Geomechanics and Engineering, *Vol. 9, No. 2 (2015) 207-218*
DOI: http://dx.doi.org/10.12989/gae.2015.9.2.207

Study on the response of circular thin plate under low velocity impact

Hashem Babaei *, Tohid Mirzababaie Mostofi [a] and Majid Alitavoli [b]

Department of Mechanical Engineering, Faculty of Engineering, University of Guilan, Rasht PO BOX 3756-41635, Iran

(*Received February 24, 2015, Revised April 03, 2015, Accepted April 20, 2015*)

Abstract. In this paper, forming of fully clamped circular plate by using low velocity impact system has been investigated. This system consists of liquid shock tube and gravity drop hammer. A series of test on mild steel and aluminum alloy plates has been done. The effect of varying both impact load and the plate material on the deflection are described. This paper also presents a simple model to prediction of mid-point deflection of circular plate by using input-output experimental data. In this way, singular value decomposition (SVD) method is used in conjunction with dimensionless number incorporated in such complex process. The results of obtained model have very good agreement with experimental data and it provides a way of studying and understanding the plastic deformation of impact loads.

Keywords: circular plate; deformation; impact; low velocity

1. Introduction

High and medium rate technique metal forming was tolerably outreach. These progresses had considerable benefits over conventional metal forming that include the ability to use single-sided dies, reduced spring back, and improved formability (Gerdooei and Dariani 2008).

Drawbacks of the conventional forming processes can be compensated by the Forming plate process as a new attractive technology However, there is still technical difficulties in industrial production owing to some setbacks such as the high manufacturing outlay, billet reheating and an impotence to produce large parts (Kim *et al.* 2008).

One of the conducted investigations in forming plates by using drop hammer is related to the researches of Cirak *et al*. (2007). They examine the plastic deformation of copper plate under the pressure wave that created by an aluminium tube with 1.3 meter length and 32 millimeter internal diameter and the 74.1 kilogram per square meter average mass per unit area which is released with 22.29 meter per second initial speed.

One of the modern fields of study is metal forming with impulsive load, generated non-explosively in a liquid. More safety and further control are the advantages of forming with

*Corresponding author, Assistant Professor, E-mail: ghbabaei@guilan.ac.ir; hghbabaei@gmail.com
[a] Ph.D. Student, E-mail: t.m.mostofi@gmail.com; tmostofi@phd.guilan.ac.ir
[b] Associate Professor, E-mail: tavoli@guilan.ac.ir

http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

liquid shock waves in contrast with forming via using blast loads (Kosing and Skews 1998).

A great number of practical and theoretical research projects have been dedicated to the quantity of deformation in plate forming process. The amount of deformation is restricted by the happening of plastic instability in the form of localized necking or wrinkling (Safikhani *et al.* 2008).

The benefits of fracture prediction in metal forming are useful to evaluate the possibility of forming process. Numerous criteria for ductile fracture have been presented for the prediction of fractures during several past years, especially when the fracture occurs without the formation and expansion of necking (Zheng *et al.* 2007).

Mechanical elements in many engineering operations might be subjected to external dynamic loads by an object or to pressure pulses resulted in impact. These involve inertial effect, finite deformation and non-linear behavior of the materials. The accessible analytical models pertain to solids of simple geometric shape, boundary condition and applied load in which simplifying hypotheses of the problem are also commonly used to allow the analytical treatment (Babaei 2014). A primarily issue which have been considerably posed is that of a fully-clamped metallic circular plate subjected to transverse impact loads.

Modeling of processes and system identification using input–output data has always attracted many researchers' attentions. Obviously, system identification techniques are applied in large fields of research in order to model and predict the behaviors of unknown and/or very complex systems based on given input–output data. Theoretically, it is necessary to learn the explicit mathematical input–output relationship accurately to attain a proper model of system (Astrom and Eykhoff 1971).

The aim of this study is to gain further understanding of the metal plate behavior subjected to liquid shock loading by testing ferrous and nonferrous metal plates, including the effect of transfer energy to test specimen and plate thickness. Besides, a new no dimensional modeling approach using input-output data and SVD is employed to derive some closed form mathematical formulation of mid-point deflections of plates in such complex processes.

2. Experimental tests

2.1 Experimental procedure

The test specimens were 250 mm by 250 mm and varied in thickness from 1.0 mm to 3.0 mm. Plate specimens were not subjected to heat treatment before being used in impact loading condition. The specimens were clamped in a frame, comprising of two (250 mm × 250 mm) frames made from 20 mm thick mild steel plating. The frames had a 100 mm diameter hole. Each specimen had a circular exposed area with a diameter of 100 mm. The front clamp is connected to a vertical water shock tube, which has a total length of 0.4 m. The shock tube consists of a stainless-steel tube with an outside diameter of 116 mm and an inner diameter of 100 mm. The inside of the tube is honed smooth.

For the high-speed metal forming in a liquid shock tube, the potential energy of drop-weight testing system is used. After dropping of a hammer with weight of 70.4 Kg, the potential energy stored is converted into kinetic energy of hammer and after incidence hammer to piston, piston rapidly moving and caused to compressed water and then the high-pressure leads to deformation of plate. Schematic corresponding to experimental set-up is given in Fig. 1.

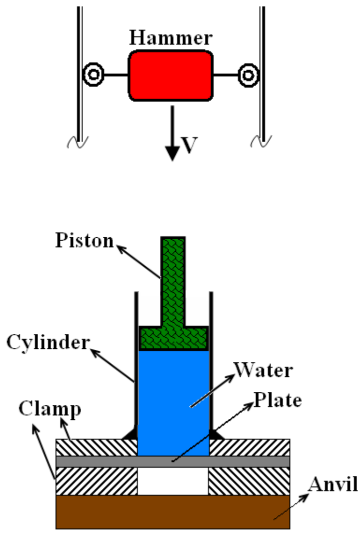

Fig. 1 Schematic of experimental arrangement

Table 1 Summary of tensile test results on steel and aluminum

Material	Nominal thickness (mm)	Mean average yield stress (MPa)	Mean average ultimate stress (MPa)
Steel	1.0, 2.0	289.2	474.5
Aluminum	1.0, 2.0, 3.0	126.7	152.3

2.2 Mechanical properties of the plate materials

Tensile test specimens were prepared from commercially aluminum alloy 1100 and mild steel (St1300) with different thicknesses. To include the effect of anisotropy on the values of yield stress, from each sheet two specimens were cut in longitudinal and transverse directions. The results of tensile test on specimens cut longitudinally show no significant difference of those cut transversely. The mean average values of yield stress and ultimate tensile stress are calculated for each material for different thickness. A summary of the properties of the different plate materials is given in Table 1.

2.3 Experimental results and discussion

For the first set of tests, stand-off distance of hammer was held variation from 1.0 m to 3 m for steel plates and 0.1 m to 1.0 m for aluminum plates, for each material and different thickness, the effect of stand-off distance of hammer or the transferred energy was investigated. The experimental details are given in Table 2.

By comparison of the profile of the plates tested at higher stand-off distance of hammer was observed to be more dome than those plates tested at lower stand-off distance. The effect of the hydrodynamic pressure upon the central portion of the plate will become increasingly greater, relative to the distance of outer edges and increasingly, smaller to the angle of incident between the hydrodynamic pressure and the plate at the outer edges. Photograph corresponding to deformed

Table 2 Results of hydrodynamic loading tests

Test No.	Material	Plate thickness (mm)	Stand-off distance of hammer (cm)	Mid-point deflection (mm)
1	Steel-1300	1	100	16
2	Steel-1300	1	125	17.8
3	Steel-1300	1	150	19.8
4	Steel-1300	2	100	11.9
5	Steel-1300	2	200	14.4
6	Steel-1300	2	300	18.7
7	Al-1100	1	10	8.2
8	Al-1100	1	20	11.5
9	Al-1100	1	30	14.4
10	Al-1100	2	30	5.8
11	Al-1100	2	50	11.3
12	Al-1100	2	65	15
13	Al-1100	3	50	8.5
14	Al-1100	3	65	9.3
15	Al-1100	3	100	15.3

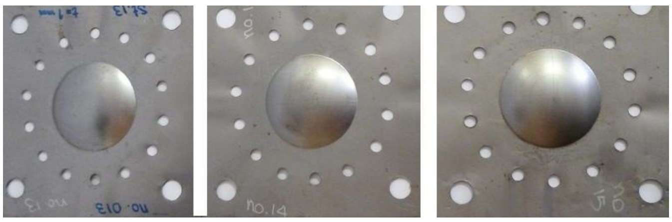

Fig. 2 Photographs of typical plates

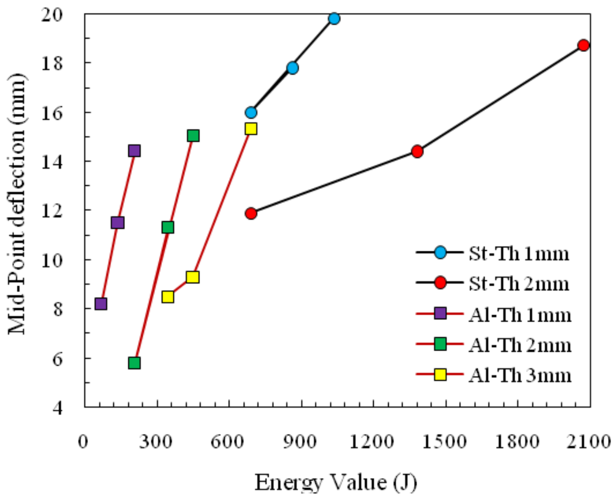

Fig. 3 Graph of measured mid-point deflection versus energy

plate with different deflection are given in Fig. 2, which mid-point deflection increases from left to right.

The graphical representation of mid-point deflection versus transferred energy for steel and aluminum plates is shown in Fig. 3. In this graphs, effect of thickness is included. The mid-point deflection increases with increasing transferred energy for each material and plate thickness.

3. Non-dimensional analysis

3.1 Effect reinforcement

Non-dimensional analysis has been performed for the prediction of plastic deformation of plates subjected to impulsive loading by various researchers. (Nurick and Martin 1989a, 1989b, Johnson 1972, Jones 1989, Jacob *et al*. 2007, Gharababaei *et al*. 2010)

In references (Nurick and Martin 1989a, 1989b, Johnson 1972, Jones 1989) modified non-dimension numbers have been employed for both uniformly and locally loaded circular plates, based on Johnson's damage number (Johnson 1972).

In the work reported (Jacob *et al*. 2007), the modified non-dimensional impulse parameter have been proposed for circular plates which was validated using data from tests on mild steel plates at various stand-off distances.

$$\frac{\delta}{H} = 0.425\phi_{cS} \tag{1}$$

where $\frac{\delta}{H}$ is mid-point deflection thickness ratio of circular plate and ϕ_{cS} is the modified non-dimensionally impulse which its relation has been given by Jacob, Nurick and Langdon (2007).

In this work, a novel approach of modelling using input-output experimental data pairs is presented for prediction of deflection-thickness ratio of circular plates subjected to impulsive loading. For this purpose, singular value decomposition (SVD) method is used in conjunction with non- dimensional numbers incorporated in the process.

The formal definition of modelling is to find a function $\hat{f}$ so that can be approximated instead of actual one f order to predict output $\hat{y}$ for a given input vector $x = (x_1, x_2, x_3,..., x_n)$ as close as possible to its actual output y. Therefore, given M observation of multi-input single-output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \ldots x_{in}) \quad i = 1, 2, 3, ..., M \tag{2}$$

It is now possible to obtain $\hat{f}$ to predict the output values $\hat{y}_i$ for any give input vector

$$x_i = (x_{i1}, x_{i2}, x_{i3}, \ldots x_{in}) \tag{3}$$

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots x_{in}) \quad i = 1, 2, 3, ..., M \tag{4}$$

The problem is now to determine $\hat{f}$ so that the square of difference between the actual output and the predicted one is minimized, i.e.

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots x_{in}) - y_i\right]^2 \to Min \tag{5}$$

In the dimensionless modelling, however, a dimensionless set, $\pi = (\pi_0, \pi_1, \pi_2,..., \pi_k)$ rather than the set of real physical variables $\{y, x\} = \{y, x_1, x_2, x_3,..., x_n\}$ is employed to obtain $\hat{f}$, i.e.

$$\hat{\pi}_{0i} = \hat{f}(\pi_{1i}, \pi_{2i}, \pi_{3i}, \ldots, \pi_{ki}) \quad i = 1, 2, 3,..., M \tag{6}$$

The problem is to determine $\hat{f}$ such that

$$\sum_{i=1}^{M} \left[\hat{f}(\pi_{i1}, \pi_{i2}, \pi_{i3}, \ldots, \pi_{in}) - \pi_{0i}\right]^2 \to Min \tag{7}$$

3.1 Static non-dimensional analysis

In order to construct such independent dimensionless numbers for modelling of deflection W_0 of the circular plates under hydrodynamic loading, stand-off distance of hammer H (m), plate thickness h (m), hammer weight m (Kg), initial plate area A_0 (m^2), radius of deformed plate R (m), material density ρ (Kg/m^3), elasticity modulus of plate E (Pa), density of water ρ_w (Kg/m^3) and static yield stress σ_y (Pa) have been taken into account. From the set of such input-output number, $K = 4$ independent dimensionless numbers can be constructed (Nurick and Martin 1989b, Johnson 1972), according to three main dimensionless (M, L, T) as follows

$$\pi_0 = \frac{W_0}{h} \tag{8a}$$

$$\pi_1 = \frac{H}{h} \tag{8b}$$

$$\pi_2 = \frac{m\sqrt{gH}}{\pi R^2 h\sqrt{\sigma_y \rho}} \tag{8c}$$

$$\pi_3 = \frac{\rho_W A_0 h E}{m\sigma_y} \tag{8d}$$

So that

$$\pi_0 = \frac{W_0}{h} = f(\pi_1, \pi_2, \pi_3) \tag{9}$$

In order to use SVD the Eq. (9) can be represented as

$$\pi_0 = C \cdot (\pi_1)^{\alpha} \cdot (\pi_2)^{\beta} \cdot (\pi_3)^{\gamma} \tag{10}$$

Therefore, the problem of modelling is now to find the coefficients C, α, β and γ so that the Eq. (7) is satisfied by using the natural logarithm , Eq. (10) can be represented as a linear relation with respect to the coefficient ($\eta = \ln C$), (α), (β) and (γ) as

$$\mathrm{Ln}(\pi_0) = \eta + \alpha\mathrm{Ln}(\pi_1) + \beta\mathrm{Ln}(\pi_2) + \gamma\mathrm{Ln}(\pi_3) \tag{11}$$

Consequently a system of M Linear algebraic equation with K = 4 unknown of the above mentioned coefficients is now constructed based on M input-output experimental data pairs

$$\left\{\begin{array}{c} \eta+\alpha\zeta_{11}+\beta\zeta_{12}+\gamma\zeta_{13}=\zeta_{10} \\ \eta+\alpha\zeta_{21}+\beta\zeta_{22}+\gamma\zeta_{33}=\zeta_{20} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \eta+\alpha\zeta_{M1}+\beta\zeta_{M2}+\gamma\zeta_{M3}=\zeta_{M0} \end{array}\right\} \tag{12}$$

where

$$\zeta_{ij}=\mathrm{Ln}(\pi_{ij}) \quad i=1,2,3,\ldots,M \quad j=1,2,3 \tag{13}$$

and

$$\zeta_{i0}=\mathrm{Ln}(\pi_{i0}) \quad i=1,2,3,\ldots,M \tag{14}$$

Such system of linear equations in which $M >> K = 4$ can be represented as

$$A=XY \tag{15}$$

In which

$$X=[\eta\ \ \alpha\ \ \beta\ \ \gamma]^T \tag{16}$$

$$Y=[\zeta_{10}\ \ \zeta_{20}\ \ \cdots\ \ \zeta_{M0}]^T \tag{17}$$

and

$$A=\begin{bmatrix} 1 & \zeta_{11} & \zeta_{12} & \zeta_{13} \\ 1 & \zeta_{21} & \zeta_{22} & \zeta_{23} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \zeta_{M1} & \zeta_{M2} & \zeta_{M3} \end{bmatrix} \tag{18}$$

SVD method is used to solve Eq. (15). Generally, SVD is the method for solving most linear least-squares problems that some singularities may exist in the normal equations.

The SVD of a matrix, $A \in R^{M\times 4}$, is a factorisation of the matrix into the product of three matrices, column-orthogonal matrix $U \in R^{M\times 4}$, diagonal matrix $W \in R^{M\times 4}$ with non-negative elements (singular values), and orthogonal matrix $V \in R^{M\times 4}$ such that

$$A=UWV^T \tag{19}$$

The problem of optimal selection of vector of the coefficients in Eq. (15) is firstly reduced to finding the modified inversion of diagonal matrix W in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal X are calculated using the following relation

$$X=V\left[\mathrm{diag}\left(\frac{1}{w_j}\right)\right]U^TY \tag{20}$$

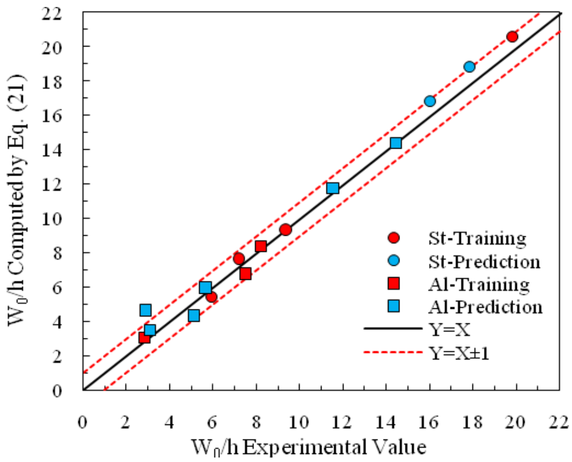

Fig. 4 Confidence limit of the Predicted mid-point deflection-thickness ratio versus the experimental data in Eq. (21)

n order to obtain a simple model for deflection – thickness ratio of thin circular plates under hydrodynamic loading (Eq. (10)), the experimental data converted into a dimensionless data table based on relations Eqs. (8a)-(8d) and the natural logarithms Eqs. (13) and (14). The unknown coefficients for $K = 4$ involved in simple model represented by Eq. (10) can now be determined by SVD approach.

The corresponding values of parameters are found as $C = 0.382$, $\alpha = 0.416$, $\beta = 0.153$ and $\gamma = -1.06$. Hence, the model can now be given as

$$\frac{W_0}{h} = 0.382\left(\frac{H}{h}\right)^{0.416}\left(\frac{m\sqrt{gH}}{\pi R^2 h\sqrt{\sigma_y \rho}}\right)^{0.153}\left(\frac{\rho_w A_0 hE}{m\sigma_y}\right)^{-1.06} \tag{21}$$

To validate the present model, Fig. 4 shows comparison between the theoretical prediction of the midpoint deflection thickness ratio (using Eq. (21)) and the experimental values. It is clear from Fig. 4 that results obtained from model are in good agreement with the experimental results. The confidence envelope for Eq. (21) is reported as 93% for ±1 displacement–thickness ratio.

3.2 Dynamic non-dimensional analysis

In this section, the dynamic yield, σ_d rather than the static yield stress, σ_y is used for non-dimensional modelling of deflection W_0. Similarly, other parameters such as material strain rate and the effect of stand-off distance are taken into account so that the dimensionless analysis is carried out as

$$\pi_0 = \frac{W_0}{h} \tag{22a}$$

$$\pi_1 = \frac{H}{h} \tag{22b}$$

$$\pi_2 = \frac{m\sqrt{gH}}{\pi R^2 h\sqrt{\sigma_d \rho}} \tag{22c}$$

$$\pi_3 = \frac{\rho_w A_0 h E}{m\sigma_d} \tag{22d}$$

So that

$$\pi_0 = \frac{W_0}{h} = f(\pi_1, \pi_2, \pi_3) \tag{23}$$

In order to use SVD the Eq. (23) can be represented as

$$\pi_0 = C \cdot (\pi_1)^{\alpha}, (\pi_2)^{\beta}, (\pi_3)^{\gamma} \tag{24}$$

The dynamic yield stress is given by

$$\sigma_d = \lambda \sigma_y \tag{25}$$

For determining value of λ, the ratio between static and dynamic yield stresses, the well-known Cowper–Symonds constitutive equation is used. Therefore, Eq. (25) can be rewritten in the form

$$\frac{\sigma_d}{\sigma_y} = \lambda = 1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{\frac{1}{q}} \tag{26}$$

Where $\dot{\varepsilon}$ is the strain rate, q and D are material constants. The constant values for mild steel and aluminium are given in Table 3.

As an estimate of strain rate $\dot{\varepsilon}$, the equation as given by Jacob *et al*. (2007) is used as

$$\dot{\varepsilon} = \frac{V_0 W_0}{3\sqrt{2}R^2} \quad (\text{where } V_0 = \sqrt{2gH}) \tag{27}$$

Therefore, the above equation can be rewritten as

$$\dot{\varepsilon} = \frac{W_0\sqrt{gH}}{3R^2} \tag{28}$$

Substituting Eqs. (27) and (28) into Eq. (26) yields to

Table 3 Summary of material constants in Eq. (26) for steel and aluminum (Jones 1989)

Material	D (s^{-1}) In Eq. (26)	q In Eq. (26)
Steel	40.4	5
Aluminum	6500	4

$$\frac{\sigma_d}{\sigma_y} = \lambda = 1 + \zeta\left(\frac{W_0}{h}\right)^{\frac{1}{q}} \tag{29}$$

where

$$\zeta = \left(\frac{h\sqrt{gH}}{3R^2 D}\right)^{\frac{1}{q}} \tag{30}$$

Hence the dynamic yield stress σ_d is determined using Eq. (29). The unknown coefficients involved in Eq. (24) can now be determined by SVD approach .The corresponding values of parameters are found as as $C = 0.411$, $\alpha = 0.238$, $\beta = 0.439$ and $\gamma = -0.914$.

Hence, the model can now be given as

$$\left(\frac{W_0}{h}\right) = 0.411\left(\frac{H}{h}\right)^{0.238}\left(\frac{m\sqrt{gH}}{\pi R^2 h\sqrt{\sigma_d \rho}}\right)^{0.439}\left(\frac{\rho_w A_0 hE}{m\sigma_d}\right)^{-0.914} \tag{31}$$

To validate the present model, Fig. 5 shows comparison between the theoretical prediction of the midpoint deflection thickness ratio (using Eq. (31)) and the experimental values. Also, it is clear from Fig. 5 that results obtained from model are in good agreement with the experimental results. The confidence envelope for Eq. (31) is reported as 93% for ±1 displacement–thickness ratio.

Evidently, it can be observed that the models obtained in this paper have much less RMSE compared with the other non-dimensional models reported in references. Moreover RMSE of dynamic non dimensional model (Eq. (31)) is less than static non-dimensional model (Eq. (21)). It is evident that the strain rate has significant effect on RMES of the models. According to Table 4

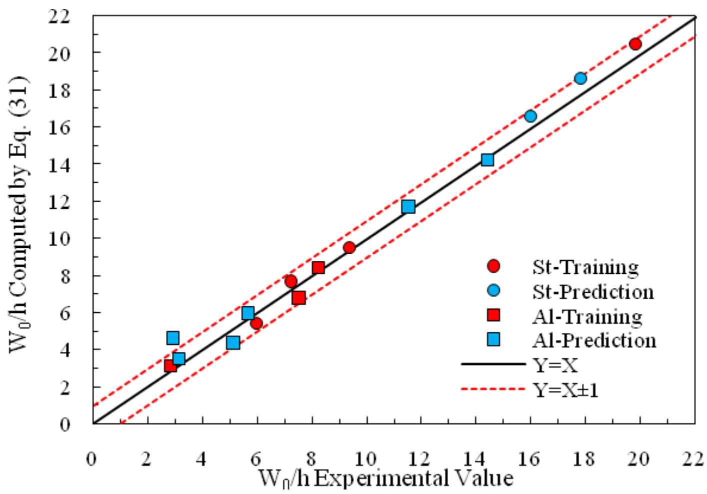

Fig. 5 Confidence limit of the Predicted mid-point deflection–thickness ratio versus the experimental data in Eq. (31)

Table 4 Comparisons of root mean squares of errors (RMSE)

Material	RMSE	
	The obtained model of this work (Eq. (21))	The obtained model of this work (Eq. (31))
Steel	0.6817	0.5604
Aluminum	0.7121	0.7061
Sum of errors	0.7001	0.6517

and the values of RMSE of each material, it can be observed that the response of aluminium is different under the effect of high strain rate deformation.

4. Conclusions

The experimental results in this paper represent the behaviour of thin circular steel and aluminium plates subjected to liquid shock loading by drop hammer. One of innovations of this paper is use of drop hammer system in experimental section. Generally, drop hammer is one of the most useful instruments that is used in the sheet forming, powder compression and energy absorbers. In this system, forming process occurs by using the amount of energy that derived from the resources which can release the hammer. In other words, the pressure of the shock wave is applied in to the segment in a controlled process. The results can provide an insight to predict the relationship between mid-point deflections and applied shock load. Obtained results delivers a good realization of relation of mid-point deflection and hydrodynamic load, while mechanical features of material, stand-off distance of the hammer from the target and thickness of the plate changes. However, the process is very simple, costly and low-risk but the quality of deformation of plates is as other processes of this kind like explosive forming with this difference that in this process hydrodynamic pressure causes domical and totally uniform deformation due to the low speed.

- Obtained experimental results are derived by mid-point deflection parameter in which the transferred energy, material and thickness of the plate is totally obvious.
- The non-dimensional modelling approach presented in this paper using singular value decomposition, leads to some closed form mathematical equations which have much less RMSE compared with those previously proposed in the literature. Moreover, it has been shown that the obtained model can successfully predict the deflection compared with that of actual experimental values.

References

Astrom, K.J. and Eykhoff, P. (1971), "System identification, a survey", *Automatica*, **7**(2), 123-162.

Babaei, H. (2014), "Prediction of Deformation of Circular Plates Subjected to Impulsive Loading Using GMDH-type Neural Network", *Int. J. Eng. (IJE)*, TRANSACTIONS A: Basics, **27**(10), 1635-1644.

Cirak, F., Deiterding, R. and Mauch, S.P. (2007), "Large-scale fluid-structure interaction simulation of viscoplastic and fracturing thin-shells subjected to shocks and detonations", *Comput. Struct.*, **85**(11), 1049-1065.

Gerdooei, M. and Dariani, B.M. (2008), “Strain-rate-dependent forming limit diagrams for sheet metals”, *P. I. Mech. Eng. B-J. Eng.*, **222**(12), 1651-1659.

Gharababaei, H., Nariman-Zadeh, N. and Darvizeh, A. (2010), “A simple modelling method for deflection of circular plates under impulsive loading using dimensionless analysis and singular value decomposition”, *J. Mech.*, **26**(3), 355-361.

Jacob, N., Nurick, G.N. and Langdon, G.S. (2007), “The effect of stand-off distance on the failure of fully clamped circular mild steel plates subjected to blast loads”, *Eng. Struct*., **29**(10), 2723-2736.

Johnson, W. (1972), *Impact Strength of Materials*, Edward Arnold, London, UK.

Jones, N. (1989), *Structural Impact*, Cambridge University Press, Cambridge, UK.

Jacob, N., Nurick, G.N. and Langdon, G.S. (2007), “The effect of stand-off distance on the failure of fully clamped circular mild steel plates subjected to blast loads”, *Eng. Struct.*, **29**(10), 2723–2736.

Kim, T., Kim, H., Bae, J., Lee, S. and Kang, C. (2008), “Semi-solid die forging of al6061 wrought aluminum alloy with electromagnetic stirring”, *P. I. Mech. Eng. B-J. Eng.,* **222**(9), 1083-1095.

Kosing, O.E. and Skews, B.W. (1998), “An investigation of high-speed forming of circular plates in a liquid shock tube”, *Int. J. Impact. Eng.*, **21**(9), 801-816.

Nurick, G.N. and Martin, J.B. (1989a), “Deformations of thin plates subjected to impulsive loading - A review; Part I - Theoretical considerations”, *Int. J. Impact. Eng.*, **8**(2), 159-170.

Nurick, G.N. and Martin, J.B. (1989b), “Deformation of thin plates subjected to impulsive loading - A review; Part II - Experimental studies”, *Int. J. Impact. Eng.*, **8**(2), 170-186.

Safikhani, R., Hashemi, R. and Assempour, A. (2008), “The strain gradient approach for determination of forming limit stress and strain diagrams”, *P. I. Mech. Eng. B-J. Eng.,* **222**(4), 467-483.

Zheng, T., Zhang, S.H., Sorgente, D., Tricarico, L. and Palumbo, G. (2007), “Approach of using a ductile fracture criterion in deep drawing of magnesium alloy cylindrical cups under non-isothermal condition”, *P. I. Mech. Eng. B-J. Eng.*, **221**(6), 981-986.

CC