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Enhanced vertical diffusion coefficient at upper layer of suspended sediment concentration profile

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Abstract. Assume fluid eddy viscosity in the vertical direction is parabolic. Sediment particles diffuse with the given fluid eddy viscosity. However, when the vertical diffusion coefficient profile is computed from the suspended sediment concentration profile, the coefficient shows lager values than the fluid mixing coefficient values. This trend was explained by using two sizes of sediment particles. When fine sediment particles like wash load are added in water column the sediment mixing coefficient looks much larger than the fluid mixing coefficient.

Keywords: vertical diffusion coefficient; suspended sediment concentration profile; sediment mixing coefficient; fluid mixing coefficient

1. Introduction

The relationship between fluid mixing coefficient and suspended sediment mixing coefficient has been poorly understood up to the present due to difficulty in measurement of relevant physical elements. The fluid mixing coefficient is normally obtained from turbulent eddy viscosity.

The turbulent eddy viscosity can be computed by using a turbulence model, chosen from several levels of models. Considering one-dimensional flows near a flat wall, mixing length hypothesis is known to describe the fluid turbulence structure near the wall quite well despite of the simplicity of the approach.

$$\tau = \varepsilon_t \frac{du}{dz} \tag{1}$$

where τ is the shear stress, ε_t is the turbulent eddy viscosity, u is the turbulence-mean fluid velocity parallel to the wall, z is the direction normal to the wall. When the shear stress is obtained from the turbulent velocity component (u', v', w'), then the eddy viscosity can be obtained from Eq. (1).

The vertical diffusion coefficient of any solvent should be identical to the fluid mixing coefficient, if only the solvent does not influence the flow field. However, the suspended sediment particles

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may not respond to the fluid flow due to the finite size and a different density of the sediment particles. The suspended sediment concentration is often relatively larger near the bed, especially when the particle sizes are large. The vertical diffusion coefficient of the suspended sediment concentration deviates from that of the fluid near the sea bed, where the sediment concentration is high, and the fluid flow cannot move the sediment particles easily.

When the sediment particles are extremely small, e.g. like silt or clay, the sediment is often called mud, or cohesive sediment. The mud particles are so small that the particles tend to flocculate influenced by electric or chemical actions of the mud material. The flocculation influences not only settling velocity but also the fluid flow behavior, and the vertical diffusion coefficient of the suspended sediment concentration.

These physical effects include the difference of a fluid particle and a sediment particle, and the influence of the sediment particles on the fluid turbulence structure, often expressed by introducing a simple coefficient as follows

$$\varepsilon_s = \beta \varepsilon_t \tag{2}$$

where ε_s is the vertical diffusion coefficient of the suspended sediment, and β is the scale parameter containing the physical effect. β is known to be slightly smaller than 1.0.

Regardless of the above physical modification of the vertical diffusion coefficient due to the sediment existence in water column, the vertical diffusion coefficient can be distorted by sediment size distribution, if the vertical sediment diffusion coefficient is computed from the profile of the suspended sediment concentration. This erroneous distortion of the vertical diffusion coefficient of the suspended sediment concentration is concerned in this paper.

2. Apparent vertical diffusion coefficient of suspended sediment

Assume a uniform steady current with suspended sediment. The sediment particles are assumed uniform-sized at this stage. Then, the setting velocity of the sediment particles is constant.

Steady uniform currents with free surfaces have shown parabolic distribution of the vertical eddy viscosity of the fluid (van Rijn 1993). This trend has been explained by involving a mixing length hypothesis for the turbulence in water column.

$$\frac{\tau}{\rho} = \varepsilon_t \frac{du}{dz} \tag{3}$$

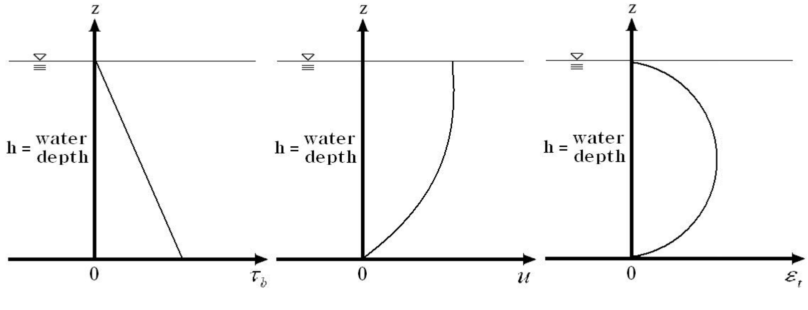

Fig. 1 Vertical distribution of shear stress, velocity, and fluid diffusion coefficient in water column

$$\varepsilon_t = l^2\left|\frac{du}{dz}\right| \tag{4}$$

$$l = \kappa z\sqrt{1-\frac{z}{h}} \tag{5}$$

$$\tau = \tau_b\left(1-\frac{z}{h}\right) \tag{6}$$

where ρ is the fluid density, l is the mixing length, κ is the von Karman constant, τ_b is the shear stress at the bed, and h is the water depth. Then

$$\varepsilon_t = \kappa u_{*,b} z\left(1-\frac{z}{h}\right) \tag{7}$$

where u_* is the friction velocity which is the root of the shear stress over the fluid density, and $u_{*,b}$ is the friction velocity at the bed. The derivation of Eq. (7) involves an ad-hoc approximation on the mixing length in Eq. (7) and the definition of the eddy viscosity in Eq. (4). In order to derive the theoretical equation of suspended sediment concentration, the distribution of diffusion coefficient has been assumed as a parabolic by Rouse (1937), as linear in the near-bed layer, and a constant in the outer layer by Coleman (1969, 1970), as triangular by Bhattacharya (1971), and recently as combined parabolic and constant by van Rijn (1984, 1987). However, the parabolic distribution of the turbulent eddy viscosity in the vertical direction has also been backed up by many direct measurements of the turbulent flow velocities, and is widely accepted (Lane *et al*. 1949, Bagnold 1966, Bogardi 1974, Ashida and Fujita 1986, Dyer and Soulby 1988, McLean 1991, van Rijn 1993, Camenen and Larson 2007, Dey and Papanicolaou 2008).

$$\frac{\partial c}{\partial t} + (w-w_f)\frac{\partial c}{\partial z} - \frac{\partial}{\partial z}\left(\varepsilon_s\frac{\partial c}{\partial z}\right) = 0 \tag{8}$$

is reduced to a steady form as Eqs. (9) and (10)

$$w_f\frac{\partial c}{\partial z} + \frac{\partial}{\partial z}\left(\varepsilon_s\frac{\partial c}{\partial z}\right) = 0 \tag{9}$$

$$w_f\, c + \varepsilon_s\frac{\partial c}{\partial z} = 0 \tag{10}$$

where c is the suspended sediment concentration, w is the fluid velocity in the z direction, w_f is the settling velocity of the sediment particles, z is the coordinate in the upward vertical direction, and ε_s is the vertical sediment diffusion coefficient in the z direction. When suspended sediment concentration profile is known, the vertical diffusion coefficient distribution of the suspended sediment concentration can be obtained from the suspended sediment concentration profile as follow

$$\varepsilon_s = -\frac{w_f c}{\partial c/\partial z} \tag{11}$$

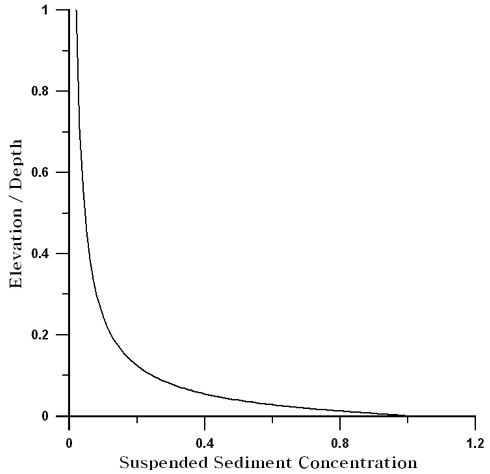

Fig. 2 Typical concentration profile of coarse sediment

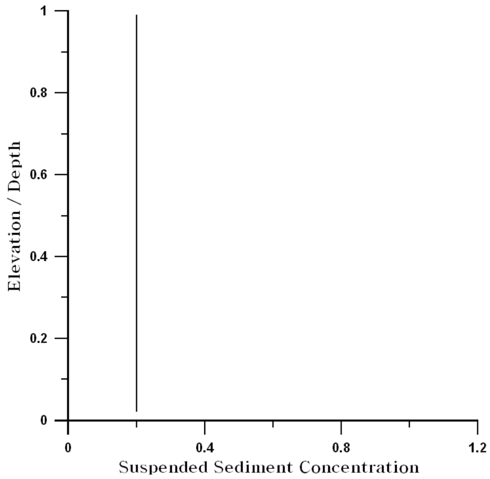

Fig. 3 Typical concentration profile of fine sediment

An analytical solution exists for a parabolic distribution of the vertical diffusion coefficient of the suspended sediment concentration, that is

$$c = c_a\left(\frac{a}{h-a}\frac{h-z}{z}\right)^{w_f/(\kappa u_*)} \tag{12}$$

where c_a is the suspended sediment concentration at the reference level, and a is a reference level from the sea bed.

Now assume that the suspended sediment is composed of two sizes. One size may represent the median diameter, and the other may represent the wash load. The suspended sediment concentration is also divided into two, c_1 and c_2 for the two sizes, respectively. Then, Eq. (12) represents c_1 (see Fig. 2) and c_2 represents the additional concentration for the wash load. The settling velocity of the very fine sediment particle approaches zero, and the concentration is constant through the water column (see Fig. 3) as follows

$$c_2 = constant \tag{13}$$

The total suspended sediment concentration is the sum of the two concentrations (see Fig. 4)

$$c_1 = c_1 + c_2 \tag{14}$$

If the vertical diffusion coefficient in computed from the total suspended sediment concentration ($c_1 + c_2$) with the setting velocity of the median diameter

$$\varepsilon_{s,a} = -\frac{w_f c}{dc/dz} = \varepsilon_s\left\{1 + \frac{c_2}{c_a}\left(\frac{a}{h-a}\frac{h-z}{z}\right)^{-w_f/(\kappa u_*)}\right\} \tag{15}$$

The apparent diffusion coefficient ($\varepsilon_{s,a}$) becomes larger than the true sediment diffusion coefficient

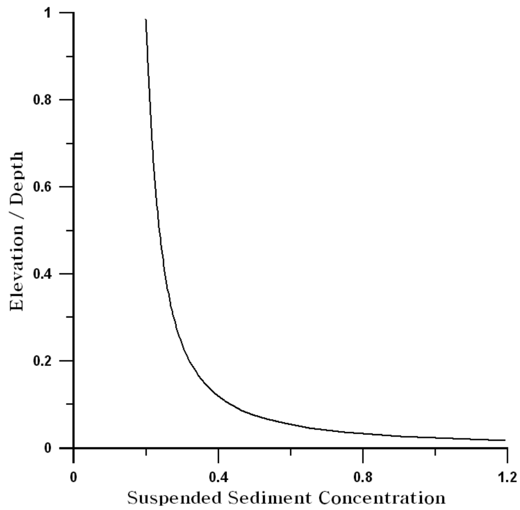

Fig. 4 Typical concentration profile of two sediment sizes

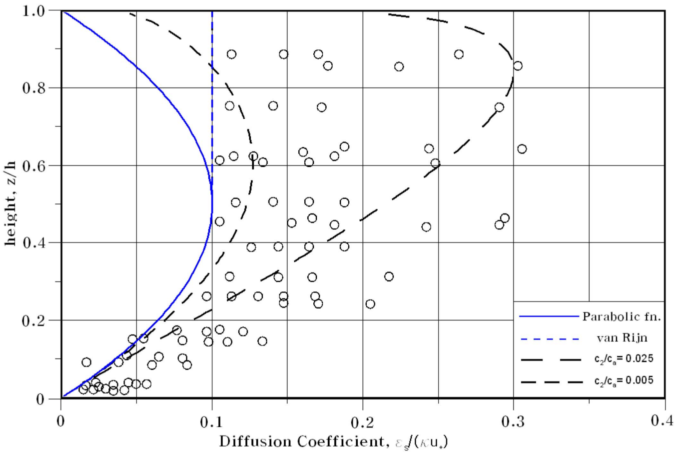

Fig. 5 Composition of computed and measures sediment diffusion coefficient (w_f / (κu_*) = 0.833)

(ε_s) as the height goes up. The deviation amount depends on the ratio between the wash load concentration and the reference level concentration, and the ratio between the setting velocity and the friction velocity.

The enhancement of the diffusion coefficient of the suspended sediment concentration was compared with measurements. The diffusion coefficient profiles were computed from the suspended sediment concentration values with the settling velocity of the coarse sediment.

The diffusion coefficients of the suspended sediment concentration are surprisingly larger than those of the fluid itself, see Fig. 5. A combination of a parabolic function at the lower half depth and a constant value above the middle height of the water depth to compensate the wide gap between the fluid and sediment mixing coefficient at high level, which in also shown in Fig. 5. However, Fig. 5 demonstrates that the excess of the diffusion coefficient of the suspended sediment concentration relative to the fluid eddy viscosity was not properly adjusted by van Rijn`s (1993) suggestion. The curves of Eq. (15) in Fig. 5 seems explaining the gap of the diffusion coefficient of

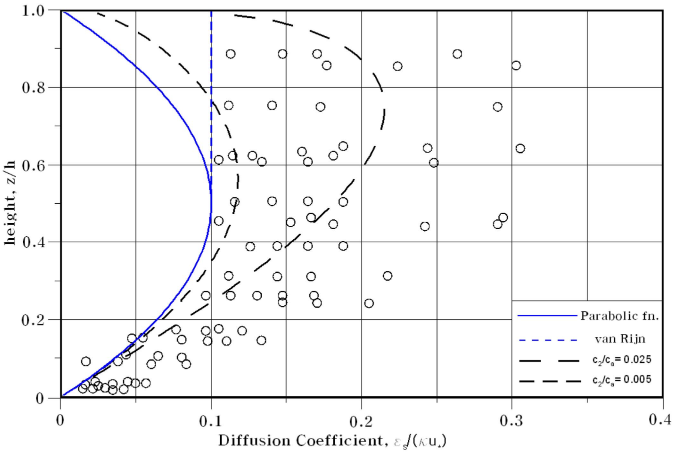

Fig. 6 Composition of computed and measures sediment diffusion coefficient (w_f / (κu_*) = 0.758)

the suspended sediment and the eddy viscosity of the fluid flow fairly well.

The difference between the sediment diffusion coefficient and the fluid diffusion coefficient is a function of the concentration ratio (c_1 / c_2), and the ratio of the settling velocity and the friction velocity, see Fig. 6. A small portion of 2.5% of the wash load leads to a large enhancement of the diffusion coefficient by about 3 times for a given settling velocity.

Sea bed material always has narrow or wide size distribution. The diffusion coefficient cannot be computed by the settling velocity of a representative particle size only. It is recommended here to deduct the wash load portion from the total suspended sediment concentration to obtain accurate diffusion coefficient profile in water column from the suspended sediment concentration.

3. Conclusions

The sediment diffusion coefficient should be identical or very close to the fluid diffusion coefficient, if the suspended sediment concentration is low enough not to hinder the fluid turbulence. Ignoring the damping effect of the sediment particles on the fluid turbulence, another factor of sediment size distribution influences the sediment diffusion coefficient. To take account of the size distribution in a simplest way, two sizes of the suspended sediment particles were considered. Additional concentration like wash load can lead to quite erroneous diffusion coefficient profile of the suspended sediment concentration. Ironically the erroneous sediment diffusion coefficients induced from two sediment sizes with settling velocity of the coarser sediment well describe Coleman`s sediment diffusion coefficient profile obtained from measured suspended sediment concentration profiles. It is recommended here that the wash load portion of the suspended sediment concentration should be removed before the total suspended sediment concentration, if the suspended sediment concentration profile is used for computation of the diffusion coefficient of the suspended sediment concentration.

Acknowledgements

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