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Study on relations between porosity and damage in fractured rock mass

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Abstract. The porosity is often regarded as a linear function of fluid pressure in porous media and permeability is approximately looked as constants. However, for some scenarios such as unconsolidated sand beds, abnormal high pressured oil formation or large deformation of porous media for pore pressure dropped greatly, the change in porosity is not a linear function of fluid pressure in porous media, and permeability can't keep a constant yet. This paper mainly deals with the relationship between the damage variable and permeability properties of a deforming media, which can be considered as an exploratory attempt in this field.

Keywords: damage; porous media; modified Darcy's law; permeability; relation

1. Introduction

Fracture rock mass is one of the common and complex rock masses encountered in hydroelectricity, mining and petroleum engineering. The strength, deformation and seepage characteristics of the fracture rock mass are critical to the design, construction and maintenance of the relevant projects.

The fluid flow in soil, sedimentary sand formation underground or petroleum reservoir is a common phenomenon. The effects of stress and fluid pressure on porous media are often neglected or regarded approximately as slightly compressible, therefore porosity is often regarded as a linear function of fluid pressure in porous media and permeability is approximately looked as constants. However, for some scenarios such as unconsolidated sand beds, abnormal high pressured oil formation or large deformation of porous media for pore pressure dropped greatly, the change in porosity is not a linear function of fluid pressure in porous media, and permeability can't keep a constant yet. Many studies have been carried out to investigate the effect of micro-structures on flow in porous media (e.g., Kim *et al*. 1987, Lemaitre and Adler 1990, Auriault and Lewandowska 1994, Ichikawa *et al*. 1999, Yang and Wang 2000, Chen *et al*. 2001, Jeong 2010, Saeed and Mohammd 2010, Akaydin *et al*. 2011, Boutt *et al*. 2011, Pradeep *et al*. 2011, Shou *et al*. 2011, Yue *et al*. 2011, Guerroudj and Kahalerras 2012, Pradhan *et al*. 2012, Hang *et al*. 2012, You *et al*.

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2013, Wang *et al*. 2014). For instance, Kim *et al*. (1987) studied Darcy's law within a representative volume element (RVE) through volume averaging theorem. A closed-form of Darcy's law was developed from the interaction of pore water and solid particles. Their method still has two issues remaining to be solved. One of them is the relationship between the micro-structures of porous media and the coefficient of permeability.

It is well known that water is ubiquitous in various types of porous media, and generally flows through the micro-cracks and micro-voids in the porous medium when there exist a hydraulic gradient. Seepage water flow is one of important factor to change the states and properties of porous media, the seepage effects can be implemented due to hydraulic loadings, physical influences and chemical reactions, etc. Whereas, porous skeleton of a solid may pose a certain resistance on the seepage flow, and show many behaviours and properties related to seepage, such as permeability, distensibility, dilapidation, and so on (Valliappan and Zhang 1996, 1999, Zhang 1999). In this study, we mainly focus on the permeability properties of a deforming media. Firstly, the relationship between porosity and damage variable is presented. Then, the modified Darcy's law is developed based on the relationship between damage and porosity of porous media, which is considered to be an extension of the Darcy's law for fluid flow and seepage in a porous medium. At last, the properties of permeability of damaged porous media are presented and discussed in this paper. This can be considered as an exploratory attempt in this field.

2. Relationship between damage and porosity

Most materials involve different types of defects like caves, pores and cracks, which are important characters of porous media and have a great influence on the physical properties of materials. This kind of characters are generally designated by the porosity of materials and described by a pore rate (porosity), defined by the percentage of the volumetric ratio between the porous volume and total volume of the specimen.

From the point of view of porous media, the damage variable can be defined as (Dai 2006)

$$\Omega = \frac{\Phi_0 - \Phi}{\Phi_0 - \Phi_s} \tag{1}$$

where Ω and Φ are defined as damage variable and porosity of porous media, respectively; Φ_0 and Φ_s are the initial porosity and final porosity, respectively. When materials are destroyed completely, that is, $\Phi = \Phi_s$, it can be concluded that $\Omega = 1$. When the material is not damaged, that is, $\Phi = \Phi_0$, it can be concluded that $\Omega = 0$.

3. Modification of Darcy's law

Coupling between fluids and solids in a porous medium results from direct interaction in pores. For example, as the volume of a pore space collapse, fluid is forced to either compress or flow out of the pore. Conversely, a fluid pressure change imparts a tangible force on the solid grain walls of the pore. The classical theory of poroelasticity is one of the noteworthy developments in continuum mechanics of multiphase media that has been successfully applied to examine time-dependent transient phenomena encountered in a wide range of natural and synthetic materials, including geomaterials and biomaterials. In classical poroelasticity, the elastic behavior of the porous skeleton is assumed to be linear, and the transport of the saturating compressible fluid through the pore space is governed by Darcy's law. However, the assumption of linear elastic behavior of the porous skeleton is recognized as a limitation when considering the applicability of classical theory of poroelasticity to a wider class of geomaterials that posses nonlinear phenomena, particularly in the constitutive behavior of the porous skeleton. A modification of the porous skeletal response to include elastoplastic constitutive behavior is one such approach that has been employed for the modeling of the poroelastic behavior of geomaterials such as soft clays and other saturated soils. In such situations, the geomaterials display distinct attributes of failure that can be described by the criteria of the initiation of failure, and constitutive laws that govern the post-failure behavior. Alternatively, in geomaterials such as soft rocks and heavily over-consolidated clays, the tendency is to display brittle elastic behavior leading to degradation in the elastic stiffness and alteration of the permeability characteristics of the porous medium. Such degradation can occur as a consequence of the development of micro-cracks and micro-voids in the porous fabric of the geomaterials while it continues to maintain its elastic character. This paper mainly deals with the relationship between the damage variable and permeability properties of a deforming media, which can be considered as an exploratory attempt in this field.

It is well known that flow within a soil profile (or system) can occur in any direction, whether vertical or horizontal. Assuming there is a seepage flow beam as shown in Fig. 1, if we ignore the inertial forces in fluid flow and the porosity of porous media is replaced by damage variable, then the equation can be obtained according to the balance of forces as follows

$$(p+\mathrm{d}p)\Omega\mathrm{d}A - p\Omega\mathrm{d}A + \rho g\Phi\mathrm{d}A\mathrm{d}L\sin\theta + F = 0 \tag{2a}$$

where dL is the length of the seepage flow beam; dA is the cross sectional area; $\rho g\Phi\mathrm{d}A\mathrm{d}L$ is the weight of pore water; F is the frictional resistance of pore channel between granules; p and p + dp are the pore pressures acting on the two ends of the seepage flow beam, respectively. Ω is the damage variable; $\sin\theta = \mathrm{d}Z/\mathrm{d}L$. As we can see from Eq. (1) that, when materials are destroyed completely, that is, $\Phi = \Phi_s$, it can be concluded that $\Omega = 1$. After substitution of $\Omega = 1$ into Eq. (2a) and performing some algebraic manipulations, Eq. (2a) can be rewritten as

$$(\mathrm{d}p + \rho g\Phi_s\mathrm{d}L\sin\theta)\mathrm{d}A + F = 0 \tag{2b}$$

when the material is not damaged, that is, $\Phi = \Phi_0$, it can be concluded that $\Omega = 0$. After substitution of $\Omega = 0$ into Eq. (2a) and performing some algebraic manipulations, Eq. (2a) can be rewritten as

$$\rho g\Phi_0\mathrm{d}A\mathrm{d}L\sin\theta + F = 0 \tag{2c}$$

Thus, it can be concluded that the equilibrium equations for the pore fluid not only relates to the soil damage status, but also the porosity status of soil.

The relationship between the total hydraulic head h and the elevation head z can be expressed as follows

$$h = \frac{p}{\rho g} + Z \tag{3}$$

Taking the derivative of Eq. (3), we can obtain

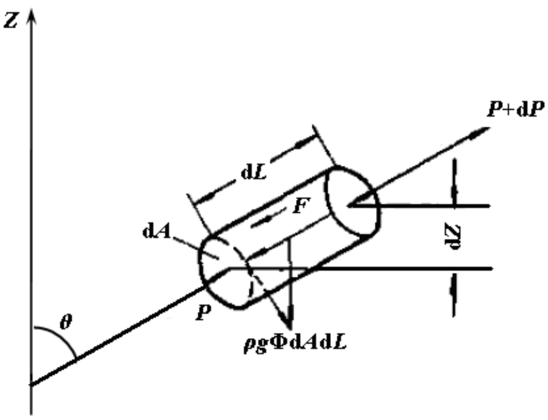

Fig. 1 Seepage flow diagram beam

$$\mathrm{d}p = \rho g(\mathrm{d}h - \mathrm{d}Z) \tag{4}$$

After substitution of Eq. (4) into Eq. (2a) and performing some algebraic manipulations, Eq. (2a) can be rewritten as

$$\frac{\mathrm{d}h}{\mathrm{d}L} - \left(1 - \frac{\Phi}{\Omega}\right)\frac{\mathrm{d}z}{\mathrm{d}L} + \frac{F}{\rho g \Omega \mathrm{d}A\mathrm{d}L} = 0 \tag{5}$$

According to the knowledge of hydraulics, the total resistance F can be expressed as (Mao 2003)

$$F = \frac{(1-\Phi)\mathrm{d}A\mathrm{d}L}{\beta d^3}\lambda \mu v^* d \tag{6}$$

where λ is the coefficient of Stokes; μ is a coefficient that takes into account the effect of neighboring spheres; v^* is the velocity of pore water, $v^* = v/\Omega$. d is the diameter of soil grains; β is the spheroid coefficient.

So far, the equation of average flow velocity v can be expressed as

$$v = -\frac{\Phi^2}{(1-\Phi)}\frac{\beta\rho g}{\lambda\mu}d^2\frac{\Omega}{\Phi}J + \frac{\Phi^2}{(1-\Phi)}\frac{\beta\rho g}{\lambda\mu}d^2\frac{\Omega}{\Phi}\left(1-\frac{\Omega}{\Phi}\right)\frac{1}{\rho g}\frac{\mathrm{d}p}{\mathrm{d}L} \tag{7}$$

where J is the hydraulic gradient and be expressed as

$$J = -\frac{\mathrm{d}h}{\mathrm{d}L} \tag{8}$$

The permeability coefficient of geological materials k can be defined as

$$k = ad^2\frac{\rho g}{\mu} \tag{9}$$

where the parameter a can be expressed as

$$a = \frac{\beta\Phi}{\lambda}(1-\Phi) \tag{10}$$

According to the above mentioned, the effective permeability coefficient of the damaged skeletal material can then be written in the following form

$$k^* = a^* d^2 \frac{\rho g}{\mu} = \Omega a d^2 \frac{\rho g}{\mu} = \Omega \frac{\beta\Phi}{(1-\Phi)} d^2 \frac{\rho g}{\mu} = k\Omega \tag{11}$$

in which

$$a^* = \frac{\beta\Phi}{\lambda(1-\Phi)}\Omega = a\Omega \tag{12}$$

After substitution of Eq. (11) into Eq. (7) and performing some algebraic manipulations, Eq. (7) can be rewritten as

$$v = -k\Omega J + k\Omega\left(1-\frac{\Omega}{\Phi}\right)\frac{1}{\rho g}\frac{\mathrm{d}p}{\mathrm{d}L} \tag{13}$$

or

$$v^* = -k^* J + k^*\left(1-\frac{\Omega}{\Phi}\right)\frac{1}{\rho g}\frac{\mathrm{d}p}{\mathrm{d}L} = -k^*\left[J-\left(1-\frac{\Omega}{\Phi}\right)\frac{1}{\rho g}\frac{\mathrm{d}p}{\mathrm{d}L}\right]k^* J^* \tag{14}$$

where J^* is called effective permeability gradient and be expressed as

$$J^* = J - \left(1-\frac{\Omega}{\Phi}\right)\frac{1}{\rho g}\frac{\mathrm{d}p}{\mathrm{d}L} \tag{15}$$

If the first part of Eq. (15) is deemed to be the contribution of general hydraulic gradient for porous media cross section, then the second part is the contribution of pore pressure gradient for porous media cross section. Introducing the effective pressure conductivity coefficient expression, $\kappa^* = k^*(1 - \Omega / \Phi)$, then Eq. (14) can be rewritten as

$$v = -k^* J + \kappa^* \left(\frac{\mathrm{d}p}{\mathrm{d}L}\right)\Big/\rho g \tag{16}$$

Eq. (16) is termed as the modified Darcy's Law.

It is obvious that the conventional Darcy's law equation, $v = kJ$, is sharply different from the modified Darcy's law equation, $v = -k^* J + \kappa^* \dfrac{1}{\rho g}\dfrac{\mathrm{d}p}{\mathrm{d}L}$, presented in this paper. The conventional Darcy's law provides only one permeability coefficient k, it is called as "conventional permeability coefficient" in this paper for the reason that it has been widely used in most literatures. However, the modified Darcy's law developed in this paper contains two material permeability constants, namely, the effective permeability coefficient $k^*$and the effective pressure conductivity coefficient κ^*, as is described in $\kappa^* = k^*(1-\Omega/\Phi) = k\,(1-\Omega/\Phi)\,\Omega$. If the fluid flows through area ΦA inside porous media, then $\Omega = \Phi$, the effective permeability coefficient equals to the conventional permeability coefficient, and the effective pressure conductivity coefficient equals to zero, that is, $k = k^*$, $\kappa^* = 0$. The modified Darcy's law of porous media given in Eq. (16) will be degenerated into the conventional Darcy's law. Conversely, if the cross sectional area is too compact and no liquid flow through it ($\Omega = 0$), we can obtain $v = 0$from Eq. (16). As we can see from the above discussion, the conventional Darcy's law is a special case of the modified Darcy's law presented in this paper.

4. Case study

In this study, unsaturated slate from Jinping Second stage hydropower station site will be tested. Based on relevant tests (Dai 2006), the initial and final porosity of the unsaturated slate is: Φ_0 = 0.003, Φ_s = 0.01.

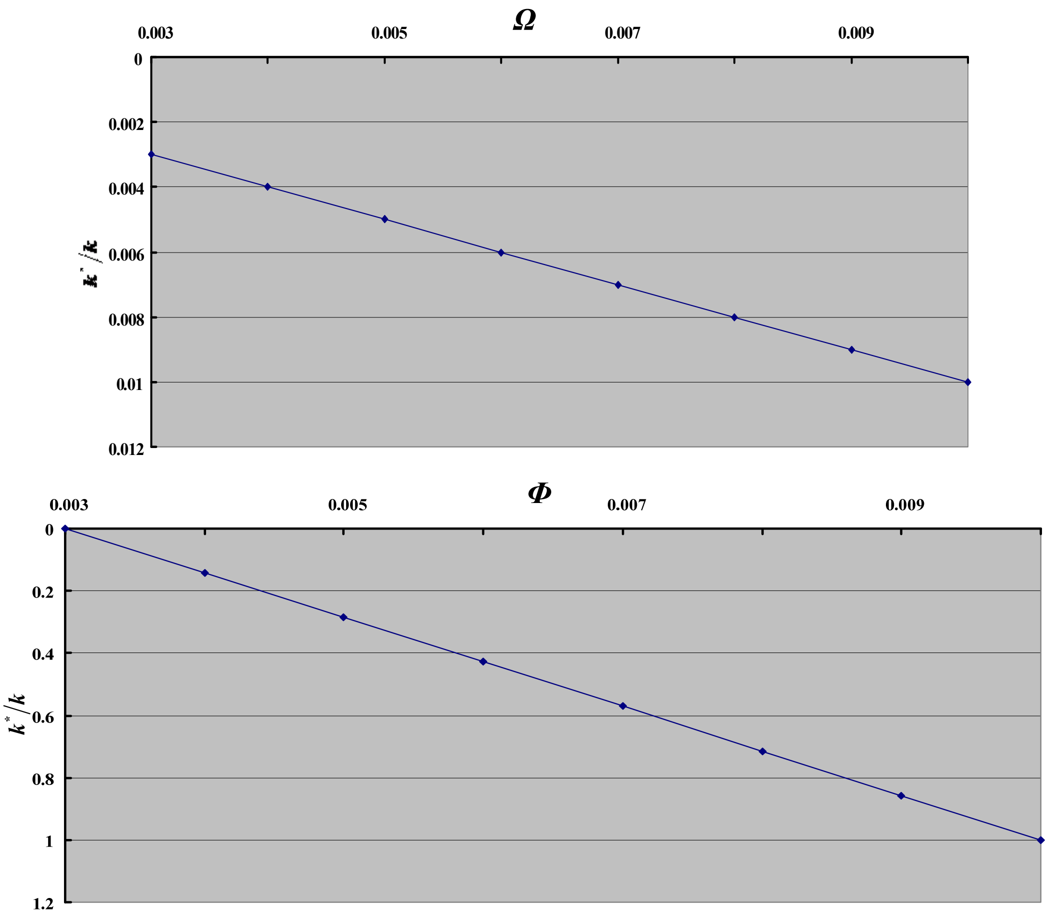

Fig. 2 Curves of the ratio k^*/k varying with Ω or Φ

According to the above mentioned, the ratio between the effective permeability coefficient k^* and the conventional permeability coefficient k of porous media can be expressed as

$$\frac{k^*}{k}=\Omega=\frac{\Phi_0-\Phi}{\Phi_0-\Phi_s}=\frac{0.003-\Phi}{0.003-0.01}=\frac{\Phi-0.003}{0.007} \tag{17a}$$

Fig. 2 shows the relationship between the ratio k^*/k, and porosity Φ or damage variable Ω. It can be seen that the influence of porosity Φ on the ratio k^*/k has the same tendency as damage variable Ω. However, the influence of porosity Φ on the ratio k^*/k is more prominent than damage variable Ω with the same value when the pore size is small enough, and the maximum ratio is about 100.

The ratio between the effective pressure conductivity coefficient κ^* and the conventional permeability coefficient k of porous media can be expressed as

$$\frac{\kappa^*}{k}=\Omega\left(1-\frac{\Omega}{\Phi}\right)=\Omega\left(1-\frac{\Omega}{0.007\Omega+0.003}\right)=\frac{\Phi-0.003}{0.007}\left(1-\frac{1}{\Phi}\frac{\Phi-0.003}{0.007}\right) \tag{17b}$$

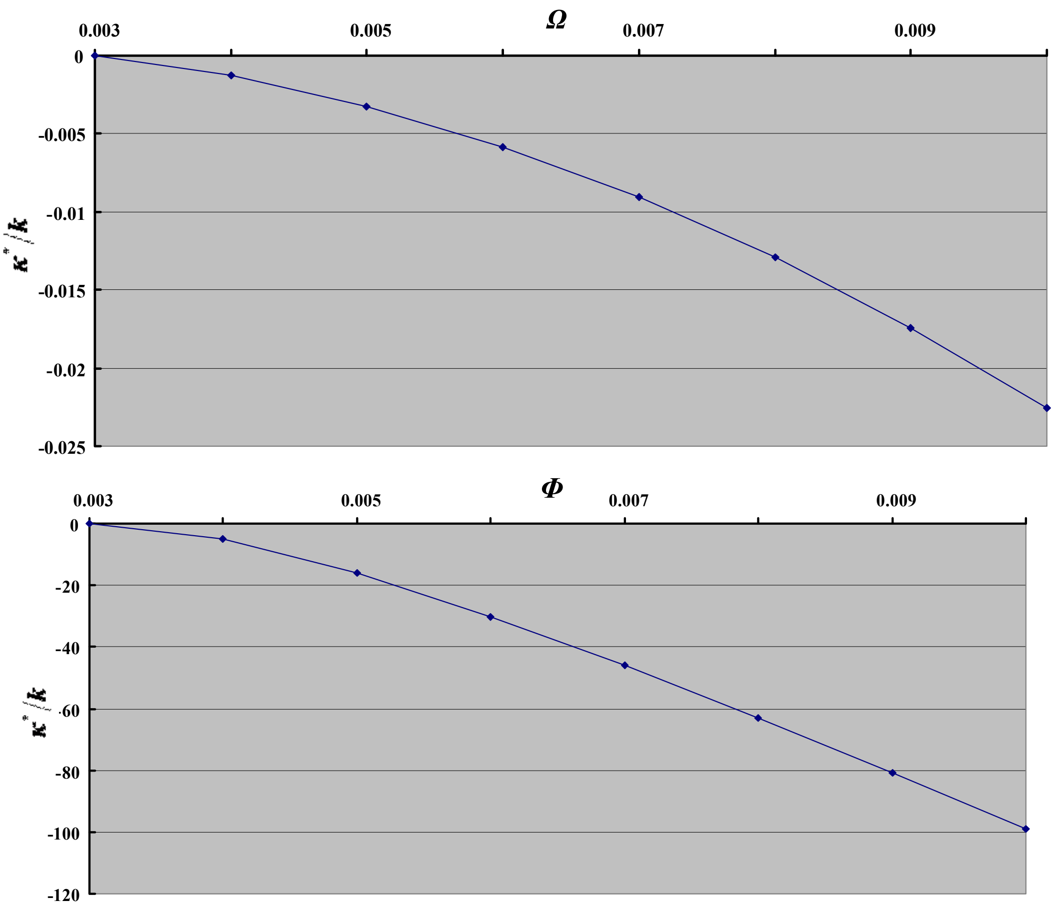

Fig. 3 Curves of the ratio κ^*/k varying with Ω or Φ

Fig. 3 shows the nonlinear relationship between the ratio κ^*/k and porosity Φ or damage variable Ω. It can be seen that the influence of porosity Φ on the ratio κ^*/k also has the same tendency as damage variable Ω. The ratio κ^*/k increases with the increasing of Φ or Ω. Similarly, the influence of porosity Φ on the ratio κ^*/k is more prominent than damage variable Ω.

The ratio between the effective pressure conductivity coefficient κ^* and the effective permeability coefficient k^* of porous medium can be expressed as

$$\frac{\kappa^*}{k} = 1 - \frac{\Omega}{\Phi} = 1 - \frac{\Phi - 0.003}{0.007\Phi} = 1 - \frac{\Omega}{0.007\Omega + 0.003} \tag{17c}$$

Fig. 4 shows the nonlinear relationship between the ratio κ^*/k^* and porosity Φ or damage variable Ω. It can be seen that the influence of porosity Φ on the ratio κ^*/k has the same tendency as damage variable Ω, but the former exhibits nonlinear change while the latter linear change. Similarly, the influence of porosity Φ on the ratio κ^*/k^* is more prominent than damage variable Ω.

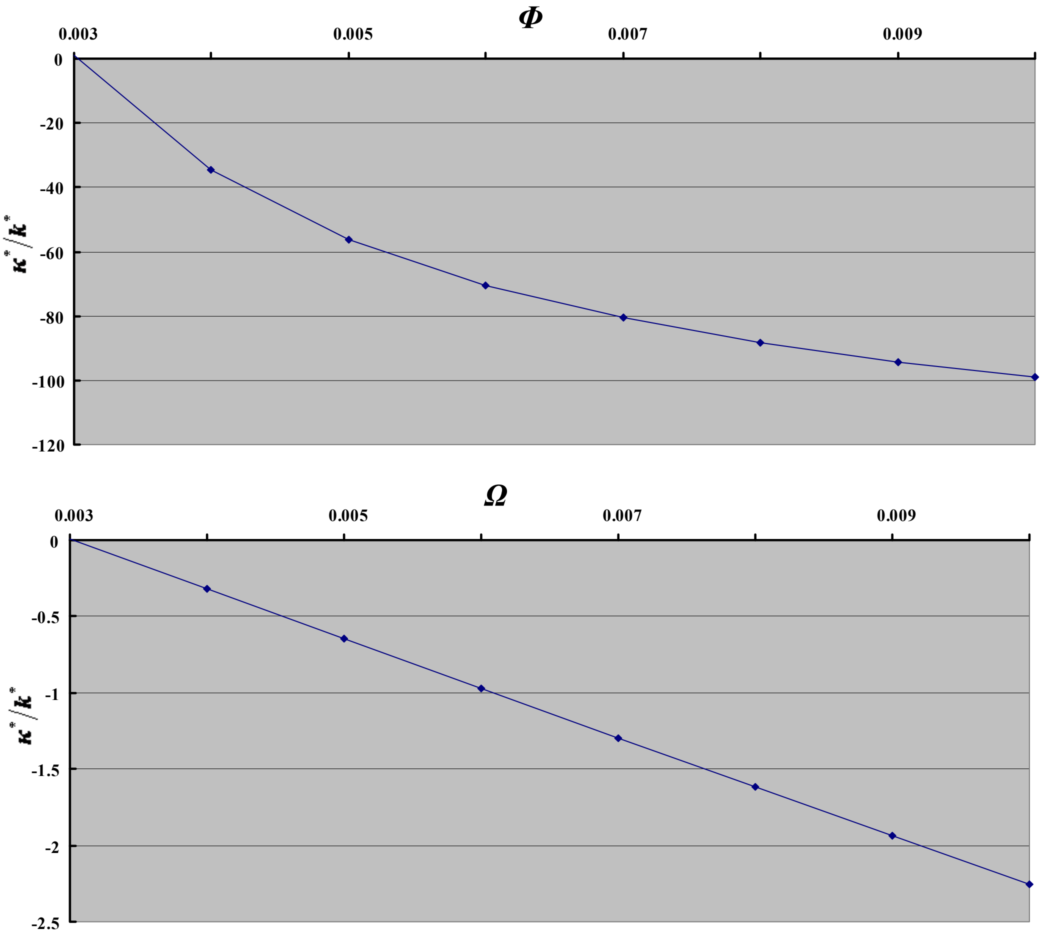

Fig. 4 Curves of the ratio κ^*/k^* varying with Φ or Ω

5. Conclusions

This paper mainly deals with the relationship between the damage variable and permeability properties of a deforming media. Firstly, the relationship between porosity and damage variable is presented. Then, the modified Darcy's law is developed based on the relationship between damage and porosity of porous media, which is considered to be an extension of the Darcy's law for fluid flow and seepage in a porous medium. At last, the properties of permeability of damaged porous media are presented and discussed in this paper. From these studies, following conclusions and understandings may be drawn:

(1) The influence of porosity Φ on the ratio k^*/k has the same tendency as damage variable Ω. However, the influence of porosity Φ on the ratio k^*/k is more prominent than damage variable Ω with the same value when the pore size is small enough, and the maximum ratio is about 100.
(2) The influence of porosity Φ on the ratio κ^*/k also has the same tendency as damage variable Ω. The ratio κ^*/k increases with the increasing of Φ or Ω. Similarly, the influence of porosity Φ on the ratio κ^*/k is more prominent than damage variable Ω.
(3) The influence of porosity Φ on the ratio κ^*/k^* has the same tendency as damage variable Ω, but the former exhibits nonlinear change while the latter linear change. Similarly, the influence of porosity Φ on the ratio κ^*/k^* is more prominent than damage variable Ω.

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