

## Optimum design of steel frames with semi-rigid connections using Big Bang-Big Crunch method

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**Abstract.** The Big Bang-Big Crunch (BB-BC) optimization algorithm is developed for optimal design of non-linear steel frames with semi-rigid beam-to-column connections. The design algorithm obtains the minimum total cost which comprises total member plus connection costs by selecting suitable sections. Displacement and stress constraints together with the geometry constraints are imposed on the frame in the optimum design procedure. In addition, non-linear analyses considering the  $P-\Delta$  effects of beam-column members are performed during the optimization process. Three design examples with various types of connections are presented and the results show the efficiency of using semi-rigid connection models in comparing to rigid connections. The obtained optimum semi-rigid frames are more economical solutions and lead to more realistic predictions of response and strength of the structure.

**Keywords:** Big Bang-Big Crunch algorithm; meta-heuristics; semi-rigid connections; steel frames; optimum design

### 1. Introduction

Despite existing major preventive factors in performing optimum design of structures such as the large number of structural required analyses and large computational costs, designers and owners have always desired to have optimal structures (Kaveh and Talatahari 2009a, b, 2010a). The optimum design of framed structures is one of the most convenient activities of research in the field of structural optimization. As it is involved in an optimization problem, the main purpose of frame optimization is to minimize the cost of a structure as an objective function considering the design or geometry requirements as constraints. The term cost may refer to the weight of structure or other economical characteristics of a structure.

On the other hand, in the current practice of optimum design of steel-framed building structures, the actual behavior of beam-to-column connections is simplified to the two idealized extremes of either fully-rigid behavior or ideally-pinned behavior. The first case implies displacement and slope continuity between the column and the beam, together with the full transfer of bending moments. The latter one, on the other hand, implies that the rotation continuity is nonexistent, and consequently, no bending moment may be transmitted to the column by the beam. Although the adoption of such idealized joint behavior greatly simplifies the analysis and design processes, the

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predicted response of the idealized structure may be quite unrealistic compared to that of the actual structure. The reason is that most connections used in current practice actually exhibit semi-rigid deformation behavior that can contribute substantially to overall structure displacements. Numerous experimental investigations on connection behavior have clearly demonstrated that a pinned connection possesses a certain amount of rotational stiffness, while a rigid connection possesses some degree of flexibility.

The semi-rigid behavior of beam to column connections has been investigated by current steel specifications such as British standard, Eurocode3 and American Institute of Steel Construction (AISC). Moreover, AISC-Load and Resistance Factor Design specification describes two types of steel construction: fully restrained (FR type) and partially restrained (PR type). PR type constructions are considered to be semi-rigid and its behavior is described by numerical and experimental studies.

Analysis and design of steel frames with semi-rigid connections and moment-rotation behavior modeling of such connections have been investigated in some studies (Lui and Chen 1986, Frye and Morris 1975, Valipour and Bradford, 2013, Gorgun and Yilmaz 2012). Determining optimum design of steel frames with semi-rigid connections and using the weight of frames as the objective has been implemented by Alsalloum and Almusallam (1995), Almusallam (1995) and Kameshki and Saka (2003). However, minimum cost design of semi-rigidly connected steel frames are investigated by Simoes (1996) and Li *et al.* (1997). As meta-heuristic methods, Genetic Algorithm (GA) (Hayalioglu and Degertekin 2005) and Harmony Search (HS) method (Degertekin and Hayalioglu 2010) are utilized to obtain the optimum designs for steel frames with semi-rigid beam-to-column connections and column bases, where the cost function includes the member plus connection costs.

Big Bang-Big Crunch (BB-BC), a relatively new meta-heuristic optimization method (Erol and Eksin 2006), relies on one of the theories of the evolution of the universe namely, the Big Bang and Big Crunch theory. In the Big Bang phase of this theory, energy dissipation produces disorder and randomness is the main feature of this phase; whereas, in the Big Crunch phase, randomly distributed particles are drawn into an order. The BB-BC optimization method similarly generates random points in the Big Bang phase and shrinks them to a single representative point via a center of mass in the Big Crunch phase. After a number of successive Big Bangs and Big Crunches, where the distribution of randomness within the search space during the Big Bang becomes smaller and smaller about the average point computed during the Big Crunch, the algorithm converges to a solution.

BB-BC was utilized to solve different engineering optimization problems. Afshar and Motaei (2011) used BB-BC to determine the optimal solution of reservoir operation problems. Parameter estimation in structural systems using BB-BC is performed by Tang *et al.* (2010). Several structural optimization examples, including space truss, dome and steel framed structures have been solved using this method by Kaveh and Talatahari (2009a, 2010b, c).

This study presents a BB-BC-based optimization method, for minimum cost design of steel frames with semi-rigid beam to column connections, wherein the aim is to minimize the constructional cost includes the utilized materials for members as well as connection costs. American Institute of Steel Construction (AISC) wide-flange (*W*) shapes are used as the available standard sections list, in the optimal design procedure. Strength constraints of AISC-LRFD (1995) specification, displacement constraint and geometry constraints for columns and beams are imposed on frames. The optimum design algorithm considers both the geometric non-linearity of the frame members and the semi-rigid behavior of the beam to column connections. The behavior

of beam to column connections are assumed to be defined by the Frye-Morris polynomial model (Frye and Morris 1975) for the calculations of the moment-rotation relationship, whereas, the column bases are supposed to be rigid.

## 2. Review on BB-BC method

The BB-BC method as developed by Erol and Eksin (2006) consists of two phases: A Big Bang phase, where candidate solutions are randomly distributed over the search space, and a Big Crunch phase, where a contraction operation estimates a weighted average of the randomly distributed candidate solutions.

Similar to other evolutionary algorithms, initial solutions are uniformly spread all over the search space in the first Big Bang. The algorithm associates the random nature of the Big Bang to energy dissipation or the transformation from an ordered state (a convergent solution) to a disordered or chaotic state (new set of candidate solutions).

After the Big Bang phase, a contraction operation is applied during the Big Crunch. This operator has several inputs but only one output, which is named as the “center of mass”, since the only output has been derived by calculating the center of mass. The term mass refers to the inverse of the objective function value,  $Mer^j$ . This representative point is denoted by  $A_i^{c(k)}$  and calculated according to

$$A_i^{c(k)} = \frac{\sum_{j=1}^N \frac{1}{Mer^j} \cdot A_i^{(k,j)}}{\sum_{j=1}^N \frac{1}{Mer^j} \cdot A_i^{(k,j)}} \quad (1)$$

where  $A_i^{(k,j)}$  is the  $i$ th component of the  $j$ th solution generated in the  $k$ th iteration;  $N$  is the population size in Big Bang phase. After the Big Crunch phase, the algorithm creates the new solutions to be used as the Big Bang of the next iteration step, by using the center of mass. This can be accomplished by spreading new off-springs around the center of mass using a normal distribution operation in every direction, where the standard deviation of this normal distribution function decreases as the number of iterations of the algorithm increases

$$A_i^{(k+1,j)} = A_i^{c(k)} + \frac{r_j \alpha_1 (A_{\max} - A_{\min})}{k+1}, \quad i = 1, 2, \dots, ng \quad (2)$$

where  $r_j$  is a random number from a standard normal distribution which changes for each candidate, and  $\alpha_1$  is a parameter for limiting the size of the search space. The allowable values for the components of the candidate solutions are restricted to the boundaries of  $A_{\min}$  and  $A_{\max}$  and  $ng$  denotes the total number of design components. In a structural optimization problem,  $A_{\min}$ ,  $A_{\max}$  and  $ng$  refer to the minimum and maximum cross-sectional areas and the total number of design variables of the structure.

## 3. Optimum design of semi-rigid frames

The total cost of a steel frame with semi-rigid beam to column connections, considering

member and connection costs, is defined by Xu and Grierson (1993) as follows

$$Cost(x) = \sum_{i=1}^{nm} W_i A_i + \sum_{i=1}^{nbm} \sum_{j=1}^2 (\beta_{ij} R_{ij} + \beta_{ij}^0) \quad (3)$$

where  $A_i$  and  $W_i$  are the  $i$ th member cross-section area and weight coefficient, respectively ( $W_i$  = material density  $\times$  member length),  $R_{ij}$  and  $\beta_{ij}$  are the connection rotational stiffness and cost coefficient, and  $\beta_{ij}^0$  is the cost of a pinned connection having zero rotational stiffness. The  $j$ -subscripts in Eq. (3) correspond to two ends of the semi-rigid beam member and  $nm$  and  $nbm$  denote the total number of members and beams of a frame, respectively.

The values of  $\beta_{ij}$  for two ends of a semi-rigid member are assumed to be equal and calculated as

$$\beta_i = \frac{0.225 W_i A_i}{S_i} \quad (4)$$

where  $S_i$  is rotational stiffness of a connection which is a estimated value depending on the stiffness of the connection, equal for the both ends of a beam and lies in the range  $2.26 \times 10^5$  kN.mm/rad to  $5.65 \times 10^8$  kN.mm/rad as suggested by Xu and Grierson (1993) and the equal value for both  $\beta_{i1}^0$  and  $\beta_{i2}^0$  is accepted to be

$$\beta_i^0 = 0.125 W_i A_i \quad (5)$$

The optimum design problem of a steel frame with semi-rigid connections has the following constraints.

The strength constraints of AISC-LRFD (1995) considering the interaction of bending moment and axial force can be formulated in the normalized form as

$$V_i^{IER} = \begin{cases} \left( \frac{P_u}{\phi P_n} \right)_i + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} \right)_i - 1.0 & \text{for } \frac{P_u}{\phi P_n} < 0.2 \\ \frac{1}{2} \left( \frac{P_u}{\phi P_n} \right)_i + \left( \frac{M_{ux}}{\phi_b M_{nx}} \right)_i - 1.0 & \text{for } \frac{P_u}{\phi P_n} > 0.2 \end{cases}, \quad i = 1, \dots, nm \quad (6)$$

where  $P_u$  and  $P_n$  are required and nominal strength of a member (tensile or compressive), respectively and  $\phi$  resistance reduction factor, which is equal to 0.9 for the member in tension and 0.85 for compressive ones. Moreover,  $M_{ux}$  and  $M_{nx}$  are notations for required and nominal flexural strength of the member about its major axis, respectively and reduction factor that corresponds to bending is denoted by  $\phi_b$  (equal to 0.9).  $nm$  is the total number of members in the frame. The nominal strength of a compressive member is calculated based on AISC-LRFD (1995) as follows

$$P_n = A F_{cr} \quad (7)$$

$$F_{cr} = \begin{cases} \left( 0.658^{\lambda_c^2} \right) F_y & \text{for } \lambda_c \leq 1.5 \\ \left( \frac{0.877}{\lambda_c^2} \right) F_y & \text{for } \lambda_c > 1.5 \end{cases} \quad (8)$$

$$\lambda_c = \frac{KL}{r\pi} \sqrt{\frac{F_y}{E}} \quad (9)$$

where  $A$  is cross-sectional area;  $F_y$  is yield stress; and  $E$  is modulus of elasticity of steel member.  $L$  and  $r$  are the member length and radius of gyration, respectively. The effective length factor, which is denoted by  $K$  in Eq. (9), is needed in stability evaluation of the columns in the frame.  $K$ -factor of columns in an unbraced semi-rigid frame is calculated using the relations proposed by Kishi *et al.* (1997).

The displacement normalized constraints including the constraints of inter-storey drift and top storey sway can be formulated in general form of

$$V_j^d = \frac{|\delta_j|}{\delta_j^u} - 1.0, \quad j = 1, \dots, m \quad (10)$$

where  $\delta_j$  is the displacement of the  $j$ th restricted displacements among the total number of  $m$ , and  $\delta_j^u$  is its allowable upper bound limit determined by the code of practice.

The other group of constraints imposed on the optimization problem in this study arises from the size adaptations of beams and columns relative to each other. This group consists of two constructional considerations: one consideration implies that flange width of a beam must be smaller than the same value for column in all joints, whereas, the other one considers the fact that the column of each storey must be larger in depth compared to its above storey column. These constraints can be formulated as (Hasançebi *et al.* 2010).

$$V_p = \frac{b_f^{bp}}{b_f^{cp}} - 1.0, \quad p = 1, \dots, nj \quad (11)$$

$$V_q = \frac{d_c^{uq}}{b_c^{lq}} - 1.0, \quad q = 1, \dots, nc \quad (12)$$

where  $b_f^{bp}$  and  $b_f^{cp}$  are the value of flange width for beam and column in node number  $p$  among the total number of  $nj$  nodes, respectively ( $nj$  is the total number of nodes of frame except the supports). The  $d_c^{uq}$  and  $d_c^{lq}$  are notations for depths of column sections of upper and lower floor in a node, respectively.  $nc$  is the total number of columns in the frame excluding ones for first storey.

The optimum design problem, considered in the present work, is a constrained problem; we can transform it into an unconstrained one using a penalty function. Here, we use the penalty function suggested by Rajeev and Krishnamoorthy (1992), so the objective function of the problem can be computed as

$$Mer(x) = Z(x) \left[ 1 + C \left( \sum_{i=1}^{nm} v_j^{IER} + \sum_{i=1}^m v_j^d + \sum_{p=1}^{nj} v_p + \sum_{q=1}^{nc} v_q \right) \right] \quad (13)$$

where  $Z(x)$  is calculated by Eq. (3);  $C$  is a penalty constant and in this paper it is equal to 1.0;  $v_i^{IER}$ ,  $v_j^d$ ,  $v_p$  and  $v_q$  are the violations of normalized interaction equation ratio, displacement, geometry considerations for beams and columns, respectively and are computed as

$$\begin{cases} \text{if } V_i^{IER} \geq 0 & \text{then } v_i^{IER} = V_i^{IER} \\ \text{if } V_i^{IER} \leq 0 & \text{then } v_i^{IER} = 0 \end{cases} \quad (14)$$

$$\begin{cases} \text{if } V_i^d \geq 0 & \text{then } v_i^d = V_i^d \\ \text{if } V_i^d \leq 0 & \text{then } v_i^d = 0 \end{cases} \quad (15)$$

$$\begin{cases} \text{if } V_p \geq 0 & \text{then } v_p = V_p \\ \text{if } V_p \leq 0 & \text{then } v_p = 0 \end{cases} \quad (16)$$

$$\begin{cases} \text{if } V_q \geq 0 & \text{then } v_q = V_q \\ \text{if } V_q \leq 0 & \text{then } v_q = 0 \end{cases} \quad (17)$$

#### 4. Nonlinear analysis of steel frames with semi-rigid connections

It is obvious that the actual complex behavior of a structure must be simplified for analysis by feasible modeling of it. Among the numerous experimental and numerical studies on the modeling of semi-rigid beam-to-column connections, the model proposed by Frye and Morris (1975) is adopted to use in this work, due to its easy-to-implement characteristic. This odd-power polynomial model is reasonably good for simulation of the nonlinear  $M - \theta$  behavior of connections and has been presented as

$$\theta = c_1(\kappa M) + c_2(\kappa M)^3 + c_3(\kappa M)^5 \quad (18)$$

where  $\theta$  is connection rotation and  $M$  is the moment acting on the connection. Parameter  $\kappa$  is the standardization factor determined by the connection type and geometry.  $c_1$ ,  $c_2$  and  $c_3$  are curve-fitting constants obtained by using the method of least squares.

For several types of beam-to-column connections, which are shown in Fig. 1, the values of the

Table 1 The Curve fitting constants and standardization parameters for Frye-Morris polynomial model

Connection type	Curve fitting constants			Standardization parameter
	$c_1$	$c_2$	$c_3$	
1	$4.28 \times 10^{-3}$	$1.45 \times 10^{-9}$	$1.51 \times 10^{-16}$	$\kappa = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
2	$3.66 \times 10^{-4}$	$1.15 \times 10^{-6}$	$1.57 \times 10^{-8}$	$\kappa = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
3	$2.23 \times 10^{-5}$	$1.85 \times 10^{-8}$	$3.19 \times 10^{-12}$	$\kappa = d^{1.287} t^{-1.128} t_c^{-0.415} l_a^{-0.694} g^{1.35}$
4	$8.46 \times 10^{-4}$	$1.01 \times 10^{-4}$	$1.24 \times 10^{-8}$	$\kappa = d^{1.5} t^{-0.5} l_a^{-0.7} d_b^{-1.5}$
5	$1.83 \times 10^{-3}$	$1.04 \times 10^{-4}$	$6.37 \times 10^{-6}$	$\kappa = d_g^{-2.4} t_p^{-0.4} d_b^{-1.5}$
6	$1.79 \times 10^{-3}$	$1.76 \times 10^{-4}$	$2.04 \times 10^{-4}$	$\kappa = d_g^{-2.4} t_p^{-0.6}$
7	$2.10 \times 10^{-4}$	$6.20 \times 10^{-6}$	$-7.60 \times 10^{-9}$	$\kappa = d^{1.5} t^{-0.5} l_t^{-0.7} d_b^{-1.1}$
8	$5.10 \times 10^{-5}$	$6.20 \times 10^{-10}$	$2.40 \times 10^{-13}$	$\kappa = d_p^{-2.3} t_p^{-1.6} t_w^{-0.5} g^{-1.6}$

constants  $c_1$ ,  $c_2$  and  $c_3$  and the parameter  $\kappa$  for each type are illustrated in Table 1 (Faella *et al.* 2000). The schematic  $M-\theta$  curves for these eight types of connections are drawn in Fig. 2 according to Chen *et al.* (1996).

In the analysis procedure of the steel frames with semi-rigid beam-to-column connections, we consider the nonlinear  $M-\theta$  behavior of semi-rigid connections, and the geometrical nonlinearity of beam-column members. In this study, the displacement method is used to analyze the structure, wherein, the stiffness matrix of the structure is constructed through assembling of the stiffness matrices of members in the global coordinates. The secant stiffness approach is applied to consider

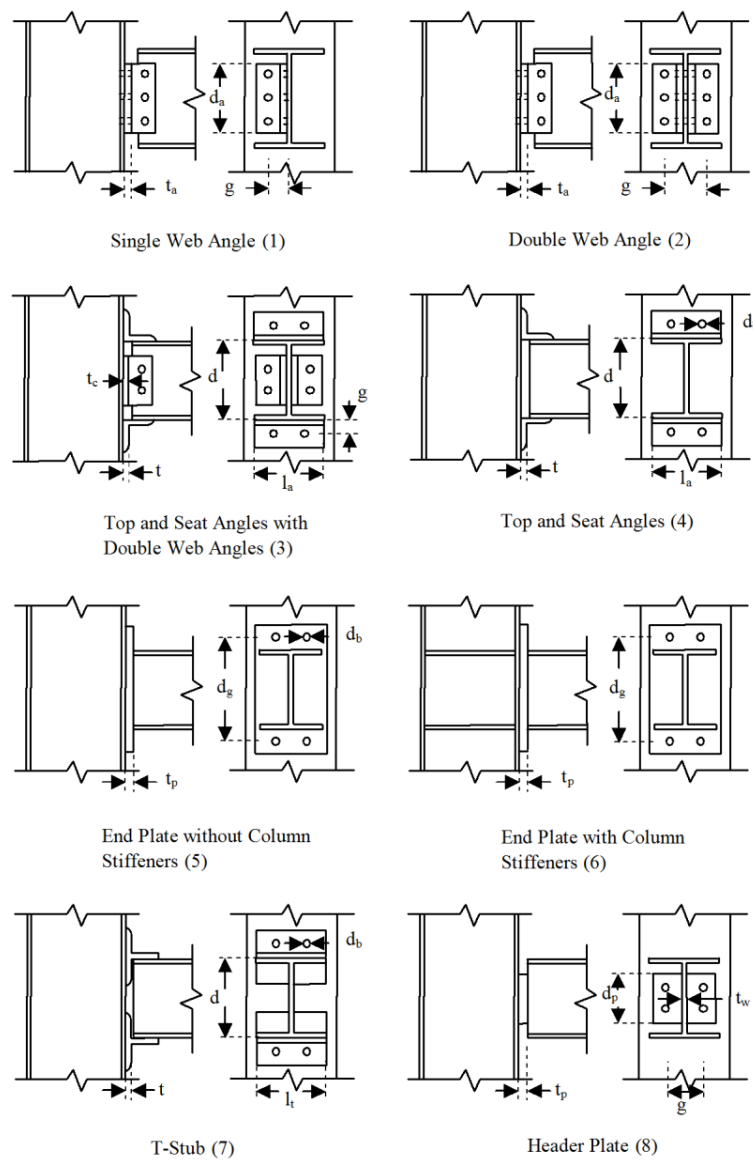


Fig. 1 Eight types of semi-rigid connections and their size parameters

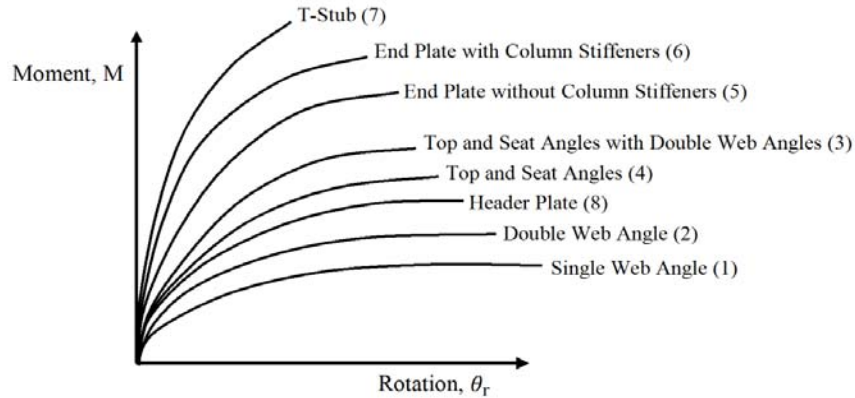
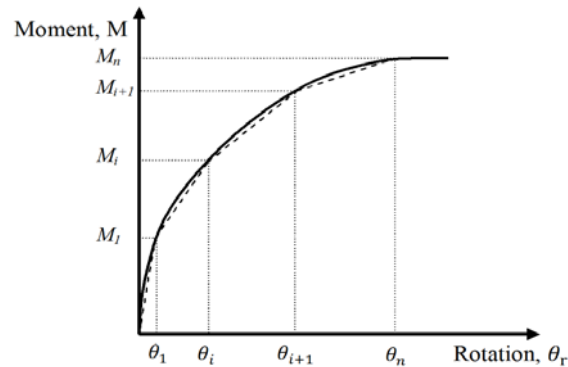
Fig. 2 The  $M - \theta$  curves for semi-rigid connections

Fig. 3 Connection secant stiffness of load increments

the semi-rigid connection stiffness nonlinearity of beam members. The connection secant stiffness corresponding to all load increments is shown in Fig. 3. In each set of iterations, convergence criterion is controlled by comparing of the difference between end forces of members with applied incremental loads so that to be smaller than a specified tolerance. A convergent solution of a load increment forms an initial estimate for the next iteration, and the iterative process continues until all load increments are considered. The solutions for all load increments are accumulated to obtain the total nonlinear responses.

## 5. Design examples

Fig. 4 depicts the flowchart of optimal design process based on the BB-BC algorithm. In this study a computer code has been developed for the optimum design procedure in MATLAB™ 7 (MathWorks, Natick, MA, USA), and then three steel frames with semi-rigid beam-to-column connections are solved. For these examples, the A36 steel grade is used for all of the members and the sections for these members are selected among a total number of 273 standard sections of American Institute of Steel Construction wide flange  $W$  shapes. The size of examples contrary to



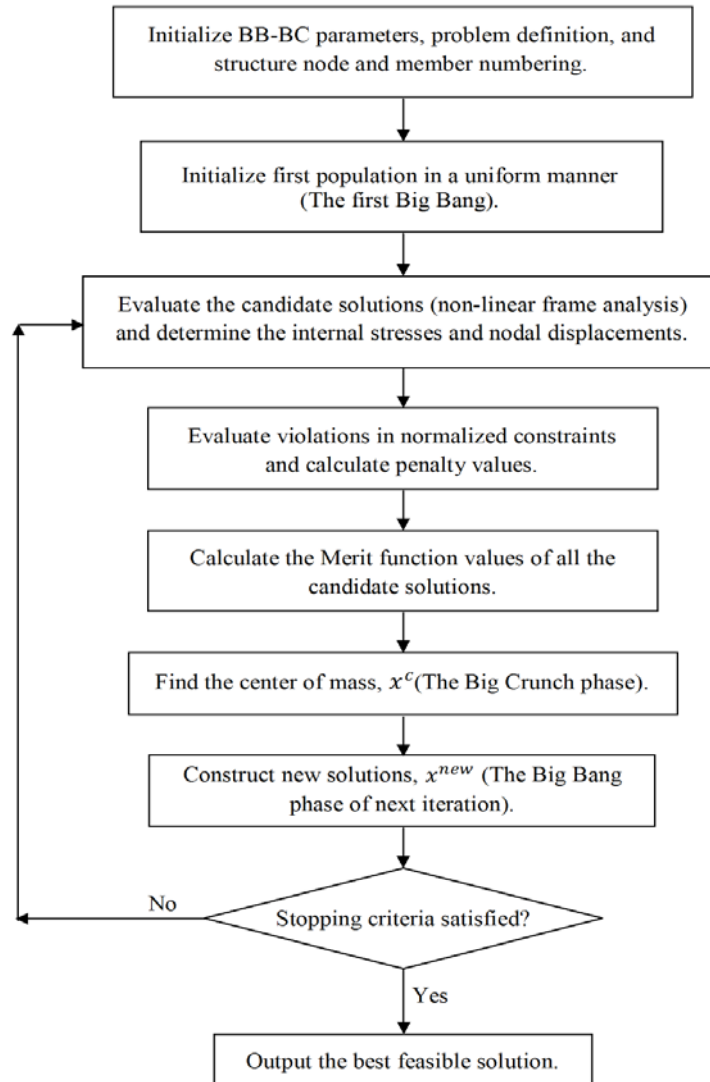


Fig. 4 Flow chart for the optimum design algorithm based on BB-BC method

Table 2 The fixed connection size parameters and rotational stiffness values

Connection type	Fixed connection size parameters (cm)			$S_i$ values in Eq.(4) (kNm/rad)
1	$t_a = 2.54$	$g = 11.43$		$85 \times 10^6$
2	$t_a = 2.858$	$g = 25.4$		$113 \times 10^6$
3	$t = 2.54$	$g = 2.54$	$g = 11.43$	$282 \times 10^6$
4	$t = 2.54$	$g = 2.858$		$226 \times 10^6$
5	$t_p = 2.54$	$g = 2.858$		$339 \times 10^6$
6	$t_p = 2.54$			$395 \times 10^6$
7	$t = 3.81$	$g = 2.858$		$452 \times 10^6$
8	$t_p = 2.54$	$g = 25.4$		$141 \times 10^6$

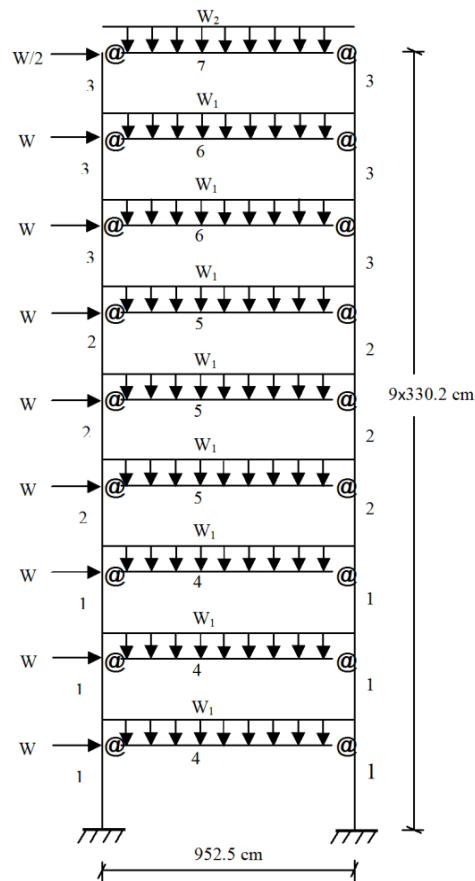


Fig. 5 Nine-storey, single-bay frame

Table 3 Optimum results of nine-storey, single-bay frame for AISC-LRFD

Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	40 × 593	36 × 487	40 × 149	40 × 183	40 × 593	36 × 210	40 × 297	40 × 362	27 × 94
2	36 × 487	30 × 116	21 × 93	40 × 149	21 × 48	33 × 354	21 × 57	36 × 529	21 × 48
3	16 × 36	27 × 281	21 × 50	27 × 281	12 × 190	27 × 368	14 × 550	14 × 426	10 × 39
4	18 × 46	14 × 53	21 × 44	18 × 46	18 × 46	18 × 46	21 × 48	21 × 44	21 × 62
5	21 × 44	18 × 46	24 × 55	21 × 48	21 × 48	21 × 44	18 × 46	18 × 46	21 × 48
6	21 × 50	14 × 53	21 × 50	18 × 46	12 × 58	14 × 53	14 × 426	18 × 46	18 × 40
7	18 × 40	12 × 58	18 × 40	18 × 40	16 × 45	18 × 46	21 × 50	18 × 40	16 × 36
Weight (kg)	38,718	32,617	14,809	23,956	30,804	33,481	43,450	44,527	11,683
Total cost (kg)	40,520	36,235	16,881	25,786	33,488	35,799	53,601	46,146	19,861
Top storey sway (mm)	56	55	66	76	54	65	44	71	73

previous studies contain a small size frame, a nine-storey single-bay frame, a median one (ten stories with four bays) and a larger one (a twenty four stories and three bays).

For each example, the eight types of connections as shown in Fig. 1 are used as semi-rigid beam-to-column connections. In order to simplification of the problem, some of the connection size parameter values required in Frye-Morris polynomial model of  $M-\theta$  curve is considered to be fixed during the optimum design procedure. These fixed values are selected according to Table 2, whereas, the values of angle length, beam height, the vertical distance between bolt groups, and web thickness of beam are calculated based on dimensions of  $W$ -shape section assigned to the beam member throughout the Big Bang phase. The last column of Table 2, gives the values of estimated rotational stiffness,  $S_i$ , for each type of semi-rigid connections. These are the case for all of the design examples considered herein. This study involves a sensitivity analysis regarding the effect of the parameter  $S_i$  on the optimum results obtained by use of the optimum design algorithm presented herein.

### 5.1 Nine-storey, single-bay frame

The geometry, member grouping and the service loading conditions for the nine-storey, one-bay frame are illustrated in Fig. 5. The applied loads  $W$ ,  $W_1$  and  $W_2$  are equal to 17.8 kN, 27.14 kN/m and 24.51 kN/m, respectively. In order to impose the fabrication conditions on the construction of the frame, the 27 members of this frame are separated to seven groups of members.

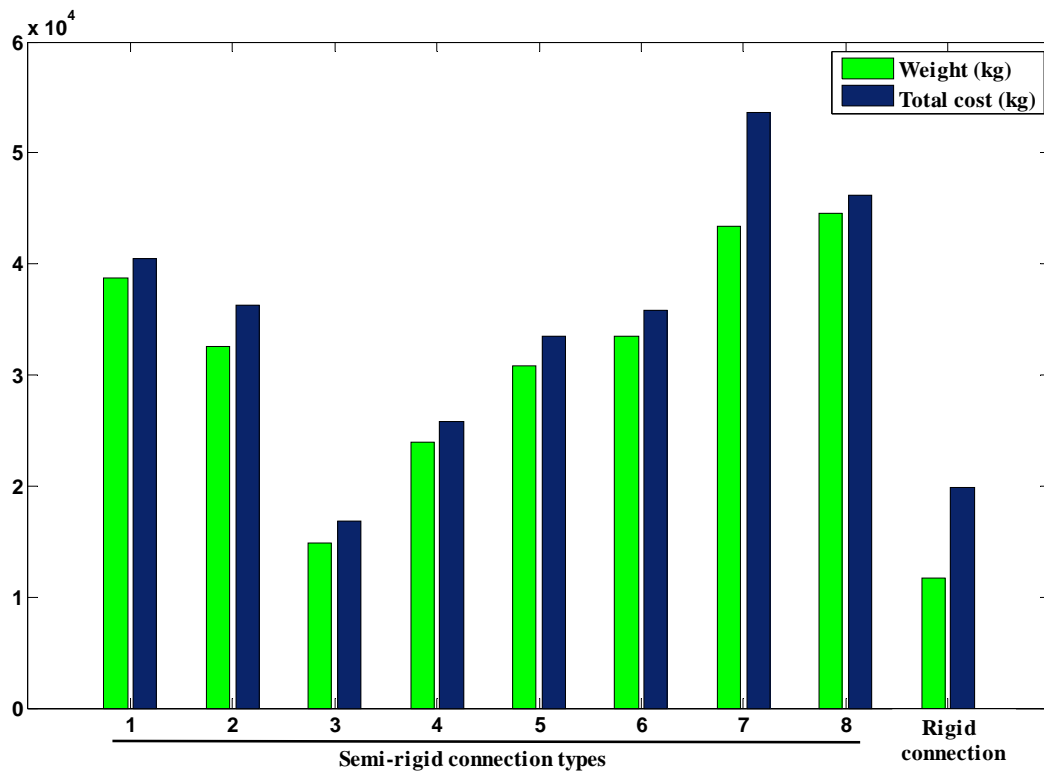


Fig. 6 Optimum results of nine-storey, single-bay frame for AISC-LRFD

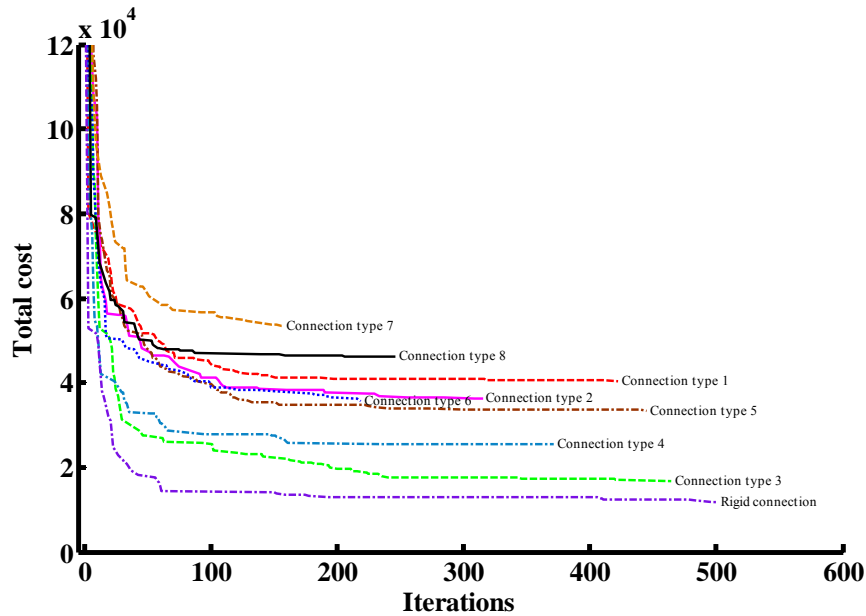


Fig. 7 Convergence history for the optimum design of nine-storey, single-bay frame

Table 4 Optimum results of nine-storey, single-bay frame for AISC-LRFD with doubled  $S_i$  values

Group no.	Semi-rigid connection types							
	1	2	3	4	5	6	7	8
1	44 × 290	30 × 116	36 × 135	40 × 149	33 × 118	36 × 487	33 × 118	40 × 264
2	36 × 135	14 × 605	30 × 99	27 × 114	27 × 94	21 × 48	30 × 99	36 × 135
3	14 × 426	14 × 426	14 × 426	14 × 550	14 × 426	14 × 550	14 × 550	27 × 539
4	21 × 44	21 × 44	24 × 55	21 × 48	21 × 44	21 × 44	18 × 46	21 × 48
5	21 × 44	21 × 50	21 × 48	24 × 55	21 × 50	21 × 48	21 × 48	21 × 48
6	27 × 114	21 × 44	21 × 44	18 × 46	21 × 44	21 × 44	14 × 550	21 × 48
7	18 × 40	16 × 45	12 × 58	18 × 40	24 × 62	18 × 40	18 × 40	33 × 130
Weight (kg)	32,704	39,715	25,932	30,240	24,956	37,769	42,760	34,982
Total cost (kg)	34,956	43,003	27,798	21,040	27,341	39,747	51,836	37,075
Top storey sway (mm)	71	73	70	74	67	50	56	74

Table 3 presents the optimum designs developed by the BB-BC algorithm for the 9-storey frame when the AISC-LRFD (1995) is selected as the code of practice. The global sway corresponding to the roof level is limited to a maximum value of 154 mm.

According to Xu and Grierson (1993), the cost of a steel member with  $W$ -section is increased by approximately 70% if its end connections are rigid jointed, so the total cost of the

rigidly-connected frame is obtained multiplying the weight value by 1.70. These optimum results are also illuminated by a bar chart in Fig. 6 to provide a good comparison of costs and weights of the frame with different types of beam-to-column connections. Fig. 7 shows the convergence histories for the optimum designs of this frame. The results presented in Table 3 and Fig. 6 show that among the semi-rigid connection types 1 through 8, the results of type 3 is the minimum cost frame compared to other types, whereas, the results of types 1, 2, 8 and 7 in cost values are greater than it for the rigidly connected frame.

To investigate the effect of the rotational stiffness values on the optimum design of frames, the nine-storey, single-bay frame is designed with doubled  $S_i$  values and the results are presented in Table 4 according to the AISC-LRFD (1995) specification. The results demonstrate that for doubled  $S_i$ , the total cost values is increased when connection types are 2, 3, 4 and 6 while it decreases when types 1, 5, 7 and 8 are used as shown in Tables 3 and 4. Although, increasing  $S_i$

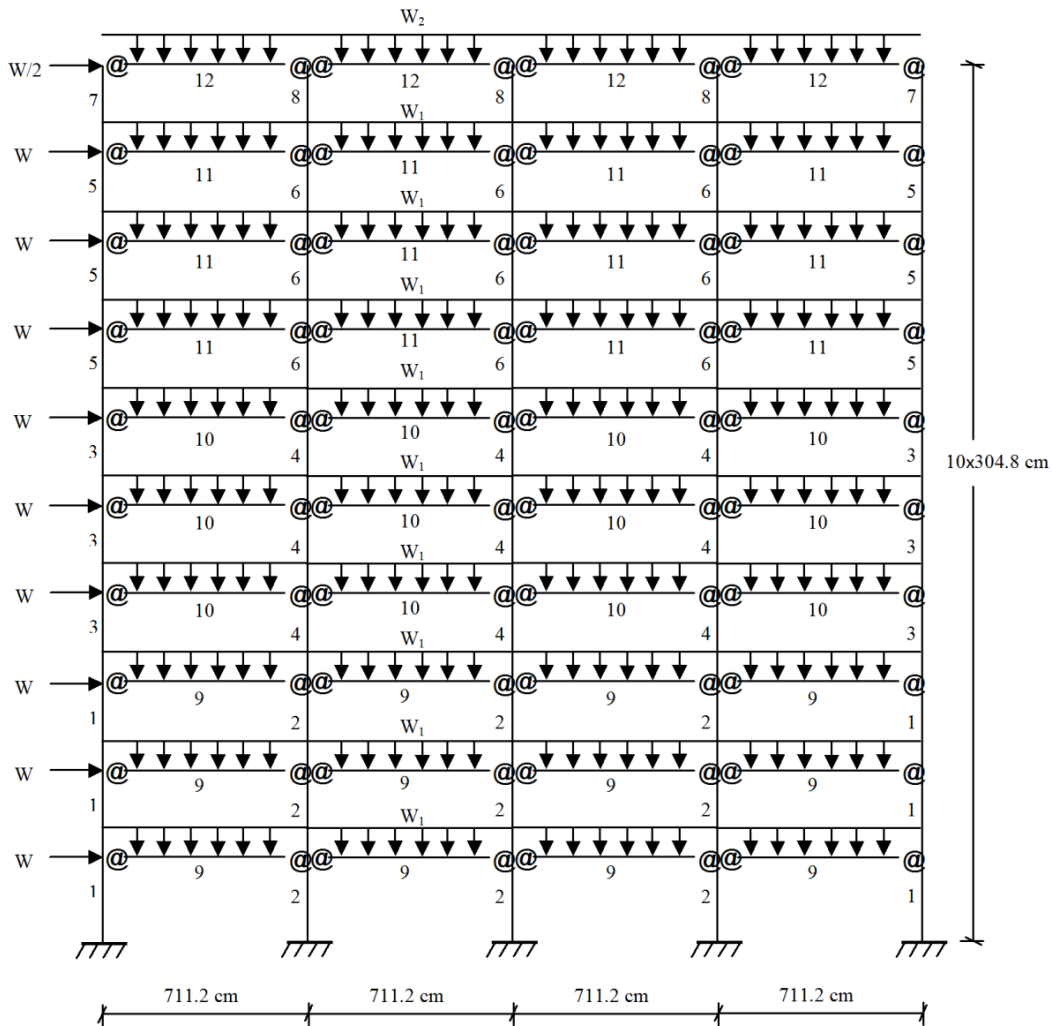


Fig. 8 Ten-storey, four-bay frame

causes in decreasing the cost of connections, this does not guaranty to improve the total cost of frame.

### 5.2 Ten-storey, four-bay frame

The second design example is a 10-storey, 4-bay frame with 90 members. Fig. 8 shows the twelve groups of members, acting loads and dimensions for this frame. The values of loads are:  $W = 44.49$  kN,  $W_1 = 47.46$  kN/m,  $W_2 = 42.91$  kN/m. The values of top storey sway for this frame is restricted to 158 mm based on the AISC-LRFD (1995) specification.

The optimum design procedure for this frame results in the  $W$ -sections, which are listed in Table 5. The last three rows of this table show the values of weights, total member plus connection costs and the top storey sways for the optimum frames with semi-rigid and rigid connections. The optimum results obtained for this example are also shown by the bar chart of Fig. 9. The convergence histories for the optimum designs of this frame are shown in Fig. 10.

Optimum solutions obtained for this frame show that the connection types 3 through 6 (connections with a medium degree of flexibility), results in frames with less total cost compared to frames with other types. Among these types, type numbers 2 (a more flexible type) and 7 (a less flexible type) leads to frames more expensive than a rigidly connected one. Meanwhile, the results show that the connection type 4 provides better interaction between different constraints imposed on the frame and results in better result.

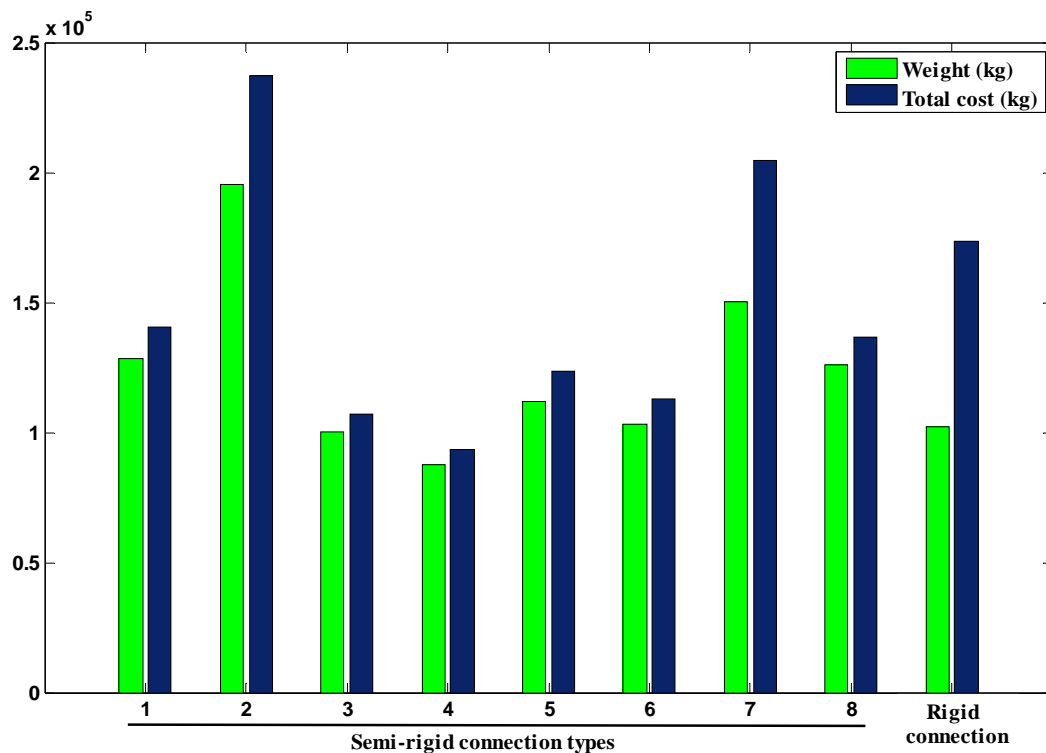


Fig. 9 Optimum results of ten-storey, four-bay frame for AISC-LRFD

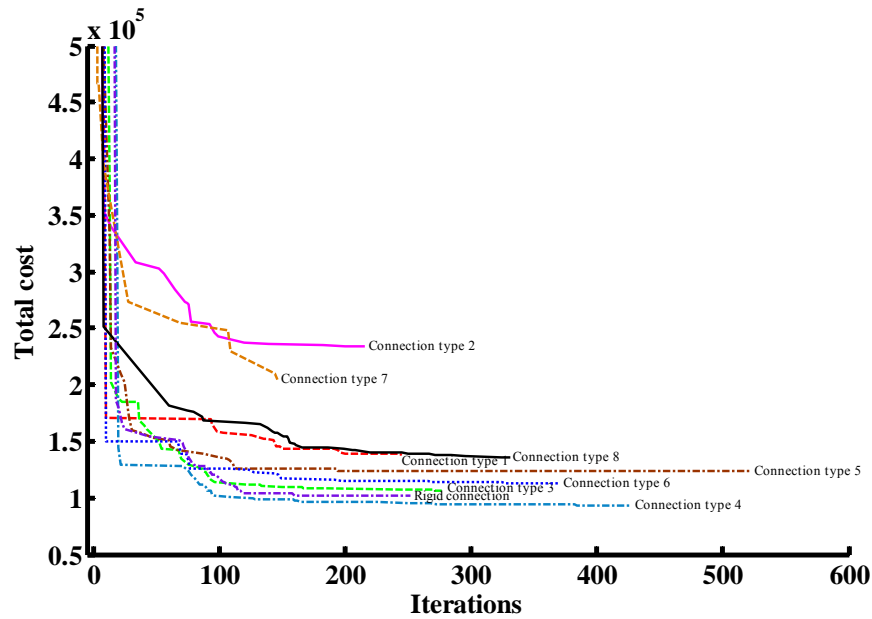


Fig. 10 Convergence history for the optimum design of ten-storey, four-bay frame

Table 6 Optimum results of ten-storey, four-bay frame for AISC-LRFD

Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	44 × 290	24 × 55	30 × 90	33 × 118	24 × 55	24 × 55	16 × 16	40 × 264	21 × 50
2	12 × 14	12 × 14	36 × 182	18 × 234	12 × 50	6 × 20	14 × 426	12 × 72	8 × 18
3	44 × 290	33 × 118	36 × 135	36 × 135	30 × 124	33 × 118	36 × 135	36 × 135	27 × 84
4	12 × 58	14 × 43	12 × 252	16 × 77	24 × 103	18 × 158	33 × 201	27 × 129	16 × 36
5	14 × 455	14 × 283	14 × 455	14 × 342	14 × 342	14 × 426	14 × 730	14 × 398	14 × 311
6	14 × 455	14 × 283	14 × 370	14 × 342	14 × 257	14 × 283	14 × 605	14 × 398	14 × 283
7	14 × 426	14 × 257	14 × 311	14 × 342	14 × 193	14 × 233	14 × 605	14 × 342	14 × 233
8	14 × 311	14 × 193	14 × 311	14 × 233	14 × 176	14 × 145	14 × 605	14 × 342	14 × 746
9	14 × 311	14 × 82	14 × 311	14 × 193	14 × 120	14 × 132	14 × 605	14 × 342	14 × 176
10	14 × 211	14 × 43	14 × 211	14 × 176	14 × 74	14 × 109	14 × 605	14 × 257	14 × 176
11	14 × 211	14 × 43	14 × 145	14 × 120	14 × 90	14 × 90	14 × 605	14 × 90	14 × 43
12	14 × 211	14 × 43	14 × 90	14 × 109	14 × 90	14 × 90	14 × 426	14 × 90	14 × 43
13	14 × 730	14 × 370	14 × 550	14 × 311	14 × 342	14 × 342	14 × 730	14 × 370	14 × 342
14	14 × 500	14 × 283	14 × 500	14 × 311	14 × 311	14 × 311	14 × 730	14 × 370	14 × 311
15	14 × 550	14 × 283	14 × 500	14 × 311	14 × 257	14 × 257	14 × 665	14 × 342	14 × 283
16	14 × 398	14 × 233	14 × 398	14 × 257	14 × 233	14 × 257	14 × 665	14 × 283	14 × 233
17	14 × 370	14 × 211	14 × 398	14 × 257	14 × 176	14 × 233	14 × 605	14 × 257	14 × 193
18	14 × 370	14 × 159	14 × 257	14 × 257	14 × 90	14 × 159	14 × 605	14 × 233	14 × 176
19	14 × 370	14 × 90	14 × 233	14 × 211	14 × 90	14 × 109	14 × 550	14 × 193	14 × 61
20	14 × 211	14 × 90	14 × 132	14 × 193	14 × 90	14 × 90	14 × 426	14 × 176	14 × 61

Table 6 Continued

Weight (kg)	371,754	139,161	236,249	211,149	140,536	150,362	359,372	297,834	137,312
Total cost (kg)	502,197	202,737	267,414	249,806	171,868	176,864	385,074	383,738	233,430
Top storey sway (mm)	204	245	170	184	237	231	240	190	238

Table 7 Performance comparing of present work with the genetic algorithm and particle swarm optimization algorithms

	PSO	GA	Present work
First example (Type 3)			
Best Result	16,881	17,698	18,056
Mean Result	17,966	18,988	20,021
Worst Result	20,125	22,656	22,365
Std	940.8	1,355.3	1,568.6
Second example (Type 4)			
Best Result	93,255	98,112	110,125
Mean Result	96,601	105,655	120,442
Worst Result	110,098	115,898	130,335
Std	3,555	5,689	8,956
Third example (Type 5)			
Best Result	171,868	185,255	200,121
Mean Result	180,655	205,265	218,356
Worst Result	210,365	240,366	242,666
Std	8,468	15,366	21,565

### 5.3 Twenty four-storey, three-bay frame

The topology, service loading conditions, four beam groups and sixteen column groups of 24-storey, 3-bay frame consisting of a total number of 168 members are shown in Fig. 11. Applied loads including point ( $W$ ) and uniformly distributed ( $W_1$  through  $W_4$ ) loads have the values of  $W = 25.628$  kN,  $W_1 = 4.378$  kN/m,  $W_2 = 6.362$  kN/m,  $W_3 = 6.917$  kN/m and  $W_4 = 5.954$  kN/m. This frame is originally designed by Davison and Adams (1974).

In the present work, together with the AISC-LRFD (1995) strength and displacement constraints, relative member size adaptation constraints are imposed on the frame during the optimum design procedure, using Eqs. (11) and (12). In this example, each of the four beam element groups may choose from all 273  $W$ -shapes, while the 16 column element groups are limited to  $W14$  sections. The top storey sway of this frame is limited to a maximum value of 456 mm. Table 6 and Fig. 12 show the optimum results obtained using the BB-BC algorithm and the convergence histories for this frame are shown in Fig. 13.



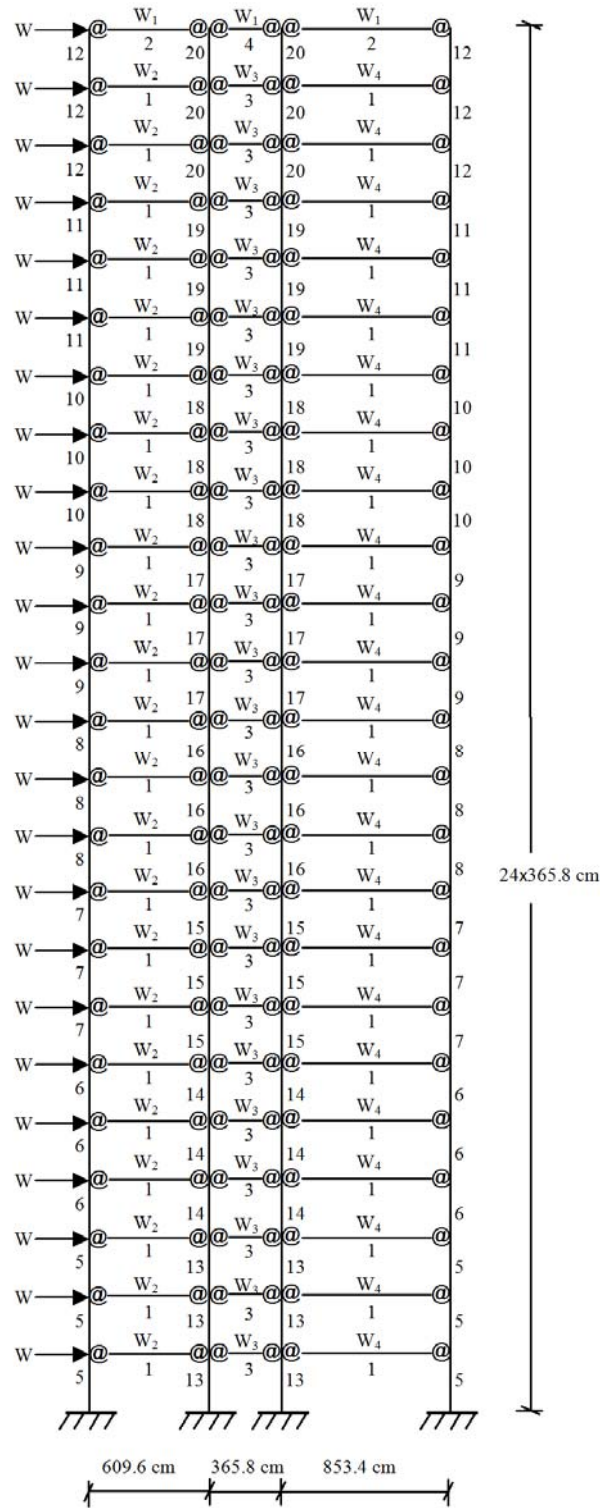


Fig. 11 Twenty four-storey, three-bay frame

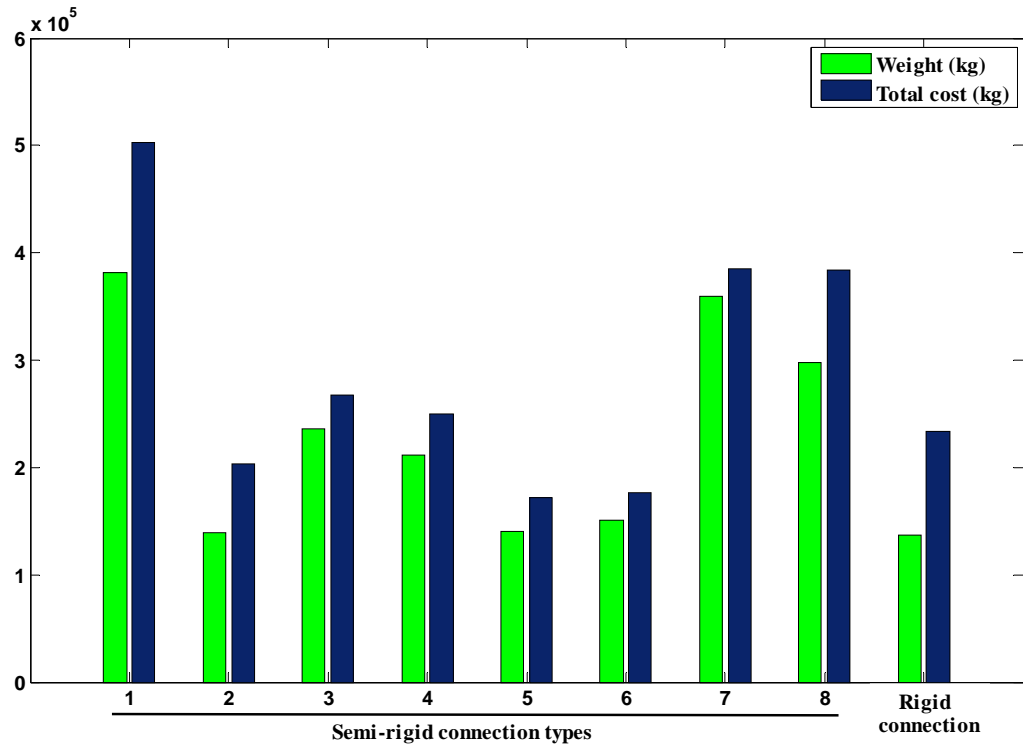


Fig. 12 Optimum results of twenty four-storey, three-bay frame for AISC-LRFD

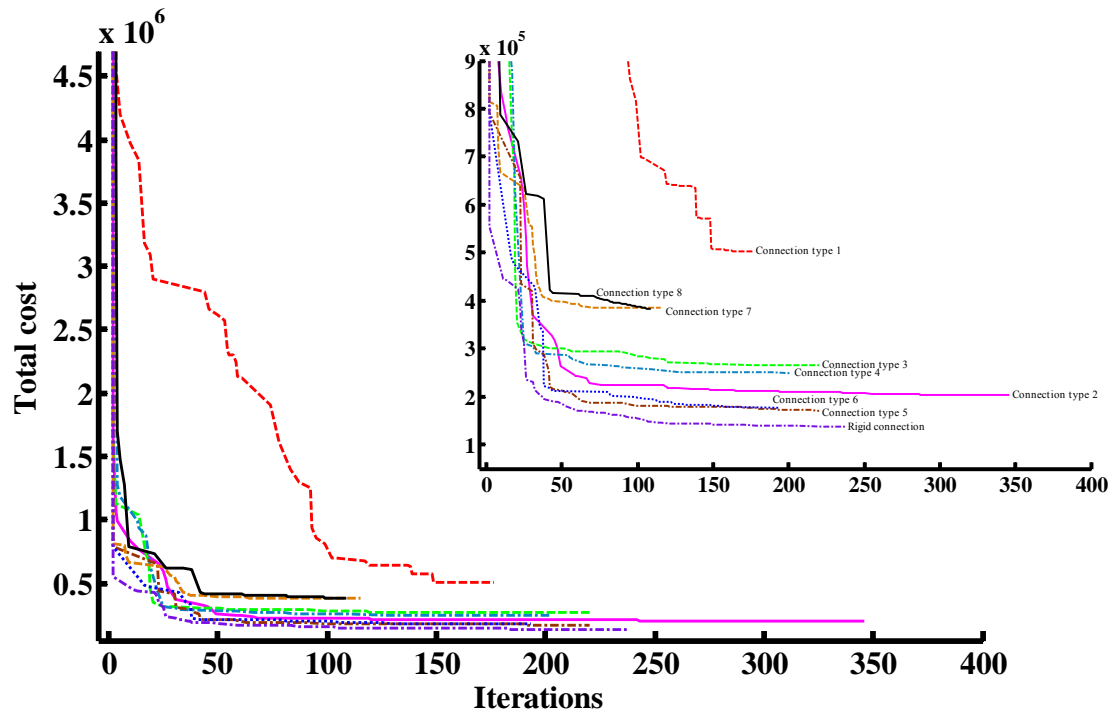


Fig. 13 Convergence history for the optimum design of twenty four-storey, three-bay frame

Results presented in Table 6 and Fig. 12 show that for 24 storey frame, connection types 1, 8 and 7 increase the total cost of the frame, whereas, the types 5 and 6 results in economic frames with 26.4% and 24.2% saves in total cost compared to a frame with rigid beam-to-column connections, respectively. In addition, comparison between optimum result of present algorithm with the results presented by Kaveh and Talatahari (2010b), shows that considering the geometry constraints for members increases the weight and as a corollary the cost of the structure; however structural point of view, the geometry constraints are necessary for practical buildings.

In order to examine the performance of the present work, the best connection of each example is selected to be solved some other well-known meta-heuristic algorithms such as the genetic algorithm (GA) and particle swarm optimization (PSO). The statistical results of 20 runs with different seeds for these methods as well as the BB-BC algorithm are presented in Table 7. It can be plainly that the present method is more accurate and reliable compared to GA and PSO. The best result of BB-BC for the first example is 5% less than the results of the GA and PSO. For the second example, the worst result of the BB-BC is better than the best result of the PSO and than the mean result of the GA. The standard deviation for the third example obtained by the present work is almost 50% less than those obtained by the GA and PSO

## 6. Conclusions

The Big Bang-Big Crunch (BB-BC) optimization algorithm is a recently proposed optimization method that relies on the Big Bang and Big Crunch theory of evolution of the universe. In this paper, a discrete Big Bang-Big crunch algorithm is presented for optimal design of non-linear steel frames with semi-rigid beam-to-column connections. The aim is to find the minimum total cost comprising utilized section as well as connection construction costs by selecting suitable sections from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange (*W*) shapes. Displacement and stress constraints of AISC-Load and Resistance Factor Design (LRFD) specification are considered as the design constraints. Also, in order to find more practical design, geometry constraints for beams and columns adaptation are imposed on the frame in the optimum design procedure. The non-linear analyses considering the  $P$ - $\Delta$  effects of beam-column members are performed during the optimization process. The nonlinear moment-rotation behavior of connections is modeled using the Frye-Morris polynomial model.

Three design examples with various types of connections are considered. Among the various types of semi-rigid connection types utilized in the design examples, connection types 3 through 6 results in more economic designs compared to other ones. Perhaps, this can be interpreted as a good interaction between stress and displacement constraints for frames with connection types 3, 4, 5 and 6 compared to other types. In other words, use of more flexible connections increases the displacements of the frame and consequently members with greater cross-section areas are required to terminate violations of displacement constraints. On the other hand, more rigid connections make algorithm assign large sections for members due to stress considerations.

To sum up, the variations in optimal results for frames with different types of connections imply that connection modeling has important effect on the optimum design of frame structures. One can conclude that the connections with a medium degree of flexibility can provide better interaction between different constraints imposed on the frame and can results in more economic frames.

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