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State-of-the-art of advanced inelastic analysis of steel and composite structures

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Abstract. This paper provides a state-of-the-art review on advanced analysis models for investigating the load-displacement and ultimate load behaviour of steel and composite frames subjected to static gravity and lateral loads. Various inelastic analysis models for steel and composite members are reviewed. Composite beams under positive and negative moments are analysed using a moment-curvature relationship which captures the effects of concrete cracking and steel yielding along the members length. Beam-to-column connections are modeled using rotational spring. Building core walls are modeled using thin-walled element. Finally, the nonlinear behaviour of a complete multi-storey building frame consisting of a centre core-wall and the perimeter frames for lateral-load resistance is investigated. The performance of the total building system is evaluated in term of its serviceability and ultimate limit states.

Key words: advanced analysis; core-braced frame; composite beams; nonlinear analysis; performance-based design; plastic hinge analysis; semi-rigid frames; steel and composite structures; tall buildings.

1. Introduction

It is envisaged that any important high-rise buildings will be stripped and retrofitted several times during the life cycle to suit the occupant requirements of the tenants and the change of building services. Concrete and post-tensioned concrete building systems tend to be relatively inflexible as they post problems for cutting out of large openings, strengthening of the structural system to accommodate additional loadings and other modifications which may be required during the life cycle of building structures. Steel structures offer the flexibility required for future modification and further redevelopment.

Modern high-rise buildings constructed in recent years have adopted composite structural systems combining the use of concrete core wall and steel framing to provide cost efficient design (Liew 2001). One of the distinctive features of composite floor construction is to design the steel beams to act compositely with the concrete floor slabs by means of the shear connectors. When building frames are subject to gravity and lateral loads, the distribution of the bending moment in the composite beams varies along the member length. In the negative moment region, the concrete in tension is cracked and the steel reinforcement in the slab may yield. In the positive moment region, a large bending moment may cause partial yielding of the steel section and crushing of the part of the concrete slab. Consequently, the flexural stiffness of beams varies along the member length. To predict accurately the limit-state behaviour of the overall system, the elastoplastic nonlinear behavior of the composite

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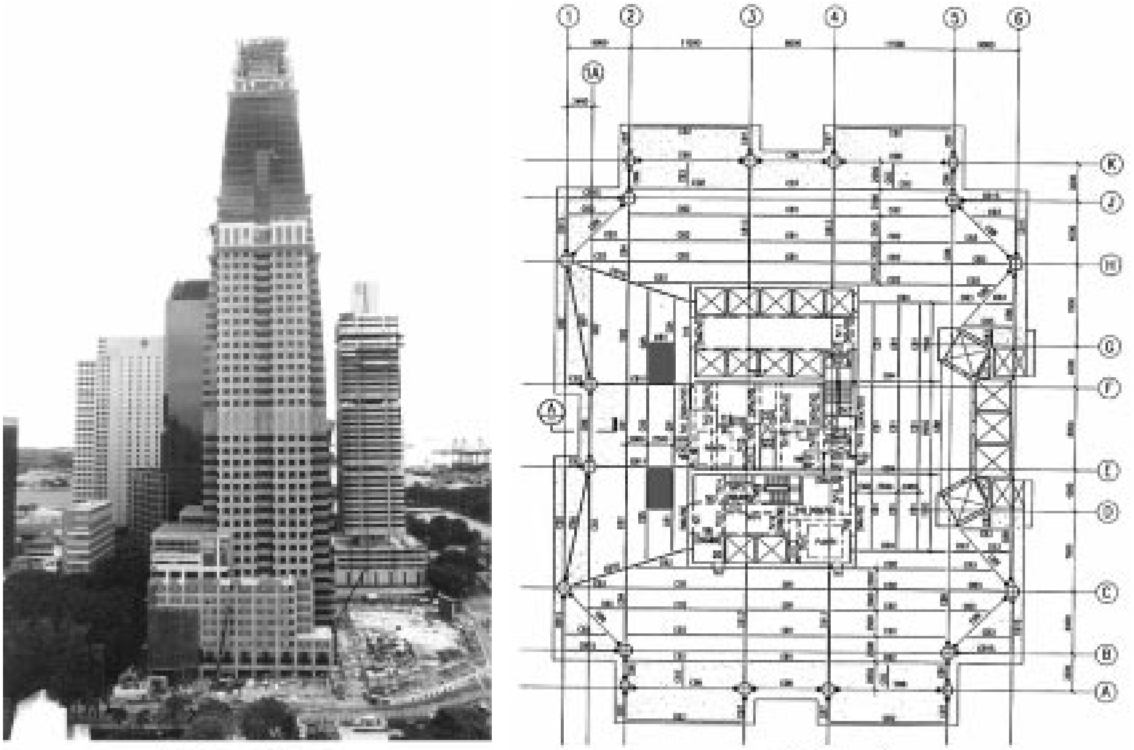

(a) Overall view (b) Plan view

Fig. 1 Steel building with internal core wall (Liew 2001)

members must be considered. The use of continuum finite elements for the analysis of composite beams is computationally too intensive for large-scale structures. The cost and effort of such a method are so great that they often prohibit analysis of a complete framework. To reduce the computational effort, a composite beam model has been proposed to provide the necessary degree of accuracy for studying the inelastic behavior of composite floor beams. The flexural stiffness of a composite beam segment can be evaluated including the effects of partial shear stud interaction between the concrete slab and the floor beams.

For high-rise buildings, concrete core walls are often used to provide torsional and flexural rigidity and strength with or without the participation from the frame system. Figs. 1a & 1b show the elevation and plan views a high-rise steel building constructed recently (2001). Conceptually, a central core with punched openings for lift access is used to provide a cantilever action for lateral load resistance. The floor framing are arranged in such a way that it distributes enough gravity loads to the core walls so that the design of the core is controlled by compressive stresses even under high lateral forces. Structural core walls provide an efficient type of lateral load resisting system up to certain height premium because of their cantilever action. However, when it is used alone, the massiveness of the core increases with height, thereby inhabiting the free planning of interior spaces, especially in the core area. The space occupied by the core walls leads to loss of overall floor area efficiency, as compared to tube system which could otherwise be used. This paper examines the nonlinear behaviour of a combined system of concrete core wall and perimeter frames to form an efficient lateral system for high-rise construction.

To model the nonlinear behaviour of the building framework, structural steel framework is modeled using an inelastic beam-column element which considers the gradual yielding of the steel sections. Beam-to-column connections are modeled as rotational spring. Structural core walls are modeled using thin-walled beam-column element; the use of one element per storey height is sufficiently accurate to capture the nonlinear behaviour of the core wall for global analysis of the total building system. The nonlinear inelastic behavior of a core-braced frame system is investigated by studying the interaction between the structural core wall and the various semi-rigid frames surrounding the core. Assessment of the building performance with respect to its load displacement behaviour, serviceability and ultimate limit states are conducted.

2. Modelling of steel framework

Structural steel frameworks can be accurately modeled by the beam-column plastic hinge element. The beam-column formulation is based on the updated Lagrangian approach as shown in Fig. 2 in which all physical quantities in C_2 configuration should be related to the last calculated configuration known as C_1. Transverse displacements of the beam-column element are calculated by using the stability interpolation functions, which satisfy the equilibrium equation of beam-column subject to end forces (Liew *et al.* 2000). The influence of axial force on transverse displacements can be accurately represented in the form of stability functions. The beam-column formulation can capture accurately the member bowing effect and initial out-of-straightness by modelling each physical member using only one beam-column element. A two-surface plasticity model as shown in Fig. 3 is adopted for representing the partial yielding of steel cross section. The initial yield surface bounds the region of elastic cross-sectional behaviour, while the bounding surface defines the state of full plastification of the cross-section. Suitable value of the surface extension parameter is selected to model the effect of initial residual stress in the cross-section. Once yielding is initiated, the yield surface will translate so that the state of cross-sectional forces remains on the yield surface during subsequent loading. This type of approach is termed as “refined” plastic hinge analysis by Liew *et al.* (1993) and Liew and Uy (2001).

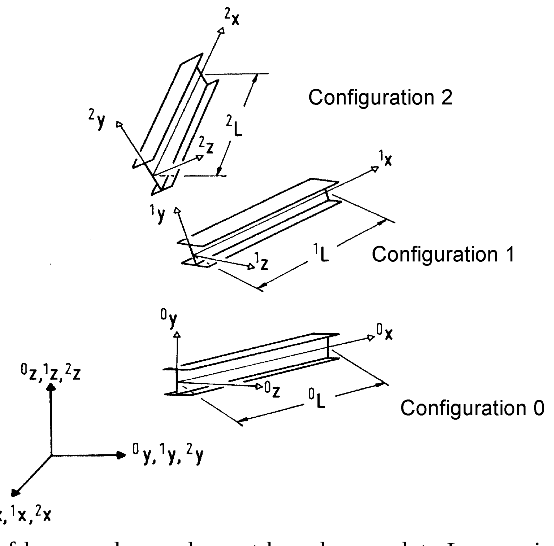

Fig. 2 Motion of beam column element based on update Lagrangian formulation

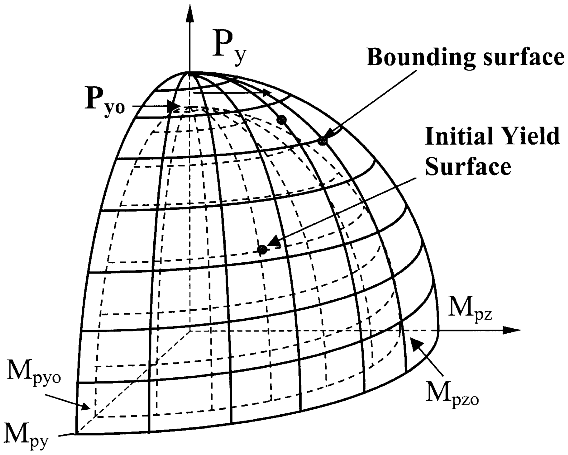

Fig. 3 Two surface plasticity model

3. Modelling of composite beams

A composite beam model proposed by Liew *et al.* (2001) has been used for studying the three-dimension (3-D) behaviour of building frames. The composite beam is subdivided into several segments to capture the varying flexural stiffness along the member length. The instantaneous flexural stiffness of a composite segment is derived using the moment curvature (M-Φ) relationships, thus discretization of cross section is not required. A closed form M-Φ relationships proposed by Li *et al.* (1993) for a composite beam section with full or partial shear connections has been adopted in the present analysis model. For the composite beam section under positive moment as shown in Fig. 4a, the M-Φ relationship is given by

$$\Phi = \frac{M}{EI}, \qquad 0 \le M \le M_y \tag{1a}$$

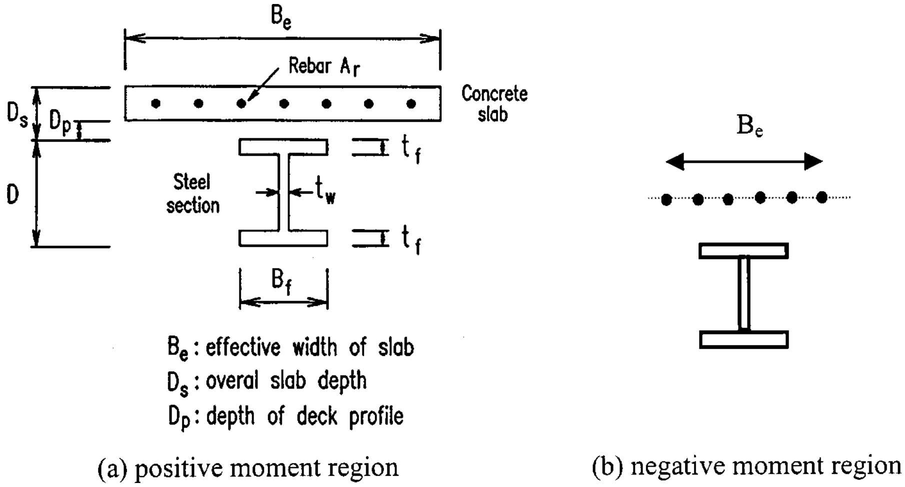

Fig. 4 Composite beam sections

$$\Phi = \frac{M}{EI} + \left(\Phi_u - \frac{M_u}{EI}\right)\left(\frac{M - M_y}{M_u - M_y}\right)^2, \qquad M_y < M \le M_u \tag{1b}$$

where EI is elastic flexural stiffness of the composite section under the positive moment; E, the elastic modulus of steel; M_y, yield moment; M_u, the ultimate moment; and Φ_u is the curvatures at the ultimate moment given as:

$$\Phi_u = 5.7\left(\frac{D}{D_s}\right)^{0.2}\frac{M_y}{EI} \tag{2}$$

in which D and D_s are the depths of the steel beam and the slab, respectively.

Numerical analysis of composite frames with partial shear-stud interaction by one element per member has been carried out by Fang *et al.* (2000). In the present approach, the elastic flexural stiffness of composite beam with partial shear interaction under the positive moment is calculated based on the code specified equation (AISC 1993):

$$EI = EI_s + \sqrt{\frac{N}{N_f}}(EI_f - EI_s) \tag{3}$$

in which EI_f = flexural stiffness of the composite beam with full shear connection; EI_s = flexural stiffness of the steel section; and N/N_f = degree of shear connection.

The instantaneous flexural stiffness of the composite beam section under the positive moment may be obtained by differentiate the moment with respect to the curvature as:

$$EI_t = EI, \qquad 0 \le m \le \alpha_y \tag{4a}$$

$$EI_t = \frac{EI}{1 + 2(\beta\alpha_y - 1)(m - \alpha_y)/(1 - \alpha_y)^2}, \qquad \alpha_y < m \le 1 \tag{4b}$$

in which $m = M/M_u$ is the moment ratio, and $\alpha_y = M_y/M_u$ is the yield moment ratio.

For the negative moment region (see Fig. 4b), it is assumed that, once cracked, the concrete slab does not contribute to the beams resistance. Applying the same rule for the positive moment region, the M-Φ relationship under the negative moment is (Liew *et al.* 2001):

$$\Phi' = \frac{M'}{EI'}, \qquad 0 \le M' \le M_y' \tag{5a}$$

$$\Phi' = \frac{M'}{EI'} + \left(\Phi_u' - \frac{M_u'}{EI'}\right)\left(\frac{M' - M_y'}{M_u' - M_y'}\right)^2, \qquad M_y' < M' < M_u' \tag{5b}$$

in which

$$\Phi_u' = 5.3\frac{M_y'}{EI'} + 2.4 \tag{6}$$

The variables with prime ($'$) have the same meaning as defined in Eq. (1), except that here they refer to the negative moment region. In determining EI', M_y', and M_u', the contribution of the effectively anchored rebars located within the effective width of the concrete slab as shown in Fig. 4b can be included and the concrete slab in tension is ignored. Eq. (5b) can be used to evaluate the flexural stiffness of the composite beam under the negative moment by differentiating M' with respect to Φ'.

Other more rigorous numerical procedure for analysing the nonlinear behaviour of mixed steel-concrete element is provided by El-Tawil and Deierlein (2001). Studies by Liew *et al*. (2001) show that the ultimate limit load of steel frames while considering the composite beam action is about 30% higher than that of the pure steel frame. The lateral stiffness can also be significantly enhanced by considering the composite beam action.

4. Modelling of building core wall

A nonlinear thin walled beam-column model proposed by Chen *et al*. (2001) has been used for inelastic analysis of core-braced frame. A storey high core wall is modeled as one thin-walled beam-column element as shown in Fig. 5a. Fig. 5c shows a generic thin-walled beam-column element which has an additional warping degree-of-freedom over the beam-column element at each end. The local coordinate is chosen with axis x lying on the shear center axis, and y and z axes paralleling to the principal $\bar{y}$ and $\bar{z}$ axes. The shear center of the core is selected as the reference point so that appropriate transformation matrices can be established to relate the kinematics relationships between the point of contact between beams and columns with the core wall. For example in Fig. 5b, the displacements at node "d" of the core wall may be related to those at the shear center "D" in the local coordinate. Detailed derivation of the transformation matrices is given in Chen *et al*. (2000).

Material nonlinearity of the core wall is modeled approximately by using the concentrated plastic hinge approach. The yielding of the core wall section would depend on the combined action of compression, biaxial bending, torsion and warping. However, in the proposed formulation, the torsional shear and warping effects has been ignored in the plasticity formulation. It is necessary to model these factors if the core wall is subject to significant torsional and warping deformation. Such situation is best avoided by selecting an appropriate shape and size of the core walls and an appropriate location for lateral load resistance.

5. Modeling of semi-rigid connections

Beam-to-column connections in building frameworks can be subject to various force combinations such as moment, shear, axial force, torque, bi-moment. However, with the present of floor slab, the deformation of the connection is predominately in the plane of the beam web and the effect associated with shear and axial force on the connections can often be decoupled. Therefore, the current connection analysis focuses primarily on the in-plane behaviour.

The common way to model a connection is to characterize its moment rotation behaviour. In this respect, the four-parameter power model proposed by Hsieh and Deierlein (1991) has been adopted:

$$M = \frac{(K_e - K_p)\,\theta}{\left[1 + \left|(K_e - K_p)\,\theta/M_0\right|^n\right]^{1/n}} + K_p\theta \qquad (8)$$

in which K_e is the initial stiffness of connection, K_p is the strain-hardening stiffness of connection, M_0 is a reference moment, and n is a shape parameter (Fig. 6). By differentiating Eq. (8) with respect to θ, the tangent connection stiffness can be expressed as

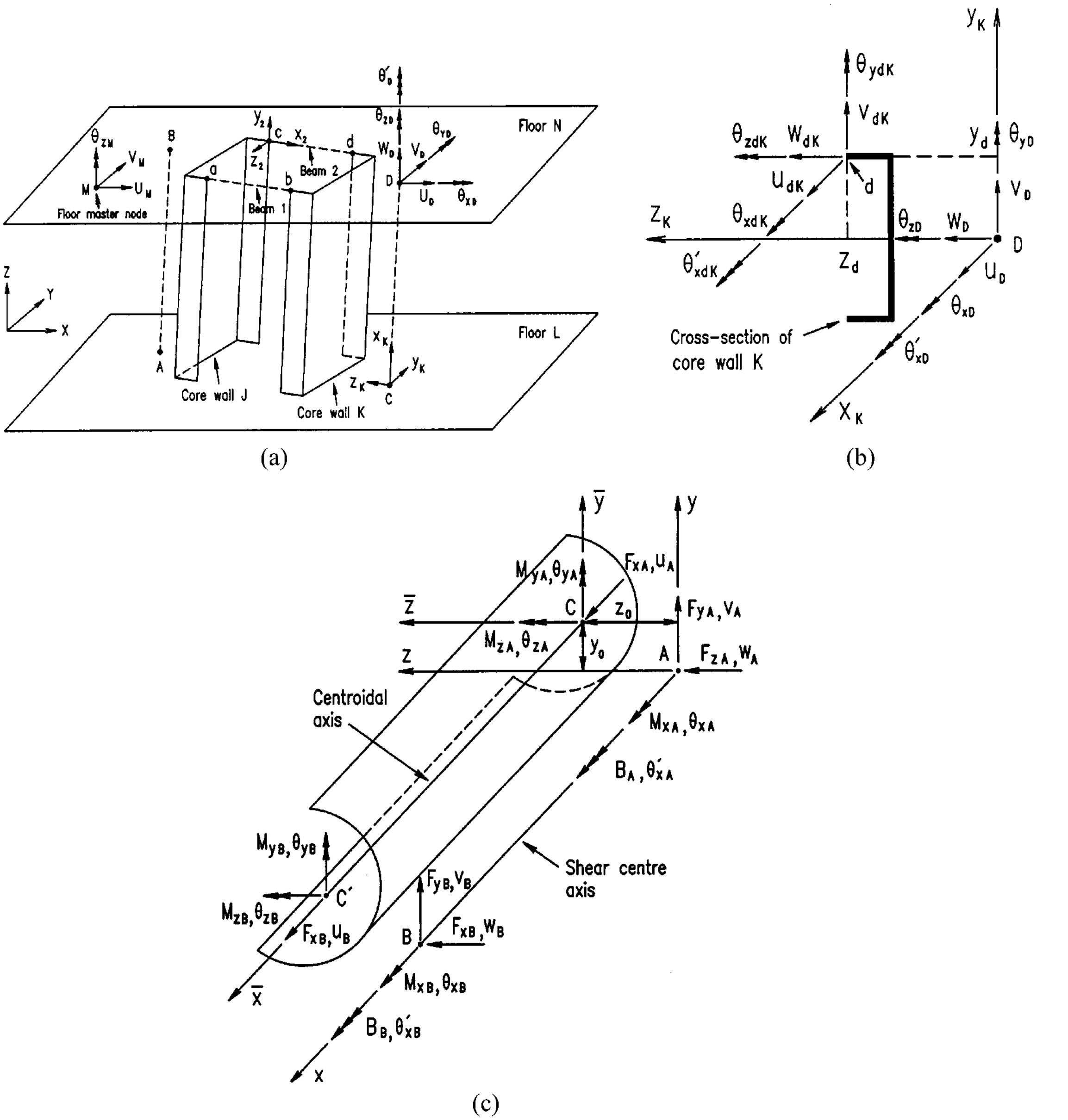

Fig. 5 (a) A building core wall between two floors, (b) Deformations of a thin-walled cross section, (c) A generic model for a thin-walled beam-column member

$$K_t = \frac{dM}{d\theta} = \frac{K_e - K_p}{[1 + |(K_e - K_p)\,\theta/M_0|^n]^{(n+1)/n}} + K_p \tag{9}$$

To allow for unloading of the connection associated with non-proportional loading and inelastic force redistribution, the tangent connection stiffness is equal to the initial stiffness K_e as shown in Fig. 6.

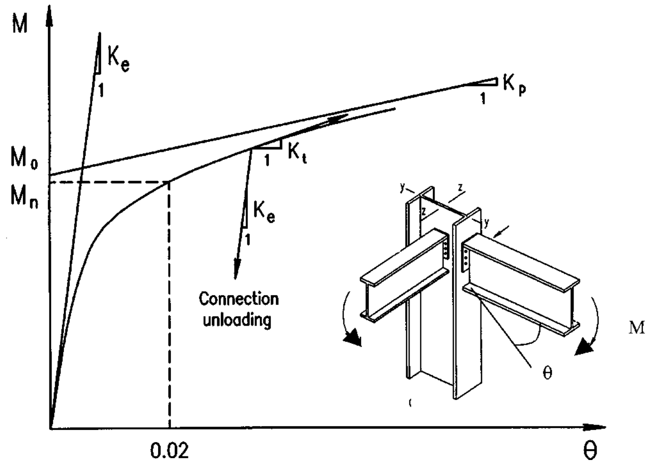

Fig. 6 Four-parameter power mode for connections

The four parameters in this connection model may be determined through curve fitting if the experimental data are available, or by using a mechanical model if the connection details are known. However, in the structural design practice, it is unlikely that specific connection details will be known during the preliminary design, and even during the final design, this information may not be available until after the structural members have been sized. Since connection flexibility will affect the structural response and therefore the required member sizes, there is a need to develop some means to account for connection behaviour during the analysis and design process before the final member sizes are selected.

For three-dimensional frame analysis, the connections are modeled separately by using rotational spring elements. This approach allows the relative torsional and flexural rotations between the member end and the connection, but does not allow for relative translational displacements. It can model the torsional and both major- and minor-axis flexibility. However, in the present study, only the relative rotations about the major axis of beam section are allowed at the semi-rigid connections. This is because the torsional and out-of-plane effects of semi-rigid connections are not significant due to the present of floor slab.

When an advanced analysis is carried out, it is important to check that the connections have enough ductility to allow the force redistribution between joints and structural members. The design issues of connection stiffness, strength, and ductility based on the connection classification approach can be found in Liew *et al.* (1993b). The analysis and design method for frames utilizing semi-rigid composite connections can be found in Leon (1990) and Viest *et al.* (1997).

6. Nonlinear analysis of core-braced multi-storey frame

Figs. 7 and 8 show the plan and isometric views of a 24-storey core-braced frame with “strong” and “weak” core walls, respectively. The total height of the building frame is about 88 m. The wall thickness of the “strong” core is 406 mm, and the “weak” core-wall is 254 mm. Concrete lintel beams with depth of 1.219 m are rigidly connected to the two *C*-shaped core wall sections to enhance its resistance against torsion. All steel members are A36 steel. All floor plates are assumed to be rigid in plane to

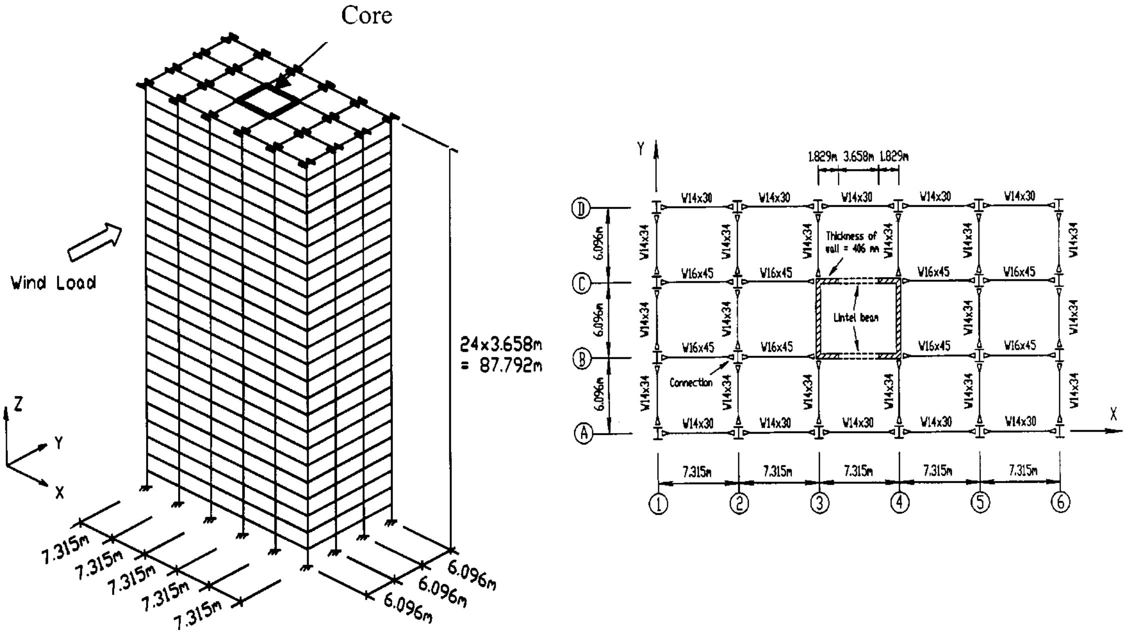

Fig. 7 Multistorey frame with "strong" core

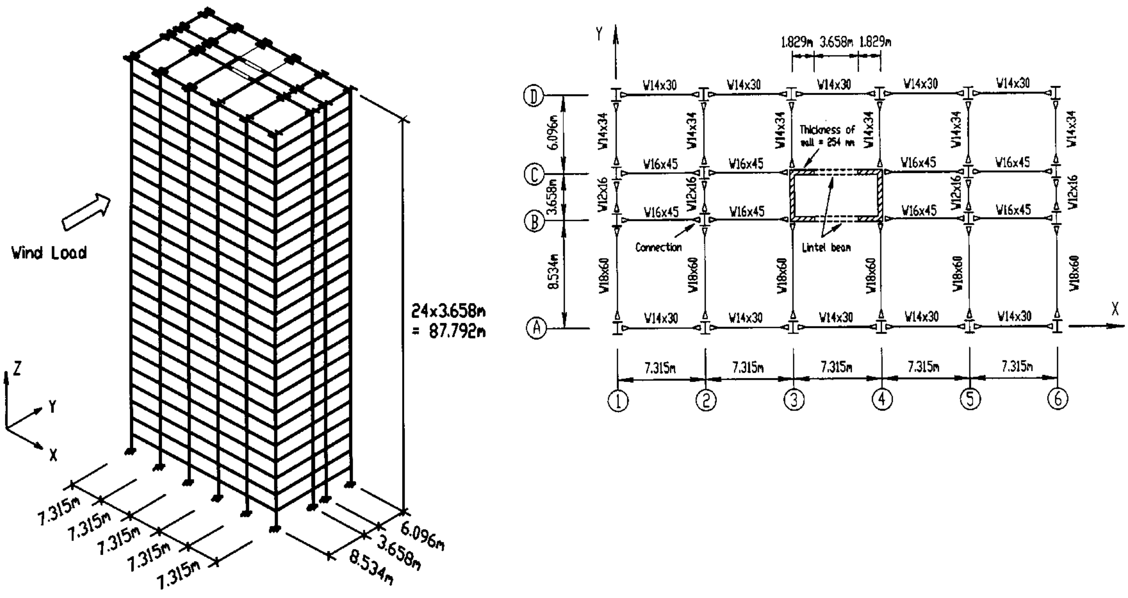

Fig. 8 Multistorey frame with "weak" core

account for the diaphragm action of concrete slabs. Elastic modulus of concrete is 23,400 N/mm^2, and compressive strength of concrete is 23.4 N/mm^2. The structure is analysed for the most critical load combination of gravity loads, 4.8 kN/m^2, and wind loads, 0.96 kN/m^2, acting in the *Y* direction.

Each steel column is modeled as one plastic hinge beam-column element, and each beam as four

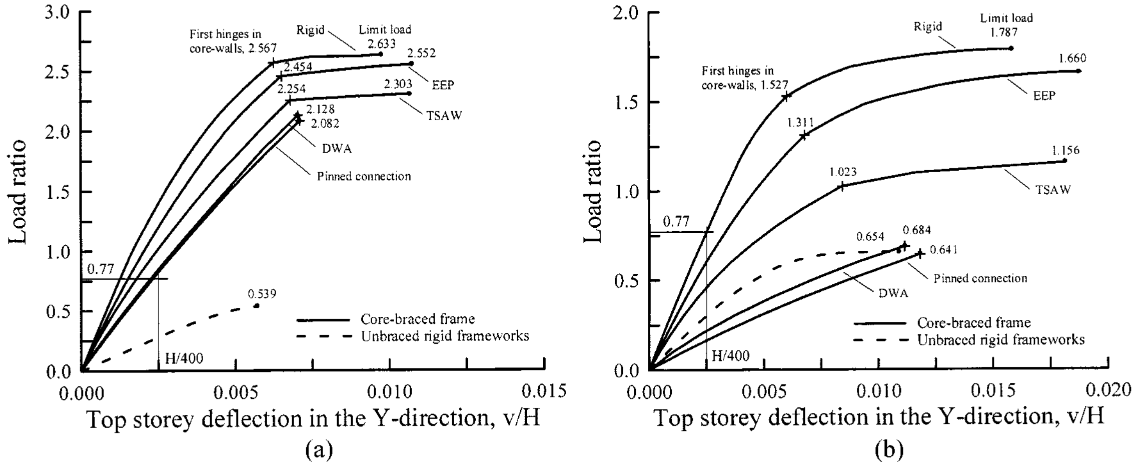

Fig. 9 Nonlinear inelastic analysis of core-braced frames (a) frame with "strong" core walls, (b) frame with "weak" core walls

plastic hinge beam-column elements. The core wall section is modeled using two *C*-shaped sections inter-connected by lintel beams. Each *C*-shape core wall is modeled using one thin-walled element within a storey height. The floor beams are assumed to be connected to the centre dimensions of the concrete core wall. The finite width (thickness) of the core is not modeled in the analysis. This is because the half thickness of the core wall is only 128 mm, which is about 2% the beam length. For this particular problem, the effect of finite wall thickness will not affect the computed results significantly. However, if the wall thickness is significant compared to the beam length, the finite thickness of the core wall may be modeled using a rigid link element to reflect the "real" behaviour of a core-braced frame. The plastic moment resistance of the core wall section has been reduced to approximately account for the tensile cracking and they are 1.57×10^5 kNm for the strong core and 4.8×10^4 kNm for the "weak" core wall.

6.1. Core-braced frames with "strong" core walls

Inelastic analyses are first carried out on the core-braced frame with "strong" core walls. If the steel frameworks are pin-connected to the concrete core, the whole building relies on the core to provide the lateral resistance. The system reaches its limit of resistance at a load ratio of 2.082 when a plastic hinge forms at the bottom of the core walls as shown in Fig. 9a. If the present of core wall is ignored, the unbraced rigid frame can provide lateral load resistance up to a load ratio of 0.539 as shown in Fig. 9a. The limit load of the unbraced rigid framework is about 26% of the limit load of the central core walls. In the case of "strong" core, the lateral load resistant of the core wall is very much higher than that of the rigid frameworks.

To study the connection effects in core-braced frames, inelastic analyses are carried out for core-braced frames with "DWA", "TSAW", "EEP" and rigid connections. The properties of "DWA", "TSAW" and "EEP" connections are given Table 1. The four-parameter of the power model for semi-rigid connections are evaluated based on the statistic values obtained from the connection test database, therefore there is no need to know the connection details in order to perform the analysis (Hsieh and

Table 1 Parameters and values of M_n/M_{pb} for connections subjected to in-plane bending

Connection type	Connection name	M_0' = M_0/M_n	K_e' = K_e/M_n	K_p' = K_p/M_n	n	M_n/M_{pb}	
						At the beam framing about the major axis of column	At the beam framing about the minor axis of column
SWA	Single web angle connection	0.98	110	10.6	1.44	0.023	0.011
DWA	Double web angle connection	1.03	301	5.0	1.06	0.053	0.026
TSAW	Top and seat angle connections with double web angles	0.94	363	6.9	1.11	0.467	0.233
TSA	Top and seat angle connections	1.05	448	7.3	0.80	0.302	0.151
EEP	Extended end plate connection without column stiffeners	0.97	309	5.5	1.20	1.040	0.520
EEPS	Extended end plate connection with column stiffeners	1.04	221	2.2	1.42	1.371	0.680
FEP	Flush end plate connection without column stiffeners	1.06	218	3.6	1.23	0.378	0.189
FEPS	Flush end plate connection with column stiffeners	1.06	314	4.5	1.03	0.429	0.214
HP	Header plate connection	0.91	142	13.2	1.20	0.078	0.0390

Table 2 Comparison of limit loads and elastic lateral stiffness of core-braced frames with "strong" core walls

Connection types	Limit load	Initial lateral stiffness
DWA (Double web angle connection)	102%	104%
TSAW (Top and seat angle connection with double web angles)	111%	131%
EEP (Extended end plate without column stiffener)	123%	154%
Rigid connection	126%	185%

*All % values are compared with the core-braced frame with pinned connections.

Deierlein 1991).

As shown in Table 2, the limit load and elastic lateral stiffness of core-braced frame with "DWA" connections are respectively 1.02 times and 1.04 times of those of the core-braced frame with pinned connections. The use of "DWA" connections does not increase the limit load and lateral stiffness significantly. If "TSAW", "EEP" and "rigid" connections are used, the surrounding frameworks will provide additional resistance to lateral loads, and the limit loads are increased by 11%, 23% and 26%, respectively. Because the core-braced frame mainly relies on the "strong" core walls to provide lateral resistance, the frames collapse soon after the cross section capacity of the core wall is reached. It is also found that the lateral stiffness of the building can be enhanced by 31% and 54% if "TSAW" and "EEP" connections are used (see Table 2).

Serviceability checks are also performed on the core-braced frames. All the floor beams satisfy the deflection requirement of span/360 under the service live load. It is observed from Fig. 8(a) that the core-braced frames with "strong" core walls satisfies the lateral drift requirement, (storey height)/400, for service wind load. Hence, several connections can be used if "strong" core walls are used for lateral load resistance. Since the pinned connection is the cheapest for construction, they are preferred for frames braced by the "strong" core walls.

6.2. Core-braced frames with "weak" core walls

Nonlinear analyses are also carried out on core-braced frames with "weak" core walls. Core-braced frame with pinned connections collapses at load ratio of 0.641 as shown in Fig. 9(b). The limit load of the pure steel frameworks is 0.654, which is 2% higher than that of the core-braced frame with pinned connections. The elastic lateral stiffness of steel frameworks is 40% higher than that of the RC core.

Inelastic analyses are carried out for core-braced frames with "DWA", "TSAW", "EEP" and rigid connections. It is found that the use of "DWA" connections does not increase the limit load and lateral stiffness of core-braced frame significantly. If "TSAW", "EEP" and rigid connections are used, first hinges will form in the core walls at much higher load ratios. The frames have much strength reserve beyond the first plastic hinge, and the limit loads are respectively 1.8 times, 2.59 times and 2.79 times those of the core-braced frame with pinned connections. It is found that the lateral stiffness can be respectively enhanced to 3.06 times, 4.4 times and 5.71 times those of core-braced frame with pinned connections if "TSAW", "EEP" and rigid connections are used (see Table 3). This is because the steel frameworks are much stiffer and stronger that the "weak" core. For core-braced frames with "weak" core walls, the strength and stiffness increase due to the use of semi-rigid connections is much more significant than core-braced frames with "strong" core walls.

The combined use of core wall and rigid frame satisfies the deflection limit requirement for lateral drift. But the core-wall when used together with other semi-rigid frames does not satisfy the lateral drift requirement. It can be concluded from the study that if "weak" core design is adopted, it is necessary to provide proper connection detailing in order to satisfy the requirements for strength and serviceability limit states.

6.3. Second order effects in the core-braced frame

If first-order elastic analyses are carried out on the "strong" core frame where the connections are designed as pin joints, the frame reaches a limit load ratio of 2.213 based on the plastic

Table 3 Comparison of limit loads and elastic lateral stiffness of core-braced frames with "weak" core walls

Connection types	Limit load	Initial lateral stiffness
DWA (Double web angle connection)	107%	120%
TSAW (Top and seat angle connection with double web angles)	180%	306%
EEP (Extended end plate without column stiffener)	259%	440%
Rigid connection	279%	571%

*All % values are compared with the core-braced frame with pinned connections.

capacity check on the core wall section. From the second-order inelastic analysis, first plastic hinge form at the base of core walls at load ratio of 2.082. Since the surrounding steel frameworks relies on the building core to provide lateral stability, the limit load is reduced by 9% and the lateral deflection is increased by 10% due to the lean column effect and the second-order effect due to the cantilever action of the concrete core. For the core-braced frame with "weak" core, the second-order effects also reduce the limit load by 9%, and increase the lateral deflection by 10%. Hence it is important to account for the geometrical nonlinear effects in the analysis of core-braced frames.

7. Conclusions

This paper summarises the various numerical models developed for advanced inelastic analysis of large building framework consisting of composite beams, steel columns, semi-rigid connections and concrete core walls. All the numerical models developed are based on the basic theory of beam-column approach using the finite element formulation. They can be easily implemented into any existing software. Illustrative design examples have been presented to show the benefits of using advanced analysis for assessing the performance of the total framework. The interaction between the concrete core wall and the surrounding semi-rigid frames has been studied and the performance of the building consisting of frames with various connection flexibility is investigated. For buildings with "strong" core, connections in steel frameworks can provide strength and stiffness reserve for core-braced frames to resist the lateral loads. For frames with "weak" cores, it was observed that the proper choice of connections in steel frameworks could provide an optimum balance between the dual functions of buildability and functionality.

Advanced analysis with detailed modelling of member and component behaviour provides a better insight into the global behaviour of the structure and the strength and stability interactions between the structural system and its components. By using advanced analysis in conjunction with the limit states design specifications, engineers can base their design on realistic estimate of the nonlinear behaviour to achieve the performance objectives such as cost effectiveness, buildability, system ductility and uniform factors of safety.

The continuous advance in computer software and hardware capability will bring the methods of advanced analysis of composite frames into wider circulation. Future code development will inevitably include advanced analysis methods and the user guidance is needed. Advanced analysis becomes more useful and applicable if the design is based on the performance criteria rather than the prescriptive methods. With the development of performance-based criteria, the reliance on advanced analysis methods becomes apparent as structural forms, member sizes, loading requirements, and non-linear effects can be quickly and accurately examined.

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