Investigation of rotational characteristics of column 'PINNED' bases of steel portal frames

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Abstract. Most of the portal frames are designed these days by the application of plastic analysis, with the normal assumption being made that the column bases are pinned. However, the couple produced by the compression action of the inner column flange and the tension in the holding down bolts will inevitably generate some moment resistance and rotational stiffness. Full-scale portal frame tests conducted during a previous research program had suggested that this moment can be as much as 20% of the moment of resistance of the column. The size of this moment of resistance is particularly important for the design of the tensile capacity of the holding down bolts and also the bearing resistance of the foundation. The present research program is aiming at defining this moment of resistance in simple design terms so that it could be included in the design of the frame. The investigation also included the study of the semi-rigid behaviour of the portal frames. A series of column bases with various details were tested and were used to calibrate a finite element model which is able to simulate the action of the holding down bolts, the effect of the concrete foundation and the deformation of the base plate.

Key words: column base; holding down bolts; flexibility; portal frame.

1. Introduction

Steel portal frames, similar to most other structures, tend to be designed almost independent of the foundation condition, mainly because most practicing engineers cannot readily appreciate or quantify this interaction. While the design of column bases of most of the multi-storey frame structures is governed by the large axial forces, column bases in portal frames are subjected to a relatively larger lateral shear (Bresler & Lin 1959). Though there have been some studies recently, the interaction between the soil/foundation block/structural frame is probably the least understood aspect of the whole building. An on-going project was designed to investigate the effect of foundation to the overall behaviour of steel portal frames following the series of full-scale tests. The research program was intended to divide into three phases, aiming to quantify the rotational and moment capacities of the column bases in order to check their effects on the overall frame behaviour and to recommend a suitable design for column base details. The first part, which is to be reported in this paper, was to look into the effect of various geometric parameters of the column bases such as the thicknesses of the base plates, column sizes, and size and length of holding down bolts. The study consists of a series of laboratory testing and computational modelling.

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Fig. 1 Bending moment diagrams of the three test portal frames at failure (all values in kNm)

2. Current design method

In the design of a typical portal frame, it is generally assumed that the bases are "pinned" for purposes of analysis, i.e. column does not transfer any moment to the foundation. A typical base consists of a base plate fillet welded to the end of the column member. The base is then attached to the concrete block by means of holding down bolts, anchored within the block. The normal detail for a "pinned" base is to locate two holding down bolts along the neutral axis of the column, one on either side of the web in an attempt to simulate a "pinned" base with the minimum cost. After completion of alignment the plate is grouted into position.

In order to comply with the regulations with respect to fire hazards, column must be capable of remaining standing even when the roof has collapse during a fire. One of the interpretations to this regulation is to cause the pinned base to have a certain amount of fixity by placing four H. D. bolts within the depth of the column section (see Fig. 2). This should further increase the moment



Fig. 2 "Pinned" column bases

capacity of the column base.

- The failure modes associated with this type of column base includes:
- Bending failure of baseplate about and around the bolt head leading to early yielding;
- Holding down bolts extension under direct tension and bending leading to rupture of the H.D. bolts.

The configuration of the column base is mainly related to the dimensioning of the holding-down bolts and the baseplate.

2.1. Design of the baseplates

The thickness of the baseplate beneath the column is designed, according to the existing design codes (BS5950:Part 1:1990), based on the bending of the projection of the plate beyond the column. In the present context, there is little problem in this regard as the projection is normally small. Instead, the local deformation around to the holding-down bolts within the depth of the web, when bending about the inner column flange should be more critical and the thickness of the baseplate should therefore be determined accordingly. The maximum bending moment in the column base should be the result of the interaction between the local stiffness and the overall stiffness of the frame.

2.2. Design of the holding down bolts

Since the bases were designed as "pinned", and there is little overall uplifting in the columns in general conditions, there is no design rule in the exiting design codes to estimate the possible tension force that may develop in the holding-down bolts. The size of these bolts are therefore normally determined largely by the applied shear forces (Morris and Plum 1995).

In a previous research program, three three-dimensional full-scale pitched-roof portal frames of spans 12 m, 12 m and 25 m respectively were tested. In additional to normal vertical load applied from the roof as in all the three frames, one of the columns in the second frame was also subjected to a horizontal load. In all cases, the columns, designed with "pinned bases", were built as mentioned above except that the concrete blocks were rest on floor. Table 1 shows the bending moment measured in the column just before the frames failed. Only the second frame failed with a plastic hinge formed near to the column head (Engel 1990, Liu 1988). The dimensions and the bending moments at failure are shown in Fig. 1. Though designed and constructed as "pinned", the bases had inevitably attracted some moments. Such moments might be about 20% of the column moment capacity (Liu 1988) and have to be resisted by the coupled generated by the bearing compression of the base plates against the concrete blocks and the tension developed in the bolts. The direct tension forces possibly developed in the holding down bolts are not parameters in deciding the configuration of the column base, in particularly, the choice of the plate thickness and the size of bolts (Redwood 1992). Since the rupture of the brittle holding-down bolts may be fatal, the tension

	Column size	Height (m)	Bending Moment near to column head (kNm)	Bending Moment at column base (kNm)
Frame 1	203×133×25UB	3.7	58	13.5
Frame 2	305×165×40UB	2.7	185	35
Frame 3	406×178×54UB	3.65	323	64

Table 1 Column bending moment in full-scale frames

capacity of the bolts should be able to support the maximum moment the column base would experience in the life of the portal frame.

3. Scope of study

The investigation may be divided into the following parts of activities:

- Experimental study of simple "pinned" base with single bolt arrangement A simply test is set up to examine the principle factors relating to the behaviour of the column base.
- Calibration of an existing finite element model against the test data In order to extend the scope of investigation, a numerical model is set up and is to calibrate against the test data based on the simple experimental set-up.
- Parametric study with the finite element models.
- Proposal of a simple design rule.

Based on the deformed shape of the base plate obtained from the finite element modelling, analytical formulae are proposed for estimating the moment capacity of the column bases with single- or two-bolt arrangement. The required minimum bolt capacity can then be determined.

In the present investigation, the base of the concrete block footing is assumed to be rigid. This is normally invalid, as there may be movement in the soil and the water table below and around the concrete footing. The effect of this should normally release some of the bending moment that may be found in the current study. However, such assumption warrants a conservative estimate of the possible moment capacity of the column base.

4. Experimental setup

The objective of the isolated column base tests was primarily to calibrate the finite element model. The main feature in the set-up was to ensure that the numerical model was able to reveal a sufficiently accurate interaction between the column base plate and the concrete block. The column of size 457×191×67UB (S355) in the arrangement was laid horizontally for the convenience of load application. It was loaded as a simple cantilever. The whole column base was rest on a $500 \times$ 1200×1500 concrete block. The whole set-up was geometrically symmetrical about the bottom of the concrete block as shown in Fig. 3. A pair of one-metre long M24 holding-down bolts went through the two concrete blocks and held the two sides in position. The type of HD bolts used in the tests was of higher strength Grade 8.8 with an ultimate strength of 845 N/mm². The material properties were given in Table 2. It was assumed that such arrangement of the H.D. bolts was equivalent to as if they were fixed at 500 mm below the base plates. For simplicity, the base plates were directly rested on concrete surface whether than grout. The H.D. bolts were strain-gauged at 500 mm to monitor the bolt forces. Three strain-gauges were positioned at 120° to each others in the same cross-section in order to measure the direct tension force and bending moments in the bolts. The rotations of the column bases were also measured by a pair of pendulum type rotationgauges placed at 250 mm from the plate. To simulate the relative size between the bending moment and the shear force, a point load was applied at a distance of 2 m from the base plate. The loadings were applied gradually to the cantilevers until the rotational behaviour became fairly non-linear to avoid any possible failure in the holding-down bolts.



Fig. 3 Details of experimental set up

	Yield stress (N/mm ²)	Modulus of Elasticity (N/mm ²)	Ultimate strength (N/mm ²)
Flange	348.20	187710	500.00
Web	401.00	189365	526.37
HD bolts	675.00	195200	845.00
Concrete	f_{cu} =30 N/mm ²	28500	

Table 2 Summary of material properties

5. Finite element modelling

A well-developed finite element package was previously developed (Liu 1988) particularly for the analysis of the full-scale portal frame tests. It was also proved to be very successful for the analysis of various types of connections (Liu and Morris 1991a, 1991b). In the finite element model, the steel columns were descretised with iso-parametric 8-noded shell elements, in which the material non-linearity was modelled by the Von-Mises plasticity theory. The concrete blocks were refined with 8-noded brick elements. Link interface elements were placed in between the two components in order to determine whether or not they were in contact. A compression force should have been transmitted when the components were in contact, otherwise the force became null. The holding down bolts were modelled by line elements following exactly the stress-strain characteristic, which was obtained from a separated tension test. In particular, the contact between the line element and the brick elements, which simulated the concrete footing, was detected by the use of the link



Fig. 4 Finite element mesh for the column base analysis

interface elements in a similar way as between the concrete block and the base plate. The bonding between the HD bolts and the concrete would quickly vanish after once or twice of loading and unloading. Therefore, shear transfer between the bolts and the concrete component was not included. The bolts were free to extend in tension from the beginning of the loading. Also, the pre-loads in the bolts, of about 25 kN, were ignored in the theoretical model as this would lead to little effect on the moment capacity.

The part of the concrete blocks beyond the tension flange of the column was not modelled in order to reduce the problem size. Due to symmetry about the web plate of the column, only half of the assembly was modelled. The base of the concrete block was assumed to be fixed. A finite element mesh which shows the three dimensional view of the concrete block and the column (in horizontal) is shown in Fig. 4. One of the crucial factors that can determine the accuracy the model is the effect of the base plate. Two thicknesses were used in the test, 12 mm and 20 mm representing two possible stiffnesses of the same column base.

6. Result of comparison

The moment-rotation characteristic and the bolt force vs. applied bending moment curves obtained from the F.E. models and the tests were plotted in Fig. 5a and 5b respectively for the two different thicknesses of base plates. The comparison was excellent except that the F.E. models depicted a slightly stiffer behaviour. This is mainly due to the in-accurate assessment of the compression stiffness of the concrete block. However, it is interesting to note that, though there is a difference in the stiffnesses between the two cases, the bolt forces do not differ a lot. The column base with a thicker base plate rotated about the toe of the base plate, i.e., about 220 mm from the centroid. The bolt force would therefore be,

$$P_{bolt} = \frac{1}{20.22m} \frac{M_{app}}{20.22m} = 2.27M_{app} \tag{1}$$



Fig. 5 Comparison between experimental and F.E. modelling

where M_{app} is the applied bending moment at the column base. This agrees very well with the results obtained for the 20 mm case from the tests and F.E. modelling.

7. Moment-rotation characteristics

Further computational analysis were carried out to examine the effect of various geometric parameters. Column bases with single bolt (i.e., Fig. 2a) and two bolts (i.e., Fig. 2b) on one side of the web were both considered. The computational models were analysed up to a complete collapse, mainly due to bolt failure. Figs. 6a and 6b shows the effect due to a variation of the base plate and the diameter of bolts respectively on the rotational behaviour of the column bases. In general, a full range moment-rotation curve consists of four parts could be found.

STAGE 1: The first part is the elastic regions where every component remains elastic. However, the behaviour is not linear, as a result of the moving centroid of the reaction from the concrete block due to the prying action. The diameter of the holding down bolts affects directly the initial elastic rotational stiffness as shown in Fig. 6b.

STAGE 2: With high strength HD bolt and the slight prying action in the cases of thinner plate, the elastic portion is followed by a static growth in moment of resistance due to an extensive flexural yielding in the base plate.

STAGE 3: The base plate loses all the flexural stiffness and the prying action disappear. Thereafter, the tensile membrance action of the base plate is able to support a further increase in the bolt force until the behaviour comes to a final stage.

STAGE 4: The bolts eventually fail.

There are a few points noted from this study:

- The moment carrying capacity at the STAGE 2 and the in-plane membrane stiffness in STAGE 3 are basically not affected by the bolt size but largely by the thickness of the base plate;
- STAGE 3 in the two-bolt cases are much shorter and less well-defined;
- Bolt size does not affect the rotational characteristics but may affect slightly the commencement



Fig. 6 Eextended study on effect of base plate thickness and bolt size

of each stage;

- Bolt fracture was not included in the modelling, but such failure may be fatal;
- For a safe design of the column base, it should be sure that the bolt should fail after the base plate has yielded completely.

A more flexible, thinner base plate, of thickness 12 mm, was able to bend and part of it would be in contact with and compress against the concrete block. The prying action increased the forces in the bolts. However, after the bolts had extended further and the plate had yielded extensively, the prying action faded away and hence the bolt forces returned to a similar level as that of the thicker plates. Since the centroids of the couple formed by the tension force in the bolts and the compression force by the reaction should normally be very close to the compression toe of the column, the bolt forces were fairly independent of the base plate thickness and bolt size. Fig. 7 show that the prying action increased the bolt forces by about 20% for thinner base plate.

Maximum prying actions, i.e., the difference between the actual bolt forces and the "estimated" bolt forces (Eq. 1), were found mainly at the beginning of STAGE 2 and larger in the thinner plate cases. In the two-bolt cases, prying actions were more severe at the bolts nearer to the line of rotation. Fig. 7 also shows the effect of plate thickness on the actual bolt forces in the two-bolt cases. Similar to the single-bolt bases, the maximum prying actions were found at the commencement of STAGE 2. However, in both arrangements, the prying actions were able to vanish almost completely at the end of STAGE 2 after the base plates had yielded extensively.

8. Yield line model of the baseplate with single holding-down bolt

Since fracture failure of holding down bolts may lead to fatal collapse of the portal frame, any design should therefore avoid such failure from happening. Capacity of the column bases should be limited by the yielding capacity of the base plate (i.e., end of STAGE 2) and the H.D. bolt should have capacity large enough to accommodate yielding of the plate.

Yield line model has been used widely to estimate the capacities of connections, especially those



Fig. 8 Yield line model for single-bolt column base

involve end plates (Horne and Morris 1985). From the stress contour plotting of the base plate with single bolt at the end of STAGE 2 shown in Fig. 8a, it is obvious that there is a double curvature yielding in the vicinity of the bolt head. A simplified model is that there is a pair of concentric yield-lines, one hogging and one sagging, see Fig. 8b. The hogging yield line consists of a straight portion starting from the edge of the base plate and joining with a semi-circle centered at the outside rim of the bolt head and radius up to the line of the welding. The sagging yield line has similar geometry but the radius of the semi-circle is equal to the diameter of the bolt head. The part of the plate inside the inner yield line is flat and resting on the concrete block, but without any

prying action.

The total length of the yielding lines (sagging and hogging) on one side of the web is

$$l_{yield} = 4\left(m - \alpha - \frac{1}{2}\phi\right) + \pi\left(\alpha + \frac{3}{2}\phi\right)$$
(2)

where

 α = the distance between the edge of the welding to the centre of the bolt head

m = distance between the edge of the welding and the edge of the base plate

 ϕ = diameter of the bolt

The rotation about the yield line is $\frac{c}{\alpha - \frac{1}{2}\phi} \sqrt{\frac{15}{t_p}}\vartheta$ where c = distance between the centre of the bolts

head and the line of rotation (assumed at the toe of the plate), ϑ is the rotation of the column base and t_p is the thickness of the baseplate. The factor $\sqrt{\frac{15}{t_p}}$ is included to account for the effect of plate thickness.

The total virtual work done by the yield lines is

$$m_p l_{yield} \vartheta \frac{c}{\alpha - \frac{1}{2} \phi} \sqrt{\frac{15}{t_p}}$$
 (3)

where $m_p = p_y \frac{t_p^2}{4}$ is the moment capacity per metre width of the plate.

By assuming that the bolts remain elastic, there is negligible virtual work done by the extension of the bolt. The only external work done is due to the rotation by the column base bending moment M. The work done is $M\vartheta$. Therefore the column base moment capacity (M_{plate}) at which the base plate fails is

$$\boldsymbol{M}_{plate} = 2 \frac{m_p c}{\alpha - \frac{1}{2} \phi} \sqrt{\frac{15}{t_p}} \left(\pi \left(\alpha + \frac{3}{2} \phi \right) + 4 \left(m - \alpha - \frac{1}{2} \phi \right) \right)$$
(4)

and the corresponding maximum tension force in the bolts which are further away from the point of rotation is (estimated bolt force)

$$P_{bolt} = \frac{M_{plate}}{2c} \tag{5}$$

The predicted values based on this Yield Line model are also shown in Figs. 8 and 9 for comparison. It should be reminded that the "pinned" based portal frame has been designed assuming that the column base has a null bending moment. The estimation of moment capacity of the base plate should therefore be useful only for the design of the baseplate and the holding-down bolts.

9. Yield line model of the baseplate with two holding-down bolts

Fig. 9a also shows the principal direct stress contour of the base plate with two holding down bolts. The vicinity of the bolts near to the outer flange of the column bend more, as expected. From

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Fig. 9 Yield line model for two-bolt column base

the stress distribution, a yield line mechanism can be obtained and is shown in Figure 9b. Based on a similar derivation as given in the previous section, the moment capacity of the column base could be modelled by the following formula:

$$\boldsymbol{M}_{plate} = 2 \frac{m_p(c_1 + c_2)}{\alpha - \frac{1}{2}\phi} \sqrt{\frac{15}{t_p}} \left(\pi \left(\alpha + \frac{3}{2}\phi \right) + 4 \left(m - \alpha - \frac{1}{2}\phi \right) \right)$$
(6)

where c_1 and c_2 ($c_2 > c_1$) are the distances of the rows of bolts from the line of rotation. Assuming there is no more prying action in the plate at this point, the corresponding tension force in the bolts, which are further away from the line of rotation, is

$$P_{bolt,2} = \frac{M_{plate}}{2} \frac{c_2}{c_1^2 + c_2^2} \tag{7}$$

Table 5 compares these "theoretical" yield moments of the column bases with various plate thicknesses. Note that the original moment capacity of the column concerned was about 520 kNm. The moment capacity of the column bases can range from about 10%, in the case of single-bolt and thin plate, to around 50%, in the case with two bolts and thicker plate, of the column capacity. If this is taken into account of the design of the portal frame, the load carrying capacity of the frame could be increased tremendously.

10. Further parametric study on rotational stiffnesses

Fig. 10 summaries the effect on the elastic rotational stiffness due to the HD bolt diameter and the



Fig. 10 Factors affecting rotational stiffness of column bases

Table 3 Rotational stiffness for 20 mm base plate (Test result = 7800 kNm/rad)

	Rotation (×10 ⁻³ radian)	Single Bolt force (kN)	equivalent eccentricity (mm)	Stiffness (kNm/rad)	Flexibility (µrad/kNm)
Case 1	0.805	98.1	152.9	35104.0	28.49
Case 2	1.040	68.2	220.0	28837.6	34.68
Case 3	2.110	74.7	201.0	14221.4	70.32
Case 4	2.683	68.2	220.0	11181.6	89.43
Case 5	3.381	69.0	217.3	8873.3	112.70

Table 4 Rotational stiffness for 12 mm base plate (Test result = 5300 kNm/rad)

	Rotation (×10 ⁻³ radian)	Single Bolt force (kN)	Equivalent eccentricity (mm)	Stiffness (kNm/rad)	Flexibility (µrad/kNm)
Case 1	0.960	59.8	125.4	15624.1	64.00
Case 2	1.660	34.1	220.0	9032.6	110.71
Case 3	1.820	38.2	196.5	8248.8	121.23
Case 4	2.458	33.9	221.4	6102.6	163.86
Case 5	2.711	36.8	204.0	5533.2	180.73

base plate thickness. The length of the holding-down bolts considered in the analyses was 500 mm. In the range of consideration, the variations seem to be fairly linear except the effect of baseplate thickness on the two-bolt bases. For the single-bolt cases with bolt diameter 24 mm, the stiffness could vary from 4500 kNm/rad in very thin plate such as 10 mm to a theoretical limit of 17500 kNm/rad when the baseplate becomes very thick. However, when the bolt diameter is large, the stiffness would depend mainly on the thickness of the baseplate. The rotational stiffness of the thin column baseplate of thickness of 12 mm would approach a theoretical limit of about 12000 kNm/rad when the bolt becomes infinity rigid.

The rotational stiffnesses obtained above were resultant of the flexibilities of various components

Base plate thickness	Single-bolt base	Two-bolt base
10 mm	41 kNm (7.9% M _{pc})	82 kNm (15.7%)
12 mm	54 kNm (10.4%)	108 kNm (20.8%)
15 mm	75 kNm (14.4%)	150 kNm (28.8%)
20 mm	116 kNm (22.3%)	232 kNm (44.6%)

Table 5 Comparison of moment capacity (M_{pc} , column capacity = 520 kNm)

constituting the column bases. This includes the concrete block, the base plate, the holding down bolt and part of the column itself. The objective of the following study was to establish and quantify the effect of each of these components in the column base on its rotational flexibility of the column base and to understand the relative importance. Five different cases were considered:

Case 1: "Rigid" concrete block with "rigid" HD bolts - to look at the bending of the base plate subjected to the existence of a rigid base;

Case 2: No concrete block, but the base rotates about the toe with "rigid" HD bolts;

Case 3: "Rigid" concrete block with normal HD bolts;

Case 4: No concrete block, but the base rotates about the toe with normal HD rigid bolts;

Case 5: Normal concrete block and HD bolts

"Rigid" concrete block or "rigid" HD bolts were modelled by imposing very large values for the respective Youngs Modulus.

The analysis was confined to the single-bolt cases only. A few points could be drawn from this study:

- (a) The F.E. analyses were carried out until the base plates started to yield. For the cases where the thicknesses of base plates were 20 mm, the average rotational stiffness up to 30 kNm were noted; whereas in the cases of 12 mm, the stiffnesses up to 15 kNm were recorded. The results are tabulated in Tables 3 and 4.
- (b) Comparing cases 4 and 5, the flexibility due to compression of the concrete block is 23.27×10⁻⁶ rad/kNm for the 20 mm case and 16.87×10⁻⁶ rad/kNm when the thickness if 12 mm. This is because the bearing area for the thinner plate is much larger than the thicker one.
- (c) The difference between cases 2 and 3 shows that the flexibility due to extension of bolts are consistently about 55×10⁻⁶ rad/kNm from both thickness cases. This agrees very well with the elastic flexibility obtained by simple calculation (58×10⁻⁶ rad/kNm) assuming all other components rigid.
- (d) The flexibility due to base plate deformation is expected to dominate the difference between the two cases. The flexibility due to the bending of the 12 mm plate together with the endportion of the column is found to be 110.71×10^{-6} rad/kNm and that of the 20 mm plate is only 34.68×10^{-6} rad/kNm.

The rotational stiffness of the column itself was about 220000 kNm/rad over a length of 250 mm. The "pinned" bases with single holding-down bolt on each side of the column web have rotational stiffnesses of about 2.5% to 4% of that of the column and those with two bolts have stiffnesses of about 5% to 15% depending on the thickness of the base plate. With such a small stiffness locally, it should not affect extensively the overall rigidity of a portal frame. However, this may affect the moment distribution and determine the maximum moment the column base may experience under the design ultimate load on the whole portal frame.

11. Conclusions

This paper has presented a simple test to study the rotational behaviour of the two "pinned" column bases subjected to large bending moment as normally found in single storey portal frames. The results from a finite element modelling compared very well with the experimental evidences in terms of the moment-rotation characteristics and the direct tension forces in the holding-down bolts. It was then followed by two sets of limiting parametric studies. A few design parameters have been considered and their effects on the rotational stiffness been examined. They include the thickness of the base plate, bolt size and the stiffness of the concrete block. The moment capacity of the column base as a whole was determined by a yield line model assuming that the base plate has yielded extensively. This follows the requirement that the strength of the bolts is sufficiently high. This is important because any bolt failure may be fatal to the whole frame stability.

The contribution of flexibility by the concrete block is about 20% on the 20 mm thick base plate whereas that on the 12 mm thick base plate is only 8%. The stress distribution within the toe region is very complex. It requires further investigation. A factor, which is not considered here, is the reduced effective column section. The tensile stress transmitted from the bolts would diffuse gradually into the column. The effective stiffness of the column at the plate-column junction could probably be halved the normal value and thereby increases the rotational flexibility.

It is also not included in this part of the research the behaviour of the underlying soil. Any moment reversal could produce differential settlement causing possible rotation of the foundation block. This might lead to a reduction of the column base moment. This is the reason that, while it is essential to quantify the possible stiffness and the moment capacity of the column base for their detail design, it is not recommendable to take this into account when designing the portal frame.

References

Bresler, B. and Lin, Y.Y. (1959), Design of Steel Structures, John Wiley & sons, N.Y.

- Engel, P. (1990), "The testing and analysis of pitched roof portal frames", Ph D Thesis, University of Salford.
- Horne, M.R. and Morris, L.J. (1985), *Plastic Design of Low-Rise Frames*, Constrado Monographs, Collins Professional and Technical Books, U.K.
- Liu, T.C.H. (1988), "Theoretical modelling of steel portal frame behaviour", Ph D Thesis, University of Manchester.
- Liu, T.C.H. and Morris, L.J. (1991a), "The development of a shear hinge and the effect on connection flexibility", *Proc. Of the Asian-Pacific Conf on Computational Mechanics*, Hong Kong.
- Liu, T.C.H. & Morris, L.J. (1991b), "The effect of connection flexibility on portal frame behaviour", Int Workshop on Connections in Steel Structures, AISC/Eurcom, Pittsburgh.
- Morris, L.J. and Plum, D.R. (1995), *Structural Steelwork Design to BS5950*, Longman Scientific & Technical, 2nd Edition, U.K.
- Redwood, R. (1992), "Anchor bolt forces for moment-resisting column bases", Canadian J. Civ. Eng., 19, 178-181

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