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Pareto optimum design of laminated composite truncated circular conical shells

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Abstract. This paper deals with multiobjective optimization of symmetrically laminated composite truncated circular conical shells subjected to external uniform pressure load and thermal load. The design objective is the maximization of the weighted sum of the critical buckling load and fundamental frequency. The design variable is the fibre orientations in the layers. The performance index is formulated as the weighted sum of individual objectives in order to obtain optimal solutions of the design problem. The first-order shear deformation theory (FSDT) is used in the mathematical formulation of laminated truncated conical shells. Finally, the effect of different weighting factors, length-to-radius ratio, semi-cone angle and boundary conditions on the optimal design is investigated and the results are compared.

Keywords: laminated composite truncated conical shells; multiobjective optimization; frequency; buckling

1. Introduction

The conical shells are often used as transition elements between cylinders of different diameter and/or end closures and sometimes as stand-alone components in various engineering applications such as tanks and pressure vessels, missiles, spacecraft, submarines, nuclear reactors and jet nozzles. Therefore, these shells may be regarded as elementary shell geometry together with cylinders and spheres. The potential of using the directional dependence of composite properties in designing tailored structures to improve structural performance together with their high specific strength/stiffness, damping properties and low coefficient of expansion especially in the fibre direction has received increasing attention in the recent years. However, the analysis of such structures is a complex task, compared with conventional single layer metallic structures, because of the exhibition of coupling among membrane, torsion and bending strains and discontinuity of the mechanical characteristics along the thickness of the laminates.

On the other hand, the structures are often are subjected to in-plane, external loads and thermal loads which may cause buckling. In addition, the vibration can be problematic when the excitation frequency coincides with the shell's resonance frequency. Such loadings may occur at different times under in-service conditions, necessitating a design approach which is capable of taking in to account these various loading conditions.

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Comprehensive works on the buckling of laminated conical shells structures have been reported in the literature. For example, Sofiyev and Kuruoglu (2011) investigated the non-linear buckling behavior of cross-ply laminated orthotropic truncated conical shells under axial load. Sofiyev and Karaca (2009) studied the free vibration and buckling of laminated homogeneous and non-homogeneous orthotropic truncated conical shells under lateral and hydrostatic pressures. Shadmehri *et al.* (2012) proposed a semi-analytical approach to obtain the linear buckling response of conical composite shells under axial compression load. Patel *et al.* (2008) studied the postbuckling characteristics of the angle-ply laminated composite conical shells subjected to the torsion, the external pressure, the axial compression, and the thermal loading considering uniform temperature change using the semi-analytical finite element approach.

Thermal buckling analysis of laminated conical shells has received limited attention in the literautre. For example, Patel *et al.* (2005) studied thermoelastic postbuckling behavior of cross-ply laminated composite conical shells under presumed uniform temperature distribution. Singh and Babu (2009) examined the sensitivity of randomness in material parameters on the thermal buckling of conical shells embedded with and without piezoelectric layer.

A comprehensive survey of the early works dealing with free vibration analysis of laminated composite conical shells can be found in the literature. For example, Tripathi *et al.* (2012) presented the sensitivity of randomness in material parameters on linear free vibration response of conical shells. Civalek (2007) studied a numerical study on the free vibration analysis for laminated conical and cylindrical shell. The analysis was carried out using Love's first approximation thin shell theory and solved using discrete singular convolution method. Tong (1993) obtained directly for the Donnell-type governing equations of the free vibration of composite laminated conical shells, with orthotropic stretching-bending coupling using a particularly convenient coordinate system, a simple and exact solution. Sivadas and Ganesan (1991) studied effects of thickness variation on natural frequencies of laminated conical shells by using a semi-analytical finite element method. Dey and Karmakar (2012) investigated the effect of rotational speeds on free vibration characteristics of delaminated twisted graphite–epoxy cross-ply composite conical shells employing finite element method.

Research on the subject of structural optimization of laminated composite conical shells has been reported by few investigators. Fares *et al.* (2004) presented minimization problem of the dynamic response of composite laminated truncated conical shells with minimum expenditure of force using design and control optimization. Hu and Ou (2001) maximized fundamental frequencies of laminated composite truncated conical shells using sequential linear programming method. Kabir and Shirazi (2008) investigated optimum laminate configuration for minimum weight of filament-wound laminated conical shells subject to buckling load constraint. Goldfeld *et al.* (2005) studied optimum laminate configuration for the maximum buckling load of filament-wound laminated conical shells. Blom *et al.* (2008) optimized fibre-reinforced composite conical shells with given geometry and material properties for maximum fundamental frequency.

On the other hand, multiobjective optimization of composite laminated truncated conical shells has not been investigated by authors until now. In this study, three different problems are combined as the weighted sum of individual objectives in order to obtain multiobjective optimization solutions to fill this gap. The design objective is the maximization of the weighted sum of the critical buckling load and fundamental frequency. The design variable is the fibre orientations in the layers. The performance index is formulated as the weighted sum of individual objectives in order to obtain optimal solutions of the design problem. The first-order shear deformation theory (FSDT) is used in the mathematical formulation of laminated truncated conical shells. Finally, the effect of different weighting factors, length-to-radius ratio, semi-cone angle and boundary conditions on the optimal design is investigated and the results are compared.

2. Basic equations

Consider a laminated circular conical shell as shown in Fig. 1, in which h denotes the thickness of the shell. A set of the conical coordinates (meridional (x), circumferential (θ) and normal (z) coordinates) is located on the middle surface. R_1 and R_2 are the radii of the cone at the small and large edges, respectively. α is the semi-cone angle of the cone and L is cone length along the meridional direction.

The displacement field of the plate based on the first order shear deformation theory is given by the following expressions

$$\begin{aligned} u(x,\theta,z) &= u_o(x,\theta) + z\psi_x(x,\theta) \\ v(x,\theta,z) &= v_o(x,\theta) + z\psi_\theta(x,\theta) \\ w(x,\theta,z) &= w(x,\theta) \end{aligned} \tag{1}$$

The stress-strain relations for a single lamina in a conical shell are given by

$$\begin{pmatrix} \tau_{\theta z} \\ \tau_{xz} \end{pmatrix}_{(k)} = \begin{pmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{pmatrix}_{(k)} \begin{pmatrix} \gamma_{\theta z} \\ \gamma_{xz} \end{pmatrix} \tag{2}$$

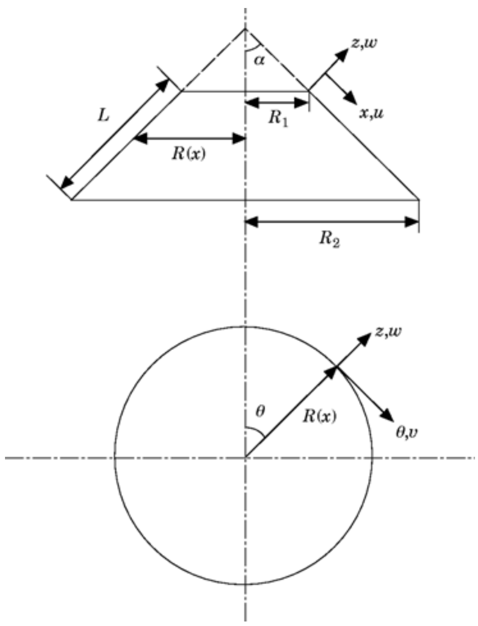

Fig. 1 Geometry of a laminated conical shell and cross sectional view of thickness of the laminated composite truncated conical Shell

$$\begin{pmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\end{pmatrix}_{(k)}=\begin{pmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{pmatrix}_{(k)}\begin{pmatrix}\varepsilon_x-\alpha_x\Delta T\\ \varepsilon_\theta-\alpha_\theta\Delta T\\ \gamma_{x\theta}-\alpha_{x\theta}\Delta T\end{pmatrix} \tag{3}$$

where $\bar{Q}_{ij}$ is the transformed reduced stiffnesses, which can be expressed in terms of the orientation angle and the engineering constant of the material. α_x, α_θ, $\alpha_{x\theta}$ are the coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

The kinematics relations in terms of the conical coordinates x, θ and z can be expressed as

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\\ \gamma_{xz}\\ \gamma_{\theta z}\end{pmatrix}=\begin{pmatrix}\partial_x & 0 & 0\\ \sin\alpha/R & \partial_\theta/R & \cos\alpha/R\\ \partial_\theta/R & \partial_x-(\sin\alpha/R) & 0\\ \partial_z & 0 & \partial_x\\ 0 & \partial_z-(coz\,\alpha/R) & \partial_\theta/R\end{pmatrix}\begin{pmatrix}u\\ v\\ w\end{pmatrix} \tag{4}$$

where R denotes the radius of the cone at any point along the meridional direction and is given by $R = R_1 + x\sin\alpha$.

The stress resultants $\{N\}$, stress couples $\{M\}$ and transverse shear stress resultants $\{Q\}$ are

$$\begin{Bmatrix}N_x\\ N_\theta\\ N_{x\theta}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\end{Bmatrix}dz\,,\quad \begin{Bmatrix}M_x\\ M_\theta\\ M_{x\theta}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\end{Bmatrix}z\,dz\,,\quad \begin{Bmatrix}Q_x\\ Q_\theta\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\tau_{xz}\\ \tau_{\theta z}\end{Bmatrix}dz \tag{5}$$

2.1 Finite element formulation

In this study, nine noded Lagrangian rectangular shell elements having five degrees of freedom per node are used for the finite element solution of the laminated conical shells. The interpolation function of the displacement field is defined as

$$\begin{pmatrix}u\\ v\\ w\\ \psi_x\\ \psi_\theta\end{pmatrix}=\sum_{i=1}^{n}\Phi_i d_i \tag{6}$$

where d_i and Φ_i are the nodal variables and the interpolation function, respectively. Following the standard procedure of the finite element formulation, the stability condition is obtained as

$$\left([K_b+K_s]-\lambda[K_g]\right)\{d\}=0 \tag{7}$$

where $[K_b]$, $[K_s]$, and $[K_g]$ are the bending stiffness, shear stiffness and geometric stiffness matrices, respectively. These matrices can be expressed as follows

$$
\begin{aligned}
[K_b] &= \int_A [B_b]^T [D_b][B_b] dA \\
[K_s] &= \int_A [B_s]^T [D_s][B_s] dA \\
[K_g] &= \int_A [B_g]^T [D_g][B_g] dA
\end{aligned}
\tag{8}
$$

where

$$
[D_b] = \begin{bmatrix} A_{ij} & 0 \\ 0 & D_{ij} \end{bmatrix}, \quad [D_s] = \begin{bmatrix} k_1^2 A_{44} & A_{45} \\ A_{45} & k_2^2 A_{55} \end{bmatrix}, \quad [D_g] = \begin{bmatrix} \overline{N}_1 & \overline{N}_{12} \\ \overline{N}_{12} & \overline{N}_2 \end{bmatrix}
\tag{9}
$$

A_{ij} and D_{ij} $(i, j = 1, 2, 6)$ denote extensional stiffnesses and bending stiffnesses, respectively. A_{ij} and D_{ij} can be calculated as follows

$$
(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij} (1, z^2) dz \quad (i, j = 1,2,6)
\tag{10}
$$

$$
(A_{44}, A_{45}, A_{55}) = \int_{-h/2}^{h/2} (\overline{Q}_{44}, \overline{Q}_{45}, \overline{Q}_{55}) dz
\tag{11}
$$

The lowest eigenvalue of the homogeneous system (7) yields the critical buckling load. Calculating the critical buckling temperature of buckling due to thermal load is two stage processes. For a specified rise ΔT in temperature the thermal loads are computed and a linear static analysis is carried out to determine the thermal stress resultants. These stress resultants are then used to compute the geometric stiffness matrice, which subsequently used in Eq. (7), to determine the least eigenvalue, λ, and the associated mode shape. The critical buckling temperature , T_{cr}, is calculated as follows

$$
T_{cr} = \lambda \Delta T
\tag{12}
$$

The free vibration problem of the shell becomes as follows

$$
([K_b + K_s] - \omega^2 [M])\{d\} = \{0\}
\tag{13}
$$

where $[M]$ is the mass matrice. The mass matrice can be obtained as follows

$$
[M] = \int_A [N^t][m][N] dA
\tag{14}
$$

where m is the intertia matrice. Eq. (13) is a set of homogeneous linear equations in the unknown displacements $\{d\}$. For non-trivial solution, the determinant is equal to zero and the eigenvalues correspond to natural frequencies of the laminated plates. The subspace iteration method is used for the frequency analysis.

3. Optimization problem

The optimization problem is formulated in order to find the best orientation angles of fibres in the laminated truncated conical shells so that to simultaneously maximize the critical external pressure buckling load, critical thermal buckling load and fundamental frequency with the laminate configurations. The multiobjective design index, MODI, can be describes as follows

$$\mathrm{MODI} = \eta N^* + \xi T^* + \mu \varpi^* \tag{16}$$

where η, ξ and μ are the weighting factors summing the three objective functions with η , ξ , $\mu \geq 0$, $\eta + \xi + \mu = 1$. As the weighting factors are varied, the emphasis of the optimization problem is shifted among various objectives resulting in compromise solutions. The single objective designs can be obtained as special cases by setting $\eta = 1$ or $\eta = 0$. In this study, the optimization problem can be expressed as follows

$$\begin{aligned} &\text{find}: \Phi, \quad 0^\circ \leq \Phi_k \leq 90^\circ, \quad \Delta\Phi = 5^\circ \\ &\text{maximize}: \mathrm{MODI} \end{aligned} \tag{17}$$

In the all computations, the following nondimensionalized quantities are used

$$N^* = \frac{N_{cr}}{N_O}, \quad T^* = \frac{T_{cr}}{T_{cro}}, \quad \varpi^* = \frac{\omega}{\omega_o} \tag{18}$$

for critical external pressure buckling load, critical thermal buckling load and fundamental frequency, respectively. The N_O, T_{cro} and ω_O are the external pressure buckling load, thermal buckling load and fundamental frequency corresponding to prescribed lamination angles $[(0°)_4]_{sym}$ for eight layered conical shells, respectively.

4. Numerical results and discussion

In this study, simply supported symmetrically laminated $(\Phi/-\Phi/\Phi/-\Phi)_s$ truncated conical shells are investigated for optimization problems. Each of the lamina is assumed to be same thickness. Numerical results are given for a typical T300/5208 graphite/epoxy material and the material properties are as below

$$\begin{aligned} &E_1 = 181\,\text{GPa},\ E_2 = 10.3\,\text{GPa},\ G_{12} = G_{13} = 7.17\,\text{GPa},\ G_{23} = 2.39\,\text{GPa}, \\ &v_{12} = 0.28,\ \rho = 1600\,\text{kg/m}^3,\ \alpha_1 = 0.02\times10^{-6}\ {}^\circ C^{-1},\ \alpha_2 = 22.5\times10^{-6}\ {}^\circ C^{-1} \end{aligned}$$

In this study, effect of different weighting factors on the optimal designs is investigated for simply supported eight layered $(\Phi/-\Phi/\Phi/-\Phi)_s$ truncated conical shells (h / R_1 = 0.2 L / R_1 = 1, α = 45°). In Table 1, fifteen different combinations of weighting factors is illustrated. It is obvious that the maximum multiobjective design index, $(\mathrm{MODI})_{max}$, occurs at a specific value of the fibre orientation and this value can be several times higher than the other MODI at other fibre orientations. In Fig. 2, performance index vs. fibre orientation for different fifteen combinations of weighting factors can be seen. As seen, the maximum $(\mathrm{MODI})_{max}$ is obtained 10.18 and the optimum fibre orientation is obtained $\Phi_{opt} = 45°$ for C4 combination. This means that, the effect of thermal buckling load is more dominant than external pressure load and fundamental frequency on the multiobjective design. On the other hand, the minimum $(MODI)_{max}$ occurs for C3 combination. That is, external pressure load causes minimum multiobjective design index. In Table 2, the $(MODI)_{max}$ and the optimum fibre orientations are given for different fifteen combinations of weighting factors. As seen, the optimum fibre orientations are mostly $\Phi_{opt} = 45°$ for different weighting factors.

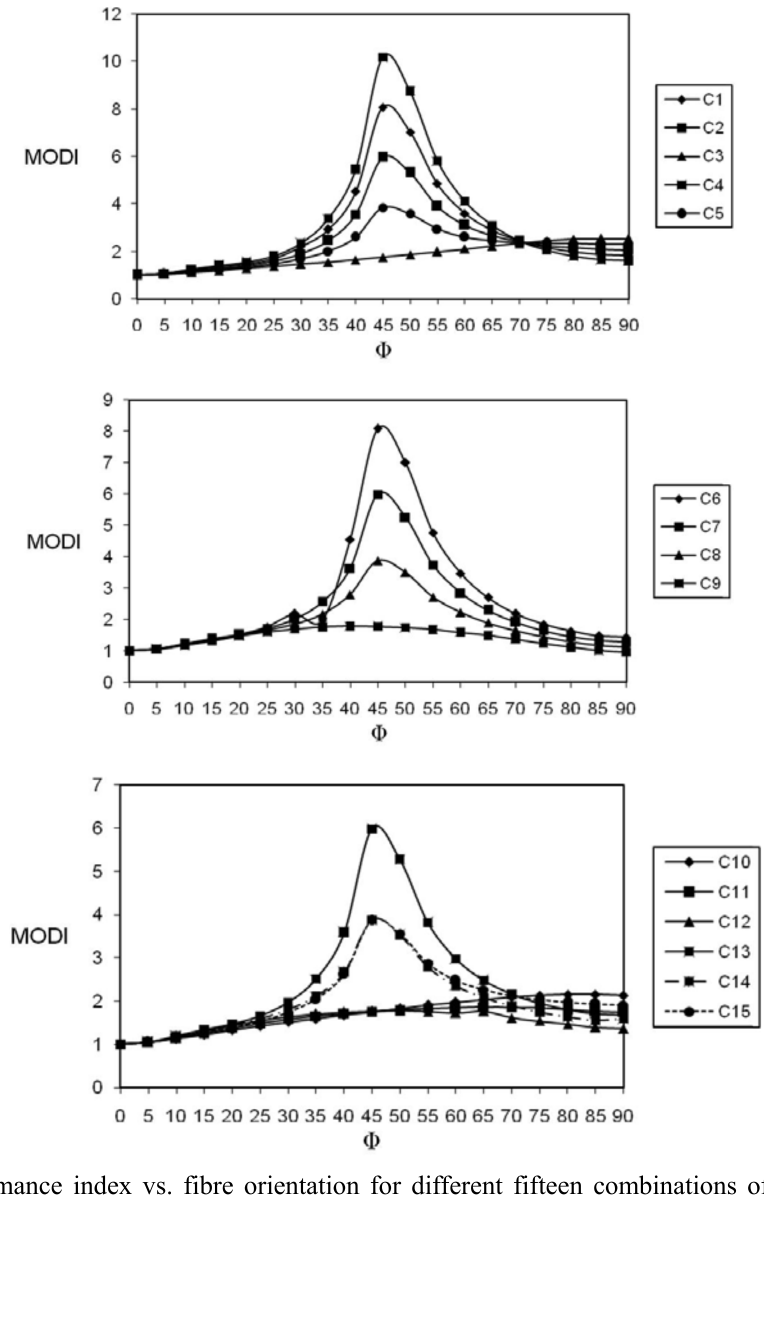

Fig. 2 Performance index vs. fibre orientation for different fifteen combinations of weighting factors

Table 1 Fifteen different combinations of weighting factors

Weighting factors	Combinations
$\eta = 0.25$, $\xi = 0.25$, $\xi = 0.75$, $\mu = 0$	C1
$\eta = 0.50$, $\xi = 0.50$, $\mu = 0$	C2
$\eta = 1.00$, $\xi = 0$, $\mu = 0$	C3
$\eta = 0$, $\xi = 1.00$, $\mu = 0$	C4
$\eta = 0.75$, $\xi = 0.25$, $\mu = 0$	C5
$\eta = 0$, $\xi = 0.75$, $\mu = 0.25$	C6
$\eta = 0$, $\xi = 0.50$, $\mu = 0.50$	C7
$\eta = 0$, $\xi = 0.25$, $\mu = 0.75$	C8
$\eta = 0$, $\xi = 0$, $\mu = 1.00$	C9
$\eta = 0.75$, $\xi = 0$, $\mu = 0.25$	C10
$\eta = 0$, $\xi = 0.50$, $\mu = 0.50$	C11
$\eta = 0.25$, $\xi = 0$, $\mu = 0.75$	C12
$\eta = 0.25$, $\xi = 0.50$, $\mu = 0.25$	C13
$\eta = 0.25$, $\xi = 0.25$, $\mu = 0.50$	C14
$\eta = 0.50$, $\xi = 0.25$, $\mu = 0.25$	C15

Table 2 The $(\mathrm{MODI})_{\max}$ and the optimum fibre orientations for different fifteen combinations of weighting factors

Combinations	$(\mathrm{MODI})_{\max}$	Φ_{opt} (°)
C1	8.07	45
C2	5.96	45
C3	2.53	85
C4	10.18	45
C5	3.85	45
C6	8.08	45
C7	5.98	45
C8	3.88	45
C9	1.78	40
C10	2.15	80
C11	1.85	65
C12	1.77	50
C13	5.97	45
C14	3.87	45
C15	3.86	45

In this study, effect of length-to radius ratio (L / R_1) on the optimal designs is investigated for simply supported eight layered $(\Phi/-\Phi/\Phi/-\Phi)_s$ truncated conical shells ($h / R_1 = 0.2$, $\eta = 0.25$, $\xi = 0.25$, $\mu = 0.50$, $\alpha = 45°$). In Fig. 3, performance index vs. fibre orientation for different L / R_1 ratios is given. As seen, the maximum $(\mathrm{MODI})_{\max}$ is obtained 9.60 for $L / R_1 = 4$. On the other hand, the optimum fibre orientation is obtained Φ_{opt} (50°). In Table 3, the $(\mathrm{MODI})_{\max}$ and the optimum fibre orientations are given for different L / R_1 ratios.

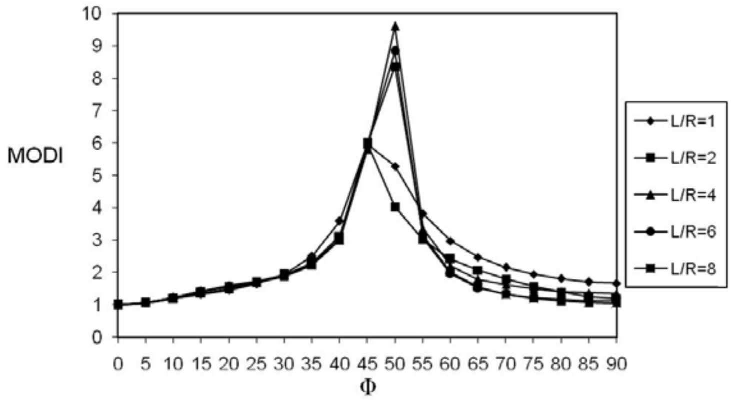

Fig. 3 Performance index vs. fibre orientation for different L / R_1 ratios

Table 3 The $(MODI)_{max}$ and the optimum fibre orientations for different L / R_1 ratios

L/R	$(MODI)_{max}$	Φ_{opt} (°)
1	5.97	45
2	5.99	45
4	9.60	50
6	8.83	50
8	8.36	50

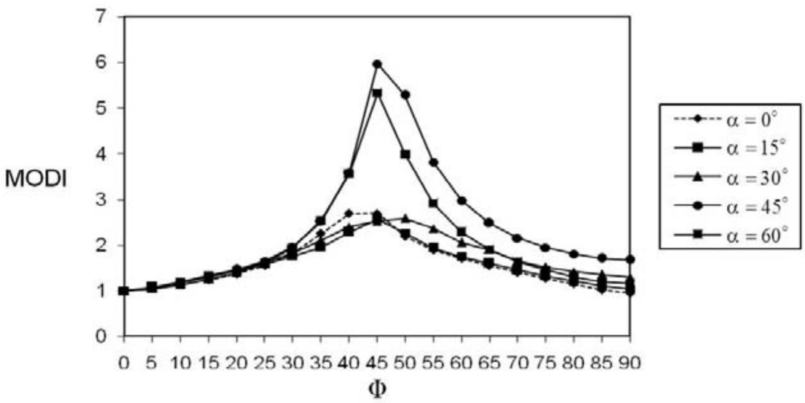

Fig. 4 Performance index vs. fibre orientation for different semi-cone angles

In this study, effect of different semi-cone angles (α) on the optimal designs is investigated for simply supported eight layered $(\Phi/-\Phi/\Phi/-\Phi)_s$ truncated conical shells (h / R_1 = 0.2, L / R_1 = 1, η = 0.25, ξ = 0.25, μ = 0.50). In Fig. 4, performance index vs. fibre orientation for different semi-cone angles can be seen. As seen, the maximum $(MODI)_{max}$ is obtained 5.97 for α = 45°. The optimum fibre orientations are obtained Φ_{opt} = 45°. In Table 4, the $(MODI)_{max}$ and the optimum fibre orientations are given for different semi-cone angles. As seen, the optimum fibre orientation is mostly Φ_{opt} = 45° regardless of semi-cone angle. One can mention that semi-cone angle has not important effect on the optimum fibre orientations.

In this study, different combinations of free (F), simply supported (S) and clamped (C) boundary conditions are considered, viz. clamped/clamped (CC), clamped/simply supported (CS), simply supported/free (SF) and clamped/free (CF). The boundary conditions are defined as below

1. (SS) boundary condition
   at $x = 0$ and $x = \mathrm{L}$, $u_O = w_O = \psi_\theta = 0$
2. (CC) boundary condition
   at $x = 0$ and $x = \mathrm{L}$, $u_O = v_O = w_O = \psi_x = \psi_\theta = 0$
3. (CS) boundary condition
   at $x = 0$, $u_O = v_O = w_O = \psi_x = \psi_\theta = 0$
   at $x = \mathrm{L}$, $u_O = w_O = \psi_\theta = 0$
4. (SF) boundary condition
   at $x = 0$, $u_O = w_O = \psi_\theta = 0$
5. (CF) boundary condition
   at $x = 0$, $u_O = v_O = w_O = \psi_x = \psi_\theta = 0$

The effect of the boundary conditions on the optimum design is investigated for eight layered $(\Phi/-\Phi/\Phi/-\Phi)_s$ truncated conical shells (h / R_1 = 0.2, L / R_1 = 1, η = 0.25, ξ = 0.25, μ = 0.50). In Fig. 5, performance index vs. fibre orientation for different boundary conditions is given. As seen, the maximum $(\mathrm{MODI})_{\max}$ is obtained 5.97 for simply supported (SS) boundary condition. The

Table 4 The $(\mathrm{MODI})_{\max}$ and the optimum fibre orientations for different semi-cone angles

α	$(\mathrm{MODI})_{\max}$	Φ_{opt} (°)
0	2.69	45
15	2.56	45
30	2.58	50
45	5.97	45
60	5.33	45

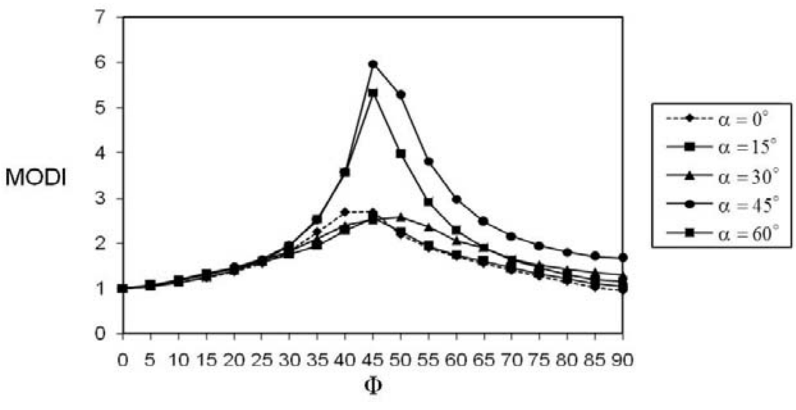

Fig. 5 Performance index vs. fibre orientation for different boundary conditions

Table 5 The $(MODI)_{max}$ and the optimum fibre orientations for different boundary conditions

Boundary conditions	$(MODI)_{max}$	Φ_{opt} (°)
(*SS*)	5.97	45
(*CC*)	3.09	45
(*CS*)	4.27	45
(*SF*)	2.98	40
(*CF*)	2.51	45

optimum fibre orientations are obtained $\Phi_{opt} = 45°$. On the other hand, the minimum $(MODI)_{max}$ is obtained for (CF) boundary condition. In Table 5, the $(MODI)_{max}$ and the optimum fibre orientations are given for different boundary conditions. As seen, the optimum fibre orientation is mostly $\Phi_{opt} = 45°$ for different boundary conditions.

5. Conclusions

In this study, a multiobjective optimization is carried out for symmetrically laminated composite truncated conical shells subjected to external uniform pressure load and thermal load. The design objective is the maximization of the weighted sum of the critical buckling load and fundamental frequency. The design variable is the fibre orientations in the layers. Results are presented for different weighting factors, length-to-radius ratio, semi-cone angle and boundary conditions. As seen from the results that, thermal buckling load has a key role on the multiobjective buckling load design of laminated conical shells. The optimum fibre orientations are mostly $\Phi_{opt} = 45°$ for different weighting factors. The maximum performance index is obtained for $L / R_1 = 4$. The maximum $(MODI)_{max}$ occurs at semi-cone angle $\alpha = 45°$. The optimum fibre orientation is mostly $\Phi_{opt} = 45°$ regardless of semi-cone angle. The maximum performance index is obtained for simply supported boundary condition. The optimum fibre orientation is mostly $\Phi_{opt} = 45°$ for different boundary conditions. Finally, it can be said from the results that, the multiobjective optimization can change the behavior of the laminated conical shell substantially. Therefore, all effects must be considered at the optimization stage of the laminates.

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CC