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3D analytical method for mat foundations considering coupled soil springs

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Abstract. The 3D numerical analysis is carried out to investigate the settlement behavior of flexible mat foundations subjected to vertical loads. Special attention is given to the improved analytical method (YS-MAT) that reflects the mat flexibility and soil spring coupling effect. The soil model captures the stiffness of the soil springs as well as the shear interaction between the soil springs. The proposed method has been validated by comparing the results with other numerical approaches and field measurements on mat foundation. Through comparative studies, the proposed analytical method was in relatively good agreement with them and capable of predicting the behavior of the mat foundations.

Keywords: soil-structure interaction; mat foundation; soil spring; coupling effect; settlement

1. Introduction

Mat foundations are usually used as a load distributing element supported by piles or directly placed on soils or rocks having sufficient load-carrying capacity. The mat foundations are cost-effective, with savings up to 20% of the total cost, compared to deep foundations (Briaud 1993).

The structure part of mat foundation can be modelled as a flexible or a rigid plate. The conventional rigid method has been used for practical design of mat foundation. This method assumes a mat to be a rigid body, which does not consider the mat flexibility and the thickness would have to be greater. Also, even very thick ones deflect when loaded by the superstructure loads (Bowles 1997). Alternatively, mat foundation can be designed as the flexible plate. The flexible theory of plates can be categorized as the thin and thick plate theory. In practice, there are two main approaches to model the soil beneath the shallow foundation. These models are known as the Winkler model and the continuum model which makes use of the FE analysis (Dutta and Rana 2002, Colasanti and Horvath 2010).

The continuum model is computationally difficult to exercise and requires extensive training because of the three-dimensional and nonlinear nature of the problem. Also the time consuming, both in modelling and computation, can be exhausting. However, the Winkler model is relatively easy and simple to exercise. For the design and analysis of the flexible mat foundation, the

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conventional spring model can be used because this model only needs one parameter to simulate the soil-structure interaction which is preferred by geotechnical engineers due to its simplicity. In this mode, the soil is related to the Winkler foundation model.

One very popular method for modeling the soil-structure interaction has its origins in the work done by Winkler in 1867. This model is considered to provide vertical reaction by a composition of independent vertical springs. In most cases, when a concentrated load applied to the surface, it must not deflect only under the load, but it also must deflect with displacements diminishing with distance in the adjacent areas. However, the Winkler model assumes that no interaction exists between adjacent points in the soil, therefore, it does not properly represent the characteristics of many practical foundations. To compensate for this limitation, several researchers have been done by considering the interaction between the spring elements (Filonenko-Borodich 1940, Pasternak 1954, Vlasov and Leontiev 1960, Selvadurai 1979, Horvath 2002, Jeong and Seo 2004, Houy *et al*. 2005, Civalek 2007, Colasanti and Horvath 2010). Kerr (1964) stated that the Pasternak foundation can be a possible mathematical model for the generalized foundation, which takes into account the effect of shear interaction among adjacent points in the foundation. Out of these two-parameter foundation models, the Pasternak foundation model is a well-used one (Guler 2004, Calim and Akkurt 2011, Maheshwari and Khatri 2012, Lee *et al*. 2014).

In order to overcome the restrictions of a conventional Winkler model, an improved analytical method (YS-MAT) of the mat foundation design using Pasternak's shear layer model has been proposed. It is intermediate in complexity and theoretical accuracy between the Winkler model and continuum model. The validity of the proposed method is tested through other analytical methods and the field measurement.

2. Proposed analytical method

2.1 Modeling of flexible mat

In the past, the rigid method has often been used for the practical design of mat foundation. This method cannot consider the mat flexibility, because this method assumes a mat to be a rigid body. Much work has been conducted to analyze the effect of mat flexibility. Typically, a plate element has been used as a mat (raft) in several numerical methods (Clancy and Randolph 1993, Poulos 1994, Zhang and Small 2000, Lal *et al*. 2007, Ayvaz and Oguzhan 2008, Darilmaz 2009). However, the plate element has only three degrees of freedom per node (z-axis displacement and x- and y- axis rotations). Therefore, it cannot consider the horizontal (membrane) behavior of a flexible mat because horizontal degrees of freedom (x- and y-axis) are excluded.

In order to consider the flexibility of mat, in this study, a flat shell element (Choi and Lee 1996, Won *et al*. 2006, Jeong and Cho 2014) was adopted. It was obtained by combining a Mindlin plate element and a membrane element with torsional degrees of freedom as shown in Fig. 1. The displacement field of plate element can be explained in terms of vertical displacement (z-axis) and rotations (x- and y- axis), and that of membrane element can be described in terms of horizontal displacements (x- and y- axis) and rotation (z- axis). Consequently, we can consider the mat flexibility by using the flat shell element.

The stiffness matrix of the flat-shell element ($k_{flat\text{ - }shell\,=\,mat}$) was constructed by combining the stiffness matrix of the plate element (k_{plate}) and that of the membrane element ($k_{membrane}$). The stiffness matrix of the plate element (k_{plate}) is represented as

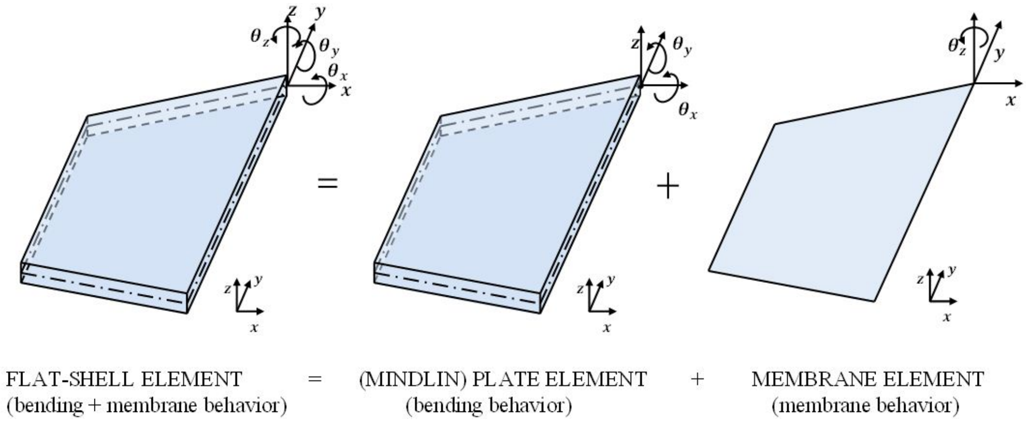


Fig. 1 Modeling of flexible mat (Flat-shell element)

$$k_{plate} = \int_V B_b^T D_b B_b \, dV + \int_V B_s^T D_s B_s \, dV \tag{1}$$

where, B_b is the bending strain matrix and B_s is the shear strain matrix. For an isotropic material, D_b and D_s are

$$D_b = \frac{Et^3}{12(1-v^2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \dfrac{(1-v)}{2} \end{bmatrix} \tag{2a}$$

$$D_s = \frac{\Psi Et}{2(1+v)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2b}$$

where, E is the Young's modulus, t is the thickness of the plate, v is the Poisson's ratio, and is the shear correction factor (5/6 for rectangular cross-section).

Next, the stiffness matrix of the membrane element $k_{membrane}$ is represented as

$$k_{membrane} = \int_\Omega \left[B_m \overline{G} \;\; \overline{R}\right]^T \cdot C \cdot \left[B_m \overline{G} \;\; \overline{R}\right] d\Omega + \frac{\gamma}{\Omega} hh^T \tag{3a}$$

$$h = \int_\Omega \left\langle b_m \; ; g \; ; r \right\rangle^T d\Omega \tag{3b}$$

$$\gamma = \frac{E}{2(1+v)} \tag{3c}$$

where, C is the constitutive modulus, γ is the shear modulus. B_m, $\overline{G}$, and $\overline{R}$ are the strain matrices representing the relationship between the displacements and the strains. b_m, g, and r are also the strain matrices for the infinitesimal rotation fields.

2.2 Soil-structure interaction

Much research has been done to find a physically close and mathematically simple representation of a soil-structure interaction. Most of the previous work began with the Winkler's model. This model is frequently referred to as a one parameter model, and it is expressed by the following equation

$$p\,(x,\, y) = k_s\, w\,(x,\, y) \tag{4}$$

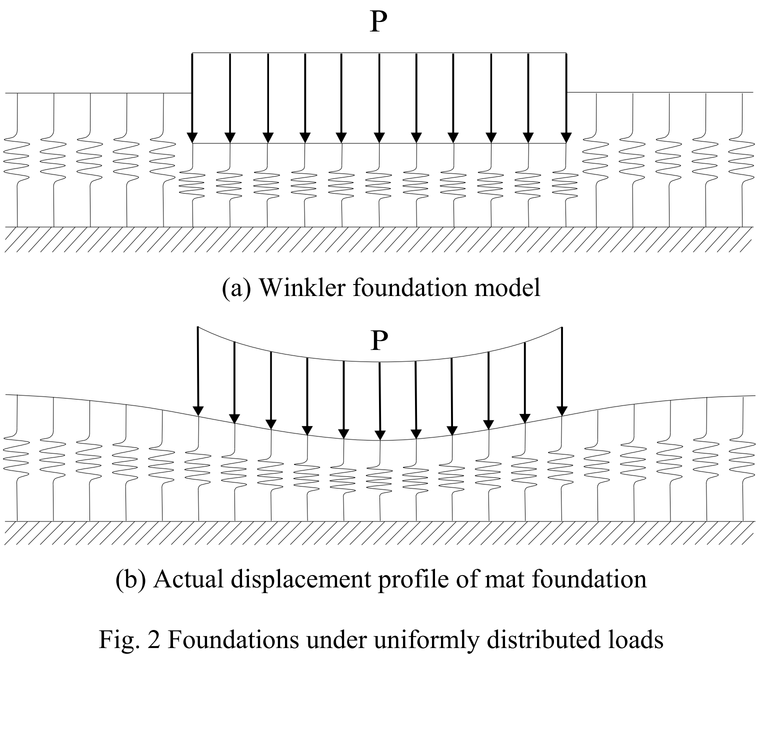
where, $p(x, y)$ is the reactive pressure of the foundation, k_s is the coefficient of subgrade reaction (Winkler foundation modulus), and $w(x, y)$ is the settlement.

This model has a displacement discontinuity between the loaded and the unloaded part of the foundation surface, therefore, this model cannot transmit the shear stresses derived from the lack of spring coupling. A demerit of the model is that it does not take into account the interaction between the soil springs. In reality, the soil surface does not show any discontinuity (Fig. 2).

In this study, Pasternak's shear layer model (1954) was incorporated for the soil spring-coupling effects. In order to introduce continuity of vertical displacements, this model assumed the shear interactions between the spring elements are existent (Fig. 3). Pasternak model provides mechanical interactions between the individual soil springs and flat shell element, and thus it shows a more realistic behavior of the soil reaction. As a result, the proposed analytical method can be represented the coupled soil-structure interactions.

2.3 Method of analysis by coupled soil resistance

As mentioned, the typically used Winkler's hypothesis ignores the effect of shear within subgrade. Thus, the continuity of the soil mass is not properly taken into account. Therefore, in



(b) Actual displacement profile of mat foundation

Fig. 2 Foundations under uniformly distributed loads

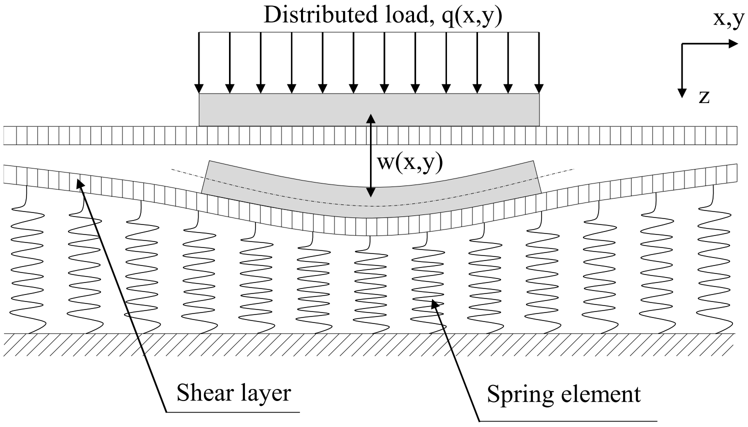


Fig. 3 Pasternak foundation model

this study, Pasternak's subgrade model was incorporated to involve the soil spring-coupling effects as

$$p(x, y) = k_s w(x, y) - k_g \nabla^2 w(x, y) \tag{5}$$

where, k_s is the coefficient of subgrade reaction (Winkler foundation modulus), k_g is the constant to consider the shear within subgrade, and ∇^2 is the Laplacian operator.

The spring-coupling effects are then incorporated in terms of the second derivative of displacement. Also, the values of k_s and k_g parameter can be determined by using a methodology given by Selvadurai (1979), as shown in Eq. (6).

$$k_s = \frac{E_s}{H_s(1+v_s)(1-2v_s)} \tag{6a}$$

$$k_g = \frac{E_s H_s}{6(1+v_s)} \tag{6b}$$

where, E_s is the elastic modulus of the subgrade soil, and v is Poisson's ratio of soil. H_s is the thickness of soil, which corresponds to the end of the influence zone for the foundation, based on Boussinesq's method (= 2.0B).

Eq. (5) can be expressed as follows

$$\{p\} = [k_s]\{w\} - [k_g]\nabla^2\{w\} \tag{7}$$

where, $\{p\}$ is the external load vector, $[k_s]$ is the individual soil stiffness matrix, $[k_g]$ is the coupled-soil stiffness matrix, and $\{w\}$ is the displacement vector.

A finite-difference approximation of Eq. (7) by taking into account the soil coupling can be written as

$$\{p\}_{i,j} = [k_s]_{i,j}\{w\}_{i,j} - [k_g]_{i,j}\nabla^2\{w\}_{i,j} \tag{8}$$

Next, using forward-, central-, and backward-difference schemes for discretizing the second-order derivative terms (soil coupling), respectively, and then gives

$$\nabla^2 \{w\}_{i,j} = \frac{1}{\Delta x^2}\left(\{w\}_{i+1,j} - 2\{w\}_{i,j} + \{w\}_{i-1,j}\right) + \frac{1}{\Delta y^2}\left(\{w\}_{i,j+1} - 2\{w\}_{i,j} + \{w\}_{i,j-1}\right) \tag{9}$$

The final governing equation for the soil stiffness (k_{soil}) is obtained by combining the stiffness of a series of individual soil springs (k_s) and the coupled-soil spring stiffness (k_g) as follows

$$\{p\}_{i,j} = [k_s]_{i,j}\{w\}_{i,j} - [k_g]_{i,j}\left\{\frac{1}{\Delta x^2}\left(\{w\}_{i+1,j} - 2\{w\}_{i,j} + \{w\}_{i-1,j}\right) + \frac{1}{\Delta y^2}\left(\{w\}_{i,j+1} - 2\{w\}_{i,j} + \{w\}_{i,j-1}\right)\right\} \tag{10}$$

Finally, the stiffness matrix of mat foundation is defined by the combination of the mat and the supporting soil. Therefore, the global stiffness matrix of a mat foundation system can be written as

$$[K] = \left[k_{flat\text{-}shell}\right] + \left[k_{soil}\right] \tag{11}$$

Idealized 3D model of present study is shown in Fig. 4.

2.4 Algorithm

An improved analytical method was proposed on the basis of modeling for mat flexibility and coupled soil springs. This analytical method has been incorporated in a computer software program called YS-MAT. For given conditions, such as the geometry, the loads, the properties of the mat foundation, and stiffness of soil springs, the displacements and internal forces (the bending moments and shear forces) on mat foundation can be calculated by YS-MAT. An algorithm of YS-MAT is shown in Fig. 5.

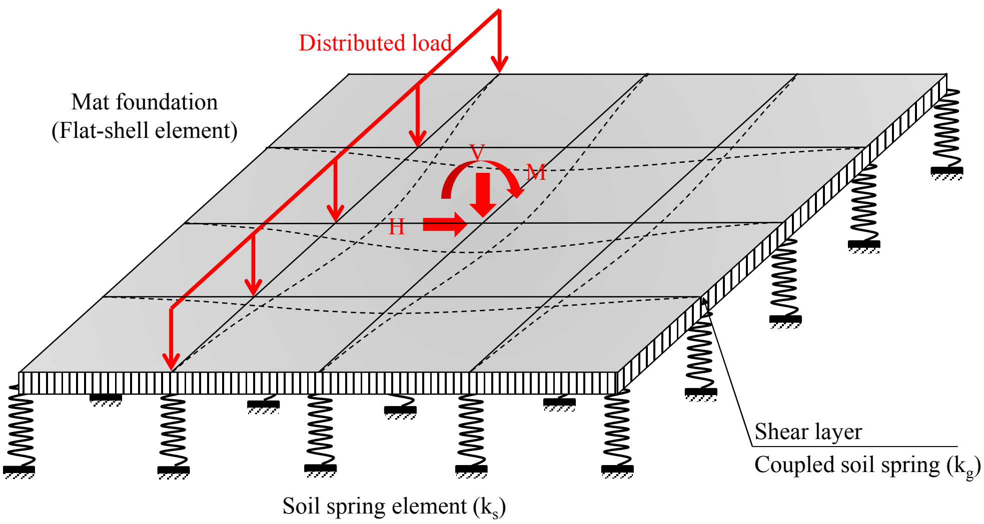


Fig. 4 Idealized 3D model for mat foundation used in proposed method (YS-MAT)

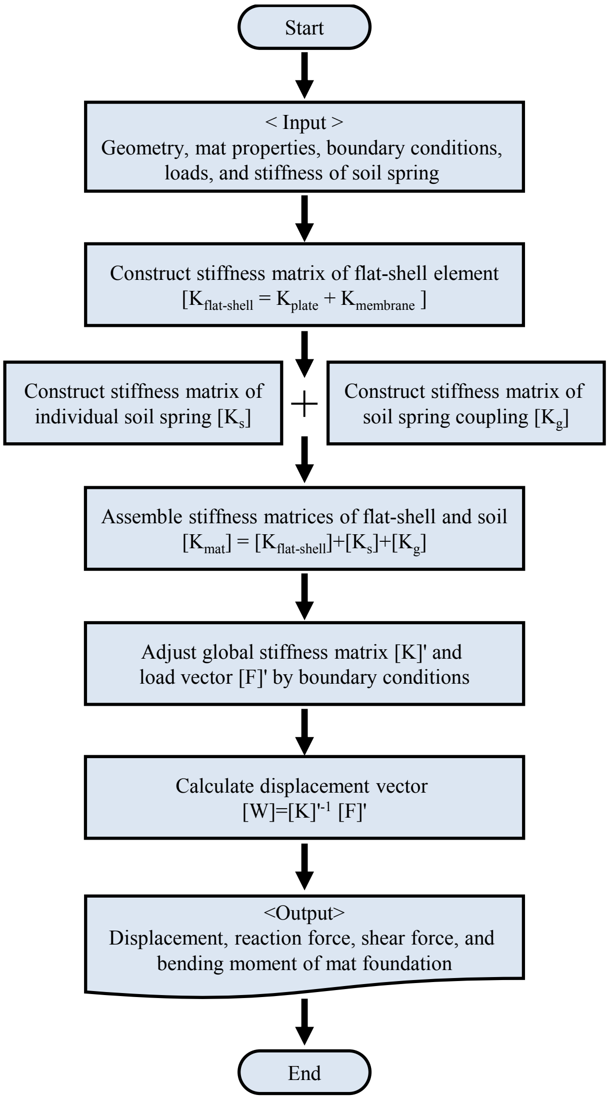
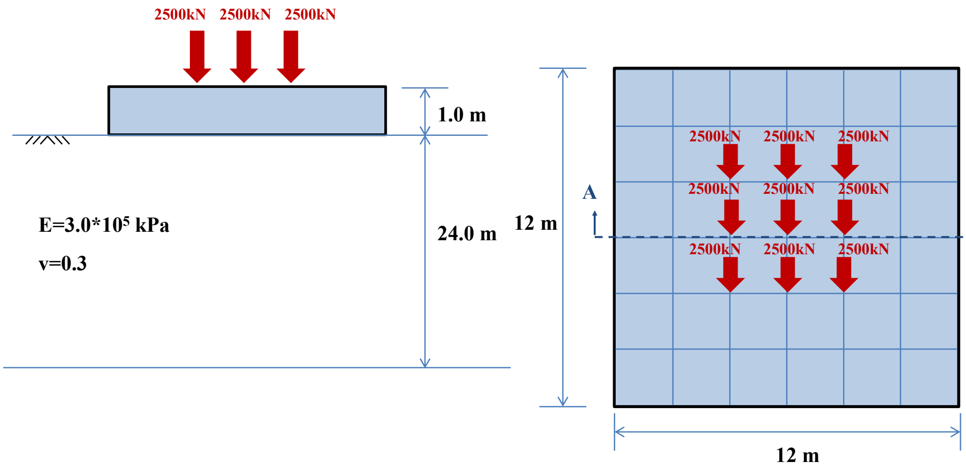


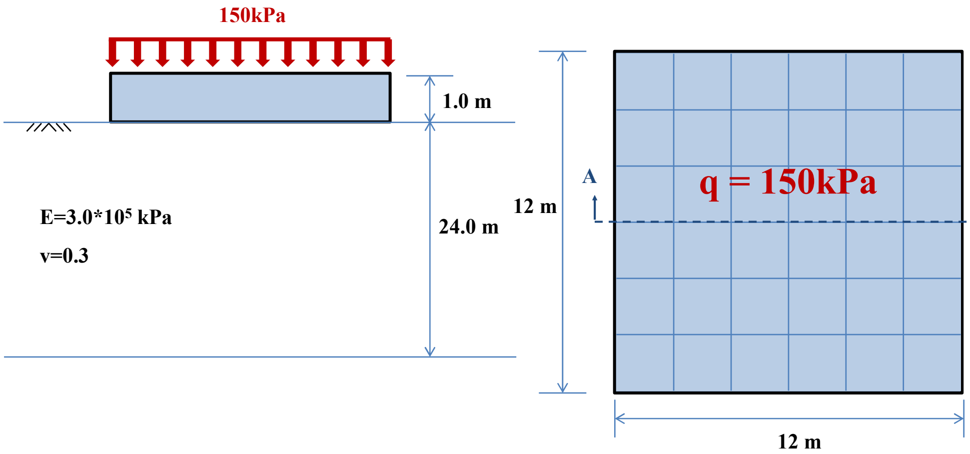
Fig. 5 Flow chart of YS-MAT

3. Validation of proposed method with numerical analysis

YS-MAT was validated against well-known existing numerical methods: finite element method (FEM), using the commercial package ABAQUS (Dassault Systemes 2012) and plate on spring analysis, using the program GEO5 Plate (Fine 2012). Validation exercises were undertaken for two different loading cases: (1) concentrated load; (2) uniformly distributed load. The response of mat foundation is presented in terms of settlement and bending moment distributions. Figures of both cases are shown in Fig. 6. A square mat of size 12 × 12 m with a thickness of 1.0 m is rested on a homogeneous soil. Table 1 shows the material properties used in this study. In the GEO5 Plate program, the mat was modelled as a plate element. Therefore, the element cannot consider the membrane behavior of a mat. However, in YS-MAT, flat shell element was used for the modeling of the flexible mat.



(a) Concentrated load



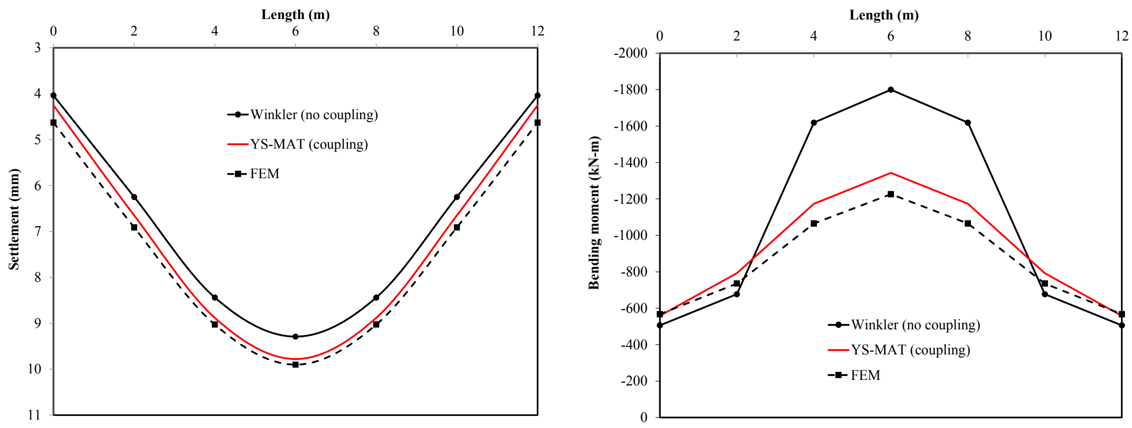
(b) Uniformly distributed load

Fig. 6 Schematic diagram of mat foundation

Table 1 Material parameters used for numerical analysis

Material	E (MPa)	v	γ (kN/m^3)	H_s (m)	Model
Mat	30,000	0.2	24	-	L.E. **
Rock	300	0.3	22	24	L.E.
	k_s (kPa/m) *		k_g (kN/m) *		YS-MAT
	24,038		923,077		

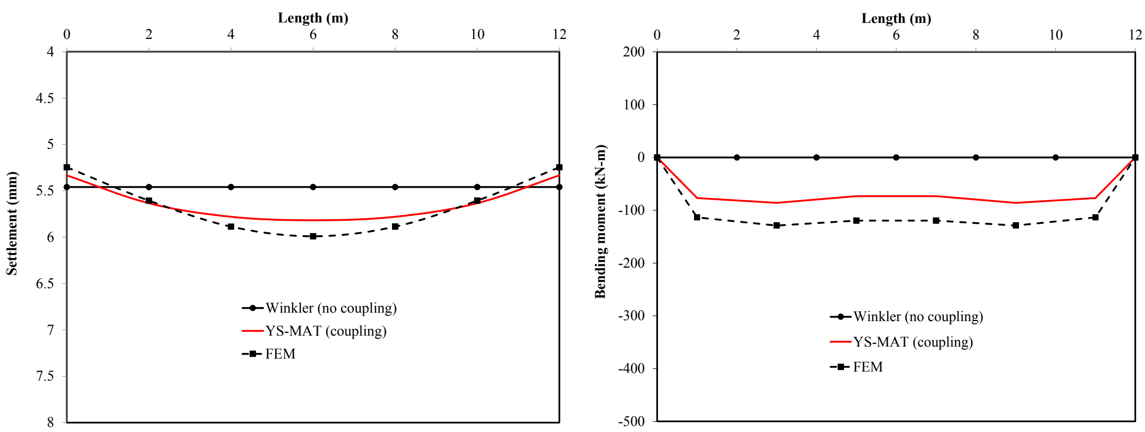
* k_s, k_g: Estimated by Eq (6)
** L.E. is linear elastic model



(a) Settlement (b) Bending moment

Fig. 7 Comparison of settlement results along centerline of mat foundation for nine concentrated loads

First numerical example is the concentrated load (Fig. 6(a)). Nine concentrated loads of P = 2,500 kN placed at the center and around the center of the mat foundation are considered symmetrically. The settlements along the centerline are presented in Fig. 7(a). It is found that the proposed analytical method shows a reasonably good agreement with other methods, and closely approaches the settlement of FEM rather than plate on spring method (Winkler analysis). Also, the bending moment distributions along the centerline are shown in Fig. 7(b). For the results of FEM and YS-MAT, the flexibility of mat is considered by using shell elements, while for that of Winkler analysis, the flexibility of mat is not considered. Additionally, the combination of the shell elements and the shear interaction between the springs is reflected in the proposed method, resulting in lower bending moment of the mat, compared to the Winker model. The proposed method using the coupling effect predicts accurately the general trend from FEM.



(a) Settlement (b) Bending moment

Fig. 8 Comparison of results along centerline of mat foundation for uniformly distributed load

Second numerical example is the uniformly distributed load (Fig. 6(b)). The settlement and bending moment are plotted in Fig. 8. The existing spring analysis method based on the Winkler foundation model gives a uniform displacement and moments equal to zero. This is because the foundation does not have any curvature due to the fact that the soil springs are not coupled to each other. On the other hand, the settlement results by the proposed method differ from that by the Winkler model which has a uniform displacement. The proposed method gives a dish-shaped settlement of the mat foundation which would be expected in a real situation, and is in good agreement with literature reviews (Straughan 1990, Vallabhan and Das 1991, Dutta and Rana 2002), because the soil springs do interact with each other. It is found that the proposed analytical method closely approaches the settlement and bending moment of FEM than Winkler analysis. Therefore, it is thought that YS-MAT can be used with some confidence in the preliminary design of mat foundations.

4. Comparison with field measurement data

Validation was also undertaken against field data form the literatures. The mat and soil properties used the same as their research reports. The measured settlement of the mat foundation reported by Johnson (1989) is compared with the predicted values from YS-MAT and FEM. The test site was located in the northwest sector of Lackland Air Force Base near San Antonio, Texas. The large mat is 33×64 m with a thickness of 1.06 m and supports the 11 story Wilford Hall Hospital. The applied uniform pressure of 115.63 kPa was applied over the whole mat area, and the mat was installed in clay soils. Fig. 9 shows the schematic figures of mat foundation and

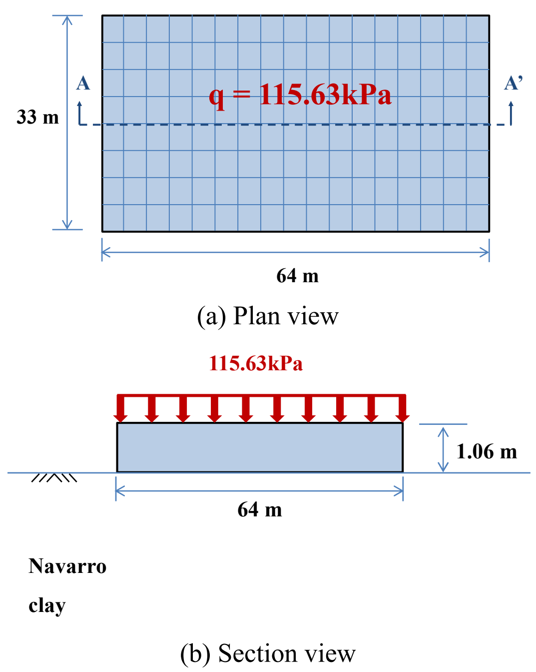


Fig. 9 Wilford Hall Hospital

Table 2 Material parameters used for a field case

Case	Material Properties										
	Type		Depth (m)	E (MPa)	v	γ (kN/m^3)	ϕ (deg.)	c (kPa)	H_s (m)	Model **
Johnson (1989)	Mat	Concrete	0 ~ 1.06	30,000	0.2	24	-	-		L.E.
	Soil	Clay	0 ~ -16.2	140.9	0.3	17.3	27	287	66	M.C
			k_s (kPa/m) *			k_g (kN/m) *				YS-MAT
			3,770			1,192,324				

* k_s: The value obtained from Johnson (1989), k_g: Estimated by Eq. (6b)
** M.C. is Mohr Coulomb elasto-plastic model, L.E. is linear elastic model

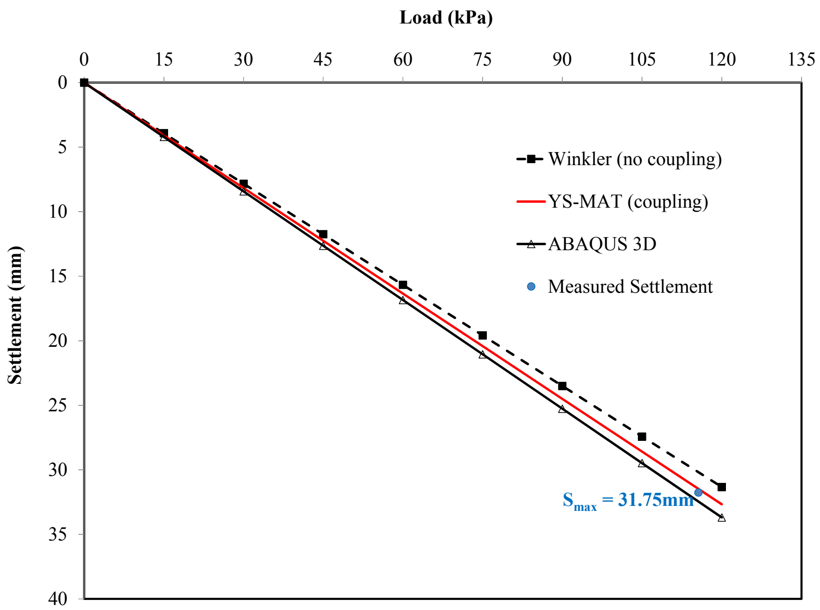


Fig. 10 Settlement behavior of large mat foundation

subsurface profile. The input parameters are summarized in Table 2.

Fig. 10 shows a series of settlement curve of the large mat foundation. The measured maximum settlement is about 31.75 mm, Winkler foundation is 30.11 mm, YS-MAT is 31.48 mm, and ABAQUS 3D is 32.46 mm. These numerical methods provide an acceptable design prediction. The proposed methodology YS-MAT approximately predicts the settlement of mat foundation when compared with the results from Winkler foundation.

5. Conclusions

The main objective of this study is to propose an improved analytical method for analyzing a mat foundation that can consider mat flexibility and soil coupling effect. Through comparisons with other numerical methods and field measurement, it is found that the proposed analytical method is in good agreement with measured data. On the basis of the findings of this study, the following conclusions are drawn:

- The analytical method is intermediate in complexity and theoretical accuracy between general three-dimensional FE analysis (ABAQUS 3D) and the conventional analysis method (Winkler spring model).
- By taking into account the mat flexibility and soil coupling, the proposed analytical method is an appropriate and realistic representation of the settlement behavior of flexible mat foundation. It provides results that are in good agreement with the field measurement and numerical analyses.
- Proposed analytical method produces a relatively larger settlement of mat foundation than the results obtained by the existing method. Also, the settlement and bending moment of mat foundation obtained by the proposed method is similar to that of ABAQUS 3D when compared with the results of the existing method.
- Compared to the results of the field measurement, the proposed method is shown to be capable of predicting the settlement of a large mat foundation. The membrane action of flat shell element and soil coupling effect can overcome the limitations of conventional method. Therefore, the proposed method could be used in the preliminary design of large mat foundation.

Acknowledgments

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References

ABAQUS (2012), Theory Manual, Version 6.12; Published by Hibbitt, Karlsson and Sorensen Inc., USA.

Ayvaz, Y. and Oguzhan, C.B. (2008), "Free vibration analysis of plates resting on elastic foundations using modified Vlasov model", *Struct. Eng. Mech., Int. J.*, **28**(6), 635-658.

Bowles, J.E. (1997), *Foundation Analysis and Design*, McGraw-Hill, New York, NY, USA.

Briaud, J.L. (1993), "Spread footing design and performance", (Contribution to a short course at the occasion of the 10th annual International Bridge Conference, Department of Civil Engineering, Texas A&M University, College Station, TX, USA.

Calim, F.F. and Akkurt, F.G. (2011), "Static and free vibration analysis of straight and circular beams on elastic foundation", *Mech. Res. Commun.*, **38**(2), 89-94.

Choi, C.K. and Lee, W.H. (1996), "Versatile variable-node flat-shell element", *J. Eng. Mech., ASCE*, **122**(5), 432-441.

Civalek, O. (2007), "Nonlinear analysis of thin rectangular plates on Winkler-Pasternak elastic foundations by DSC-HDQ methods", *Appl. Math. Model.*, **31**(3), 606-624.

Clancy, P. and Randolph, M.F. (1993), "An approximate analysis procedure for piled raft foundations", *Int. J. Numer. Anal. Method. Geomech.*, **17**(12), 849-869.

Colasanti, R.J. and Horvath, J.S. (2010), "A practical subgrade model for improved soil-structure interaction analysis: Software implementation", *Pract. Period. Struct. Des. Constr.*, **15**(4), 278-286.

Darilmaz, K. (2009), "An assumed-stress hybrid element for modeling of plates with shear deformations on elastic foundation", *Struct. Eng. Mech., Int. J.*, **33**(5), 573-588.

Dutta, S.C. and Rana, R. (2002), "A critical review on idealization and modeling for interaction among soil-foundation-structure system", *Comput. Struct.*, **80**(20-21), 1579-1594.

Filonenko-Borodich, M.M. (1940), "Some approximate theories of elastic foundation", Uchenyie Zapiski Moskovskogo Gosudarstvennogo Universiteta, *Mekhanica*, **46**, 3-18. [In Russian]

GEO 5 manual (2012), User's guide; Version 16. Fine Ltd.

Guler, K. (2004), "Circular elastic plate resting on tensionless Pasternak foundation", *J. Eng. Mech.*, **130**(10), 1251-1254.

Houy, L., Breysse, D. and Denis, A. (2005), "Influence of soil heterogeneity on load redistribution and settlement of a hyperstatic three-support frame", *Geotechnique*, **55**(2), 163-170.

Horvath, J.S. (2002), *Soil-Structure Interaction Research Project*; *Basic SSI Concepts and Applications Overview*; Report No. CGT-2002-2, Manhattan College, School of Engineering, New York, NY, USA.

Jeong, S.S. and Cho, J.Y. (2014), "Proposed nonlinear 3-D analytical method for piled raft foundations", *Comput. Geotech.*, **59**, 112-126.

Jeong, S.S. and Seo, D.H. (2004), "Analysis of tieback walls using proposed *P–y* curves for coupled soil springs", *Comput. Geotech.*, **31**(6), 443-456.

Johnson, L.D. (1989), "Design and construction of mat foundations", Miscellaneous Paper GL-89-27, US Army Corps of Engineers, Washington, D.C., USA.

Kerr, A.D. (1964), "Elastic and viscoelastic foundation models", *J. Appl. Mech.*, **31**(3), 491-498.

Lal, A., Singh, B.N. and Kumar, R. (2007), "Natural frequency of laminated composite plate resting on an elastic foundation with uncertain system properties", *Struct. Eng. Mech.*, *Int. J.*, **27**(2), 199-222.

Lee, J.K., Jeong, S.S. and Lee, J.H. (2014), "Natural frequencies for flexural and torsional vibrations of beams on Pasternak foundation", *Soil. Found.*, **54**(6), 1202-1211.

Maheshwari, P. and Khatri, S. (2012), "Nonlinear analysis of infinite beams on granular bed-stone column-reinforced earth beds under moving loads", *Soil. Found.*, **52**(1), 114-125.

Pasternak, P.L. (1954), "On a new method of analysis of an elastic foundation by means of two constants", Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvui Arkhitekture, Moscow. [In Russian]

Poulos, H.G. (1994), "An approximate numerical analysis of pile-raft interaction", *Int. J. Numer. Anal. Method. Geomech.*, **18**(2), 73-92.

Selvadurai, A.P.S. (1979), *Elastic Analysis of Soil-Foundation Interaction*, Elsevier Scientific Publishing Company, Amsterdam, Netherlands.

Straughan, W.T. (1990), "Analysis of plates on elastic foundations", Dissertation; Texas Tech University, Lubbock, TX, USA.

Vallabhan, C.V.G. and Das, Y.C. (1991), "Modified Vlassov model for beams on elastic foundation", *J. Geotech. Eng.*, **117**(6), 956-966.

Vlasov, V.Z. and Leontiev, N.N. (1960), *Beams*, *Plates and Shells on an Elastic Foundation*, USSR: Fizmatgiz, Moscow, Russia. [In Russian]

Winkler, E. (1867), *Theory of Elasticity and Strength*, (H. Dominicus), Prague, Czech Republic.

Won, J.O., Jeong, S.S., Lee, J.H. and Jang, S.Y. (2006), "Nonlinear three- dimensional analysis of pile group supported columns considering pile cap flexibility", *Comput. Geotech.*, **33**(6-7), 355-370.

Zhang, H.H. and Small, J.C. (2000), "Analysis of axially and laterally loaded pile groups embedded in layered soils", *Proceedings of the 8th Australia-New Zealand Conference on Geomechanics*, Hobart, **1**, 475-483.

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