Steel and Composite Structures, Vol. 12, No. 3 (2012) 183-205
DOI: http://dx.doi.org/10.12989/scs.2012.12.3.183

Behavior and design of steel I-beams with inclined stiffeners

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(Received September 10, 2011, Revised November 24, 2011, Accepted November 30, 2011)

Abstract. This paper presents an investigation of the effect of inclined stiffeners on the load-carrying capacity of simply-supported hot-rolled steel I-beams under various load conditions. The study is carried out using finite element analysis. A series of beams modeled using 3-D solid finite elements with consideration of initial geometric imperfections, residual stresses, and material nonlinearity are analyzed with and without inclined stiffeners to show how the application of inclined stiffeners can offer a noticeable increase in their lateral-torsional buckling (LTB) capacity. The analysis results have shown that the amount of increase in LTB capacity is primarily dependent on the location of the inclined stiffeners and the lateral unsupported length of the beam. The width, thickness and inclination angle of the stiffeners do not have as much an effect on the beam's lateral-torsional buckling capacity when compared to the stiffeners' location and beam length. Once the optimal location for the stiffeners is determined, parametric studies are performed for different beam lengths and load cases and a design equation is developed for the design of such stiffeners. A design example is given to demonstrate how the proposed equation can be used for the design of inclined stiffeners not only to enhance the beam's bearing capacity but its lateral-torsional buckling strength.

Keywords: steel design; I-beams; inclined stiffeners; lateral-torsional instability; finite element analysis

1. Introduction

To maximize their load-carrying capacity, steel beams are often oriented in such a way that the strong axis of the cross-section is perpendicular to the loading plane. When a beam is loaded in this manner, several failure modes are possible depending on its lateral unsupported length L_b. For a doubly symmetric I-shaped compact section, if L_b is less than a reference length referred to by the AISC (2011) specification as L_P, the failure mode is most likely flexural yielding. On the other hand, if L_b is larger than another reference length L_r, the failure mode is most probably elastic lateral-torsional buckling (LTB). Finally, if L_b is in between L_P and L_r, the failure mode will likely be inelastic LTB (Aghayere and Vigil 2009). The presence of LTB decreases a beam's load-carrying capacity, and the amount of decrease can be quite significant as L_b increases. For a given beam, its LTB strength can be enhanced by the use of lateral supports or doubler plates placed at strategic locations. In this study, the effect of using inclined steel stiffeners to increase the LTB strength of beams with compact section is investigated.

At present, stiffeners are almost always provided in a transverse or longitudinal direction with respect to the longitudinal axis of the beam. Transverse stiffeners are often used to strengthen beam webs

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primarily to enhance their resistance to shear and/or concentrated loads (Kim *et al*. 2007, Xie *et al*. 2008, Arabzadeh and Varmazyari 2009) while longitudinal stiffeners are primarily used to increase both the bending and shear strengths of plate girders with non-compact webs (Maiorana *et al*. 2010). The use of longitudinal stiffeners in hot-rolled beams is mostly to reinforce the cutout regions of coped beams (Yam *et al*. 2007, 2011). One objective of this paper is therefore to investigate the strengthening effect stiffeners placed in an inclined position will have on hot-rolled I-beams.

The idea of adding stiffeners to strengthen steel beams can be dated to 1961. Vlasov (1961) gave a design method for beams stiffened with web stiffeners and/or batten plates. However, their effect on the LTB behavior of beam was not studied. Heins and Potocko (1979) performed theoretical and experimental studies on the torsional responses of a plate and a box stiffened I-shaped beam. Szewczak *et al*. (1983) investigated four different stiffener types: transverse stiffeners (intermediate stiffeners), longitudinal stiffeners, cross stiffeners, and box stiffeners for wide-flange steel beams under torsional loading. In Takabatake (1988), the lateral buckling of a uniform and symmetric I-shaped beam stiffened with web stiffeners and/or batten plates was studied theoretically. This was followed by an experimental study (Takabatake *et al*. 1991) on the elastic lateral-torsional buckling behavior of an I-shaped beam with stiffeners subject to uniform bending. The results demonstrated that the lateral displacement of I-shaped beams stiffened with web stiffeners or batten plates was smaller than that of the beams without such stiffeners. It was also found that the effect of vertical stiffeners became more noticeable when they were placed closer to the end supports. Plum and Svensson (1993) investigated the effect of transverse stiffeners on the load-carrying capacity of I-beams with U-shaped stiffeners using the energy method . Chen and Das (2009) investigated the effect of using Carbon Fiber Reinforced Polymers (CFRP) on retrofitting degraded steel beams due to corrosion, while Egilmez *et al*. (2009) and Egilmez and Yormaz (2011) conducted experimental studies on the cyclic behavior of steel I-beams reinforced with glass fiber reinforced polymers (GFRP) and proposed effective methods to increase the inelastic flange and web local buckling strengths of the beams at plastic hinge locations.

Although a considerable amount of research has been conducted on stiffeners, a comprehensive literature review reveals that very few research publications are available on the behavior of beams reinforced with inclined stiffeners, especially when the behavior is in the inelastic LTB range. The primary objectives of this paper are therefore to study the elastic and inelastic behavior of I-shaped beams with inclined stiffeners and to propose a design procedure for the design of such beams.

2. Numerical modeling

Fig. 1 shows a schematic diagram of a beam with inclined stiffeners. Like other web stiffeners, they are to be welded to the web and bear against the inside faces of the flanges. To reduce the amount of uneven stresses that will be transferred to the flanges, the inclined stiffeners are to be provided in pairs in the form of a triangle on both sides of the beam web. The vertical inclination angle is denoted as θ, the distance measured from the supported end of the beam to the apex of this triangular stiffener pair is labeled l, and the stiffener width and thickness are represented by the letters b and t, respectively.

The finite element program ANSYS is used in the present study to determine the effect of inclined stiffeners on the load-carrying capacity of simply-supported steel I-beams. In developing the finite element model, initial geometric imperfections, material nonlinearity and residual stresses are all incorporated in the modeling. The flanges and web of the beam is modeled using 20-node rectangular shape solid elements SOLID95 as shown in Fig. 2. The element has a total of 60 degrees-of-freedom

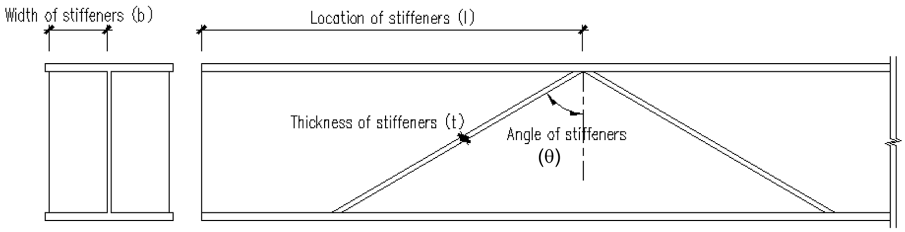


Fig. 1 Beam with inclined stiffeners

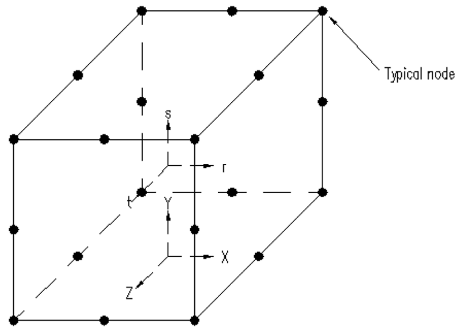


Fig. 2 A 20-node SOLID95 element

(dof) or 3 translational dof per node. The same element type with parallelepiped shape is used to model the inclined stiffeners, as shown in Fig. 3. Although plate or shell elements are often used to model I-sections in a finite element analysis, solid elements are used here because it has been found that if two elements are used to model the flanges and web in the thickness direction, more accurate results can be obtained. More importantly, because the dof of solid elements are located at the elements' corners and edges, proper merging of nodes to simulate full continuity between the edges of the stiffeners and the surfaces of the flanges and web can be achieved more easily. SOLID95 is capable of large deformation analysis, modeling finite strain as well as accounting for material nonlinearity. As far as numerical integration is concerned, there are 14 integration points associated with this element. Besides the 8 commonly used integration points, an additional 6 points are included and located close to the center of each face of the element. These additional integration points can improve the accuracy of the solution in an inelastic analysis.

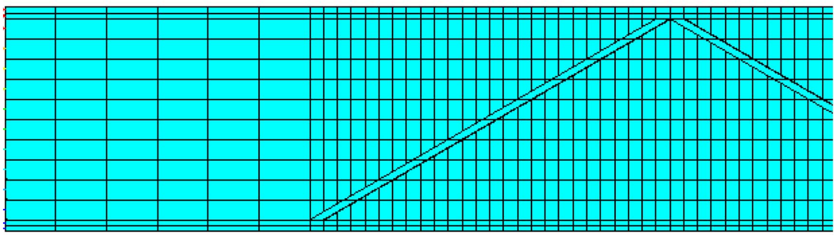
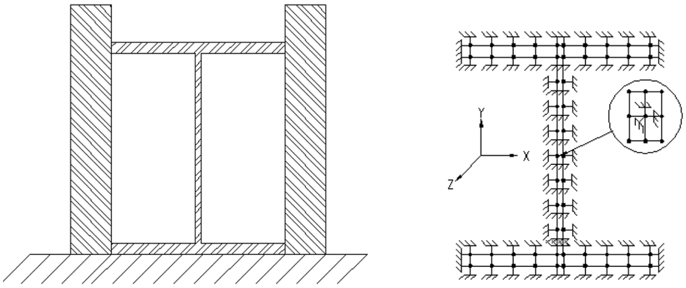


Fig. 3 Typical finite element model for a steel beam with inclined stiffeners



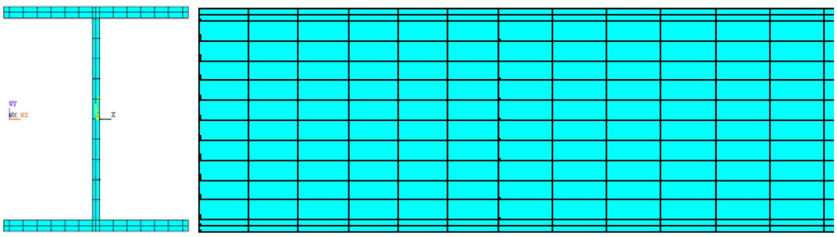
(a) Actual support condition (b) Finite element model

Fig. 4 Simply-supported boundary condition

To model the simply-supported boundary condition, the degrees-of-freedom in the x- and y-directions of all nodes at the two end sections of the beam model are restrained, but z-direction restraint is only provided to the node located in the center of the each end cross-section. This boundary condition is shown in Fig. 4. By using this specific set of restraints in the finite element model, the beam is able to undergo rotation about the x- and y-axes but is restrained against translation in the x- and y-directions as well as rotation about the z-axis at the supported ends.

2.1 Convergence and sensitivity studies

Like any finite element analysis, a convergence study is needed to determine the optimal mesh size required for obtaining reliable results. Based on a series of analyses using different mesh sizes, it was found that results were rather sensitive to the number of element used in the longitudinal direction, but they varied only slightly with the number of elements used in the transverse directions. For a simply-supported beam without inclined stiffeners subjected to equal and opposite end moments, it has been found that the theoretical critical moment can be obtained if the element to beam length does not exceed 1/80. When a change in loading condition or the introduction of inclined stiffeners is involved, a sensitivity study has shown that for beams loaded by a concentrated load at mid-span, the results stabilize and converge when the element-to-beam length in the longitudinal direction is less than or equal to 1/160. As for the uniformly distributed load case, convergence is achieved when the element-to-beam length in the longitudinal direction is less than or equal to 1/320. A typical finite element mesh is shown in Fig. 5. The meshing is more refined in the longitudinal direction than in the transverse (especially the



(a) End view (b) Elevation view

Fig. 5 Finite element mesh

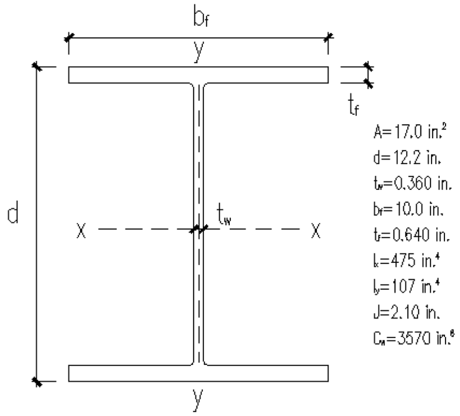


Fig. 6 Cross-section properties of a W12 × 58 section (1 in. = 2.54 cm)

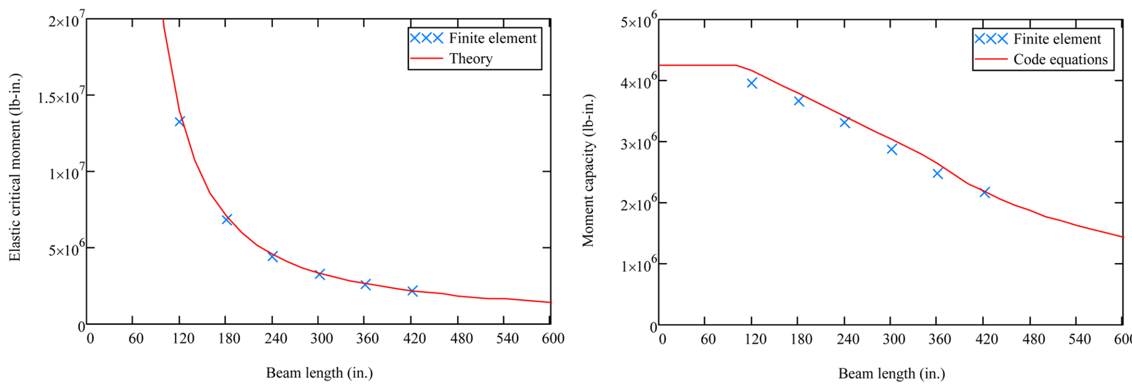
thickness) directions. When inclined stiffeners are added to the model, further refinement of the mesh in the longitudinal direction in the region of the stiffeners as shown in Fig. 3 is made to ensure nodal compatibility between the elements used to model the beam and those used to model the stiffeners.

Using the cross-section shown in Fig. 6, elastic and inelastic analyses are performed using the finite element models. The finite element results obtained for a simply-supported beam with different beam lengths subjected to a pair of equal and opposite moments applied at the ends are compared to the theoretical and code equation values, respectively, in Figs. 7(a) and 7(b). Good correlations are observed. Note that critical moment or moment capacity increases with decreasing beam length, but once the plastic moment M_p (which for this beam section is 4.32×10^6 lb-in. or 4.88×10^5 N-m) is reached, no further increase in moment capacity is possible.

2.2 Modeling for geometrical imperfections, material nonlinearity and residual stresses

2.2.1 Geometrical imperfections

Based on a study by Wang and Helwig (2005), cross-section distortion in the form of an accidental



(a) Elastic range (b) Inelastic range

Fig. 7 Results comparison (1 in. = 2.54 cm, 1 lb-in. = 0.113 N-m)

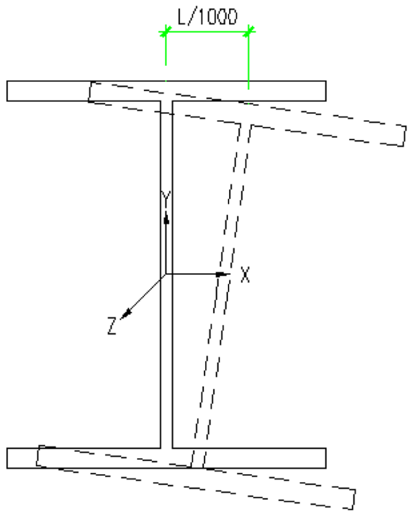


Fig. 8 Magnitude of initial geometrical imperfection at mid-span

twist that results in a lateral displacement of one flange relative to the other can be as high as $L_b/500$, where L_b is the lateral unsupported length of the beam. While cross-section distortion is not taken into consideration in the present study, member out-of-straightness in the form of a camber or sweep will be considered. According to ASTM A6/A6M - 11 (2011), the permitted variations in straightness for W and HP shapes (i.e., I-sections) with beam flange width equal to or larger than 6 inches (152 mm) and having a member length of L are given by the formula $1/8\times(L/10)$ when L is expressed in feet, or $L/960$ when L is expressed in inches. Because $L/960$ is the maximum camber or sweep allowed, a more practical value of $L/1000$ is used in the numerical analyses. Note that in accordance with the Code of Standard Practice for Steel Buildings and Bridges (AISC 2011), $L/1000$ is also the maximum out-of-straightness tolerance allowed if the I-section is to be used as a compression member.

In this study, the first lateral-torsional buckling mode obtained from an eigenvalue buckling analysis is used to account for the member overall geometric imperfections. Since only the buckled mode shape (i.e., the eigenvector) but not its actual magnitude can be obtained from an eigenvalue analysis, the mode shape is normalized so that its largest component is equal to $L/1000$ (as shown in Fig. 8), where L is the beam length. A nonlinear finite element analysis is then performed to generate the load-displacement diagram, and the collapse load is obtained as the peak point from this load-displacement plot.

2.2.2 Material nonlinearity

In this study, the steel yield stress F_y used for the beam is 50 ksi (or 345 MPa). Once the stress exceeds this value, yielding is assumed to occur. The post yield or strain-hardening material stiffness is assumed to be 5% of its elastic stiffness, i.e., $E_t = 5\%E$, where E is the elastic modulus as shown in Fig. 9. The Poisson ratio used is 0.3.

2.2.3 Residual stresses

Residual stresses, or more accurately residual stresses and strains, are locked-in stresses and strains that result from the manufacturing process (Ziemian 2010). Residual stresses in steel structure may result from several sources: (1) uneven cooling that occurs after hot rolling of structural shapes; (2) cold

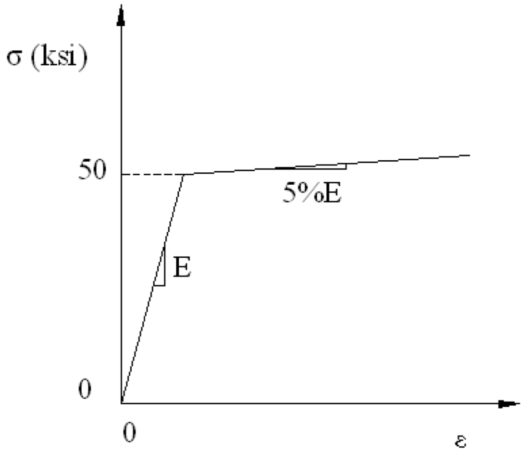


Fig. 9 Material stress-strain model

bending or cambering during fabrication; (3) punching of holes and cutting operations during fabrication; and (4) welding (Salmon *et al*. 2009). Among these sources, residual stresses induced from uneven cooling and welding are the most significant, especially for hot-rolled I-sections. Although the distribution of residual stresses in the longitudinal direction (perpendicular to the plane of cross section) is different for different cross-sections, an approximate model based on a statistical analysis is used (Chen 2008). In this study, the assumed residual stress distribution is shown in Fig. 10. Note that these stresses are self-equilibrating, i.e., the volumes of the compression and tension stress blocks are equal and distributed over the cross-section in such a way that no axial force or bending moment will be induced in the cross-section. These assumed initial stresses are incorporated into the structural model by applying them directly on the integration points of the elements.

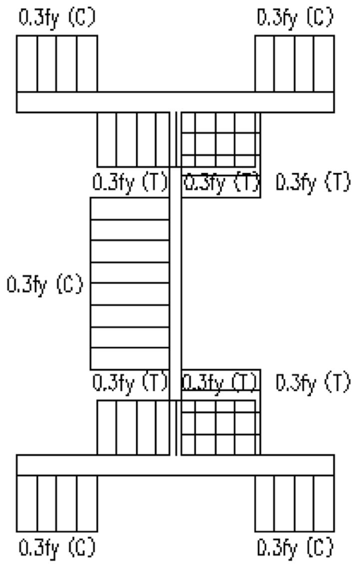


Fig. 10 Assumed residual stress pattern

3. Solution schemes

Nonlinear problems often entail the use of increment load and iterative solutions (Cook *et al*. 2002, Zienkiewicz and Taylor 2005). In ANSYS, the two primary iterative schemes used are the Newton-Raphson method and the arc-length method (Bittnar and Sejnoha 1996). In using these methods, users are required to specify convergent points. In the computational simulation of steel structures, several iterations are inevitable to arrive at these convergent points (Schafer *et al*. 2010). Because of the size of the model, large systems of simultaneous algebraic equations are generated. These equations are solved either by a direct elimination process such as the Gaussian elimination, or some variant of it with consideration of matrix sparsity; or an iterative procedure, such as the conjugate gradient method. The direct elimination method is used in the present study.

Because the number of load increments specified and the size of the substeps used within each load increment may affect the results, ANSYS provides automatic load stepping if many load increments are specified. To improve accuracy, the applied load at the first convergent point should be sufficiently small. If this is not the case, the solution may not be sufficiently accurate even when many convergent steps are used after the first substep. For the study of LTB of steel beams with inclined stiffeners, it was found that good results could be obtained if at least 10 convergent points were defined before the peak load when the arc-length method was used. To be conservative, more than 15 convergent points are used in generating the finite element results.

4. Numerical analyses

The physical model of a beam with inclined stiffeners is shown in Fig. 11. The corresponding finite element model is shown in Fig. 12. To investigate the inclined stiffeners' effect on the beam's load-carrying capacity, one hundred and fifty-nine simply-supported steel beams that covers the range of parameters given in Table 1 are analyzed. As depicted in Fig. 1, l denotes the distance measured from the beam end to the top of the inclined stiffeners pair, and θ is the vertical inclination angle of the stiffeners. For the reference cross-section shown in Fig. 6, L_p = 106 in. or 269 cm, and L_r = 359 in. or

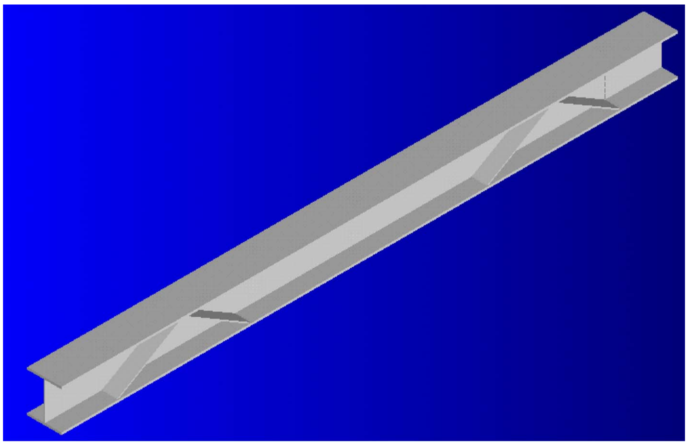


Fig. 11 Beam with inclined stiffeners

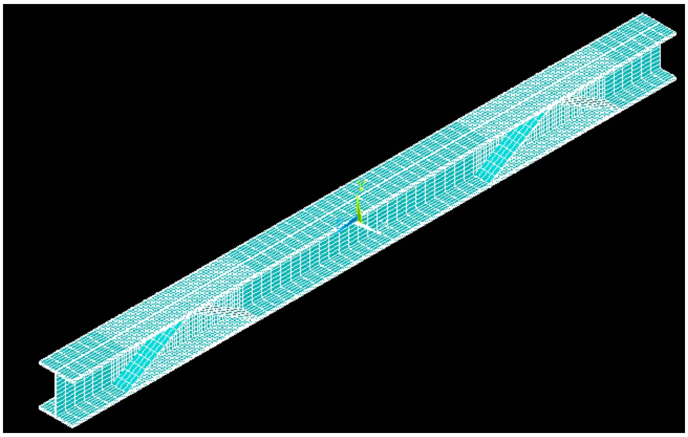


Fig. 12 Finite element model of a beam with inclined stiffeners

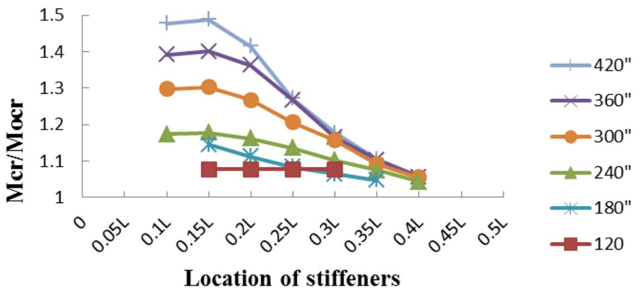
Table 1. Range of parameters used (1 in. = 2.54 cm)

Parameter	Range, values, or case
Stiffener location, l	$0.05L$ to $0.45L$ in $0.05L$ increment
Stiffener vertical inclination angle, θ	30°, 45°, 60°
Beam length, L	120 to 420 in. in 60 in. increment
Stiffener width, b	3, 3.75, 4,5, 4.8 in.
Stiffener thickness, t	0.375, 0.5, 0.625 in.
Load case	Equal and opposite end moments Uniformly distributed load Concentrated load at midspan

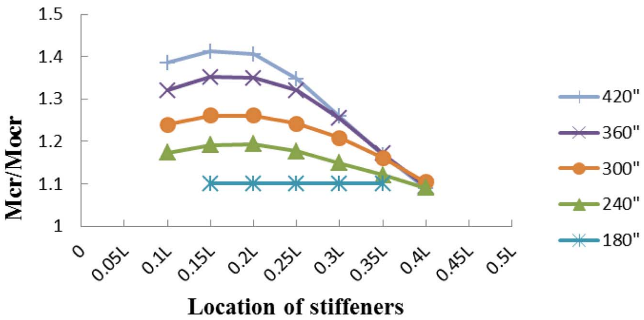
912 cm (AISC 2011), so the range of beam length L (which is equal to the lateral unsupported length L_b because the beam is laterally braced only at the supports) selected for the study will cover all three possible failure modes of flexural yielding, inelastic LTB and elastic LTB for the beam.

As discussed earlier, for each run an eigenvalue buckling analysis is first employed to obtain the fundamental lateral-torsional buckling mode shape of the beam. This mode shape is scaled to $L/1000$ at midspan to model the effect of initial geometric imperfections. Initial stresses are then applied to simulate the residual stresses present in the beam. Finally, a nonlinear collapse analysis is carried out to trace the load-deflection curve using the arc-length method and the maximum load capacity of the beam is obtained as the peak value from this load-deflection curve. In what follows, representative results are shown and discussed. The results are given in terms of the moment ratio M_{cr} / M_{ocr}, where M_{cr} is the maximum moment capacity of the beam with the inclined stiffeners and M_{ocr} is the maximum moment capacity of the beam without the inclined stiffeners. Thus, if M_{cr} / M_{ocr} is larger than 1, the load-carrying capacity of the beam will be enhanced by the use of inclined stiffeners.

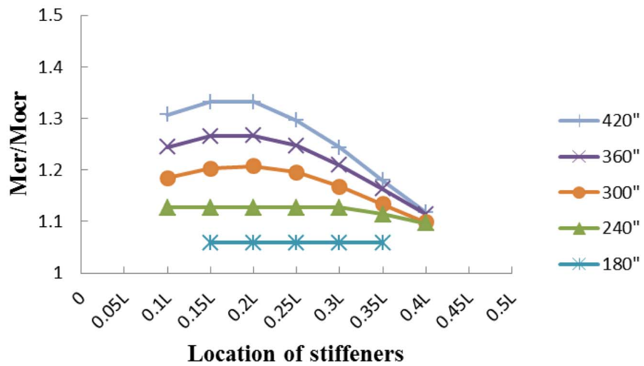
Figs. 13(a) to 13(c) show the effects of stiffeners' location and beam length on the load-carrying capacity of the beam for the three load cases given in Table 1. These plots were generated using $\theta = 60°$, $b = 4.5$ in. (11.4 cm) and $t = 0.375$ in. (9.5 mm). As can be seen, both the location of stiffeners and beam length have a noticeable effect on the beam, except for cases in which the lateral unsupported length of the beam is less than L_p when the failure mode is governed by flexural yielding and so the effect of the inclined stiffeners becomes insignificant regardless of their locations. These cases are shown as



(a) Equal and opposite end moments



(b) Uniformly distributed load



(c) Concentrated load

Fig. 13 Effects of stiffeners' location and beam length

horizontal lines in the figures.

The increase in load capacity is more pronounced when the inclined stiffeners are placed at the distance $0.1L$ to $0.2L$ from the beam end, and M_{cr}/M_{ocr} increases more rapidly for long beams. These observations are consistent with the fundamentals of stability theory (Chen and Lui 1987, Galambos and Surovek 2008): Because the rate of change in slope about the minor axis and the rate of change in the angle of twist are the highest near the simply-supported ends, placing the inclined stiffeners in these regions will be the most beneficial. As for the different load cases, it can be seen that for a given beam length, M_{cr}/M_{ocr} is the highest for the end moments case and lowest for the concentrated load case.

The effect of the stiffeners' vertical inclination angle is shown in Fig. 14. The curves in the figure correspond to $L = 240$ in. (6.1 m), $b = 4.5$ in. (11.4 cm), $t = 0.375$ in. (9.5mm) and the equal and opposite end moments load case. As expected, the increase in M_{cr}/M_{ocr} is somewhat in proportion to the

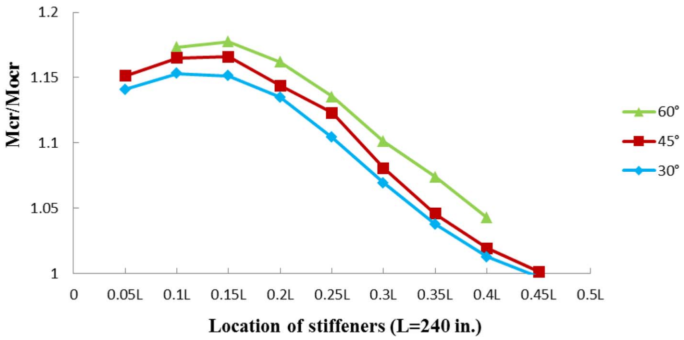


Fig. 14 Effect of stiffeners' vertical inclination angle

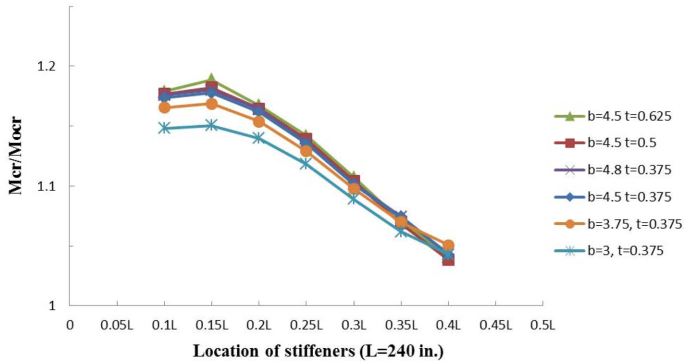


Fig. 15 Effect of stiffeners' size

magnitude of the vertical inclination angle, although this increase is not very significant when compared to the stiffeners' location.

The effect of stiffeners' size is shown in Fig. 15. The curves are shown for L=240 in. (6.1 m) and $\theta = 60^{\circ}$ and the equal and opposite moments load case. Although M_{cr} / M_{ocr} increases as the stiffeners' size increases, the increase is not very prominent when compared to the stiffeners' location.

Although the curves shown in Figs. 14 and 15 are for the equal and opposite end moments load case, similar trends (and conclusions) can be made for the uniformly distributed and concentrated load cases. Based on the results of this study, the following conclusions can be made:

(1) Except for cases when flexural yielding is the controlling failure mode of the beam, the use of inclined stiffeners increases the beam's load-carrying capacity. This increase is strongly influenced by the location of the stiffeners and the length of the beam.

(2) The increase in load-carrying capacity is the most prominent when the inclined stiffeners are placed near the beam ends, and M_{cr}/M_{ocr} is the highest when l is in the $0.1L$ to $0.2L$ range.

(3) The increase in M_{cr}/M_{ocr} becomes more pronounced as L increases.

(4) Although M_{cr}/M_{ocr} increases with the stiffeners' vertical inclination angle, this increase is not very important when compared to the effect of stiffeners' location and beam length.

(5) Slight change in the width and thickness of the inclined stiffeners will not have a significant effect on the load-carrying capacity of the beam. Therefore, as long as the stiffeners' width is extended near the outer edges of the flanges, the width and thickness effect can be neglected.

5. Design considerations

In actual application, inclined stiffeners are to be used in place of vertical stiffeners often provided at locations of concentrated applied loads or support reactions. Based on the present study, it is recommended that these inclined stiffeners be placed at a distance anywhere from $0.1L$ to $0.2L$ from the beam end with a vertical inclination angle $\approx 60^{\circ}$, so that one end of the inclined stiffeners will coincide with the line of action of the concentrated load. The width of the stiffeners is to be extended near the outer edges of the beam flanges, and its thickness should be selected so that the AISC local buckling criterion will not be violated.

Because the use of inclined stiffeners can also enhance the lateral-torsional buckling strength of beams when $L_b > L_p$, a design equation will be proposed in the following section for the design of simply-supported beams with inclined stiffeners.

5.1 Regression analysis

A design equation that synthesizes the results obtained from the present study will be derived based on a regression analysis. The proposed design equation will incorporate the optimal location ($l = 0.1L$ to $0.2L$) of and the most effective vertical inclination angle ($\approx 60^{\circ}$) for the stiffeners as well as the types of load (end moments, uniformly distributed load, concentrated load) acting on the beam and beam length. The width of the stiffeners b is to be taken approximately as $\frac{b_f - t_w}{2}$ where b_f and t_w are the beam flange width and web thickness, respectively. The thickness of the stiffeners t is to be selected so the AISC local buckling criterion of $\frac{b}{t} \le \frac{95}{\sqrt{F_y}}$, where F_y is the material yield stress in ksi, will be satisfied. The data upon which the proposed design equation is based are listed in Table 2 and plotted in Fig. 16.

5.2 Design equations

For each type of loading, it can be seen from Fig. 16 that a somewhat linear relationship exists between M_{cr} / M_{ocr} and the beam length. As a result, linear regression analyses are performed. If we define $C_{is} = \frac{M_{cr}}{M_{ocr}}$ as a coefficient that takes into consideration the increase in moment capacity due to the presence of inclined stiffeners and is a function of the beam length L, and reference lengths L_p and L_r, then:

For the equal and opposite end moments load case

$$C_{is} = \frac{0.351(L - L_p)}{(L_r - L_p)} + 1.035 \tag{1}$$

Table 2. Regression analysis data

Beam length	M_{cr} / M_{ocr}		
	Equal and opposite end moments	Uniformly distributed load	Concentrated load at midspan
120	1.077	NA	NA
180	1.144	1.101	1.058
240	1.178	1.191	1.127
300	1.303	1.261	1.203
360	1.400	1.352	1.265
420	1.489	1.412	1.322

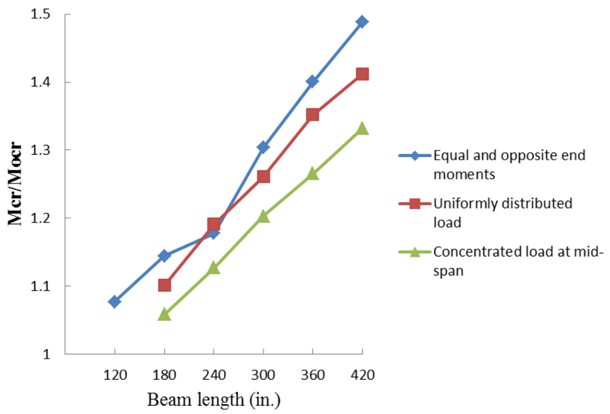


Fig. 16 Plot of data used for the regression analysis

For the uniformly distributed load case

$$C_{is} = \frac{0.325(L-L_p)}{(L_r-L_p)} + 1.012 \tag{2}$$

For the concentrated load case

$$C_{is} = \frac{0.277(L-L_p)}{(L_r-L_p)} + 0.981 \tag{3}$$

For simplicity, the above equations can be approximated by a single equation

$$C_{is} = \frac{0.35\beta(L-L_p)}{(L_r-L_p)} + 1 \tag{4}$$

where, $\beta = 1$ for beams subject to equal and opposite end moments, $\beta = 0.9$ for beams subject to a uniformly distributed load, and $\beta = 0.7$ for beams subject to a midspan concentrated load.

The R^2 value of Eq. (4) and the maximum error for each load case are summarized in Table 3 and a comparison of the results calculated using this equation and those from the finite element analysis is given in Fig. 17. As can be seen, for most cases Eq. (4) gives slightly conservative results for all load cases.

Table 3. Regression analysis results

Load case	R^2	Maximum error
End moments	0.978	3.7%
Uniformly distributed load	0.997	0.6%
Concentrated load	0.996	0.8%

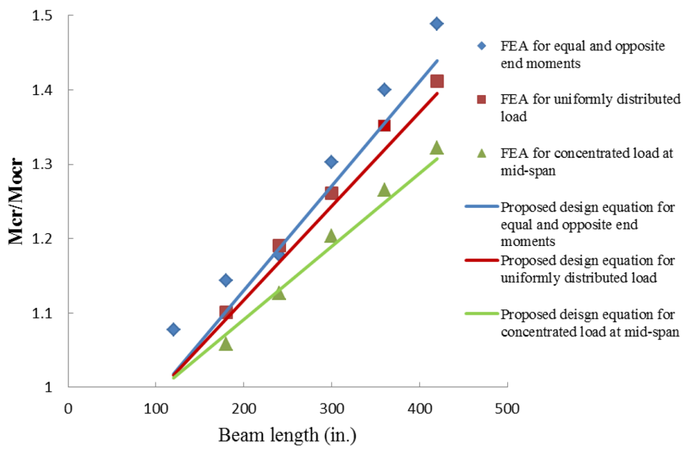


Fig. 17 Curve-fitting results comparison

5.3 Design procedure

A procedure for the design of inclined stiffeners is proposed as follows:
1. Compute the required flexural strength M_u.

2. Select a beam section so that $\phi_b M_n \approx M_u$, where ϕ_b= 0.9 is the resistance factor for bending and M_n is the design flexural strength (AISC 2011). Note that $\phi_b M_n$ can be smaller than M_u at this point.

3. Check the web yielding and web crippling criteria (AISC 2011) as follows:

<u>Web yielding:</u> $$\phi_{wy} R_n \geq R_u \tag{5}$$

where, for $x > d$, $$\phi_{wy} R_n = \phi_{wy}(5k+N)F_y t_w \tag{6}$$

and for $x \leq$ d, $$\phi_{wy} R_n = \phi_{wy}(2.5k+N)F_y t_w \tag{7}$$

in which
x = location of concentrated load or reaction from the beam end;
d = beam depth;
ϕ_{wy} = 1.0 (resistance factor for web yielding);
R_n = design strength for web yielding;
R_u = required strength for web yielding;
k = distance from outer face of flange to web toe of fillet;
N = bearing length;
F_y = yield stress of beam web;
t_w = beam web thickness.

Web crippling: $\phi_{wc}R_n \geq R_u$ (8)

where for $x \geq d/2$, $$\phi_{wc}R_n = \phi_{wc}0.8t_w^{\,2}\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \tag{9}$$

and for $x < d/2$ and $N/d \leq 0.2$, $$\phi_{wc}R_n = \phi_{wc}0.4t_w^{\,2}\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \tag{10}$$

and for $x < d/2$ and $N/d > 0.2$, $$\phi_{wc}R_n = \phi_{wc}0.4t_w^{\,2}\left[1+\left(4\frac{N}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \tag{11}$$

in which x, d, k, N, F_y, t_w are defined as before, and
$\phi_{wc} = 0.75$ (resistance factor for web crippling);
R_n = design strength for web crippling;
R_u = required strength for web crippling;
t_f = beam flange thickness.

4. Use the local buckling, compression and bearing strength criteria to design the inclined stiffeners.

Local bucking strength: $$\frac{b}{t} \leq \frac{95}{\sqrt{F_y}} \tag{12}$$

where b, t and F_y are the stiffener width, thickness and yield stress, respectively.

Compression strength: $$\phi_c P_n \geq R_{us} \tag{13}$$

where $$\phi_c P_n = \phi_c F_{cr} A_g \tag{14}$$

in which

$\phi_c = 0.9$ (resistance factor for compression);
P_n = nominal compressive strength;
R_{us} = factored compression force in the stiffeners;
F_{cr} = flexural buckling stress as given in Eq. (15) and (16);
A_g = cross-sectional area of stiffeners.

The critical stress for flexural buckling, F_{cr}, is determined as follows

For $KL_{st}/r_{st} \leq 4.71\sqrt{\frac{E}{F_y}}$, $$F_{cr} = [0.658^{\frac{F_y}{F_e}}]F_y \tag{15}$$

and for $KL_{st} / r_{st} > 4.71\sqrt{\dfrac{E}{F_y}}$, $$F_{cr} = 0.877F_e \tag{16}$$

where

F_e = elastic critical buckling stress given by $$F_e = \frac{\pi^2 E}{\left(\dfrac{KL_{st}}{r_{st}}\right)^2} \tag{17}$$

L_{st} = length of stiffeners;
r_{st} = radius of gyration of stiffeners taken about the longitudinal axis of the beam;
K = 0.75 for stiffeners welded to both the upper and lower beam flanges.

Bearing strength: $$\phi_{pb}R_n = \phi_{pb}1.8F_yA_{pb} \tag{18}$$

where

ϕ_{pb} = 0.75 (resistance factor for bearing);
F_y = yield stress of stiffeners;
A_{pb} = projected bearing area of stiffeners.

5. Compute C_{is} using Eq. (4) and ensure the design flexural strength exceeds the required flexural strength at this point, i.e., $\phi_b M_n \geq M_u$.

6. Check the shear strength of the beam.

$$\phi_v V_n = \phi_v 0.6F_yA_wC_v \geq V_u \tag{19}$$

where

ϕ_v = 1.0(resistance factor for shear of hot-rolled I-shaped members);
F_y = yield stress of beam web;
A_w = area of the beam web = dt_w;
C_v = web shear coefficient (AISC 2011).

7. Check the deflection limit of the beam.

$$\Delta_L \leq L/360 \tag{20}$$

where

Δ_L = the computed deflection under unfactored (or nominal) live load;
L = beam length.

6. Design example

The following example is used to demonstrate how the proposed procedure is applied for the design of a simply-supported beam with inclined stiffeners. The beam shown in Fig. 18 is to be designed to carry a uniformly distributed superimposed dead load of 1 k/ft (14.6 kN/m) and a live load of 2.5 k/ft (36.5 kN/m). The simply-supported span is 20 ft (6.1 m). Lateral supports are provided only at the two ends of the beam (i.e., $L_b = L$). The steel used is A992 (F_y = 50 ksi or 345 MPa) for the beam and A36 (F_y = 36 ksi or 248 MPa) for the stiffeners and the elastic modulus is E = 29,000 ksi (200 GPa).

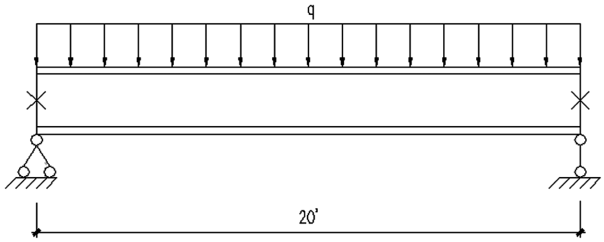


Fig. 18 Beam design example (1 ft = 0.305 m)

Solution:

1. Assume the beam weighs approximately 50 lb/ft (0.73 kN/m), the required flexural strength M_u is computed as follows

$$q_u = 1.2D + 1.6L = (1.2)(1 + 0.05) + (1.6)(2.5) = 5.26 \text{ k/ft (or 76.8 kN/m)}$$

$$M_u = \left(\frac{1}{8}\right)(5.26)(20)(20) = 263\text{k-ft(or 357 kN-m)}$$

2. Try a W14×48 section. The design flexural strength M_n for this section is computed as follows:

For a simply-supported beam under a uniformly distributed load, the lateral-torsional modification factor for non-uniform moment C_b=1.14 (AISC 2011), and for the W14 × 48 section M_p = 327 k-ft (or 443 kN-m), L_p = 6.75 ft (or 2.06 m), L_r = 21.1 ft (or 6.44 m), S_x = 70.2 in^3 (or 1,150 cm^3), k = 1.19 in. (30 mm), d = 13.8 in. (or 350 mm), t_w = 0.340 in. (or 8.64 mm), t_f = 0.595 in. (or 15.1 mm), b_f = 8.03 in. (or 204 mm).

Since $L_p < [L_b = 20 \text{ ft}] < L_r$, $M_n = C_b[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)] \leq M_p$

$$M_n = (1.14)\left[327 - (327 - (0.7)(50)(70.2/12))\left(\frac{20 - 6.75}{21.1 - 6.75}\right)\right] \leq 327$$

M_n = 244 k-ft (or 331 kN-m), and so

$\phi_b M_n = (0.9)(244) = 220$ k-ft, which is $< [M_u = 263 \text{ k-ft}]$, $\therefore$ not ok

3. Check the concentrated load criteria for web yielding and web crippling as follows:

(1) Web yielding

$$R_u = \frac{(5.26)(20)}{2} = 52.6 \text{ k(or 234 kN)}$$

Since $x = 0 \leq d = 13.8$ in. (or 350 mm)

$$\phi_{wy}R_n = \phi_{wy}(2.5k + N)F_y t_w$$

$\phi_{wy}R_n = 1.0[(2.5)(1.19) + 0](50)(0.34) = 50.6$ k(or 225 kN) which is $< [R_u = 52.6 \text{ k}]$, $\therefore$ not ok

(2) Web crippling

Since $x = 0 < d/2 = 6.9$ in. (or 175 mm) and $\frac{N}{d} = 0 \le 0.2$,

$$\phi_{wy}R_n = \phi_{wc}0.4t_w^2 \left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} = (0.75)(73.6) = 55.2 \quad \text{kips (or 246 kN) which is} >$$

$[R_u = 52.6 \text{ k}]$, $\therefore$ ok

Since both the flexural strength and the web yielding criteria are violated, we will design a pair of inclined stiffeners to enhance the beam's flexural strength as well as to resist web yielding.

4. The inclined stiffeners are designed as follows

The width of each inclined stiffeners is to be taken as $b \approx \frac{b_f - t_w}{2} = \frac{8.03-0.340}{2} = 3.845$ in., so use $b = 3.75$ in. (or 95.3 mm)

Check the local buckling criterion:

The minimum thickness to prevent local buckling is determined from Eq. (12)

Min $t = \frac{b}{95}\sqrt{F_y} = 0.236$, try $t = 0.5$ in. (or 12.7 mm)

Check the compression criterion for the inclined stiffeners

$$R_{us} = \frac{R_u}{\cos 60^\circ} = \frac{52.6}{1/2} = 105.2\text{k(or 468 kN)}$$

$$K = 0.75,\ L_{st} = \frac{1}{\cos 60^\circ}(d - 2t_f) = (2)(13.8 - 0.595 \times 2) = 25.2\text{, in. (or 640 mm)}$$

$$\frac{KL_{st}}{r_{st}} = \frac{(0.75)(25.22)}{\sqrt{\frac{1}{12}(7.84)(7.84)}} = 8.4$$, so F_{cr} as calculated from Eq. (15) is equal to 35.9 ksi (or 248 MPa)

$\phi_c P_n = \phi_c F_{cr} A_g = (0.9)(35.9)(3.92) = 126.7\text{k}$ (or 564 kN), which is $> [R_{us} = 105.2 \text{ k}]$, $\therefore$ ok

Check the bearing criterion

If a 1-in cutout is used to clear the fillet, we have

$$A_{pb} = (2)(0.5)(3.75-1) = 2.75 \text{ in.}^2 \text{ (or 1,774 mm}^2\text{)}$$

$\phi_{pb}R_n = (0.75)(1.8)(36)(2.75) = 133.7$ kips (or 595 kN), which is $> [R_{us} = 105.2 \text{ k}]$, $\therefore$ ok

5. Check that $\phi_b M_n \ge M_u$.

From Eq. (4), $C_{is} = (0.35)(0.9)\left(\frac{20-6.75}{21.1-6.75}\right) + 1 = 1.29$

$$M_n = (1.29)(1.14)\left[327 - (327 - (0.7)(50)(70.2/12))\left(\frac{20-6.75}{21.1-6.75}\right)\right] \le 327$$

M_n = 315 k-ft (or 427 kN-m), and so

$\phi_b M_n = (0.9)(315) = 283.5$ k-ft (or 384 kN-m) which is now > [M_u = 263 k-ft], ∴ ok

6. The shear strength of the beam is checked as follows

$V_u = R_u = 52.6$ k (or 234 kN)

$\phi_v V_n = \phi_v 0.6 F_y A_w C_v = (1.0)(0.6)(50)(13.8)(0.34)(1.0) = 141$ k (or 627 kN), ∴ ok

7. The deflection limit of the beam is checked as follows

$$\Delta_L = \frac{5qL^4}{384EI} = \frac{5(2.5/12)(240)^4}{(384)(29000)(484)} = 0.64 \text{ in. (or 16.3 mm) which is} < [\frac{L}{360} = \frac{240}{360} = 0.67 \text{ in.}], \therefore \text{ ok}$$

The final design showing the location and placement of stiffeners is given in Fig. 19. The principal and maximum shear stresses under factored loads obtained from a finite element analysis of this beam with the inclined stiffeners are shown in Figs. 20 to 22. As can be seen, given that F_y=50 ksi (345 MPa) for the beam and F_y=36 ksi (248 MPa) for the stiffeners, no distress in the beam flange, web or

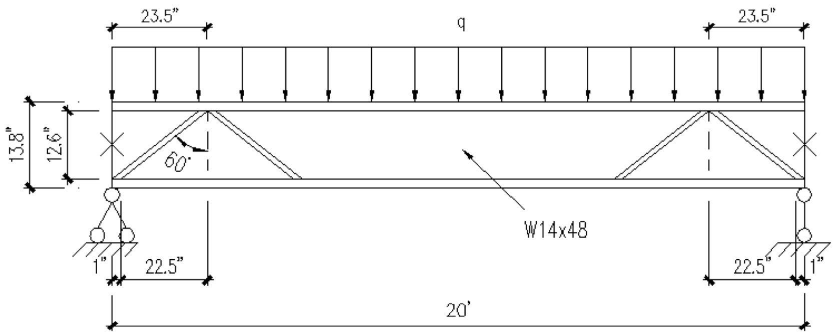


Fig. 19 Beam with inclined stiffeners (1 ft = 0.305 m, 1 in. = 2.54 cm)

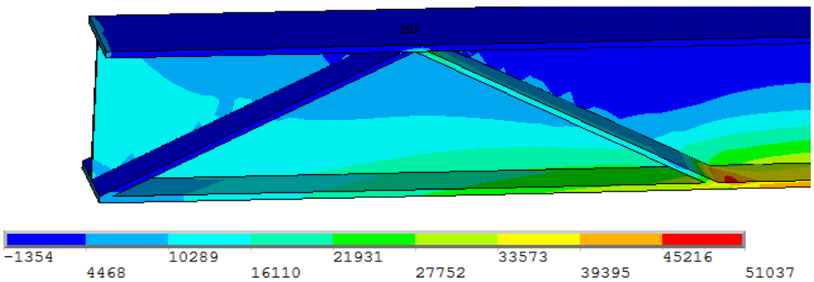


Fig. 20 Maximum tensile stress σ_1 in lb/in^2 (1 lb/in^2 = 6.895 kPa)

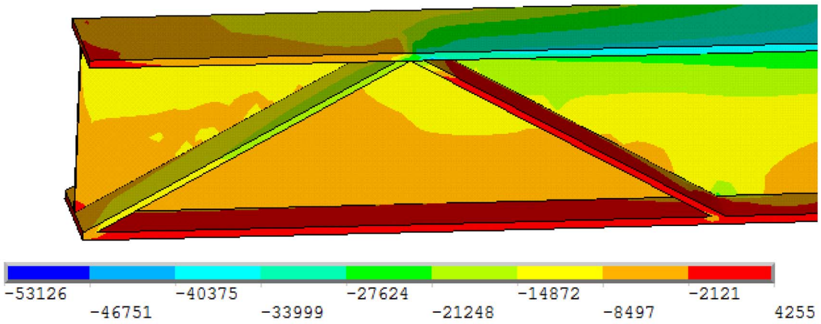


Fig. 21 Maximum compressive stress σ_3 in lb/in^2 (1 lb/in^2 = 6.895 kPa)

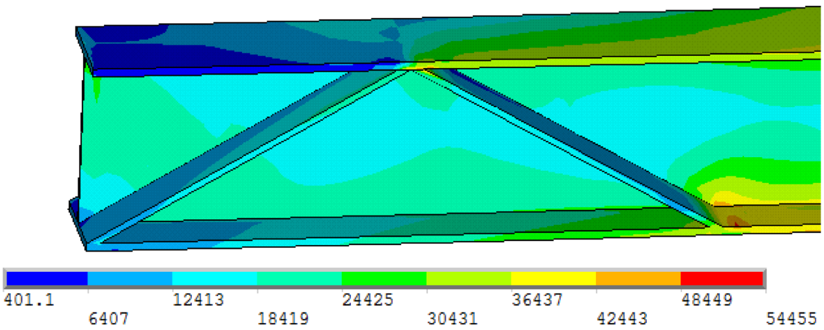


Fig. 22 Shear stress intensity $(\sigma_1 - \sigma_3) = 2\tau_{max}$ in lb/in^2 (1 lb/in^2 = 6.895 kPa)

stiffeners is observed. It should be noted that the stresses at the beam ends shown in these figures are not zero because of the presence of residual stresses.

The C_{is} factor computed using a finite element analysis is 1.38, which is higher than the value computed from Eq. (4). The higher C_{is} value from the finite element analysis is expected because Eq. (4) is on the conservative side.

7. Summary and conclusions

The effect of inclined stiffeners on the load-carrying capacity of simply-supported I-beams in the inelastic and elastic lateral-torsional buckling ranges has been investigated using 3-D finite element analysis. The finite element model not only accounts for the initial geometric imperfections and the nonlinear material property, but the presence of residual stresses as well. The finite element model is verified by comparing the results generated numerically with results computed using beam buckling theory (in the elastic range) and with results calculated using the AISC code equations (in the inelastic range). Using the finite element model, one hundred and fifty-nine beams with inclined stiffeners highlighting the effects of the inclination angle, beam length, stiffener size, stiffener location, and different loading conditions were analyzed.

The study of beams reinforced with inclined stiffeners has shown that the presence of such stiffeners offers a noticeable increase in the lateral-torsional buckling load of the beams. It has been found that the beneficial effect of the stiffeners is primarily related to their location on the beam and that a distance of $0.1L$ to $0.2L$ measured from the beam ends would be considered as optimal for the stiffeners. It has also been found that the beneficial effect becomes more prominent for longer beams and when the vertical inclination angle increases. On the other hand, the size of inclined stiffeners does not appear to affect the lateral-torsional capacity considerably, provided that the stiffeners were extended near the outer flange edges of the cross-section. The analyses were carried out for three typical load cases. Based on the optimal stiffener location and inclination angle, a design equation is proposed for use in the design of simply-supported beams with inclined stiffeners. A design procedure is then given and a design example is provided to demonstrate how the proposed procedure can be applied.

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Notations

The following symbols are used in this paper:

A_g = cross-sectional area of stiffeners;
A_{pb} = projected bearing area of stiffeners;
A_w = area of the beam web;
b = stiffener width;
b_f = beam flange width;
C_b = lateral-torsional buckling modification factor for nonuniform moment diagrams when both ends of the unsupported segment are braced;
C_{is} = lateral-torsional buckling modification factor for the presence of inclined stiffeners;
C_v = web shear coefficient;
d = beam depth;
E = elastic molulus;
E_t = tangent modulus;
F_{cr} = critical stress for flexural buckling;
F_e = elastic critical buckling stress;
F_y = yield stress;
I = moment of inertia about the strong axis;
k = distance from outer face of flange to web toe of fillet;
K = 0.75 for stiffeners welded to both the upper and lower beam flanges.
l = distance measured from beam end to top of the stiffeners;
L = member length;
L_b = lateral unsupported length;
L_p = limiting laterally unsupported length for the limit state of yielding;
L_r = limiting laterally unsupported length for the limit state of inelastic lateral-torsional buckling;

L_{st} = stiffener length;
M_{cr} = maximum moment capacity of beam with inclined stiffeners;
M_n = design flexural strength;
M_{ocr} = maximum moment capacity of beam without inclined stiffeners;
M_p = plastic bending moment capacity;
M_u = required flexural strength;
N = bearing length;
P_n = nominal compressive strength;
r_{st} = radius of gyration of stiffeners taken about the longitudinal axis of the beam;
R_n = design web yielding or web crippling strength;
R_u = required web yielding or web crippling strength;
R_{us} = factored compression force in the stiffeners;
t = stiffener thickness;
t_f = beam flange thickness;
t_w = beam web thickness;
V_n = nominal shear strength;
V_u = required shear strength;
x = location of concentrated load or reaction from the beam end;
β = load modification factor used in Eq. (4) to account for different load conditions;
Δ_L = computed deflection under unfactored (or nominal) live load;
ϕ_b = resistance factor for flexure;
ϕ_c = resistance factor for compression;
ϕ_{pb} = resistance factor for bearing;
ϕ_v = resistance factor for shear;
ϕ_{wc} = resistance factor for web crippling;
ϕ_{wy} = resistance factor for web yielding;
θ = vertical inclination angle for the stiffeners