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Practical design guidlines for semi-continuous composite braced frames

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Abstract. This paper presents a simplified approach for the design of semi-continuous composite beams in braced frames, where specific attention is given to the effect of joint rotational stiffness. A simple composite beam model is proposed incorporating the effects of semi-rigid end connections and the non-prismatic properties of a 'cracked' steel-concrete beam. This beam model is extended to a sub-frame in which the restraining effects from the adjoining members are considered. Parametric studies are performed on several sub-frame models and the results are used to show that it is possible to correlate the amount of moment redistribution of semi-continuous beam within the sub-frame using an equivalent stiffness of the connection. Deflection equations are derived for semi-continuous composite beams subjected to various loading and parametric studies on beam vibrations are conducted. The proposed method may be applied using a simple computer or spreadsheet program.

Key words: semi-rigid connections; connection stiffness; moment capacity; sub-frame analysis; plastic hinge; non-prismatic; composite beam; moment redistribution.

1. Introduction

The Eurocodes EC3 (1996) and EC4 (1994), dealing respectively with the design for steelwork and steel-concrete composite construction, both explicitly permit an alternative design approach termed "semi-continuous". This recognises that design is no longer confined to either assumptions of no rotational stiffness and negligible moment transfer or full moment capacity and full continuity. Most of the work done in the past focused mainly on the ultimate strength limit-state behaviour of composite frames. There is a need to show how the wide range of joint stiffness offered by composite connections affect beams' deflection and vibration. Some design guides (BS 5950 1990, Lawson and Gibbons 1995, Cunningham 1990, Nethercot *et al.* 1995) have proposed procedures for continuous composite design but do not consider the effects of composite connections stiffness in calculating beam and column resistance as well as beam deflection.

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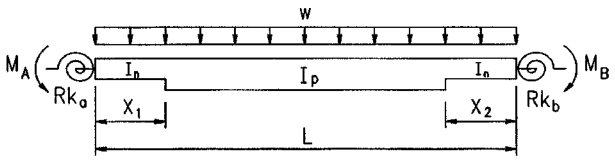

Fig. 1 Non-prismatic composite beam under uniform loading

BS 5950: Part 3 recommends that elastic global analysis of continuous composite beams be carried out assuming that for a length of 15% the span on each side of internal supports, the section properties are those of the 'cracked' section for negative moments. However, no guidelines are given as to whether this may be extended for use in semi-continuous composite beams.

A simple non-prismatic beam model (see Fig. 1) considering the effects of semi-rigid end connections is proposed for elastic design of composite braced frames. From this model, slope-deflection equations are developed to find the beam end moments. The simple beam model is then extended to a simple sub-frame model commonly used to analyse plane frames.

Using the proposed model, the design of composite beams in semi-continuous frames may be approached in the following ways:

1. In most frame analysis where it is often easier to obtain a rigid, 'uncracked' moment distribution, this study shows that it is possible to predict the amount of moment redistribution according to the stiffness of the composite connection after the rigid analysis has been performed.

2. Equations for finding maximum beam span deflection are derived based on the simple beam model. These equations incorporate the stiffness of the composite connections and the ratio of positive to negative second moment of area of the composite beam section $\beta = I_p/I_n$.

3. The use of $x = 0.15L$ as the 'cracked' length to approximate the hogging moment length in semi-continuous frames is verified through parametric studies.

4. Methods for predicting the vibration and deflection responses of composite beams are proposed. These methods are essential to ensure that all serviceability criteria are satisfied.

The proposed method provides a rigorous estimate of beam's bending moment, deflection and natural frequency taking into account the effect of cracking at the support region, flexural rigidity of the adjoining members as well as the stiffness of the connections. The deflection and natural frequency of a composite beam in a semi-rigid braced frame can be computed rapidly by using a sub-frame model. Approximate expressions are proposed for calculating the beam deflection, and thus avoiding a more complex approach of global analysis of the overall framework.

2. Composite beam model

Consider a semi-rigid composite beam model as shown in Fig. 1, the end connections are modelled as rotational springs having spring stiffness Rk_a and Rk_b at opposite ends of the beam. The beam has an 'uncracked' second moment of area, I_p and 'cracked' second moment of area I_n at lengths x_1 and x_2 measured from both ends.

When the beam is subjected to a uniformly distributed load, w, the end moments, M_A and M_B may be calculated using the slope-deflection equations:

$$M_A = -\frac{-Rk_aRk_bM_{fa} - Rk_aK_BM_{fa} - C_{AB}Rk_aK_AM_{fa}}{-Rk_aRk_b - Rk_bK_A - Rk_aK_B - K_AK_B + C_{AB}C_{BA}K_AK_B} \tag{1}$$

$$M_B = -\frac{Rk_b(C_{BA}K_BM_{fa} + Rk_aM_{fb} + K_AM_{fb})}{-Rk_aRk_b - Rk_bK_A - Rk_aK_B - K_AK_B + C_{AB}C_{BA}K_AK_B} \tag{2}$$

where M_{fa} and M_{fb} are the fixed end moments at ends A and B, respectively:

$$\begin{aligned} M_{fa} = {} & (w(L^6 + B^2x_1^6 + 6L^5Bx_2 - 15L^4Bx_2^2 + 20L^3Bx_2^3 + 3L^2(2B-3)Bx_2^4 - 6LB^2x_2^5 + B^2x_2^6 + \\ & 9Bx_1^4(L^2 + 2LBx_2 - Bx_2^2) - 16Bx_1^3(L^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3) + \\ & 9Bx_1^2(L^4 + 4L^3Bx_2 - 6L^2Bx_2^2 + 4LBx_2^3 + B^2x_2^4)))/ \\ & (12(L^4 + B^2x_1^4 + 4L^3Bx_2 - 6L^2Bx_2^2 + 4LBx_2^3 + B^2x_2^4 + 4Bx_1^3(L + Bx_2) - \\ & 6Bx_1^2(L^2 + 2LBx_2 - Bx_2^2) + 4Bx_1(L^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3))) \end{aligned} \tag{3}$$

$$\begin{aligned} M_{fb} = {} & -(Lw((2(L^3 + 3LBx_1^2 - 2Bx_2^2 + 3LBx_2^2 - 2Bx_2^3)^2)/(L^2 + 2LBx_1 - Bx_1^2 + 2Bx_2^2) - \\ & 3(L^4 + 4LBx_1^3 - 3Bx_1^4 + 6L^2Bx_2^2 - 8LBx_2^3 + 3Bx_2^4)))/(24(L^3 + Bx_1^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3) + \\ & (1 - ((L^2 + Bx_1^2 + 2LBx_2 - Bx_2^2)(L^3 + 3LBx_1^2 - 2Bx_1^3 + 3LBx_2^2 - 2Bx_2^3))/ \\ & (2(L^2 + 2LBx_1 - Bx_1^2 + Bx_2^2)(L^3 + Bx_1^3 + 3L^2Bx_2 - (3LBx_2^2 + Bx_2^3)))) \end{aligned} \tag{4}$$

K_A, K_B are the beam stiffness factors at ends A and B, respectively. They are derived as:

$$\begin{aligned} K_A = {} & 1/\left(\frac{1}{4L}(4x_1 + 4x_2 - \frac{4(-L + x_1 + x_2)}{(B+1)} - \right. \\ & (3(L^2 + Bx_1^2 + 2LBx_2 - Bx_2^2)^2)/((B+1)(L^3 + Bx_1^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3)))) \end{aligned} \tag{5}$$

$$\begin{aligned} K_B = {} & 1/((L^4 + B^2x_1^4 + 4L^3Bx_2 - 6L^2Bx_2^2 + 4LBx_2^3 + B^2x_2^4 + 4Bx_1^3(L + Bx_2) - \\ & 6Bx_1^2(L^2 + 2LBx_2 - Bx_2^2) + 4Bx_1(L^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3))/ \\ & (4L(B+1)(L^3 + 3L^2Bx_1 - 3LBx_1^2 + Bx_1^3 + Bx_2^3))) \end{aligned} \tag{6}$$

C_{AB}, C_{BA} are the carry-over factors of member AB at ends A and B, given as

$$C_{AB} = \frac{3L(L^2 + Bx_1^2 - 2LBx_2 - Bx_2^2)}{2(L^3 + Bx_1^3 + 3L^2Bx_2 - 3LBx_2^2 + Bx_2^3)} \tag{7}$$

$$C_{BA} = \frac{L^3 + 3LBx_1^2 - 2Bx_1^3 + 3LBx_2^2 - 2Bx_2^3}{2(L^3 + 3L^2Bx_1 - 3LBx_1^2 + Bx_1^3 + Bx_2^3)} \tag{8}$$

L is the length of the composite beam; x_1 and x_2 are the cracked length of the beam at the negative support regions and $B=\beta-1$ where $\beta = I_p/I_n$.

A computer program was written to calculate the 'exact' moments at the ends of the composite beam model under different kinds of loading as well as different connection flexibility. Figs. 2 to 4 show the variation of end moments with connection fixity factor under different kinds of loading for $\beta = 1.75$. The connection fixity factor is defined as the ratio of the rotation of the beam end due to an applied unit

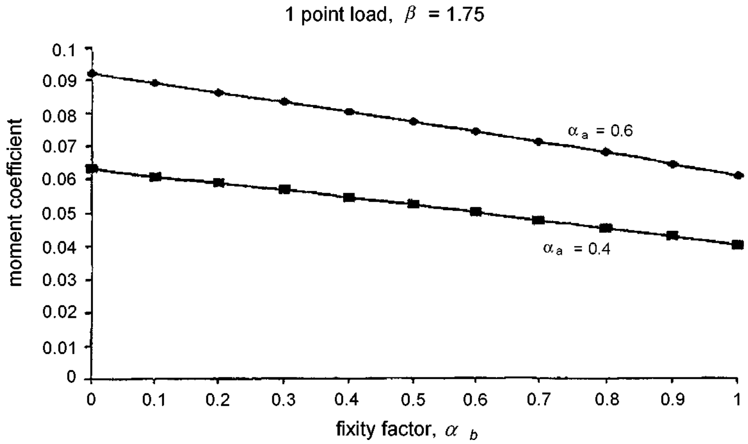

Fig. 2 Variation of moment coefficient with fixity factor-point load applied at mid span, β=1.75

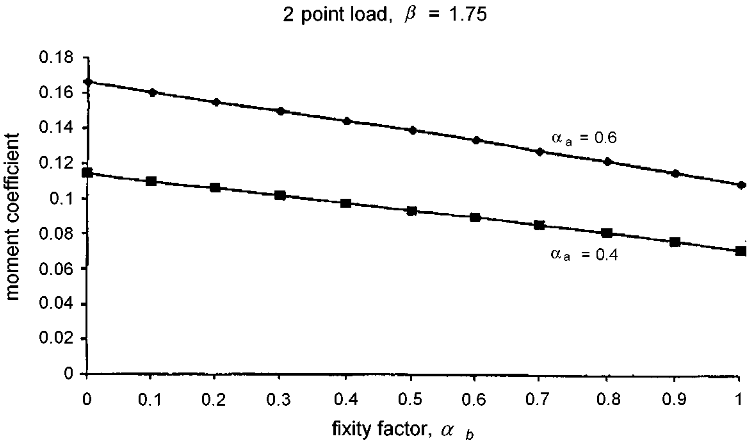

Fig. 3 Variation of moment coefficient with fixity factor-3 point loads at one-third points, β=1.75

moment divided by the rotation of the beam and connection for the same moment. The assumption made in the 'exact' analysis was that the concrete has no tensile resistance such that the lengths of negative moment (x_1 and x_2) are measured from each end to the points of contraflexure. The exact cracked lengths are calculated by locating the point of contraflexure in the beam using an iterative process. Approximate expressions, which give the end moments for different loading conditions and different fixity factors, were then derived from these results using curve-fitting techniques as follow:

For uniformly distributed loading w, the end moment at the left support is

$$M = \mu \times L^2 w \tag{9}$$

For point loads P, the end moment at the left support is

$$M = \mu \times LP \tag{10}$$

μ is the moment coefficient defined as:

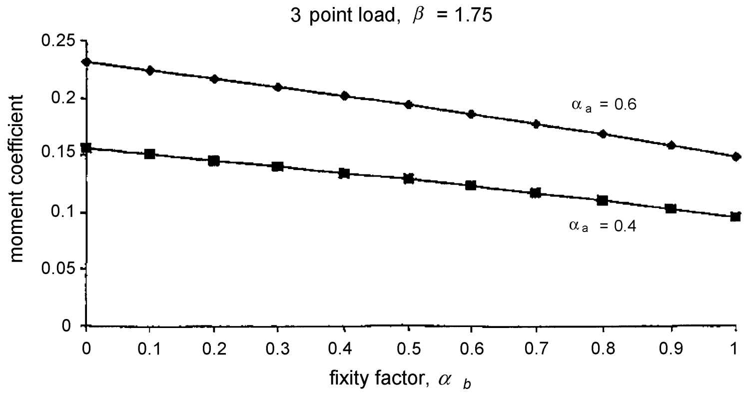

Fig. 4 Variation of moment coefficient with fixity factor-3 point loads at quarter points, β=1.75

Table 1 Moment coeffificent, μ_m

Loading	*A*	*B*	*C*	*D*	*E*
1 point	$-0.0064\beta^2+0.0376\beta-0.0672$	$0.0144\beta^2-0.855\beta+0.1681$	$0.0019\beta^3-0.0116\beta^2+0.0243\beta-0.0159$	$-0.0145\beta^2+0.0917\beta-0.2640$	$0.0020\beta^2-0.0108\beta+0.0087$
2 point	$-0.0123\beta^2+0.0702\beta-0.1228$	$0.0262\beta^2-0.1535\beta+0.3011$	$-0.0238\beta^2+0.1533\beta-0.4623$	$-0.0020\beta^2+0.0101\beta-0.0093$	$0.0033\beta^2-0.0182\beta+0.0149$
3 point	$-0.0163\beta^2+0.091\beta-0.1683$	$0.0363\beta^2-0.2107\beta+0.4218$	$-0.0030\beta^2+0.0147\beta-0.0141$	$-0.0271\beta^2+0.1810\beta-0.6240$	$0.0016\beta^2-0.0085\beta+0.0078$
Uniform	$-0.0050\beta^2+0.0279\beta-0.0479$	$0.0103\beta^2-0.0593\beta+0.1150$	$-0.0010\beta^2+0.0047\beta-0.0042$	$-0.0082\beta^2+0.0534\beta-0.1702$	$0.0013\beta^2-0.0073\beta+0.0059$

$$\mu = A(\alpha_a^2\alpha_b) + B(\alpha_a\alpha_b) + C\alpha_b + D\alpha_a + E \tag{11}$$

These expressions for *A*, *B*, *C*, *D* and *E* are shown in Table 1. The percentage difference from the ‘exact’ values are shown in Table 2. The maximum error is about 7% for the three-point load case. For other load cases the error is within 5%. Therefore, the simple beam model is considered to be adequate as it considers directly the effects of semi-rigid end connections and non-prismatic properties of a cracked composite beam. However, it is necessary to extend this beam model to a sub-frame model in which the restraining effects from the adjoining members can be considered.

3. Sub-frame model

The sub-frame shown in Fig. 5 may be used to evaluate the internal forces and deformations in each individual beam of a non-sway composite frame. It consists of the beam itself (middle span beam), the columns attached to the ends of the beam and the beams on either side if any. The column and beam-ends remote from the beam under investigation are assumed to be fixed unless the assumption of pinned ends is more appropriate.

It has two levels of equal column heights and the beams have second moment of area I_1, I_2 and I_3. They are loaded uniformly with loadings w_1, w_2 and w_3. The columns have stiffness (*EI/L*) given by S_{lct}, S_{lcb}, S_{rct} and S_{rcb}. The connections at the ends of each beam are assumed to have the same rotational stiffness $R_k = R_{ki}$, $i = 1$ to 6. The cracked lengths at the end of each beam are shown as x_1 to x_6.

Table 2 Percentage difference from actual values

Loading condition	% Difference	
	Min	Max
1 point	-1.01	2.88
2 point	-1.17	1.99
3 point	-6.98	4.71
Uniform	-6.86	4.76

Table 3 Properties of multibay sub-frames (see Fig. 5)

Frame type	Span length (m)		Loading (kN/m)	
	L_1 and L_3	L_2	w_1 and w_3	w_2
1	10	10	15	15
2	10	10	5	25
3	5	10	15	15

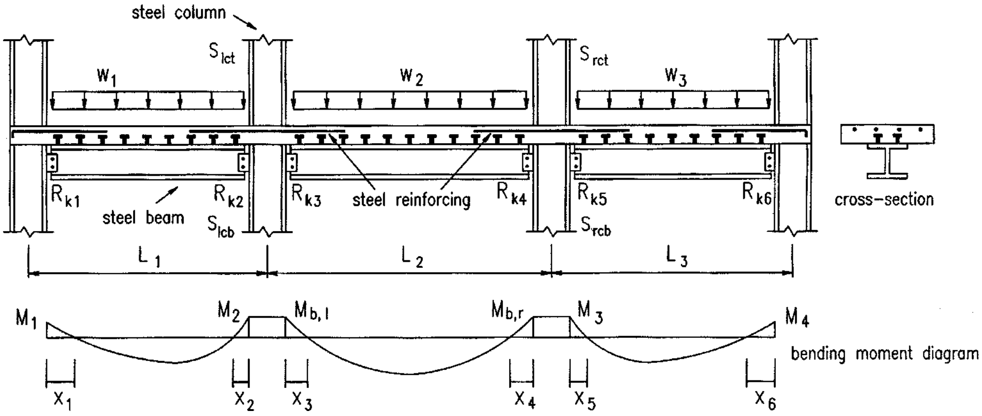

Fig. 5 General frame layout for parametric studies

Parametric studies were performed on frames subject to three types of loadings:

1. equal loading on equal spans, i.e., $w_1 = w_2 = w_3$, $L_1 = L_2 = L_3$
2. unequal loading on equal spans, i.e., $L_1 = L_2 = L_3$, $w_1 = w_3$ and
3. equal loading on unequal spans, i.e., $w_1 = w_2 = w_3$, $L_1 = L_3$.

Table 3 shows three types of frames of various span lengths subject to different loading conditions. Within each type of frame, the following parameters were studied:

1. The effect of varying connection rotational stiffness shown in Table 4.
2. The effect of using two different uncracked to cracked stiffness ratios, $\beta = I_p / I_n = 1.5$ and 2.5. For type 1 frame (equal loading, equal span), the influence of beam's second moment of area, I_p= 1500 cm^4 and 5000 cm^4, was also studied.

Elastic global analyses were performed on the frames and the support moments are compared using the following four analysis assumptions:

1. Rigid uncracked analysis - The connections (Rk_1 to Rk_6) are assumed to be completely rigid and the beam is assumed to be 'uncracked' throughout.
2. Semi-rigid uncracked analysis - The connections are semi-rigid having the rotational stiffness given in Table 4. Beam is assumed to be 'uncracked'.
3. Simplified semi-rigid cracked analysis - The connections are semi-rigid and the composite beam is assumed to have 'cracked' under negative support moments. The 'cracked' length (x_1 to x_6) is assumed to be 15% of the span length.
4. Semi-rigid cracked analysis - The connections are semi-rigid and the actual 'cracked' length under hogging support moments is used in the analysis.

The above analyses were carried out to investigate how the semi-rigid connections and 'cracked'

Table 4 Properties of semi-rigid connections

Type of connection	Rotational stiffness (kNm/rad)
A	1×10^3
B	5×10^3
C	3×10^4

length affected the redistribution of support moments. Parametric tests were conducted for 21 sets of frames with different combinations of loading, dimensions and connection stiffness as shown in Table 5. Table 6 shows only two sets of results for a sub-frame with equal loading on all spans. The complete results of 21 sets of frames are reported in Looi (1999). Tables 7 and 8 summarise the results of joint support moments from various analysis methods obtained from the parametric studies of the 21 sets of semi-continuous composite frames.

It can be observed from Table 7 that using the ‘cracked’ length as 15% of the span length in elastic analysis gives support moments which is at most 2% different from those obtained from analysis using the exact ‘cracked’ length. The results show that this difference does not vary much with the type of loading on the beam and the position of the joint in the frame. Thus, the use of 0.15L as the ‘cracked’ length is accurate enough for the elastic global analysis to calculate the support moments under different kinds of connection rotational stiffness and span loading in a sub-frame. It also suggests that there is no need to use the actual cracked length, as this would require more complex calculations. Therefore, the recommendation by BS5950 (1990) and Eurocode (1994) for the use of $x=0.15\,L$ in

Table 5 Parametric tests on frames

Parametric test no.	Frame type	Connection type	Second area of moment of beams (cm^4)	Column Stiffness EI/L	$\beta=I_p/I_n$
1	1	A	1500	1000	1.5
2	1	A	1500	1000	2.5
3	1	B	1500	1000	1.5
4	1	B	1500	1000	2.5
5	1	C	1500	1000	2.5
6	2	A	1500	1000	1.5
7	2	A	1500	1000	2.5
8	2	B	1500	1000	1.5
9	2	B	1500	1000	2.5
10	2	C	1500	1000	2.5
11	3	A	1500	1000	1.5
12	3	A	1500	1000	2.5
13	3	B	1500	1000	1.5
14	3	B	1500	1000	2.5
15	3	C	1500	1000	2.5
16	1	A	1500	1000	1.5
17	1	A	5000	1000	2.5
18	1	B	5000	1000	1.5
19	1	B	5000	1000	2.5
20	1	C	5000	1000	1.5
21	1	C	5000	1000	2.5

Table 6 Subframe analyses with w_1=w_2=w_3=15 kN/m S_{lcb}=S_{lct}=S_{rcb}=S_{rct}=1000; I_1=I_2=I_3=1500 cm^4 and $\beta = I_p/I_n$=1.5

Parametric Test No. 1: equal span length L_1=L_2=L_3=10 m				
	Elastic Global Analysis Condition			
	(1)Rigid-uncracked	(2) Semi-Rigid-Uncracked	(3) Simplfied Semi-Rigid-Cracked	(4) Semi-Rigid-Cracked
Rk_i (i=1 to 6) (kNm/rad)	Rigid	5000	5000	5000
$x_i/L_{i\ (i=1\ \mathrm{to}\ 6)}$	0	0	0.150	0.167
M_1=M_4 (kNm)	125.0	111.6 (-10.7%)	104.4 (-16.5%)	104.4 (-16.5%)
$M_2 = M_3$ (kNm)	125.0	111.6 (-10.7%)	104.4 (-16.5%)	104.4 (-16.5%)
$M_{b,l}$=M_{br} (kNm)	125.0	111.6 (-10.7%)	104.4 (-16.5%)	104.4 (-16.5%)
Parametric Test No. 11: unequal span length: L_1=L_2=5 m and L_2=10 m				
Rk_i (i=1 to 6) (kNm/rad)	Rigid	5000	5000	5000
x_1/L_1 & x_6/L_3	0	0	0.150	0.137
x_2/L_1 & x_5/L_3	0	0	0.150	0.213
x_3/L_2 & x_4/L_2	0	0	0.150	0.159
M_1=M_4 (kNm)	21.0	20.4 (3.1%)	20.2 (4.1%)	20.2 (4.0%)
M_2=M_3 (kNm)	51.7	38.3 (25.8%)	34.6 (33.0%)	34.5 (33.4%)
$M_{b,l} = M_{br}$ (kNm)	119.9	107.0 (10.7%)	100.5 (16.2%)	100.5 (16.2%)

Table 7 Comparison of joint support moment using x=0.15L and actual cracked length (See Fig. 6)

Frame	% difference of support moments between the use of 0.15 L and actual cracked length			
	End Span		Middle Span	
	joint g	joint h	joint i	joint j
1	0.01-1.5		0.01-1.5	
2	0.04-0.9	0.04-0.8	0.02-0.8	
3	0.06-2.0	0.1 -1.0	0.02-0.8	

continuous beams may be extended to semi-continuous frames.

From Table 8, it is shown that the difference between an 'uncracked' and a 'cracked' analysis ranges from 2-18% (i.e., average of 10%) for the middle span beam when subjected to the three different kinds of loading. This generally agrees well with the difference in percentage of redistribution of support moments using properties of gross 'uncracked' and 'cracked' sections given in BS5950: Part 3.

An attempt was then made to quantify the amount of moment redistribution of a joint. The sub-frame was reduced to a beam with modified connection stiffness $R_{k,mod}$ (see Figs. 7 and 8) taking into account the joint flexibility and the flexural stiffness of adjacent members:

$$R_{k,\,mod} = \frac{R_k \cdot \left(4(S_{ct}+S_{cb}) + \dfrac{12\alpha_n S_b}{4-\alpha_n\alpha_f}\right)}{R_k + \left(4(S_{ct}+S_{cb}) + \dfrac{12\alpha_n S_b}{4-\alpha_n\alpha_f}\right)} \tag{12}$$

where $S_b = EI_b/L_b$ and $S_{ct} = S_{cb} = EI_c/L_c$

I_b and L_b are the second moment of area and length of the adjacent beam,

I_c and L_c are the second moment of area and length of the column

Table 8 Comparison of 'uncracked' and 'cracked' semi-rigid analysis

Frame	% difference of support moments from uncracked to cracked semi-rigid analysis			
	End Span		Middle Span	
	joint *g*	joint *h*	joint *i*	joint *j*
1	2-18		2-18	
2	2-7	2-27	2-17	
3	0.08-4	1-27	2-17	

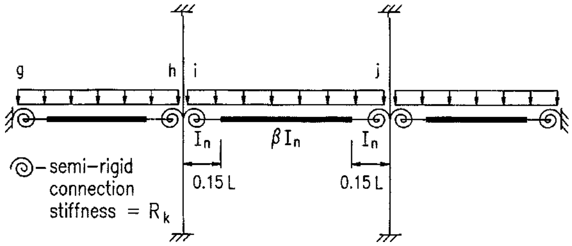

Fig. 6 Typical subframe using x=0.15L for both ends

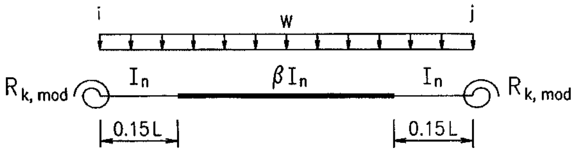

Fig. 7 Beam with modified connection stiffness for middle span beam

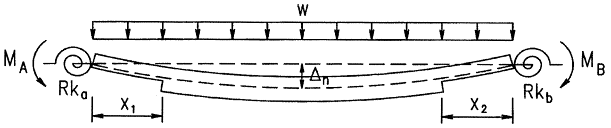

Fig. 8 Deflection model

α_n is the fixity factor for the near connection of the adjacent beam, and
α_f is the fixity factor for the far connection of the adjacent beam.

The concept of fixity factor α was introduced by Cunningham (1990) for semi-rigid prismatic beams and its use may be extended to that for non-prismatic beams. Using the value of $x = 0.15L$ as the cracked length, the fixity factor, α for the joints at the ends of the middle span beam may be expressed as:

$$\alpha = \frac{1}{\left(\dfrac{1.2}{[0.4 + 0.1557(\beta - 1)]\overline{Rk}} + 1\right)} \tag{13}$$

where $\overline{Rk} = \dfrac{R_{k,mod}L}{EI_p}$, and $\beta = I_n/I_p$.

The fixity factor, α ranges from the 0 (pinned connection) to 1 (fixed connection). For the subframe model, the factor is dependent on the stiffness of the adjoining members framing into the joint, and it incorporates the non-prismatic behaviour of the adjoining beams and the variable stiffness of the semi-rigid connection.

The fixity factor at each end of the beam was then used to determine the amount of support moment redistribution for joints *g*, *h* and *i* (see Fig. 6) under different loading conditions and frame geometry. Tables 9 to 11 show the percentage of support moment redistribution from a Rigid/Uncracked to Semi-Rigid/Cracked analysis under the three loading conditions. The results show that it is possible to predict the amount of moment redistribution of a joint in the sub-frame by calculating its fixity factor. It may also be noted that the percentage of moment redistribution for the joints of the middle span beam (joints

Table 9 Equal loading on equal span

Joints *g*, *h* and *i*	Fixity factor, α			
	0.9-0.7	0.69-0.5	0.49-0.3	0.29-0.2
	% moment redistribution			
$\beta=1.5$	12-25	26-43	44-63	64-75
$\beta=2.5$	23-37	38-52	53-69	70-80

Table 10 Unequal loading on equal span

		Fixity factor, α			
		0.9-0.7	0.69-0.5	0.49-0.3	0.29-0.2
		% moment redistribution			
Joint *g*	$\beta=1.5$	2-13	14-31	32-54	55-68
	$\beta=2.5$	9-22	23-39	40-60	61-73
Joint *h*	$\beta=1.5$	26-36	37-55	56-73	74-82
	$\beta=2.5$	32-48	49-63	64-78	79-85
Joint *i*	$\beta=1.5$	9-25	26-42	42-62	63-74
	$\beta=2.5$	20-34	35-50	51-68	69-78

Table 11 Equal loading on unequal span

		Fixity factor, α			
		0.9-0.7	0.69-0.5	0.49-0.3	0.29-0.2
		% of moment redistribution			
Joint *g*	$\beta=1.5$	2- 8	9-25	26-49	50-64
	$\beta=2.5$	2-15	16-33	34-56	57-69
Joint *h*	$\beta=1.5$	20-31	32-52	53-72	73-82
	$\beta=2.5$	37-45	46-61	62-77	78-85
Joint *i*	$\beta=1.5$	10-25	26-43	44-63	64-74
	$\beta=2.5$	21-35	36-51	52-68	69-78

i) is fairly insensitive to the kind of loading. Table 10 (unequal loading on equal span) shows that it is possible to account for pattern loading using this method as well.

As a comparison, Table 4 of BS5950:Part 3 only provides maximum values of moment redistribution and does not take into consideration the stiffness of the semi-rigid composite connection. This method, however, gives a range of moment redistribution based on the fixity factor (stiffness) of the joint.

4. Implications for beam design

The *Simplified Method* described in BS5950:Part 3 cl 5.2.2 does not give clear guidance on how it may be applied to beams with semi-rigid joints. It gives no room to vary the amount of moment redistribution or the effects of pattern loading having already allowed for them in the coefficients for redistribution of moments.

The proposed method has advantages over the Simplified Method in BS5950:Part3 as it does not have restrictions on variation of beam span and loading. Pattern loading may also be included directly in the analysis and a more realistic range of values for moment redistribution is possible. It is also able to determine the contribution of the stiffness of adjoining members to the beam in a sub-frame whereas the Simplified Method may only be used for continuous beams.

The *Elastic Analysis Method* described in BS5950:Part 3 cl 5.2.3 uses the gross uncracked properties of the beam and performs a moment redistribution with values not exceeding the maximum percentages in Table 4 (BS5950). If the cracked section is used, the amount of moment redistribution is reduced. Much is then left to the experience of a design engineer as to how much moment redistribution is really required. This method also does not consider the semi-rigid effect of composite connections.

The advantages of the proposed model are that it incorporates both the semi-rigid behaviour of the connections and the uncracked/cracked properties of the composite beam directly into elastic analysis. The method also allows the amount of moment redistribution to be determined more realistically based on the geometry and flexural rigidity of the adjoining members as well as the stiffness of the connections.

Using the proposed model, the design of a composite beam in a semi-continuous braced frame begins by first isolating the design beam in question together with adjoining members to a similar sub-frame shown in Fig. 5. Elastic Global Analysis is performed on the sub-frame assuming *rigid* connections and *uncracked* beam properties (This may be helpful to engineers who are more familiar with rigid analysis or have computer programs that only perform rigid analysis on frames). The sub-frame is then reduced to a beam with modified connection stiffness. The semi-rigid connections and cracked beam properties are introduced through the fixity factor of the joint. The support moments may then be redistributed by an amount based on the fixity factor. The design procedure will be illustrated using an example given in Section 8.

If the design engineer has a spreadsheet program available, they may compute the support moments directly by including the semi-rigid connections and uncracked/cracked section properties using the simplified composite beam model in the analysis.

5. Beam deflection

In calculating deflections, BS5950:Part 3 cl 6.1.3.5 states that the gross uncracked composite section should be used throughout and the percentage redistribution of moments at the serviceability limit state are assumed to be the same as that at the ultimate limit state. The code again gives no guidance for the effect of semi-rigid joints on deflection. It also does not account for any increase in deflection likely to take place due to cracked section properties at each end of a composite beam bending under negative support moments. Moreover the use of the same percentage of moment redistribution for both ultimate and serviceability limit states raises the question of whether this is realistic and practical, albeit it may be conservative in some cases.

The subframe model proposed in Section 3 is further extended for use in deflection calculations. The advantage of the proposed model is that it eliminates the uncertainties inherent in the BS5950: Part 3 for calculating beam's deflection. Fig. 8 shows a non-prismatic composite beam with semi-rigid connections under uniform loading. Δ_n denotes the maximum deflection of the beam. A computer program has been developed to compute the "exact" maximum deflection of a semi-continuous non-prismatic beam of different β ratios and end fixity factors subjected to different loading conditions. The results are shown in Figs. 9 to 12. Approximate expressions to find maximum deflections have been

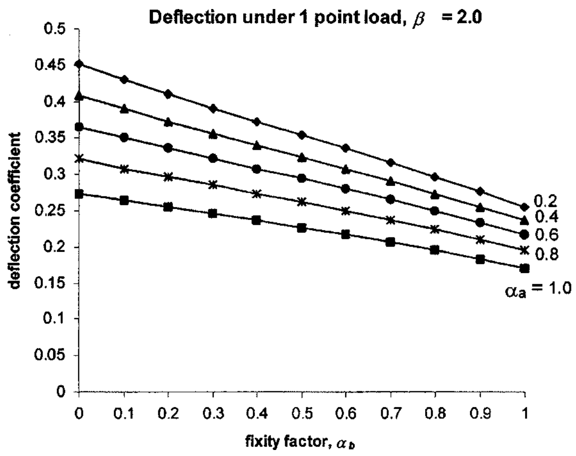

Fig. 9 Maximum deflection under 1 point load

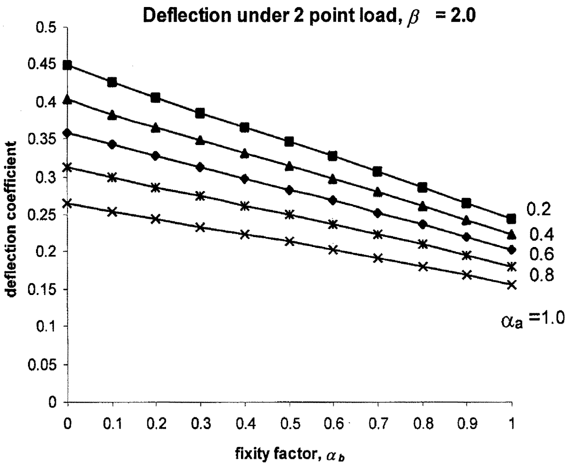

Fig. 10 Maximum deflection under 2 point load

derived from these "exact" results by using a curve fitting technique. The expressions for beam deflection are given in Table 12. The deflection coefficients may be expressed as:

$$\rho = \boldsymbol{F}.(\alpha_a.\alpha_b) + \boldsymbol{G}.(\alpha_a + \alpha_b) + \boldsymbol{H} \tag{14}$$

The constants F, G and H are expressed in terms of the β ratio as shown in Table 12. These constant terms are obtained by curve-fitting the "exact" curves generated by the computer program as shown in Figs. 9 to 12. The maximum error is found to be within ±4% for beams with different β ratio (see Table 13).

For example, consider a composite beam of stiffness EI_p/L under a single point load with a value of $\beta = I_p/I_n = 2.0$, and connection fixity factors: α_a=1.0, α_b=0.1, the deflection constants evaluated from Table 12 are F = 0.1118, G = -0.2137 and H = 0.4901. The deflection coefficient from Eq. (14) may be evaluated as $F.(\alpha_a.\alpha_b) + G.(\alpha_a + \alpha_b) + H$ = 0.26621. The 'exact' value is 0.26428, which gives an error of 0.73%. Therefore, maximum deflection = 0.26621× $PL/48\ EI_p$ = 0.005546 PL/EI_p.

The maximum deflection of a composite beam in a semi-rigid braced frame can be computed rapidly by using a sub-frame model. By converting the sub-frame into an equivalent end-restrained beam as described in Section 3, the approximate expressions given by Eq. (12) and Table 12 may be used for calculating the beam deflection, and thus avoiding a more complex approach of global analysis of the overall framework.

6. Vibrations of semi-continuous composite beams

To study the effects of beam vibration, a structural model developed herein employs a beam finite element approach which incorporates the varying beam stiffness and is augmented with semi-rigid joints by invoking linear springs at the beam ends. The general structural model uses discrete masses, which are referred to as lumped masses, to represent appropriate fractions of the dead load assigned to the nodes. These nodes are placed at regular intervals along the length of the beam as illustrated in Fig. 13. The lumped masses were determined as a proportion of the self-weight of the composite beam.

In the analysis the short-term beam stiffness is utilised considering the beam as either cracked or uncracked. In the positive moment region, the concrete slab is primarily in compression. The concrete

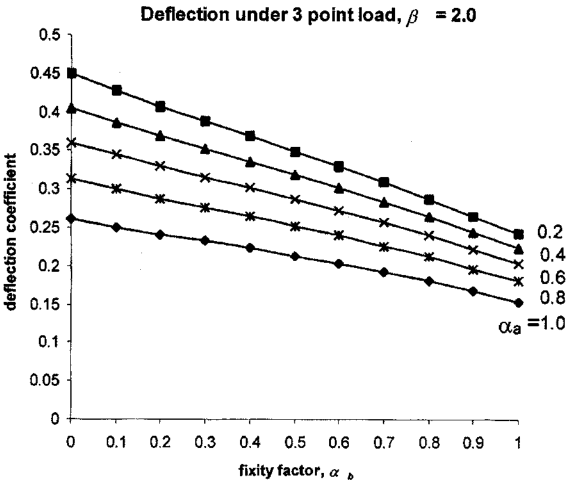

Fig. 11 Maximum deflection under 3 point load

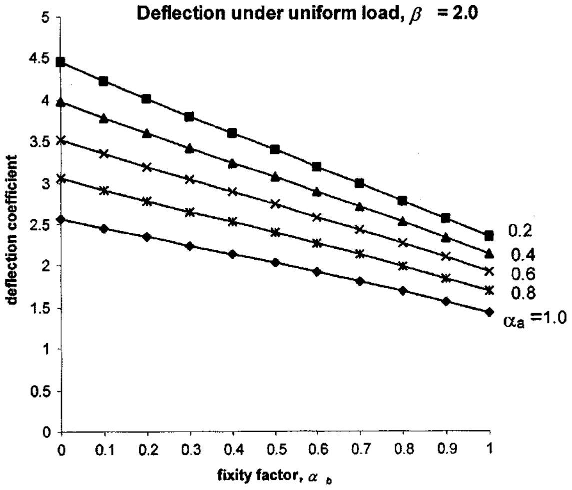

Fig. 12 Maximum deflection under uniform load

Table 12 Maximum deflection of semi-continuous composite beam with different β ratio

Loading condition	Mid-span deflection	*F*	*G*	*H*
1 pt	$\rho \times PL/48\ EI_p$	$0.0897\beta^2$-0.4645β+0.6820	-$0.1214\beta^2$+0.6468β-1.0217	$0.1748\beta^2$-0.9693β+1.7295
2 pt	$\rho \times PL/29.177\ EI_p$	$0.0883\beta^2$-0.4603β+0.6816	-$0.1231\beta^2$+0.6583β-1.0462	$0.1741\beta^2$-0.9659β+1.7234
3 pt	$\rho \times PL/EI_p$	$0.0924\beta^2$-0.4782β+0.7062	-$0.1244\beta^2$+0.6656β-1.0620	$0.1719\beta^2$-0.9563β+1.7192
Uniform	$\rho \times wL^4/384\ EI_p$	$0.1605\beta^2$-1.0219β+2.6054	-$0.1327\beta^2$+0.9164β-3.6217	$0.0248\beta^2$-0.1381β+5.0833

Table 13 Percentage error from "exact" values

Loading condition	% difference	
	Minimum	Maximum
1 pt	-2.72	2.22
2 pt	-2.49	2.63
3 pt	-3.84	3.03
Uniform	-2.20	2.89

is assumed to be uncracked and the composite beam stiffness is equal to I_p. In the negative moment region, the concrete slab is in tension and cracked; the stiffness of which is taken as I_n. The beam stiffness throughout both regions is calculated using the transformed section approach, where the beam is converted to an equivalent area of a steel section. The reinforcing steel in the negative moment region may also be included in this calculation. The regions over which the positive and negative flexural rigidities are applied are based on the recommendations of serviceability provisions prescribed in Eurocode 4 (PrENV 1994[2]) and these are illustrated in Fig. 13.

A parametric study is undertaken here to consider the effects of cracking, reinforcement percentage (in terms of fixity factor) and span length on the vibration response of semi-continuous composite beams. The parametric study has been conducted on beams which have been designed for typical office floor loading according to Standard Australia[9] in a gravity loaded multi-storey braced frame. The beams thus

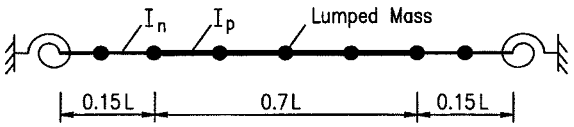

Fig. 13 Structural model for dynamic analysis of semi-continuous composite beam

satisfy both the serviceability and strength limit states of Australian Standards for composite beam design[10] with appropriate modifications to account for semi-rigid and partial strength joints.

6.1. Effect of cracking

A comparison of the natural frequencies for cracked and uncracked beam sections was undertaken. The fundamental natural frequency ratio, ϕ, is computed as shown in Table 14, using the following equation:

$$\phi = \frac{f_{cracked}}{f_{uncracked}} \tag{15}$$

where $f_{cracked}$ = the fundamental natural frequency which includes cracking in the negative support region; and $f_{uncracked}$ = the fundamental natural frequency which ignores cracking in the negative support region.

The results in Table 14 illustrate that the inclusion of cracking in the negative support region does not have any significant effect on the fundamental natural frequency. The effects of cracking influence the fundamental natural frequency by at most 10% for the cases considered. The effects of cracking were considered for all of the remaining analyses considered in this section.

6.2. Effect of fixity factor

Assuming an uncracked length of 0.15 L, the fixity factor for the joints at the beam's ends may be expressed by Eq. (13). The factor depends on the stiffness of adjoining members framing into the joints, the rotation stiffness of joints and the non-prismatic stiffness properties of the members. In beam design, the end fixity can be increased by using a stiffer beam-to-column connection or by increasing the reinforcement steel in the slab. In Fig. 14, the effect of fixity factor was considered for beam lengths varying from 6 m to 14 m. The results indicate that the natural frequencies of a semi-continuous beam with $\alpha = 0.2$ can be increased by up to about 10% compared to a simply supported beam with $\alpha = 0$. Thus by considering the effect of end fixity in the design of composite beams, one can provide a more accurate determination of their deflection and vibration responses, and in many occasions, can lead to more cost effective design.

6.3. Effect of span length

The most important parameter that influences the natural frequencies of semi-continuous composite beams is the beam length. The effect of span length on natural frequency for the fundamental mode is illustrated in Fig. 15. This shows that the fundamental natural frequency decreases as the span length increases. The change in curvature in Fig. 15 between a span length of 10 m and 12 m is due to an increase in the number of bolts required in the connection. Whilst a reduced natural frequency might be expected as the span length increases, the stiffness of the connection increases due to an increased in number of bolts. Since the bolt number increases discretely, a smooth transition of the curves in Fig. 15 does not occur.

Table 14 Effects of cracking on beam natural frequency

Fixity Factor, α	ϕ = Fundamental natural frequency ratio				
	Span length, L				
	6 m	8 m	10 m	12 m	14 m
0.0	1.0	1.0	1.0	1.0	1.0
0.0625	0.99	0.97	0.98	0.99	0.99
0.0875	0.98	0.97	0.98	0.98	0.99
0.115	0.97	0.96	0.97	0.97	0.98
0.150	0.96	0.96	0.96	0.97	0.97
0.195	0.94	0.94	0.95	0.96	0.96
0.225	0.93	0.93	0.94	0.95	0.95

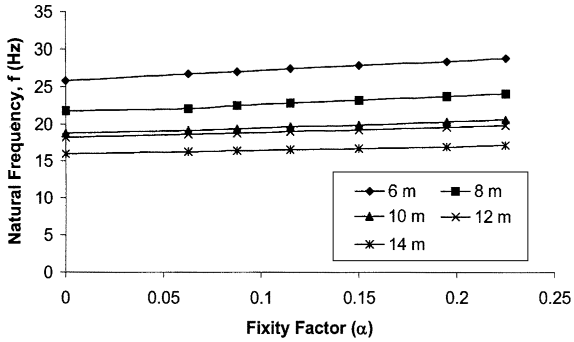

Fig. 14 Effect of reinforcement percentage on beam natural frequency

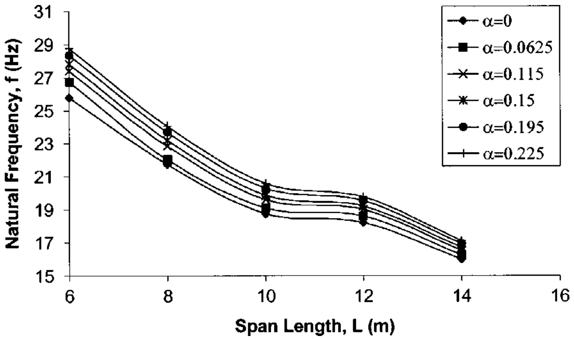

Fig. 15 Effect of span length on beam natural frequency

Typical span lengths of composite beams used in construction range from 8 m to 14 m. It is therefore important to note that for such span lengths, the natural frequencies of the beams are in the range of 16 to 24 Hz, which is above the 15 Hz limit suggested by the Australia Standard (1996). Therefore, vibration sensitivity is not considered to be a critical issue for the design of semi-continuous beams. However, an extrapolation of the curves in Fig. 15, would suggest that the natural frequencies for beams exceeding 14 metres would drop below 15 Hz and hence their resonance effects would need to be checked more closely according to the imposed loading conditions. This may include considering the effects of damping provided by partitions and other parts of the structure.

7. Design example

A typical plan layout of a five-storey, three-bay braced frame is shown in Figs. 16a and 16b. The beam to be designed is 9 m long supported by columns of 4 m height. The beam spacing is 7 m. Grade 50 steel and Grade 30 lightweight concrete are used.

Metal decking of height 50 mm is running perpendicular to the beam. Depth of concrete slab is 130 mm. Shear stud of diameter 19 mm and as-welded height 100 mm is used. The construction dead load is S_{cdl} = 2.20 kN/m^2, live load S_{cll} = 1.0 kN/m^2, and the factored construction load is F_{cl} = 1.4 S_{cdl} + 1.6 S_{cll} = 32.76 kN/m. The dead loads at the full composite stage are 2.91 kN/m^2, and the corresponding live load is 3.9 kN/m^2. The factored uniformly distributed load on girder is W_f = 72.17 kN/m.

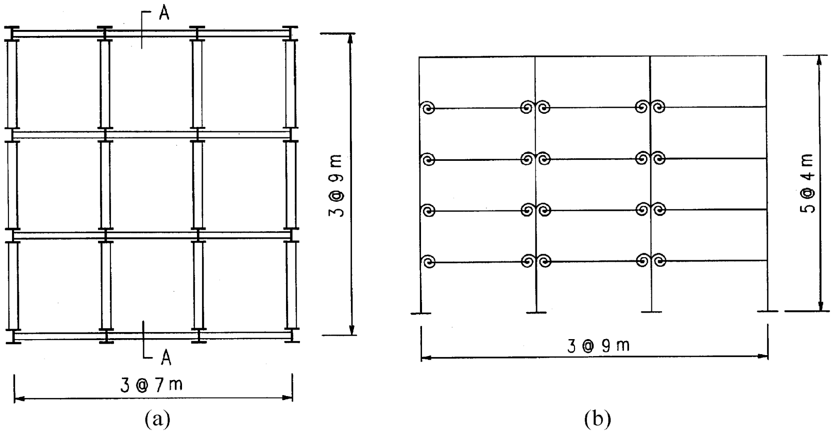

Fig. 16 (a) Typical floor plan, (b) Side elevation of a semi-rigid composite frame (section *A-A*)

In the preliminary sizing, the beam is first designed as simply supported. The free bending moment during construction is M_{cf} = 334.09 kNm. The free bending moment at full design load is $M_{u,comp}$ = 731.41 kNm. A UB 457×152×60 Grade 50 is the lightest section that can resist these moments. For this section, designing for full composite action, the plastic moment capacity is M_{pc} = 830.96 kNm (>731.41, Plastic Neutral Axis (PNA) in concrete slab). The required number of shear connectors for full composite action (assuming two connectors per trough) is 76. This may be redesigned for partial shear connection using 40 shear connectors and the moment capacity in this case will be 742.8 kNm (>731.41 kNm, PNA in steel flange). At construction stage, moment capacity of steel beam is M_s = 454 kNm (>334.09 kNm). The deflection under unfactored imposed loads with partial shear composite action and unpropped construction is calculated as 24.4 mm (<L/360 = 25 mm). For propped construction, the deflection may be reduced to 19.73 mm (< 25 mm).

The design axial loads on the external and internal columns at ground floor are 1624 and 3248 kN respectively. The following sections are chosen for external column - UC 203×203×86 Grade 50 (P_{cy}=2182 kN, M_{bs} = 317 kNm), and internal column - UC 254×254×132 Grade 50 (P_{cy} = 4077 kN, M_{bs} = 645 kNm).

From Lawson and Gibbons (1995), the maximum slab reinforcement percentage is 3% so that the column does not require stiffening. The moment resistance of the composite connection is chosen to be M_{cc} = 500 kNm (for reinforcement = 3%). The initial stiffness of the composite connection is calculated to be about 30,000 kNm/rad based on procedures given in Lawson and Gibbons (1995). The second moment of area of the ‘cracked’ composite beam calculated according to BS 5950:Part 3:Section 3.1:1990 in Appendix B.3.2 for positive moment is I_p = 70997 cm^4 and that for negative moment is I_n = 34631 cm^4. The beam stiffness ratio is therefore $\beta = I_p / I_n$ = 2.05.

A moment distribution or sub-frame analysis is carried out using the appropriate composite beam, column and connection stiffness.

7.1. Based on composite beam model

If the composite beam model is used, the beam end moments may be obtained from Eq. (9). The modified fixity factor is calculated from Eq. (13) as α = 0.41 assuming that x=0.15L for cracked length. The moment coefficient μ = 0.03676. Therefore, the beam end moment, M_A = 0.03676×L^2 w = 214.9 kNm. The beam span design moment is 516.5 KNm.

7.2. Based on subframe model

From Table 9 and fixity factor $\alpha = 0.41$, the joints would have a support moment redistribution of 56%. From a rigid, 'uncracked' analysis of the sub-frame, the joints are found to have a support moment of 504.84 kNm. Using the moment redistribution obtained from Table 8, the moment is reduced to 0.41×504.8 = 207.01 kNm. Therefore, the midspan moment in beam = $wL^2/8$-207.0 = 524.0 kNm which is less than M_{pc} = 742.8 kNm, the moment capacity of composite beam from partial shear action calculated earlier.

For serviceability requirement, service load, $w_{LL} = (LL_{comp}$ + additional dead load)×beam spacing = 32.27 kN/m. Maximum deflection coefficient from Eq. (14) and Table 12 is $\rho = 3.217$. Therefore the maximum deflection is $3.217 \times w_{LL}L^4/384\ EI_p$ = 12.5 mm (< $L/360$ = 25 mm).

8. Conclusions

This paper showed how the connection stiffness could be easily included in analysis and design using the beam-end spring model. The proposed method is shown to be more rigorous and it can be implemented using a spreadsheet or computer program. From the parametric studies, it was shown that the recommended use of x = 0.15L as 'cracked' length by BS 5950:Part 3 for continuous beams may be extended to semi-continuous composite braced frames in elastic analyses to find support moments. Therefore, there is no need to use the actual cracked length, as this would require more tedious calculations. From the results, it was also shown that an equivalence of 10% redistribution of the 'uncracked' support moments might be acceptable. The simplified beam model may also be used to predict the amount of moment redistribution in a sub-frame based on the stiffness of the connections.

Using the proposed model, the effects of composite connection stiffness in moment redistribution, deflection and vibration were taken into account. Comparisons were then made with the existing codes of practice recommended by BS5950 and the proposed model showed that it has advantages over them.

The use of semi-rigid composite connections instead of commonly assumed pinned connections was found to improve the performance of composite beams and could potentially allow increased spans for structural systems. The effects of including cracking were considered and found to be quite important in modelling the dynamic response of these beams. The effects of end restraints were also considered and typical increases of about 10% in the fundamental natural frequency were achieved by adopting semi-rigid connections with modest amounts of reinforcing steel in the slab. The most important parameter to affect the vibration response of semi-continuous composite beams was span length and it was concluded that the use of these beams for spans up to 16 metres would provide acceptable vibration performance in building floors. Further research is required into the serviceability behaviour of semi-continuous composite beam systems. In particular, full-scale tests on the dynamic behaviour of semi-continuous composite beams would allow calibration of the proposed model. In addition to this, the effects of partial interaction and partial shear connection need to be investigated to assess the effects of these aspects on the dynamic response on semi-continuous beams.

References

ENV. (1996), 1993-1-1: 1992 Eurocode 3: Design of Steel Structures Part 1.1: General rules and rules for buildings, as published by the European Committee for Standardisation (CEN). Eurocode 3: Part 1.1, *Revised Annex J: Joints in Building Frames,* The Steel Construction Institute, May.

PrENV. (1994), 1994-1-1 Eurocode 4. Design of Composite Steel and Concrete Structures, Part 1.1-General Rules and Rules for Buildings. Brussels: European Committee for Standardisation.

British Standard Institution (1990), BS 5950: Part 3: Section 3.1, "Structural use of steelwork in building",

British Standard Institute.
Lawson, R.M. and Gibbons, C. (1995), "Moment connections in composite construction: interim guidance for end-plate connections", *The Steel Construction Institute (SCI) Technical Report SCI Publication*, 143.
Cunningham, R. (1990), "Some aspects of semi-rigid connections in structural steel-work", *Struct. Eng.*, **68**(5), 85-92.
Nethercot, D.A. Li, T.Q. and Choo, B.S. (1995), "Required rotations and moment redistribution for composite frames and continuous beams", *J. Const. Steel Res.*, **35**: 121-163.
Looi, R.K.L. (1999), "Practical design procedures for semi-continuous braced frames", M. Eng Thesis, Department of Civil Engineering, National University of Singapore, 216 pp.
Uy, B. and Belcour, P.F.G. (2000), "Serviceability of semi-continuous composite beams in buildings", *Proceedings of the ASCCS-6 Conference*, Los Angeles, USA, 623-630.
Standards Association of Australia (1989), *AS1170.1-1989, SAA Loading Code, Part 1: Dead and live loads and load combinations*.
Standards Association of Australia (1996), *AS2327.1-1996, Australian Standard, Composite Structures, Part: Simply supported beams*.

Notation

M_R	: Moment capacity
M_P	: Plastic moment capacity of connecting beam
M_A	: Moment at left end of beam
M_B	: Moment at right end of beam
R_k	: Connection secant stiffness
I_b	: Beam second moment of area
L_b	: Length of beam
α	: Fixity factor
$\overline{R}_{kN}$	: Stiffness ratio
L_E	: Column effective length
K	: Column effective length factor
I_{eqv}	: Equivalent moment of inertia
S_b	: Stiffness EI/L of beam
S_{lct}, S_{lcb}	: Stiffness EI/L of top, left and bottom, left columns respectively
S_{rct}, S_{rcb}	: Stiffness EI/L of top, right and bottom, right columns respectively
$R_{k,mod}$	: Modified connection stiffness
I_p	: Second moment of area of cracked beam under positive moments
I_n	: Second moment of area of cracked beam under negative moments
β	: Ratio of positive to negative second area of moment for a cracked beam
ρ	: Deflection coefficient of composite beam m_m - Moment coefficient for composite beam
x_i	: Cracked length measured from left end of first span
K_{AB}	: Stiffness factor of left to right end of middle span
K_{BA}	: Stiffness factor of right to left end of middle span
C_{AB}	: Carry over factor of left to right end of middle span
C_{BA}	: Carry over factor of right to left end of middle span
M_{cf}	: Free bending moment during construction
$M_{u,comp}$	: Free bending moment at full design load
M_{cc}	: Connection moment capacity
M_{pc}	: Moment capacity of composite beam
M_s	: Moment capacity of steel beam
SC	