

Partitioned analysis of nonlinear soil-structure interaction using iterative coupling

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Abstract. This paper investigates the modelling of coupled soil-structure interaction problems by domain decomposition techniques. It is assumed that the soil-structure system is physically partitioned into soil and structure subdomains, which are independently modelled. Coupling of the separately modelled partitioned subdomains is undertaken with various algorithms based on the *sequential iterative Dirichlet-Neumann sub-structuring method*, which ensures compatibility and equilibrium at the interface boundaries of the subdomains. A number of mathematical and computational characteristics of the coupling algorithms, including the convergence conditions and choice of algorithmic parameters leading to enhanced convergence of the iterative method, are discussed. Based on the presented coupling algorithms a simulation environment, utilizing discipline-oriented solvers for nonlinear structural and geotechnical analysis, is developed which is used here to demonstrate the performance characteristics and benefits of various algorithms. Finally, the developed tool is used in a case study involving nonlinear soil-structure interaction analysis between a plane frame and soil subjected to ground excavation. This study highlights the relative performance of the various considered coupling algorithms in modelling real soil-structure interaction problems, in which nonlinearity arises in both the structure and the soil, and leads to important conclusions regarding their adequacy for such problems as well as the prospects for further enhancements.

Keywords: soil-structure interaction; nonlinear analysis; domain decomposition; iterative coupling; adaptive relaxation

1. Introduction

There are many examples of functionally distinct physical systems interacting with each other, where finding a solution of any one system is impossible without considering the interaction with the other systems. Such systems are categorized as “Coupled Systems” (Zienkiewicz and Taylor 1991). One of the generic classes of coupled systems in the field of Civil Engineering relates to modelling of soil-structure interaction problems where, due to the interaction between soil and structure subdomains at the interface, neither domain can be solved separately from the other without undue simplification, especially in the nonlinear range of response.

In principle, there are two possible approaches for dealing with coupled problems, namely: *direct (monolithic / simultaneous)* and *domain decomposition (partitioning)* techniques (Felippa *et al.* 2001).

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Although coupled modelling of soil-structure interaction problems may be achieved using a *monolithic* treatment, with the whole problem modelled as a single computational entity, a *partitioned* treatment, with different partitioned domains modelled as separate computational entities amongst which interaction effects are communicated (Fig. 1), offers major benefits in the context of nonlinear soil-structure interaction. Such benefits include: i) allowing field-specific discretisation and solution procedures that have proven performance for each partitioned domain, ii) facilitating the reuse of existing nonlinear analysis software with all the resource savings that this brings, and iii) enabling parallel computations through problem partitioning (Lai 1994).

Partitioned analysis is mainly carried out by using *staggered* or *iterative coupling* methods. The staggered approach is particularly suited to transient dynamic analysis where the governing equations of the partitioned domains are solved independently at each time step using predicted boundary conditions at the interface (either force or displacement), obtained from previous time step(s) by a predictor.

Application of the staggered approach to coupled mechanical systems has attracted significant research interest over the past years, mainly in the context of fluid structure interaction analysis (Piperno 1997, Huang and Zienkiewicz 1998, Farhat and Lesoinne 2000). An example application of the staggered approach in the context of soil-structure interaction analysis is that of Rizos and Wang (2002). The authors developed a partitioned method for soil-structure interaction analysis in the time domain through a staggered solution, where a standard Finite Element Method (FEM) model, representing the structure domain, was coupled to a Boundary Element Method (BEM) model, representing the soil domain as an elastic half-space. Further to this, O'Brien and Rizos (2005) considered the application of the staggered approach for the simulation of high speed train induced vibrations.

This staggered approach is illustrated in Fig. 2 for a problem involving two physical partitions. In the context of soil-structure interaction, domain Ω_B stands for the soil, while Ω_T represents the structure. The prediction stage typically stands for prediction of displacements from the soil model at the soil-structure interface, while the substitution stage typically stands for the substitution of the reaction forces from the structural model into the soil model at the same interface. This approach is clearly approximate, since the predicted interface displacements, obtained using displacements/velocities/accelerations from previous steps, are invariably different from the displacements evaluated following force substitution into the soil model, thus violating the compatibility conditions at the interface.

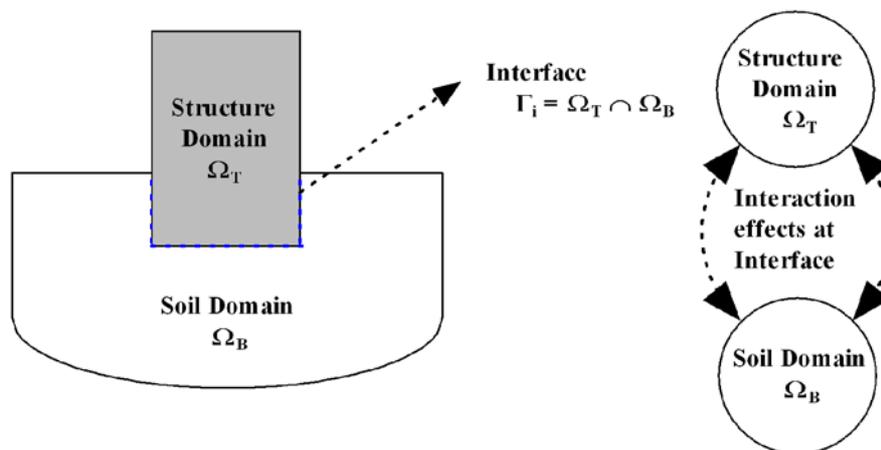


Fig. 1 Partitioned treatment of soil-structure interaction

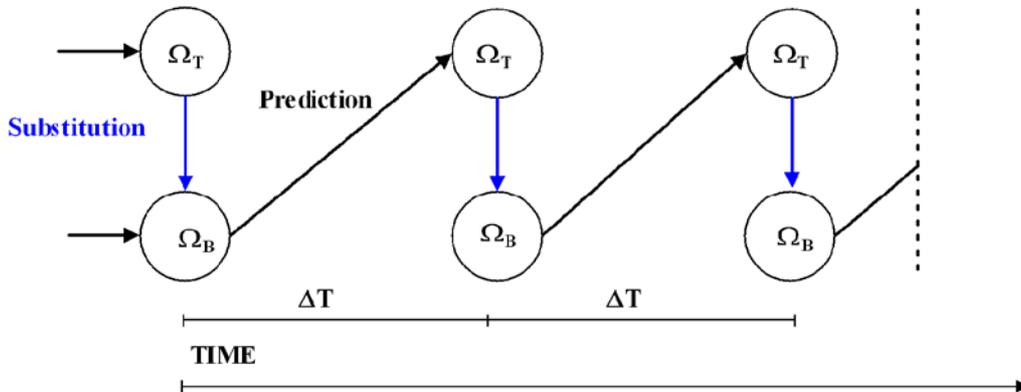


Fig. 2 Staggered coupling approach

As a result, the staggered approach should be used with great care, since its stability is conditional on the size of time step. Stability and accuracy issues related to the staggered approach typically demand excessively small time steps, rendering this scheme computationally prohibitive for many coupled problems. This has led to the development of coupling algorithms that are stable and accurate for a wider range of time step size, which is mainly achieved by introducing corrective iterations, hence the name *iterative substructuring methods* (Quarteroni and Valli 1999). It is worth mentioning that iterative coupling approaches, in addition to the aforementioned enhancement in stability and accuracy of the staggered approach, facilitate parallel computing through problem partitioning which could lead to much greater computational efficiency.

Fig. 3 shows a soil-structure interaction coupled system, decomposed into the soil and structure subdomains, treated by an iterative coupling algorithm. The governing equations of the partitioned domains are solved independently at each load increment (or time step), using predicted boundary conditions (either force or displacement) at the interface. These predicted boundary conditions are then successively updated using corrective iterations, until convergence to equilibrium and compatibility is achieved at the interface and within the partitioned domains. Frequently, a relaxation of the iteratively updated boundary conditions is augmented to the iterative coupling algorithms in order to improve the convergence characteristics, hence the term *interface relaxation* (Marini and Quarteroni 1989).

An example application of the sequential Dirichlet-Neumann iterative coupling approach for coupling BEM and FEM in elastostatic analysis was given by Elleithy *et al.* (2001). Later, Elleithy, and Tanaka (2003) presented an overview of a variety of interface relaxation schemes for coupling BEM and FEM models in elastostatic analysis. Based on these techniques, Hagen and Estorff (2005) presented a hybrid approach for transient dynamic investigation of dam-reservoir-soil problem, where the formulation was based on coupling BEM and FEM in the time domain.

Despite the significant potential benefits of iterative coupling, a major issue relates to whether convergence to equilibrium and compatibility at the interface can always be achieved through successive iterations. In this respect, work by El-Gebeily *et al.* (2002) on static coupling of BEM-FEM domains and that of Estorff and Hagen (2005) on coupling of BEM-FEM in dynamic analysis have shown that convergence is considerably influenced by the employed combination of the partitioned subdomain mesh density, material properties and the adopted relaxation scheme. Although the superiority of the partitioned approach, as highlighted before, has been recognised more recently for coupled modelling of soil-structure interaction, there remain significant technical challenges related to algorithmic and

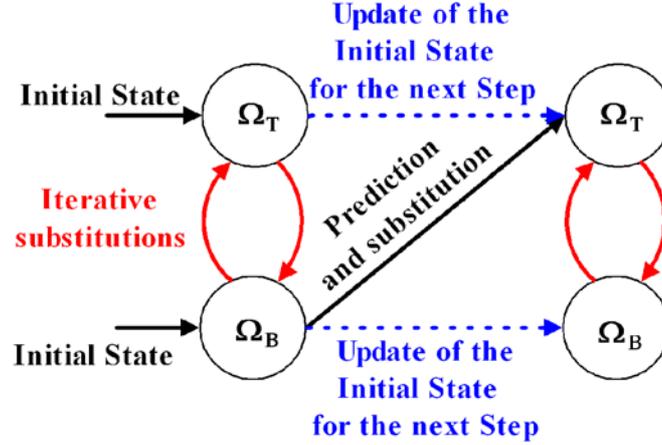


Fig. 3 Schematic of sequential iterative coupling approach

computational issues, particularly with respect to the convergence of iterative coupling methods.

Here the convergence characteristics of algorithms based on the sequential Dirichlet-Neumann iterative coupling method are investigated for soil-structure interaction problems. The efficiency of employing a relaxation scheme to enhance the convergence characteristics and the effect of nonlinearity in the partitioned domain on these characteristics is demonstrated through a detailed case study. Furthermore, it is proposed that the performance of iterative coupling methods for nonlinear soil structure interaction problems may be effectively enhanced through the use of an *adaptive* relaxation scheme. Similar approaches have been introduced by Funaro *et al.* (1998) for iterative coupling of partitioned second-order elliptic problems and Wall *et al.* (2007), focusing on fluid-structure interaction. In contrast with the traditional relaxation scheme, in which the relaxation parameter is typically evaluated by trial and error, an adaptive relaxation scheme offers improved prospects for achieving convergence and computational efficiency in complicated large scale nonlinear problems. The evaluation of such prospects and the comparison against conventional relaxation schemes are therefore primary objectives of this paper, particularly considering soil-structure interaction problems with nonlinearity in both structure and soil.

2. Soil-structure interaction partitioning

Before some variant coupling algorithms are introduced, it is beneficial to illustrate the domain decomposition strategy and discrete representation of the partitioned subdomains in coupling algorithms. Discretisation of a linear coupled soil-structure system can be shown in general as:

$$[\mathbf{K}]\{\mathbf{U}\} = \{\mathbf{F}\} \quad (1)$$

where $[\mathbf{K}]$, $\{\mathbf{U}\}$ and $\{\mathbf{F}\}$ represent the global stiffness matrix, the displacement vector and external force vector of the coupled soil-structure interaction system.

For a partitioned treatment of a coupled nonlinear soil-structure interaction problem, a common practice, which allows field-specific discretisation and solution procedures, would be to decompose the coupled system into two subdomains according to their physical and material properties, namely soil and structure subdomains. Here it is assumed that the soil-structure interaction coupled system is partitioned into soil (Ω_B) and structure (Ω_T) subdomains as shown in Fig. 1, where each subdomain is independently

discretised by FEM.

Eq. (1) can then be re-written in its general form as:

$$\begin{bmatrix} K_{11}^T & K_{12}^T & 0 \\ K_{21}^T & K_{22}^T + K_{22}^B & K_{21}^B \\ 0 & K_{12}^B & K_{11}^B \end{bmatrix} \begin{Bmatrix} U_T^T \\ U^i \\ U_B^B \end{Bmatrix} = \begin{Bmatrix} F_T^T \\ F_{ext}^i \\ F_B^B \end{Bmatrix} \quad (2)$$

where vectors $\{U_X^X\}$ and $\{F_X^X\}$ correspond to displacements and external loads respectively, for the non-interface degrees of freedom in subdomain Ω_X , while vectors $\{U^i\}$ and $\{F_{ext}^i\}$ correspond to displacements and external loads, respectively, for the interface degrees of freedom. Henceforth, for the sake of simplicity, it is assumed that there is no external load applied at the interface of the coupled system (i.e. $\{F_{ext}^i\} = 0$).

According to this partitioning strategy, the soil and structure subdomains can be formulated independently as in the following, where, without loss of generality, the response of individual subdomains is assumed to be linear elastic.

Governing equilibrium conditions for partitioned structure subdomain:

$$\begin{bmatrix} K_{11}^T & K_{12}^T \\ K_{21}^T & K_{22}^T \end{bmatrix} \begin{Bmatrix} U_T^T \\ U_T^i \end{Bmatrix} = \begin{Bmatrix} F_T^T \\ F_T^i \end{Bmatrix} \quad (3)$$

Governing equilibrium conditions for partitioned soil subdomain:

$$\begin{bmatrix} K_{11}^B & K_{12}^B \\ K_{21}^B & K_{22}^B \end{bmatrix} \begin{Bmatrix} U_B^B \\ U_B^i \end{Bmatrix} = \begin{Bmatrix} F_B^B \\ F_B^i \end{Bmatrix} \quad (4)$$

where vectors $\{U_X^i\}$ and $\{F_X^i\}$ correspond to displacements and external loads for the interface degrees of freedom in subdomain Ω_X , respectively.

Clearly, Eqs. (3) and (4) for the soil and structure partitions cannot be solved independently of each other. This is due to existence of unknown displacements/forces ($\{U_X^i\}$, $\{F_X^i\}$) at the interface. However, by applying an iterative coupling scheme and coupling the subdomains at interface level, the partitioned problem can be solved. The proposed solution scheme couples the response of the soil and structure subdomains by enforcing explicitly compatibility and equilibrium conditions at the interface. This task is achieved by employing a sequential Dirichlet-Neumann iterative coupling algorithm. Here, it is assumed that the soil and the structure remain always in contact at the interface, while the separation and slip can be treated as a simple extension of this approach through the use of interface elements that may be considered to be either part of one of the subdomains, or part of the interface model. The partitioned structure domain is considered subject to Dirichlet (displacement) boundary conditions at the interface, whereas the partitioned soil subdomain is subjected to a Neumann (force) boundary condition at the same interface. This is purely due to the fact that in static problems only essential Dirichlet boundary conditions can be imposed on the interface of the structure subdomain, since applying the natural Neumann boundary conditions at this interface results in singularity of the equilibrium equations for the structural subdomain. The sequential Dirichlet-Neumann iterative coupling algorithm can be formally defined by the algorithmic steps presented in the following section, where a linear response is assumed, again without loss of generality.

3. Sequential Dirichlet-Neumann iterative coupling algorithm

For $n = 0, 1, 2, \dots$ (number of load/time increments)

STEP 1: At the start of each increment, the structure domain is loaded by the external forces $\{F_T^I\}_n$, while the displacements at the interface nodes, $\{U_T^I\}_n$, are prescribed in accordance with the initial conditions:

$$\{U_T^I\}_n = \{\bar{U}\}$$

where I superscript and n subscript denote the iteration number and the increment number, respectively.

STEP 2: The structural solver computes the response of the structure using Eq. (3) for $\{U_T^I\}_n$ and $\{F_T^I\}_n$:

$$\begin{aligned} \{U_T^I\}_n &= [K_{11}^T]^{-1} (\{F_T^I\}_n - [K_{12}^T] \{U_T^I\}_n) \\ \{F_T^I\}_n &= [K_{21}^T] \{U_T^I\}_n + [K_{22}^T] \{U_T^I\}_n \end{aligned} \quad (5)$$

It should be noted here that although Eq. (5) is for a linear response, the same entities can be obtained for a nonlinear response from the model of the structural subdomain.

STEP 3: The corresponding interface forces at the soil domain can be calculated by applying equilibrium:

$$\{F_B^I\}_n + \{F_T^I\}_n = 0 \quad (6)$$

STEP 4: Based on the forces $\{F_B^I\}_n$ and the external loading applied to the soil domain $\{F_B^B\}_n$, the soil solver computes the response of the soil domain using Eq. (4). for $\{U_B^I\}_n$ and $\{U_B^B\}_n$:

$$\begin{Bmatrix} \{U_B^B\}_n \\ \{U_B^I\}_n \end{Bmatrix} = \begin{bmatrix} K_{11}^B & K_{12}^B \\ K_{21}^B & K_{22}^B \end{bmatrix}^{-1} \begin{Bmatrix} \{F_B^B\}_n \\ \{F_B^I\}_n \end{Bmatrix} \quad (7)$$

It again noted here that although Eq. (7) is for a linear response, the same entities can be obtained for a nonlinear response from the model of the soil subdomain.

STEP 5: If convergence to compatibility at the interface of partitioned domains has been achieved the solution will proceed to the next time step:

$$\text{If } \|\{U_B^I\}_n - \{U_T^I\}_n\| \leq \varepsilon, \text{ then } n = n + 1, \text{ and go to STEP 1}$$

STEP 6: If convergence to compatibility has not been achieved, the new estimation of the displacements will be applied to the structure domain:

$$\begin{aligned} \text{If } \|\{U_B^I\}_n - \{U_T^I\}_n\| > \varepsilon, \text{ then} \\ \{U_T^I\}_n^{I+1} &= (1 - \alpha_I) \{U_T^I\}_n + \alpha_I \{U_B^I\}_n \\ I &= I + 1 \text{ and go to STEP 2} \end{aligned} \quad (8)$$

where α is a real positive *relaxation parameter* that can improve convergence of the iterative scheme.

4. Iterative coupling with *constant* relaxation

Despite the significant potential benefits of iterative coupling, a major concern relates to convergence of the method to satisfy compatibility at the soil-structure interface after successive iterations. In this

respect, the use of a relaxation scheme as in Eq. (8) can enhance convergence, where it is assumed here that the relaxation parameter is constant during the coupling iterations (i.e., $\alpha_1, \dots, \alpha_I = \alpha$). There are several issues regarding the applicability of such a technique, namely: i) determination of the range of suitable relaxation parameters for the specific problem under consideration in order to achieve convergence, and ii) selection of the optimum relaxation parameter in order to achieve maximum computational efficiency. The convergence characteristics of the method can be established by considering the sequential Dirichlet-Neumann algorithmic steps presented in the previous section. Using Eqs. (5), (6) and (7) the interface displacements of the soil subdomain at iteration I , $\{U_B^i\}_n^I$, can be written as:

$$\{U_B^i\}_n^I = -[\lambda](U_T^i)_n^I + [C] \quad (9)$$

where,

$$[\lambda] = [K_C^B]^{-1}[K_C^T] \\ [C] = -[K_C^B]^{-1}[K_{21}^B][K_{11}^B]^{-1}\{F_B^B\} - [K_C^B]^{-1}[K_{21}^T][K_{11}^T]^{-1}\{F_T^T\} \quad (11)$$

and,

$[K_C^T] = ([K_{22}^T] - [K_{21}^T][K_{11}^T]^{-1}[K_{12}^T])$: Condensed stiffness matrix of the structure subdomain corresponding to the interface degrees of freedom.

$[K_C^B] = ([K_{22}^B] - [K_{21}^B][K_{11}^B]^{-1}[K_{12}^B])$: Condensed stiffness matrix of the soil subdomain corresponding to the interface degrees of freedom.

Substituting Eq. (9) for K successive iterations using a constant relaxation parameter, the compatibility default at the interface of the decomposed soil-structure system can be obtained by the following difference equation:

$$\{U_B^i\}_n^K - \{U_T^i\}_n^K = ([\mathbf{I}] - \alpha([\lambda] + [\mathbf{I}]))^K (\{U_B^i\}_n^1 - \{U_T^i\}_n^1) \quad (12)$$

In view of Eq. (12), the convergence of the presented coupling algorithm can now be related to the eigenvalues of matrix $([\mathbf{I}] - \alpha([\lambda] + [\mathbf{I}]))$, where the matrix $([\mathbf{I}] - \alpha([\lambda] + [\mathbf{I}]))^K$ can be regarded as the k^{th} error reduction factor. For the above successive iteration process to converge for any initially prescribed value, it is necessary and sufficient that all eigenvalues of the matrix $([\mathbf{I}] - \alpha([\lambda] + [\mathbf{I}]))$ be less than one in modulus, that is:

$$|1 - \alpha(\mu_i + 1)| < 1, i = 1, \dots, n \quad (13)$$

where, μ_i is the i^{th} eigenvalue of matrix $[\lambda]$.

Assuming that eigenvalues of $[\lambda]$ are positive, which is true for a positive definite $[\lambda]$, the above condition simplifies to:

$$|1 - \alpha(\mu_{\max} + 1)| < 1 \quad (14)$$

where, μ_{\max} is the largest eigenvalue of $[\lambda]$.

Considering Eq. (14) it can be clearly shown that for all possible partitioned soil and structure subdomains, there exists a range of relaxation parameters that guarantees convergence to compatibility of the coupling algorithm:

$$0 < \alpha < \frac{2}{1 + \mu_{\max}} \leq 2 \quad (15)$$

Moreover, the best choice of α is that for which (Fox 1964):

$$1 - \alpha_{opt}(\mu_{max} + 1) = -1 + \alpha_{opt}(\mu_{min} + 1) \quad (16)$$

where, μ_{min} is the smallest eigenvalue of matrix $[\lambda]$.

Therefore, the optimum relaxation value that ensures the convergence and holds the best convergence rate can be defined as:

$$0 < \alpha_{opt} = \frac{1}{1 + \mu_{av}} = \frac{1}{1 + \frac{\mu_{max} + \mu_{min}}{2}} \leq 1 \quad (17)$$

However, in large multi degree of freedom problems finding the optimum relaxation parameter based on Eq. (17) will require solving for the eigenvalues of $[\lambda]$, which is not readily available using black box field solvers. Therefore, the optimum constant relaxation parameter is usually found by a process of trial and error for every case under consideration.

Eqs. (10), (15) and (17) clearly indicate that convergence of the Dirichlet-Neumann iterative coupling method depends on the stiffness ratio of the partitioned soil and structure domains. In linear analysis, the stiffness matrix remains constant at all computational steps. However, for nonlinear analysis the effective stiffness depends on the deformation state. This change of the stiffness of the partitioned domains will have a significant effect on the convergence characteristics of the coupling method at the interface level, where the optimum relaxation parameter will change over the load/time increments. Accordingly, ensuring convergence and computational efficiency in large scale nonlinear problems, where the optimum relaxation parameter is to be determined over the full range of response by trial and error, would be very difficult if not impossible. Instead, such a scenario requires a dynamic change of relaxation parameter to ensure optimal convergence and associated computational efficiency. This leads to the concept of adaptive relaxation, where the relaxation parameter is determined during coupling iterations, using error minimization techniques. With adaptive relaxation, the use of trial and error for determining the relaxation parameter is avoided, whilst leading to significant improvement in the convergence rate of iterative coupling, as discussed hereafter.

5. Iterative coupling with *adaptive* relaxation parameter

Consideration is given here to an iterative coupling algorithm utilising an adaptive relaxation parameter that changes during successive iterations. In this respect the relaxation parameter is obtained, for a specific coupling iteration, from minimizing the compatibility and equilibrium default at the next iteration by using the compatibility and equilibrium default history of the previous iterations.

By assuming that different relaxation parameters are used for various iterations, Eq. (12) can be extended to:

$$\{U_B^i\}^{I+1} - \{U_T^i\}^{I+1} = ([\mathbf{I}] - \alpha_I([\lambda] + [\mathbf{I}])(\{U_B^i\}^I - \{U_T^i\}^I) \quad (18)$$

Allowing a change in the relaxation parameter for iterations ($I > 1$), the proposed method defines the adaptive relaxation parameter α_{I+1} based on minimizing the following function:

$$\Omega(\alpha_{I+1}) = \left\| ([\mathbf{I}] - \alpha_{I+1}([\lambda] + [\mathbf{I}])(\{U_B^i\}^I - \{U_T^i\}^I) \right\| \quad (19)$$

Using Eq. (18), Eq. (19) simplifies to :

$$\Omega(\alpha_{I+1}) = \left\| \frac{\alpha_{I+1}}{\alpha_I} \{E^{I+1}\} - \frac{\alpha_{I+1}}{\alpha_I} \{E^I\} + \{E^I\} \right\| \quad (20)$$

where:

$$\{E^I\} = \{U_B^i\}^I - \{U_T^i\}^I \quad (21)$$

Minimizing $\Omega(\alpha_{I+1})$ with respect to α_{I+1} leads to:

$$\frac{d}{d(\alpha_{I+1})} \left\| \frac{\alpha_{I+1}}{\alpha_I} \{E^{I+1}\} - \frac{\alpha_{I+1}}{\alpha_I} \{E^I\} + \{E^I\} \right\| = 0 \Rightarrow \quad (22)$$

$$\alpha_{I+1} = \frac{\alpha_I (\|\{E^I\}\|_2^2 - \{E^{I+1}\} \cdot \{E^I\})}{\|\{E^I\} - \{E^{I+1}\}\|_2^2} \quad (23)$$

With:

$$\frac{d^2 \Omega}{d(\alpha_{I+1})^2} = \|\{E\}^2\|_2^2 + \|\{E\}^1\|_2^2 - 2\{E\}^2 \{E\}^1 = \|\{E\}^1 - \{E\}^2\|_2^2 > 0 \quad (24)$$

where, in view of Eq. (24), the solution in Eq. (23) corresponds to the minimization of $\Omega(\alpha_{I+1})$ in Eq. (20).

Eq. (23) can be further simplified to :

$$\alpha_{I+1} = \frac{(\|\{\Delta U_T^{I+1}\}\|_2^2 - \{\Delta U_T^{I+1}\} \cdot \{\Delta U_B^{I+1}\})}{\|\{\Delta U_T^{I+1}\} - \{\Delta U_B^{I+1}\}\|_2^2} \quad (25)$$

where:

$$\{\Delta U_T\}^{I+1} = \{U_T^i\}^{I+1} - \{U_T^i\}^I, \{\Delta U_B\}^{I+1} = \{U_B^i\}^{I+1} - \{U_B^i\}^I \quad (26)$$

The automatic choice of relaxation parameter by Eq. (25) or (23) can be used for successive evaluation of the interface displacements using the iterative coupling algorithm for $I > 1$ (STEP 6). For the first iterative stage ($I = 1$) the relaxation parameter can be chosen as an arbitrary real value. Although the choice of the first relaxation parameter does not have influence on the convergence rate, a choice which is close to the optimal value of Eq. (17) will result in better error reduction of the first coupling iteration, and hence to a fewer coupling iterations with adaptive relaxation. This method enhances the constant relaxation scheme, as it avoids the trial and error for establishing the optimum relaxation parameter for the specific coupled system under consideration. Furthermore, the computational cost of determining the adaptive relaxation parameter is quite small, as evidenced by the simple vector algebra in Eqs. (23) and (25).

6. Simulation environment for nonlinear soil-structure interaction

Coupled modelling of nonlinear soil-structure interaction is considered here through the coupling of two powerful FEM codes, ADAPTIC (Izzuddin 1991) and ICFEP (Potts and Zdravkovic 1999), that have been developed at Imperial College London for advanced nonlinear structural and geotechnical analysis, respectively. ADAPTIC and ICFEP run on separate processors as independent black box solvers, where the task of communication and synchronization between the two individual codes is

achieved via an interface program that implements the iterative coupling method with constant and adaptive relaxation. In this respect, the interface program manages the retrieval, manipulation and passing the necessary data between the two field programs during coupled analysis. A schematic diagram and a flowchart of the developed simulation environment are shown in Fig. 4.

7. Convergence studies

The theoretical aspects of the Dirichlet-Neumann (D-N) iterative coupling algorithms discussed above show that the convergence characteristics of the constant relaxation scheme are highly sensitive to the partitioned domain parameters, specifically the condensed stiffness of the partitioned subdomains at the interface. Here, these convergence findings of the coupling algorithms are demonstrated through an illustrative example. The plane strain problem of Fig. 5(a) is considered, which is discretized using eight noded quadrilateral elements. The presented system is partitioned into two subdomains, namely Ω_T and Ω_B , where each partitioned domain has 9 nodes at its interface as shown in Fig. 5(b). The resulting partitioned problem is treated by the D-N iterative coupling technique. As pointed out before, prescribing Neumann boundary condition on Ω_T will result in singularity of the matrices; therefore, the Ω_T and Ω_B partitioned subdomains are treated by Dirichlet and Neumann boundary conditions, respectively.

The proposed problem is analysed using the partitioned treatment with constant relaxation for different stiffness ratios of the partitioned subdomains Ω_T and Ω_B . This is achieved by analysing the

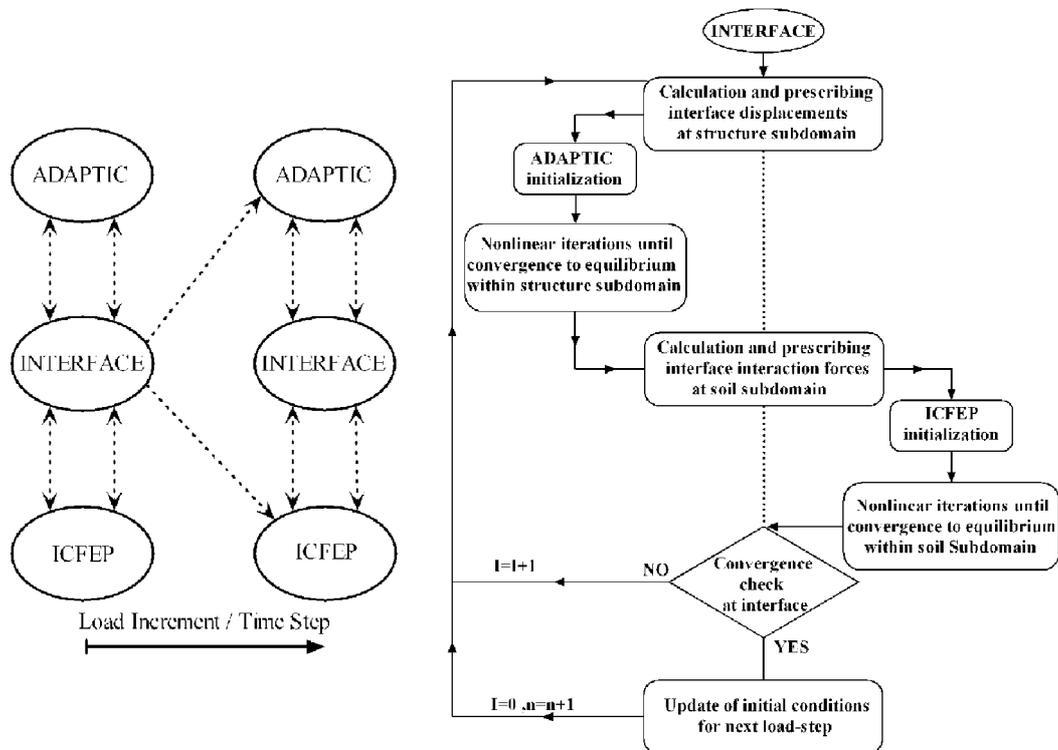


Fig. 4 Coupling procedure

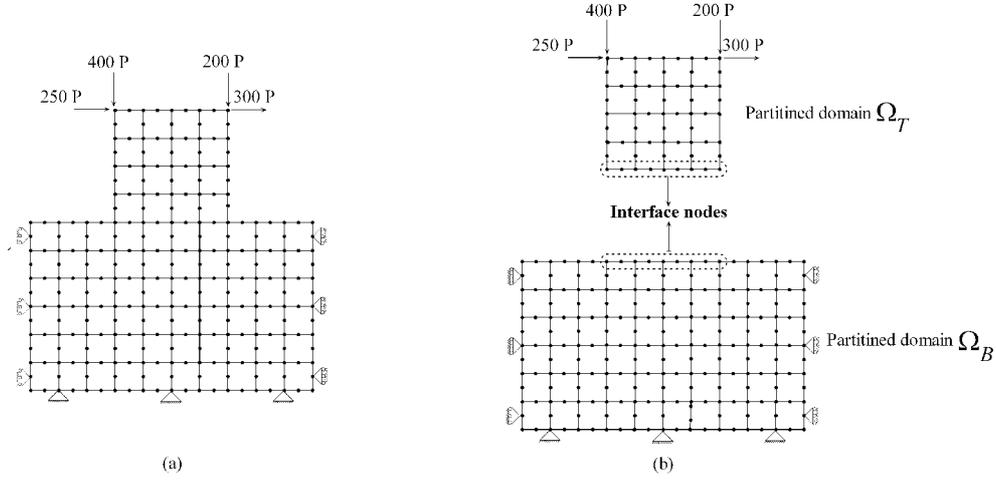


Fig. 5 (a) Plain strain example, (b) Problem partitioning

problem with different ratios of the respective elastic moduli ($E_{\Omega_T}/E_{\Omega_B}$). The results obtained from coupled partitioned analysis match very well with those obtained from the monolithic treatment within the prescribed compatibility tolerance. A list of the analysed models, including the considered elastic modulus ratios, range of suitable relaxation parameters and the optimum relaxation parameter associated with the least computational cost, is presented in Table 1. These values have been obtained by a process of trial and error for each model, where the relaxation parameter is chosen as a real value in the range of $]0,2]$ according to Eq. (15).

Fig. 6 shows the number of coupling iterations required for various constant relaxation parameters for each of the considered coupled systems, where convergence is assumed at a tolerance of $1 \times 10^{-3}L$ for the compatibility defaults, with L being the characteristic element size. The results confirm that the convergence behaviour is significantly influenced by the stiffness ratios of the partitioned subdomains.

Considering Fig. 6, the optimum relaxation parameter for each model can be easily defined as the one corresponding to the minimum number of iterations. The fact that the optimum relaxation parameter varies and depends on the partitioned model stiffness ratios has been discussed earlier, and is confirmed by the obtained results. The sensitivity of the convergence rate and value of the optimum relaxation parameter with respect to the problem parameters is even more critical for cases where the stiffness of the domain treated by Dirichlet boundary condition is relatively higher than that of the other domain treated by Neumann boundary condition.

In general, as this stiffness ratio increases the optimum relaxation parameter tends to smaller

Table 1 Range of suitable and optimal relaxation parameter for different

Model	$E_{\Omega_T}/E_{\Omega_B}$	Range of relaxation	Optimum relaxation
M1	8.0	(0-0.148]	[0.13-0.14]
M2	4.0	(0-0.275]	[0.25-0.27]
M3	2.0	(0-0.484]	[0.4-0.45]
M4	1.0	(0-0.79]	[0.6-0.7]
M5	0.5	(0-1.12]	[0.7-0.9]
M6	0.2	(0-1.52]	[0.9-1]

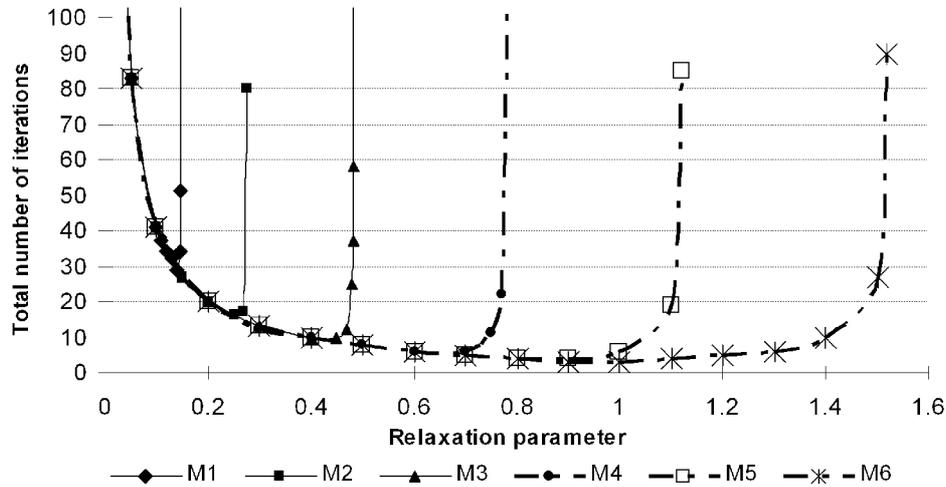


Fig. 6 Influence of relaxation parameter on convergence properties

values, while the range of an applicable relaxation parameter significantly reduces. Moreover, the convergence rate is considerably reduced, leading to a significant increase in computational cost.

The above coupled models are also analysed using the adaptive relaxation technique with the same prescribed tolerance, where a comparison between the number of coupling iterations using optimum constant relaxation and adaptive relaxation is provided in Table 2.

These results clearly indicate that the adaptive relaxation method is far superior to the constant relaxation scheme, since not only does it avoid the process of trial and error for finding the relaxation parameter, but it also enhances the convergence rate of the coupling method significantly. This fact is further demonstrated in Fig. 7 for the most critical case with regard to convergence, namely Model M1, where it is clear that adaptive relaxation method achieves the prescribed tolerance on compatibility defaults with far fewer iterations than with optimum or non-optimum constant relaxation.

8. Case study: Nonlinear soil-structure interaction

It has been long been recognized that the nonlinear behaviour of the soil and structure has a significant influence on the soil-structure interaction analysis (Jardine *et al.* 1986, Noorzaei *et al.*

Table 2 Number of required coupling iterations with adaptive and optimum relaxation

Model	Optimum relaxation	Adaptive relaxation
M1	29	8
M2	16	7
M3	10	6
M4	6	5
M5	4	3
M6	3	3

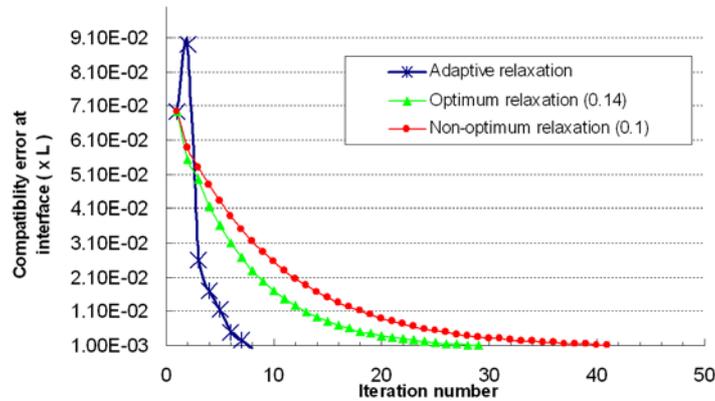


Fig. 7 Error reduction for different relaxation schemes

1995). Here, the developed coupled environment utilising ADAPTIC and ICFEP is employed to study the nonlinear soil-structure interaction of a plane frame resting on a soil subjected to ground excavation, where nonlinear elasto-plastic constitutive behaviour of the soil, as well as geometric and material nonlinearity of the structure, are taken into account. The chosen problem represents a typical urban situation, where ground works can often induce significant movements to the soil-structure system, such a problem typically requiring consideration of nonlinearity in both the soil and structure for an accurate assessment of potential damage.

The objective of this study is to: i) establish the applicability and efficiency of the presented coupling algorithms for nonlinear soil-structure interaction analysis, ii) evaluate the convergence sensitivity of the coupling algorithms to the algorithmic parameters, and iii) assess the need for further enhancements in the coupling algorithms in view of their achieved convergence characteristics.

A four-storey, two-bay steel plane frame is considered, supported by soil subjected to ground excavation as shown in Fig. 8, for which the geometric and material properties and the loading conditions are provided in Table 3. The left hand side boundary of the problem is an axis of symmetry.

According to the partitioned treatment, the considered soil-structure system is partitioned physically into two subdomains, soil and structure, where each subdomain is discretised separately according to its characteristics.

The frame structure is modelled with ADAPTIC using cubic elasto-plastic 2D beam-column element (Izzuddin and Elnashai 1993), which enables the modelling of geometric and material nonlinearity. The frame is discretised using 6 and 12 elements per member for columns and beams, respectively, and the material behaviour is assumed to be bilinear elasto plastic with kinematic strain hardening.

The soil subdomain is modelled with ICFEP, using an elasto-plastic Mohr-Coulomb constitutive model, with parameters chosen to represent the behaviour of London clay, with $\phi' = 22^\circ$, $c' = 20$ kPa and dilation angle $\nu = 11^\circ$. The stiffness, in terms of the Young's modulus E , was chosen to vary with depth from 10,000 kPa at the ground surface. The non-linear solution procedure employed for analysing the soil subdomain is based on Modified Newton-Raphson technique, with an error controlled sub-stepping stress point algorithm. The soil continuum is discretised using 8-noded isoparametric quadrilateral elements.

In the coupled nonlinear analysis using ADAPTIC and ICFEP, both constant and adaptive relaxation methods are employed, where the excavation in the soil and loads on the structure are applied in

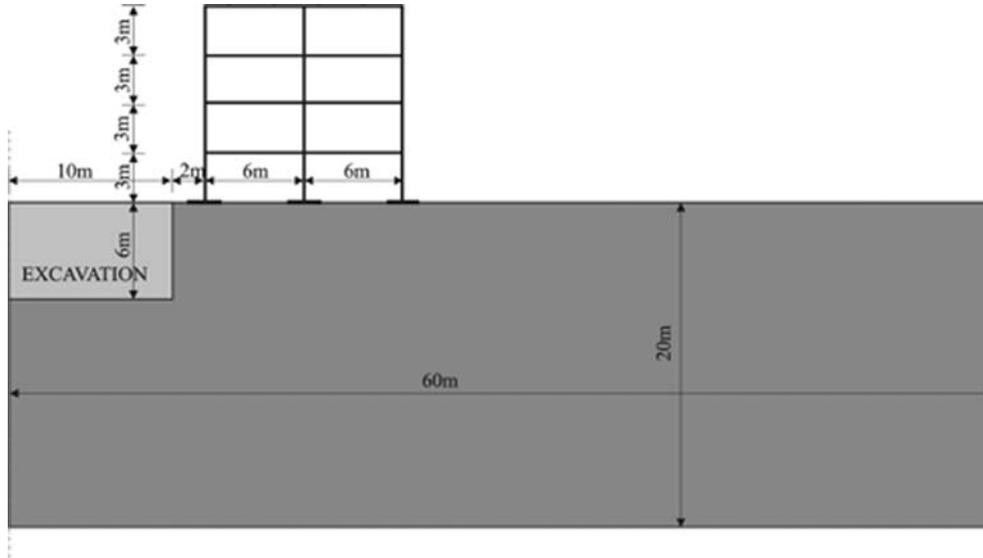


Fig. 8 Soil-structure interaction of plane frame resting on soil subject to ground excavation

Table 3 Geometric and material properties and loading of the coupled frame-soil problem

Structure subdomain	Size/Material properties
All beams	Steel - Length = 6.0 m - Cross section = 0.1×0.1
All columns	Steel - Length = 4.0 m - Cross section = 0.2×0.2
Elastic modulus of steel	210×10^6 kN / m ²
Yield stress	300×10^3 kN / m ²
Loading	Beams at all storey level : 30 kn / m
Soil subdomain	Size/Material properties
Angle of shearing resistance (ϕ')	22°
Cohesion	20 kPa
Young's modulus	Vary with depth from 10,000 kPa at the ground surface
Foundation beam	Concrete - Length = 1.0 m - Second moment of area = 0.0142 m ⁴
Elastic modulus of concrete	300×10^3 kN / m ²
Excavation	Width = 20 m - Depth = 6 m - Distance from the structure = 2 m
Effective out-of-plane depth	1 m

one increment. Table 4 presents the range of constant relaxation parameters which guarantee convergence, the optimum relaxation parameter and the number of coupling iterations required for convergence to the prescribed tolerance of $1 \times 10^{-3}L$ for both adaptive and constant relaxation schemes, where L is the characteristic element size which is equal to 1 m.

Fig. 9 illustrates the convergence characteristic of the coupling scheme for both adaptive and constant relaxation for the considered coupled problem. It is clear that the convergence rate of the iterative scheme using constant relaxation is so sensitive to the chosen relaxation parameter, rendering its selection a very difficult task, as evidenced by the significant increase in number of iterations between ($\alpha=0.23$) and ($\alpha=0.235$). These results also demonstrate the superiority of adaptive relaxation scheme, which achieves much faster convergence than the constant relaxation scheme, whether optimal

Table 4 Convergence characteristics of constant and adaptive relaxation schemes

Relaxation range	Optimum relaxation	Number of iterations with optimum relaxation	Number of iterations with adaptive relaxation
(0,0.235]	0.23	36	20

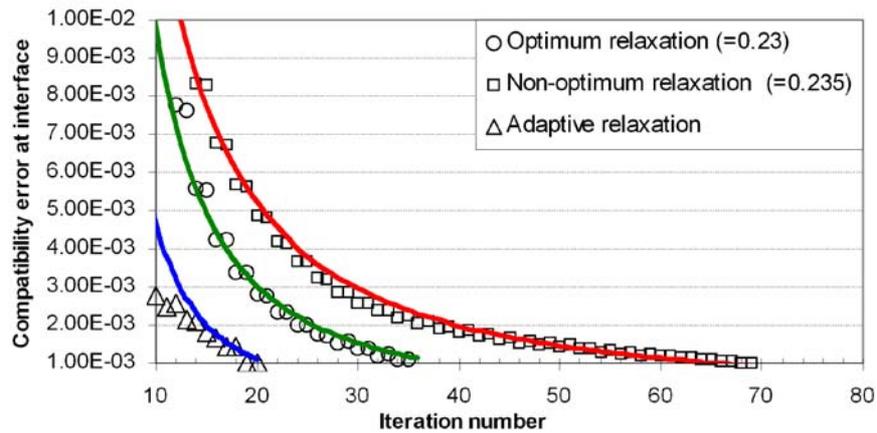


Fig. 9 Convergence of different relaxation schemes

or not. It is, however, worth observing that the adaptive relaxation scheme still requires 20 iterations, which is relatively large in comparison with what would be necessary in a typical monolithic treatment, thus highlighting the need for further enhancement in iterative coupling algorithms.

Notwithstanding the need for improved coupling algorithms, the potential benefits of the developed simulation environment in the practical assessment of nonlinear soil-structure interaction problems can be readily demonstrated by considering the results of this case study.

The frame structure with its loading conditions imposes loading on the soil, which in turn deforms due to cumulative action between these loads and the excavation. As a consequence, the soil deformations beneath the structure transmit back additional deformations and corresponding forces to the structure. This interactive process is continued until the whole coupled system reaches a compatible equilibrium state. In the following, the deformation and stress states of the coupled problem are presented.

The vertical deformation profile of the soil surface with respect to the distance from the excavation edge obtained from coupled analysis is given in Fig. 10, where the three troughs correspond to the locations of the corresponding footings. Clearly, the level of vertical displacements, of the order of 10cm, requires the structural analysis model to account for geometric nonlinearity. On the other hand, the horizontal movement of the excavation wall is presented in Fig. 11, where the maximum horizontal displacement is also of the order of 10 cm. A vector plot of the displacements at the various points of the soil subdomain is also shown in Fig. 12. The absolute magnitudes of these vectors are not important, it is their relative magnitude that shows the mechanism of ground deformation. Only part of the original mesh is shown around the excavation for clarity. It is evident from the vectors underneath each of the three footings that they all experience rigid tilting, indicated previously in Fig. 10. However, the footing nearest to the excavation has the smallest tilting, as its deformation is also dominated by the horizontal movement towards the unsupported excavation.

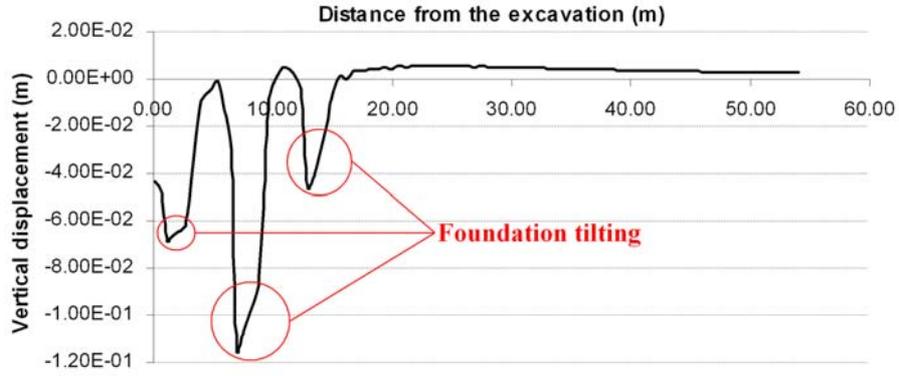


Fig. 10 Vertical displacement of the soil surface

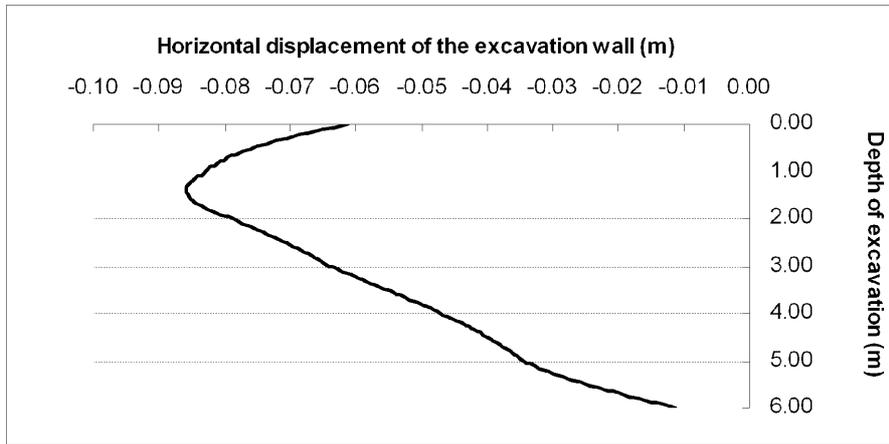


Fig. 11 Horizontal displacement of the excavation wall

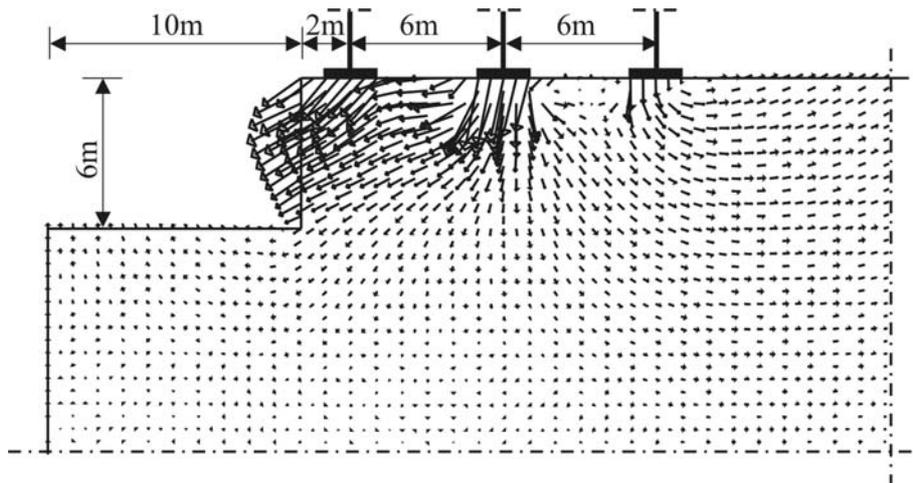


Fig. 12 Vectors of displacements in soil subdomain

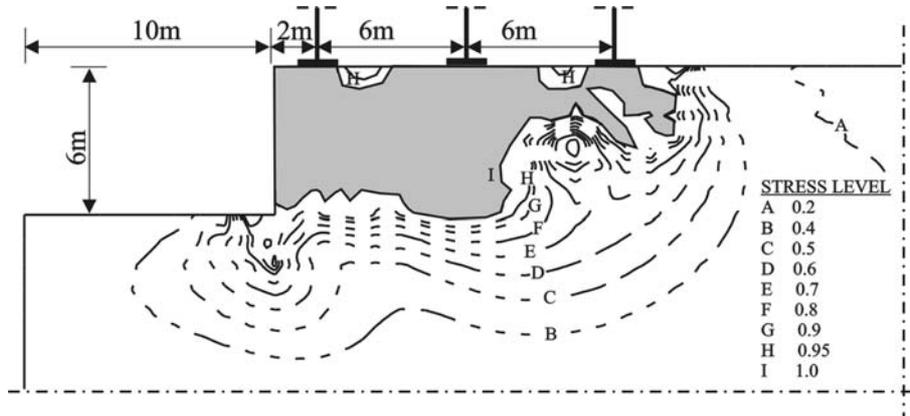


Fig. 13 Contour plot of stress levels in soil and plasticity induced in soil subdomain

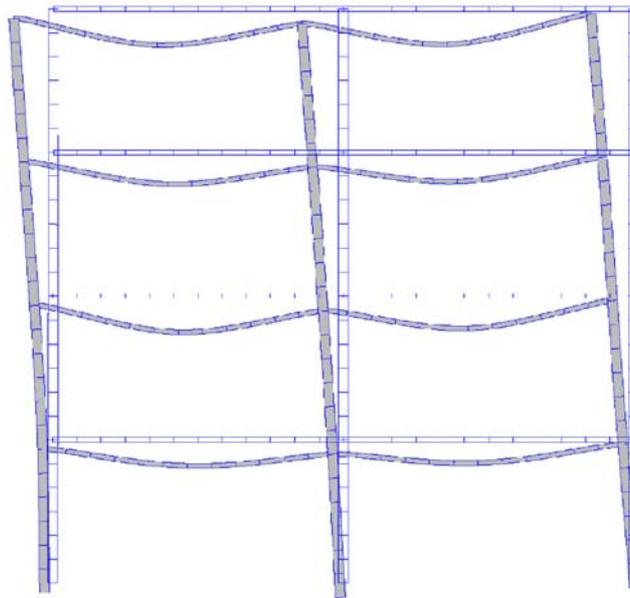


Fig. 14 Deformed shape of frame (Displacement scale: 3.0)

Fig. 13 shows contours of the drained stress level after the full excavation to 6 m depth is achieved under the applied load on the structure. The depicted stress level is the ratio, at the same mean effective stress, of the current deviatoric stress to the deviatoric stress at failure. It therefore varies from 0 to 1, where 1 indicates full plasticity and failure. It is evident from Fig. 13 that the applied loading conditions have mobilised an extensive plastic zone underneath the building. This zone is, however, smaller and shallower under the right hand side footing, which is in agreement with the previous figures that show most of the deformation and load concentration nearer the excavation.

Finally, Fig. 14 shows the deformed shape of the structure subdomain following excavation and load application.

9. Conclusions

This paper has investigated domain decomposition methods for nonlinear analysis of soil-structure interaction problems, where particular emphasis is given to iterative coupling methods. In this respect, the overall domain is divided into physical partitions consisting of soil and structure subdomains. Coupling of the separately modelled subdomains is undertaken with various algorithms based on the sequential iterative Dirichlet-Neumann sub-structuring method, which ensures compatibility and equilibrium at the interface boundaries of the subdomains. In order to enhance the convergence characteristics of the iterative coupling method, a relaxation of the interface Dirichlet entities is employed in successive iterations. Various mathematical and computational characteristics of the coupling method, including the governing convergence rate and choice of relaxation parameter, are studied where it is demonstrated that the constant relaxation scheme is very sensitive to the stiffness ratio of the partitioned subdomains. This renders the selection of the relaxation parameter rather difficult, leading to considerable computational inefficiency, especially for realistic large-scale nonlinear problems.

An adaptive relaxation scheme is also considered for enhancing the performance of iterative coupling algorithms, where the choice of the relaxation parameter is easily guided by the iterative corrections of Dirichlet entities at the interface. Based on the presented coupling algorithms, a simulation environment has been developed, which employs two specialist programs for nonlinear structural and geotechnical analysis, ADAPTIC and ICFEP, previously developed at Imperial College London. In addition to demonstrating the key convergence characteristics of the considered coupling algorithms, the overall tool has been used in a case study involving nonlinear soil-structure interaction analysis between a plane frame and soil subjected to ground excavation. It is shown that the adaptive scheme improves the convergence characteristics in both linear and nonlinear analysis significantly, where the number of coupling iterations required for convergence is often reduced by over 50%. Moreover, the adaptive scheme has the advantage of avoiding trial and error for the selection of an optimum, even adequate, constant relaxation parameter.

Although using the adaptive scheme removes significant difficulties in the conventional relaxation iterative coupling scheme, it is proposed that further enhancement is possible, particularly for nonlinear analysis, through the use of the condensed tangent stiffness of the structure and soil model at the interface. Such enhancements are currently being investigated by the authors, where again particular emphasis is placed on coupled modelling of soil-structure interaction with nonlinearity in both physical subdomains.

Finally, it should be emphasised that the considered methods are generally applicable to the coupling of various computational procedures that are used for nonlinear structural and geotechnical analysis. In this context, these coupling methods have the potential to provide an integrated interdisciplinary approach which combines the advanced features of both structural and geotechnical modelling for a variety of challenging problems in the field of nonlinear soil-structure interaction.

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