\sum_{i=1}^{n}[Q_e] \tag{17}$$

The dynamic equation of motion in the global form is

$$[M]\{\ddot{d}\}+[K]\{d\}=Q \tag{18}$$

2.3 The static problem

If the inertia force term of Eq. (18) is dropped and the displacement and load vectors are assumed to be time independent then the following equation of static equilibrium is obtained

$$[K]\{d\}=\{Q\} \tag{19}$$

The above equation is solved by the Gauss elimination technique.

2.4 The free vibration problem

If the load vector of Eq. (18) is dropped the equation of free vibration is obtained as

$$[K]\{d\}+[M]\{\ddot{d}\}=0 \tag{20}$$

In Eq. (20) the displacement $\{d\}$ is a function of space and time and solved using subspace iteration algorithm.

2.5 The forced vibration problem

In Eq. (18) the global load vector $\{Q\}$ is transient in nature and solved using Newmark's method to get the dynamic responses. Displacement for $(n+1)^{\text{th}}$ time step at time $t+\Delta t$ can be obtained using the following equations

$$\lfloor \hat{K} \rfloor \{d\}_{n+1}=\{\hat{Q}\}_{n+1} \tag{21}$$

where

$$\lfloor \hat{K} \rfloor=[K]+a_0\,[M] \tag{22}$$

and

$$\{\hat{Q}\}_{n+1}=\{Q\}_{n+1}+[M]\,a_0\{d\}_n+a_1\{\dot{d}\}_n+a\{\ddot{d}\}_n \tag{23}$$

with $a_0=1/(\beta\Delta t^2)$, $a_1=a_0\,\Delta t$, $a_2=1/2\beta-1$.

The acceleration vector $\{\ddot{d}\}$ and the velocity vector $\{\dot{d}\}$ can be computed from the displacement vector $\{d\}$ for n^{th} time step at time 't' as

$$\{\ddot{d}\}_{n+1}=a_0(\{d\}_{n+1}-\{d\}_n)-a_1\{\dot{d}\}_n-a_2\{\ddot{d}\}_n \tag{24}$$

and

$$\{\dot{d}\}_{n+1}=\{\dot{d}\}_n+a_3\{\ddot{d}\}_n+a_4\{\ddot{d}\}_{n+1} \quad \text{with,} \quad a_3=(1-\alpha')\Delta t \quad \text{and} \quad a_4=\alpha'\Delta t \tag{25}$$

The values of α and β are taken as 0.5 and 0.25 respectively as proposed by Newmark.

3. Numerical problems

The validation of the proposed static bending and free vibration formulations are established by comparing the present results with those published by Reddy (1984). The comparison of static displacements and fundamental frequencies of the cross ply simply supported spherical shells are presented in Tables 1 and 2 respectively. The material and geometric properties of the spherical shells are reported with the tables as footnote. For establishing the correctness of the forced vibration formulation proposed in this paper authors solve two problems as benchmark. The first

Table 1 Nondimensional central displacements ($\overline{w} \times 10^3$) of simply supported composite spherical shell under uniformly distributed load

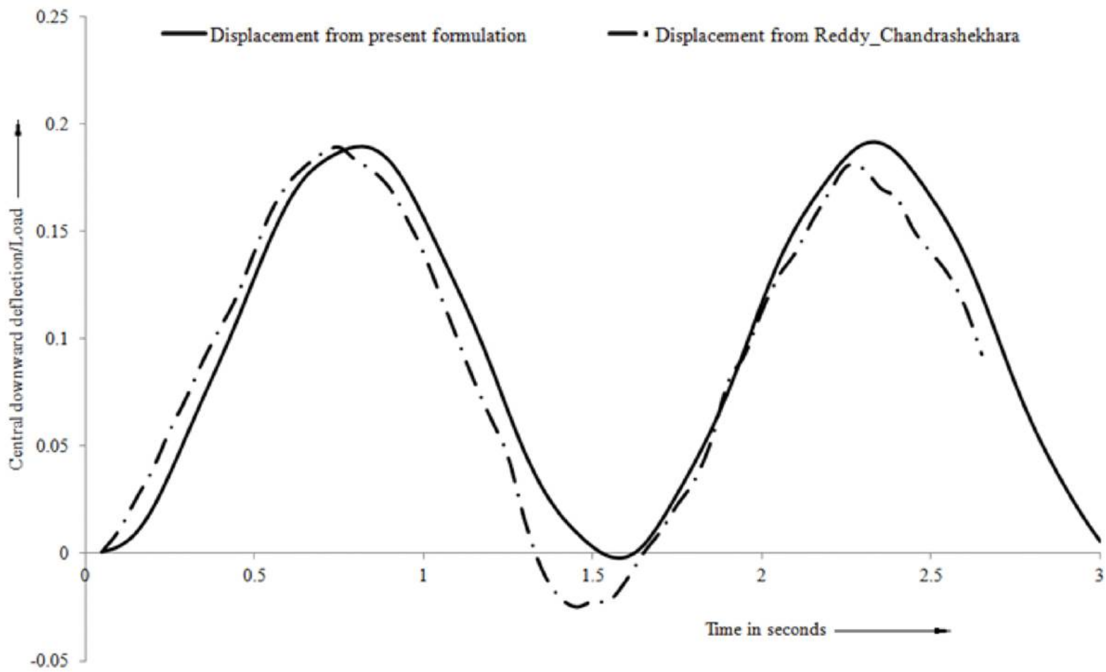
Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [2]	16.980	6.697	6.833
Present FEM (2 × 2)	8.294	5.116	4.644
(4 × 4)	16.898	6.724	6.820
(6 × 6)	16.969	6.710	6.826
(8 × 8)	17.009	6.707	6.835

[a] $a/b = 1$, $a/h = 100$, $E_{11} = 25E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\nu = 0.25$, $E_{22} = 10^6$ N/cm^2, $R/a = 10^{30}$

Table 2 Nondimensional fundamental frequencies (ϖ) of simply supported composite spherical shell

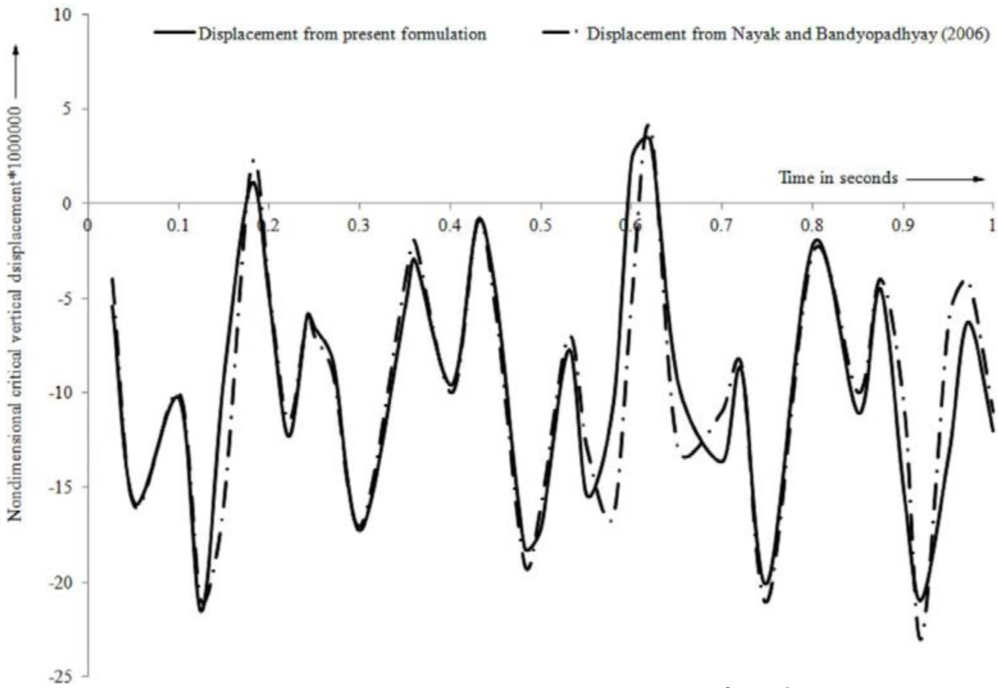
Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [2]	9.687	15.183	15.184
Present FEM (2 × 2)	14.897	15.209	15.221
(4 × 4)	9.722	15.209	15.222
(6 × 6)	9.691	15.179	15.195
(8 × 8)	9.681	15.180	15.183

[a] $a/b = 1$, $a/h = 100$, $E_{11} = 25E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\nu = 0.25$, $EZ_{22} = 10^6$ N/cm^2, $R/a = 10^{30}$



$R = 1000$ cm, $E_{11} = 25E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $E_{22} = 10^6$ N/cm^2, $\nu = 0.25$, $\rho = 1$ N-S^2/cm^6, lamination $= 0^0/90^0$

Fig. 4 Dynamic response of laminated composite spherical shell under uniformly distributed step load



$a/b = 1$, $b/h = 250$, $b/h_h = 0.15$, $h_l / h_h = 0.25$, $b = 25.0$ cm, $E = 25.491 \times 10^9$ N/m^2, $\nu = 0.15$, $\rho = 2500$ kg/m^3

Fig. 5 Dynamic response of isotropic conoidal shell under uniformly distributed step load

problem relates to the dynamic response of simply supported laminated composite spherical shell under step load of infinite duration which was solved earlier by Reddy and Chandrashekhara (1985). Authors have taken the liberty to incorporate the curvature term along *x*-axis in their strain displacement relations to work the formulation for spherical shell. The radius of cross curvature is assigned a high values (10^{30}) to make the cross curvature effectively zero. The geometric and material details of the problem are furnished with the Fig. 4 which shows the comparative response curves.

The second benchmark problem is one that was solved earlier by Nayak and Bandyopadhyay (2006) and deals with transient response of clamped isotropic conoidal shell under rectangular step load of infinite duration. Present formulation developed for composite shell is used for the isotropic material by making the elastic and shear modulii equal in all directions. For this problem also the relevant parameters are furnished with Fig. 5 shown the response curves.

Apart from the benchmark problems, the authors solved additional problems of forced vibration response of graphite-epoxy multilayered composites. The laminations include symmetric and antisymmetric, cross and angle ply stacking sequences for two different boundary conditions (Fig. 6) and three different uniformly distributed load-time histories (Fig. 7). To have a comprehensive idea about the relative performances of the shell options nondimensional values of static and dynamic displacements, nondimensional fundamental frequencies and dynamic magnification factors (henceforth referred as DMF) are presented systematically in Tables 3 and 4. The material and geometric properties of the conoidal shells for additional problems are presented with Table 3 and 4 as footnote. The magnitude of the static load is considered equal to the peak step load value

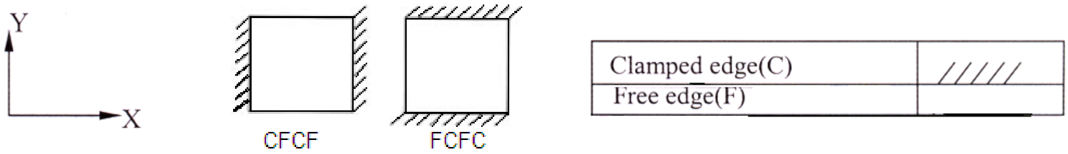
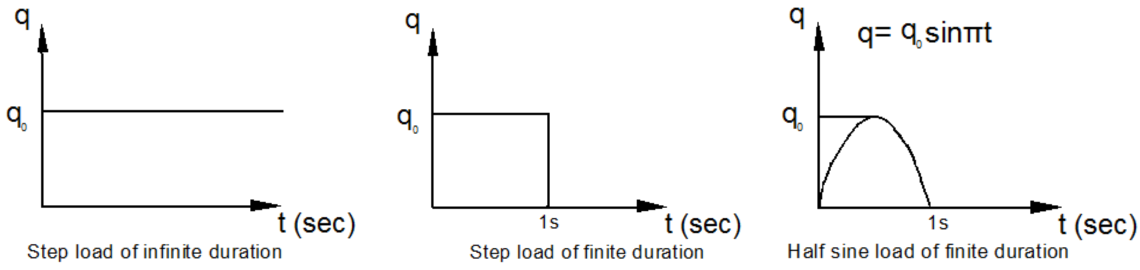


Fig. 6 Arrangement of the support conditions



(a) (b) (c)

Fig. 7 Transient load cases. (a) case I; (b) case II; (c) case III

of different load-time histories considered here. The DMF values calculated as ratios of maximum dynamic displacements to the corresponding static ones are also furnished to conclude on vulnerabilities of the shell combinations against transient loads.

4. Results and discussion

Tables 1 and 2 show good agreement of the present results with the values reported by Reddy (1984). Close agreement of static displacements in Table 1 ensure correct formulation of laminate stifness matrix in the corrent code. The results of the second benchmark problem indicated in Table 2 confirms accurate incorporation of mass matrix in the computer code. The natures of published response curves and those obtained by present approach that are shown in Figs. 4-5 establish correct incorporation of the time step integration scheme of Newmark's in the present code.

The shells are ranked from 1 to 16 in terms of static and dynamic displacements and the first rank being given to the shell option showing least displacement value. In terms of fundamental frequency the first rank goes to the shell exhibiting the highest fundamental frequency. Such ranks are provided in parentheses in Tables 3 and 4 and are very helpful to understand the relative behaviors of shell combinations comprehensively.

Two practical boundary conditions that are considered here involve a set of opposite edges being clamped (straight edges for FCFC and curved edges for CFCF boundary condition) and other set of opposite edges being free (straight edges for CFCF and curved edges for FCFC boundary condition). Naturally a practicing engineer may be interested to explore the relative performances of shells in terms of these two edge conditions as they have equal number of support movement locked but arranged in different manner. It is interesting to note from Table 3 that for

Table 3 Values of nondimensional static displacement, nondimensional fundamental frequency and DMF of composite conoidal shells for different laminations and boundary conditions

Lamination (Degree)	Boundary condition	Nondimensional downward static displacement	Nondimensional fundamental frequency	Dynamic magnification factors		
				Load case I	Load case II	Load case III
0 / 90	CFCF	0.00069276 (13)	43.68 (12)	2.47 (10)	2.47 (10)	1.08 (15)
	FCFC	0.000075639 (4)	43.25 (14)	2.47 (11)	2.47 (11)	1.04 (5)
0 / 90 / 0	CFCF	0.00078165 (15)	43.70 (11)	2.38 (5)	2.38 (4)	1.06 (10)
	FCFC	0.000058068 (1)	85.37 (1)	2.53 (12)	2.87 (16)	1.03 (3)
45 / -45	CFCF	0.00080401 (16)	42.36 (15)	2.33 (3)	2.33 (2)	1.05 (7)
	FCFC	0.000366430 (7)	38.96 (16)	2.44 (8)	2.44 (7)	1.05 (8)
45 / -45 / 45	CFCF	0.00069466 (14)	46.08 (10)	2.41 (7)	2.41 (5)	1.10 (16)
	FCFC	0.000369990 (8)	43.28 (13)	2.39 (6)	2.42 (6)	1.07 (12)
0 / 90 / 0 / 90	CFCF	0.000536680 (9)	50.90 (5)	2.34 (4)	2.34 (3)	1.02 (1)
	FCFC	0.000072675 (2)	60.73 (3)	2.31 (1)	2.44 (8)	1.02 (2)
0 / 90 / 90 / 0	CFCF	0.00064587 (12)	47.42 (9)	2.45 (9)	2.45 (9)	1.07 (13)
	FCFC	0.000073215 (3)	81.90 (2)	2.61 (16)	2.61 (15)	1.03 (4)
45 / -45 / 45 / -45	CFCF	0.00059939 (10)	50.60 (6)	2.31 (2)	2.31 (1)	1.05 (9)
	FCFC	0.000351320 (5)	51.33 (4)	2.60 (15)	2.60 (14)	1.04 (6)
45 / -45 / -45 / 45	CFCF	0.00062955 (11)	49.35 (7)	2.58 (14)	2.58 (13)	1.07 (14)
	FCFC	0.000357110 (6)	48.22 (8)	2.53 (13)	2.53 (12)	1.06 (11)

$a/b = 1$, $a/h = 100$, $h_h / a = 0.2$, $h_l / h_h = 0.25$, $E_{11} = 25E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\nu_{12} = 0.25$, $\rho = 100$ N-sec^2/m^4

Values in parentheses indicate ranks for respective shell actions

any given lamination the shells with straight edges clamped exhibit lower values of static displacements. The conoidal shell is relatively strong along the curved direction compared to the straight beam direction. Along arch direction, bending rigidity combines with axial rigidity to contribute to the stiffness of the shell. Hence a proper balance in stiffness may be achieved by locking the degree of freedoms along straight boundaries of the shell and this is why the FCFC boundary condition shows less static displacements then the CFCF one, as in CFCF boundary condition, the degree of freedoms along straight boundaries are free. When these two edge conditions are compared in terms of fundamental frequency, no such unified trend is evident. These establishes the fact that on observing a particular shell option stiffer than another in terms of deflection one cannot form an idea about their relative dynamic behavior and a free vibration analysis is absolutely necessary.

It is also interesting to observe that for FCFC boundary condition cross ply laminates show relatively better performances than the angle ply ones. This means in order to get the static displacement restrained an engineer's choice will be FCFC shells with cross ply laminations. The above conclusion is further reinforced by the fact that in terms of fundamental frequency also cross ply shells are better choices than angle ply laminations.

Table 4 Values of non-dimensional dynamic displacement of composite conoidal shell for different lamination and boundary condition

Lamination (Degree)	Boundary condition	Non-dimensional dynamic displacement		
		Load case I	Load case II	Load case III
0 / 90	CFCF	0.001709907 (14)	0.001709907 (14)	0.000748884 (13)
	FCFC	0.0001871060 (3)	0.0001871060 (3)	7.869650E-05 (4)
0 / 90 / 0	CFCF	0.001861472 (15)	0.001861472 (15)	0.000831947 (15)
	FCFC	0.0001467400 (1)	0.0001666780 (1)	5.963620E-05 (1)
45 / -45	CFCF	0.001874397 (16)	0.001874397 (16)	0.000845103 (16)
	FCFC	0.0008943230 (6)	0.0008943230 (5)	0.0003829420 (7)
45 / -45 / 45	CFCF	0.001674276 (13)	0.001674276 (13)	0.000762697 (14)
	FCFC	0.0008853630 (5)	0.0008955810 (6)	0.0003960180 (8)
0 / 90 / 0 / 90	CFCF	0.0012555590 (9)	0.0012555590 (9)	0.0005488560 (9)
	FCFC	0.0001680090 (2)	0.0001773890 (2)	7.44650E-05 (2)
0 / 90 / 90 / 0	CFCF	0.001583404 (11)	0.001583404 (11)	0.000693711 (12)
	FCFC	0.0001910040 (4)	0.0001910040 (4)	7.546840E-05 (3)
45 / -45 / 45 / -45	CFCF	0.001383673 (10)	0.001383673 (10)	0.000629455 (10)
	FCFC	0.0009146110 (8)	0.0009146110 (8)	0.0003662540 (5)
45 / -45 / -45 / 45	CFCF	0.001625347 (12)	0.001625347 (12)	0.000670493 (11)
	FCFC	0.0009043670 (7)	0.0009043670 (7)	0.0003781450 (6)

$a/b = 1$, $a/h = 100$, $h_h / a = 0.2$, $h_l / h_h = 0.25$, $E_{11} = 25E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\nu_{12} = 0.25$, $\rho = 100$ N-sec^2/m^4

Values in parentheses indicate ranks for respective shell actions

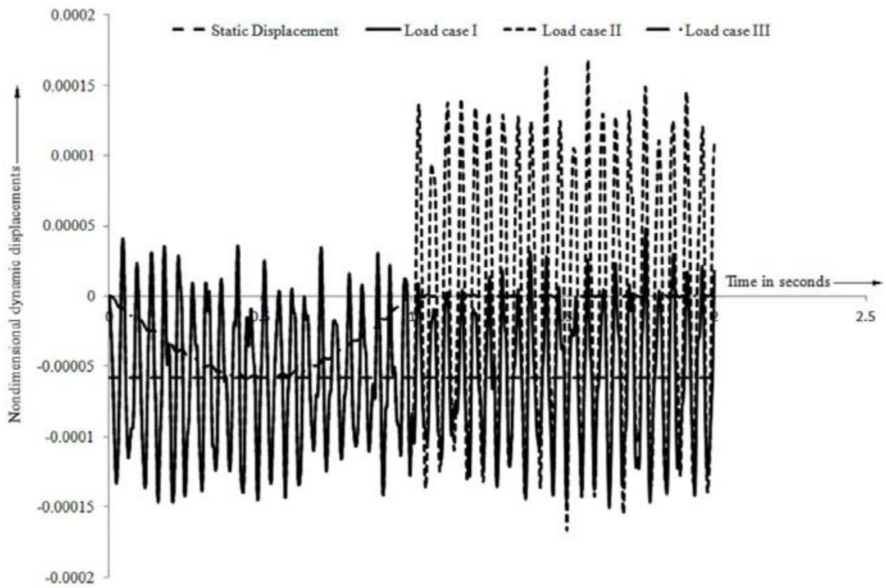


Fig. 8 Response curve for conoidal shell at $x = 0$ m, $y = 0.5$ m
Boundary condition: FCFC; Lamination: $0^0 / 90^0 / 0^0$

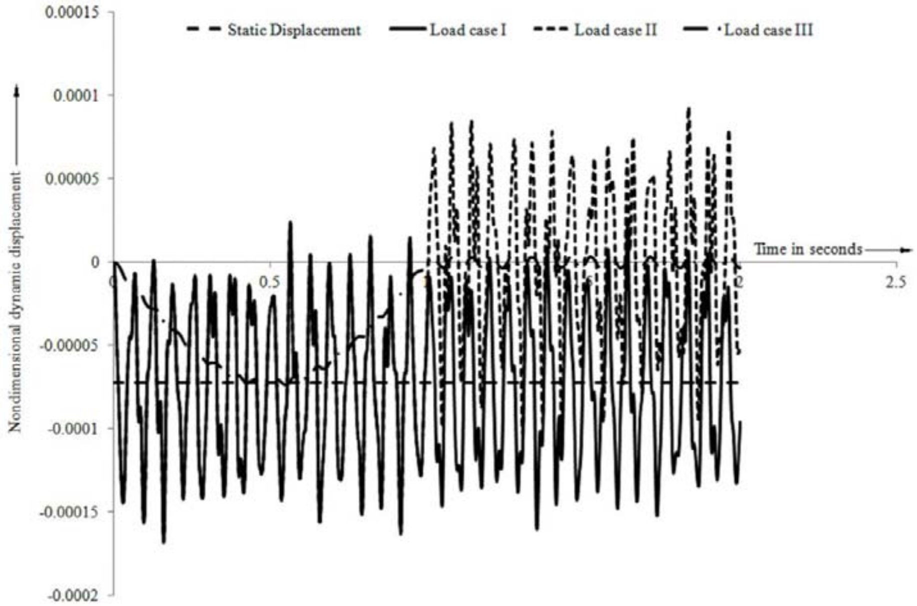


Fig. 9 Response curve for conoidal shell at x = 0 m, y =0.5 m
Boundary condition: FCFC; Lamination: 0^0 / 90^0 / 0^0 / 90^0

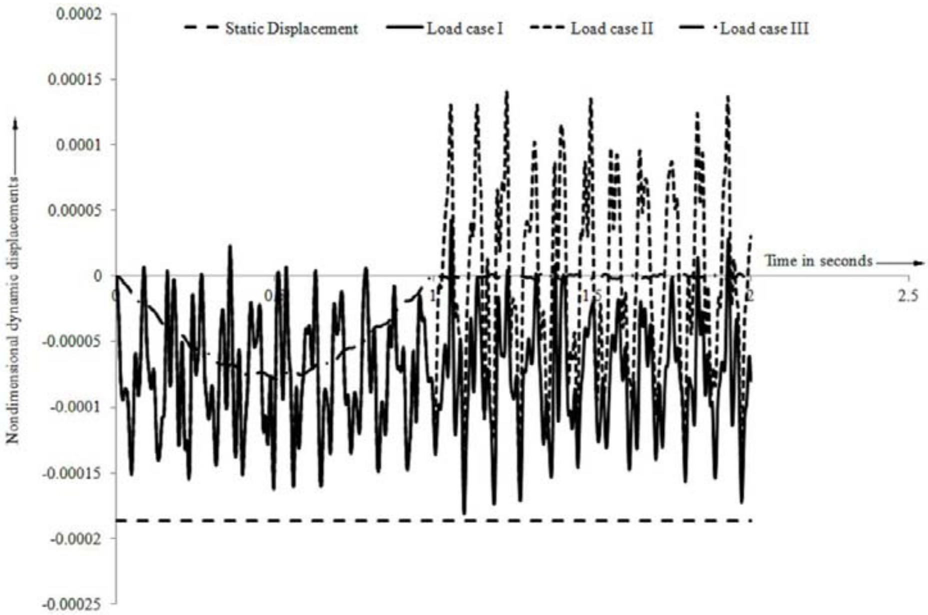


Fig. 10 Response curve for conoidal shell at x = 0 m, y = 0.5 m
Boundary condition: FCFC; Lamination: 0^0 / 90^0

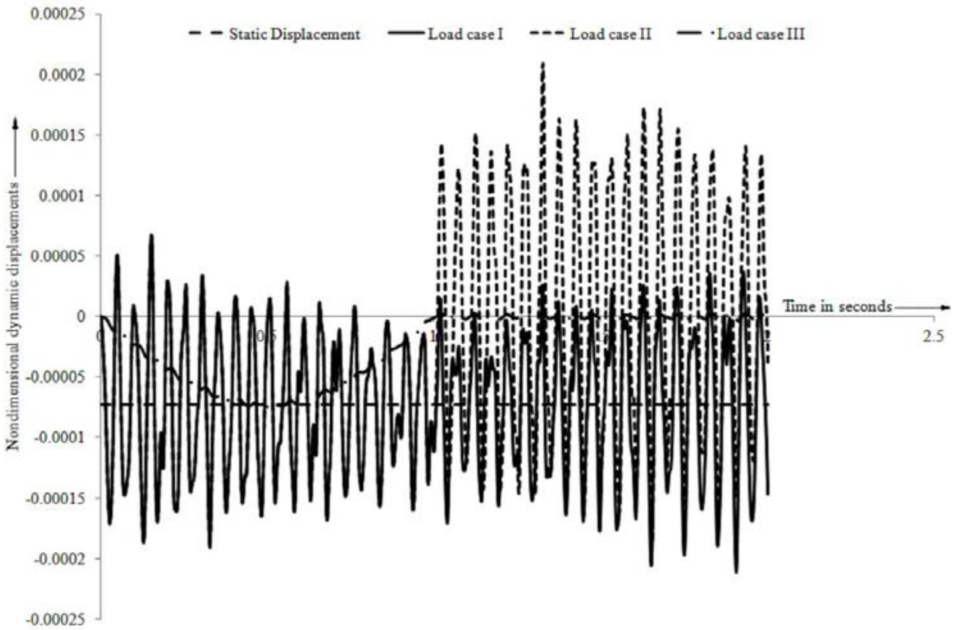


Fig. 11 Response curve for conoidal shell at x = 0 m, y = 0.5 m
Boundary condition: FCFC; Lamination: 0^0 / 90^0 / 90^0 / 0^0

With the dynamic displacements studied from Table 4 the relatively superior performances of FCFC cross ply shells is re-established. The dynamic responses of the shells are studied upto 2 seconds duration. This is required to observe the displacement responses that continue to act even after the loads are removed at the end of 1second for load cases II and III. It is established from the results furnished in Table 3 that DMF values are nearly equal for load cases I and II for almost all the shell options and these values are more than 2. In load case III both the application and withdrawn of pressure are gradual and DMF values are marginally greater than 1. Based on ranks of nondimensional dynamic displacement values in Table 4 best six cases are selected for more detailed dynamic analysis. Time variation of dynamic displacements for all the load cases including the static displacement are presented graphically in Figs. 8-13. Dynamic displacements are studied at the location of maximum static displacements and their respective x and y coordinates are presented with the figures. The difference of shell responses under load cases I and II is that for load case I the dynamic displacement tend to converge to static ones with passage of time and for load case II the shell vibrates at almost its fundamental frequency (free vibration) when the load is withdrawn as shown in Figs. 8-13.

The ranks of the shell options in terms of static displacement and dynamic displacement for load case III are identical, whereas such ranks for load case I and II are drastically different, though the peak values for all the transient load cases are equal. Theses show that the mode of load application is more important than its actual magnitude. Dynamic responses against rapidly applied or withdrawn loads are much more severe than gradually applied and withdrawn (quasi static) loads. Dynamic displacements in the above figures are observed to oscillate at different

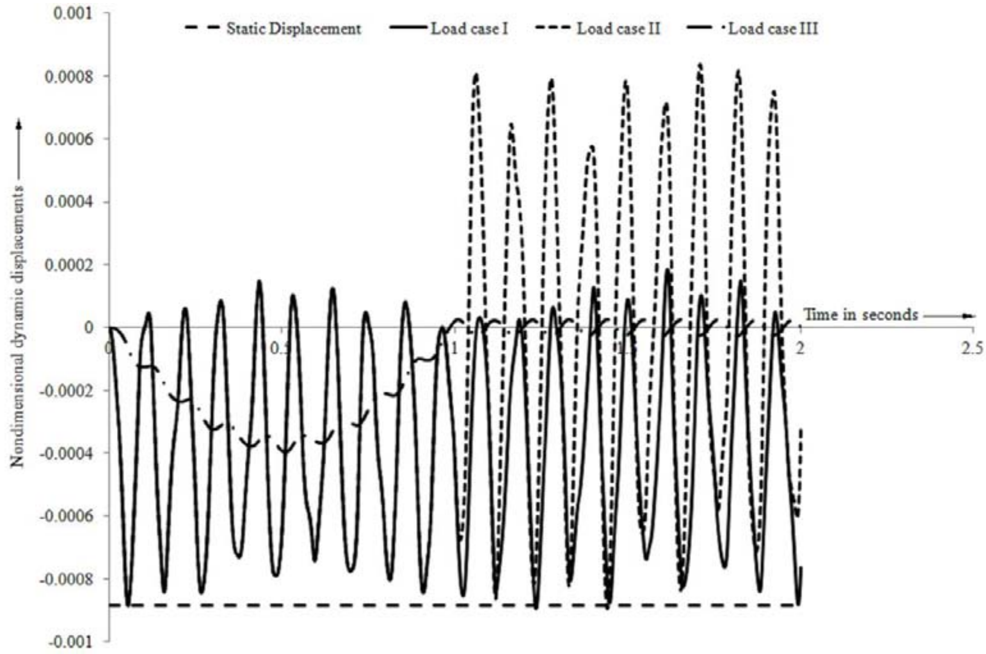


Fig. 12 Response curve for conoidal shell at x = 0 m, y = 0.44 m
Boundary condition: FCFC; Lamination: 45^0 / -45^0 / 45^0

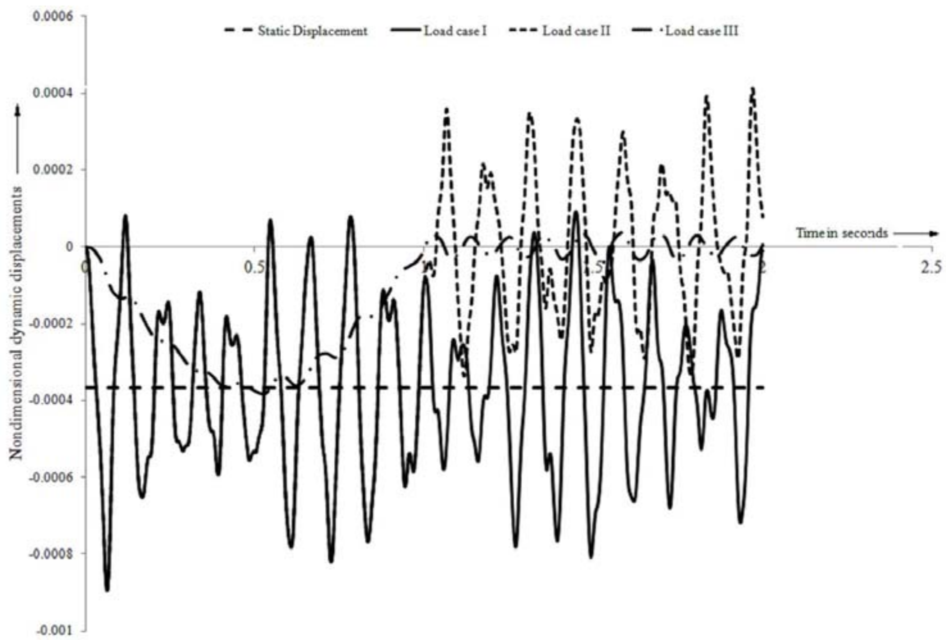


Fig. 13 Response curve for conoidal shell at x = 0 m, y = 0.5 m
Boundary condition: FCFC; Lamination: 45^0 / -45^0

points of time for all the transient load cases, resulting in even upward displacements. This observation brings out the importance of forced vibration analysis as static simplification of dynamic problems using a suitable factor cannot account for the reversal of dynamic displacements and hence stresses.

Another very interesting observation is that the points of occurrence of maximum static and maximum dynamic displacement may not match at a few places. Response curves in Figs. 10 and 12 show that dynamic displacements for all the load cases either marginally or do not exceed the static one but Table 3 shows that for all the conditions DMF greater than unity. This is because the DMF considered in present study is the ratio of the maximum dynamic displacement to the maximum static displacement irrespective of their point of occurrence and the response curves are plotted taking the static and dynamic displacement at the point of occurrence of maximum static displacement. Although in the previous cases it is found that some correlation exists between boundary condition and lamination for static displacement and fundamental frequency but such relationships cannot be formulated so far the variation of dynamic magnification factor is concerned.

5. Conclusions

The comparative study of composite conoidal shells for different boundary conditions and laminations reveal that cross ply laminated FCFC shell is the best choice from all the static bending, free and forced vibration points of view among the shell combinations considered here. Conclusions drawn by analyzing relative performances of shells for static analysis cannot provide an idea about the behavior of those shells under dynamic conditions and to get that a separate vibration analysis is needed. A detail dynamic analysis under different transient load-time histories reflect that dynamic responses are magnified than the static ones and for that matter time variation of transient loads are more important than their magnitudes. The dynamic displacements are not only magnified by the transient loads but also they show a reversal nature in them. Hence, the static simplification of a dynamic problem using a suitable factor does not give the full picture of the dynamic behavior of the shell. It is also important note that the point of occurrence of maximum static and dynamic displacement may not be same in a shell. Hence dynamic analysis at the point of maximum static displacement will underestimate the maximum dynamic displacement and stresses.

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Notations

The following symbols are used in this paper

A	Area of the shell.
a,b	Length and width of shell in plan.
D	Flexural rigidity matrix of the conoidal shell.
d_e	Element displacements.
d	Global displacements.
E_{11}, E_{22}, E_{33}	Young's moduli.
G_{12}, G_{13}, G_{23}	Shear moduli.
h_h, h_l	Higher and lower heights of conoidal shell.
h	Shell thickness.
M_x, M_y, M_{xy}	Moment resultants per unit length.
N_x, N_y, N_{xy}	Force resultants per unit length.
np	Number of plies in a laminate.
n	Number of elements.
Q_x, Q_y	Transverse shear resultants per unit length.
q_0	Peak value of the transient load cases.
R_{xy}	Radius of cross curvature of conoidal shell.
R_y	Radius of curvature of conoidal shell along y axis.
R	Radius of the spherical shell.
u, v, w	Degree of freedoms along x, y and z directions respectively.
$\overline{w}$	Nondimensional transverse displacement of shell $\left[= wE_{22}h^3/\left(qa^4\right)\right]$
$\ddot{u}$	Global acceleration vector.
x,y and z	Global co-ordinates of the laminate.
1,2 and 3	Local co-ordinates of a lamina.
α,β	Rotational degrees of freedom about y and x axes respectively.
$\varepsilon_x, \varepsilon_y$	Inplane strains.
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	Shear strains.
θ	Fiber orientation with respect to the x axis of the shell.
ν_{ij}	Poisson's ratio.

ξ, η	Isoparametric coordinates of isoparametric elements.
ρ	Mass density of material.
$\sigma_x,\ \sigma_y$	Inplane stresses.
$\tau_{xy},\ \tau_{xz},\ \tau_{yz}$	Shear stresses.
$k_x,\ k_y\ k_{xy}$	Curvatures of the element.
ω	Fundamental frequency.
$\overline{\omega}$	Nondimensional fundamental frequency $\left[= \omega a^2 \left(\rho / E_{22} h^2\right)^{1/2}\right]$