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Probability-based design charts for stone column-improved ground

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Abstract. A simplified probability-based design charts for stone column-improved ground have been presented based on the unit cell approach. The undrained cohesion (c_u) and coefficient of radial consolidation (c_r) of the soft soil are taken as the most predominant random variables. The design charts are developed to estimate the diameter of the stone column or the spacing between the stone columns by employing a factored design value of c_r and c_u so as to satisfy a specific probability level of the target degree of consolidation and/or a target safe load that needs to be achieved in a specified timeframe. The design charts can be used by the practicing engineers to design the stone column-improved ground by considering consolidation and /or bearing capacity of the improved ground.

Keywords: bearing capacity; consolidation; design charts; probability; stone column-improved ground; uncertainty

1. Introduction

Stone column is one of the most effective techniques to improve the soft soil strata by increasing the bearing capacity and reducing the settlement of the soil. It also acts as a drainage path to accelerate the consolidation of clay. In recent years, many studies have been carried out to understand the behaviour of foundations reinforced with stone columns (Madhav and Vitkar 1978, Balaam and Booker 1981, Alamgir *et al*. 1996, Poorooshasb and Meyerhof 1997, Lee and Pande 1998, Muir-Wood *et al*. 2000, Ambily and Gandhi 2007, Elshazly *et al*. 2007, Krishna *et al*. 2007, Black *et al*. 2007, Deb 2008, Bouassida *et al*. 2009, Shahu and Reddy 2011, Deb *et al*. 2011, Deb and Dhar 2011, 2013). However, most of the reported studies are based on deterministic approach.

The degree of consolidation (at a specific time period) and bearing capacity achieved by the stone column-improved ground are controlled by the soil properties and stone column dimension. Some of the soil and stone column properties are uncertain due to the variation of soil deposit, measurement error and transformation errors. Thus, it is required to develop probability based design methodology for stone column-improved ground by taking into account the uncertainty in design variables. Hong and Shang (1998) and Zhou *et al*. (1999) suggested that in case of PVD-improved ground (prefabricated vertical drain), the horizontal or radial coefficient of

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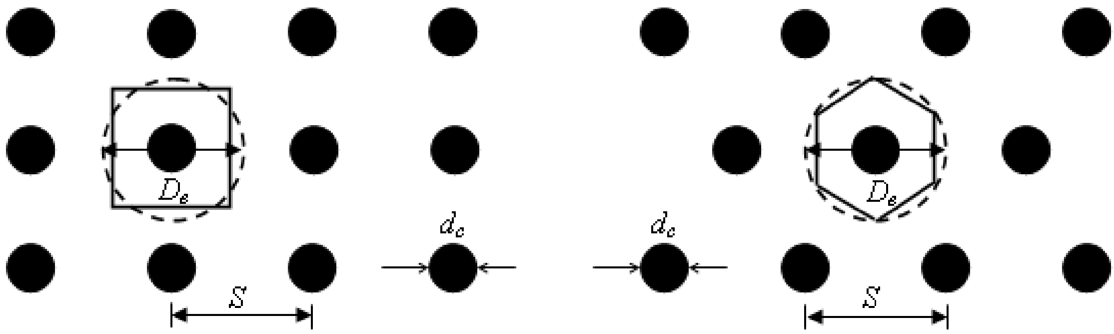
consolidation is the most important random variable affecting the degree of consolidation. It is also shown that the type of probability distribution of random variables has significant effect on degree of consolidation. Zhou *et al*. (1999) also presented a probability-based design methodology to design the PVD-improved ground for achieving a specific target reliability level of degree of consolidation in a specific time period. Alonso and Jimenez (2011) observed that for stone column-improved ground also radial coefficient of consolidation has the highest influence on the reliability results. Bari *et al*. (2013) conducted reliability analysis of soil consolidation via PVD by considering the inherent spatial variability of soil properties. Bari and Shahin (2014) proposed a simplified probabilistic method in which the inherent variability of the coefficient of consolidation of the soil is considered. An easy-to-use design procedure and charts are also provided for routine use by practitioners. Thus, it is observed that most of the reliability analysis of soil consolidation via vertical drain is conducted on PVD-improved ground. Limited studies are conducted on stone column-improved ground. The major difference between the PVD and stone columns is that stone columns have larger drained elastic modulus than the surrounding soft soil as compared to the PVD or sand drain (Han and Ye 2000). Thus, stone column is not only used to increase the consolidation rate, but also used for bearing capacity improvement of the soft soil. In the reported probability-based design methodology of PVD-improved ground, only consolidation of the improved ground is taken into consideration. However, in case of stone column-improved ground both consolidation and bearing capacity of the improved ground have to be taken into design consideration. In the present paper, probability-based design charts for stone column-improved ground have been presented according to the methodology proposed by Zhou *et al*. (1999) by considering both strength and consolidation properties of the improved ground as random variables. In the present study, the horizontal or radial coefficient of consolidation and undrained shear strength or undrained cohesion of soft soil are considered to be uncertain due to the inherent variability of soil deposits and measurement errors. The range of coefficient of variation (COV) of the uncertain variable of horizontal or radial coefficient of consolidation (c_r) value is taken as 0.1 to 1.0 according to Lee *et al*. (1983) and Zhou *et al*. (1999). The range of coefficient of variation (COV) of the uncertain variable undrained shear strength or undrained cohesion of soft soil value is taken as 0.2 to 0.5 as suggested by Baecher and Christian (2003).

2. Degree of consolidation

The simplified method developed by Han and Ye (2000) to determine the rate of consolidation of stone column-improved ground has been used in the present study. In the present study, only radial consolidation is considered to prepare the design charts as in case of stone column-improved ground for a particular period of time major contribution of total consolidation is due to radial flow. The radial rate of consolidation can be expressed as (Han and Ye 2000)

$$U_r = 1 - \frac{8}{\pi^2}\exp^{-[8/F(N)]T_r'} \tag{1}$$

where $T_r' = c_r' t/D_e^2$, a modified time factor in the radial flow; t is the time; d_c is the diameter of stone column, $F(N) = [N^2/(N^2-1)]\ \ln(N) - (3N^2-1)/(4N^2)$; $N = D_e/d_c$ diameter ratio; D_e is the diameter of the influence zone (as shown in Fig. 1); $D_e = 1.05S$ and $1.13S$ for triangular and square arrangement of stone column, respectively; S is the spacing between the stone column; c_r' is the



(a) Square arrangement (b) Traiangular arrangement

Fig. 1 Arrangements of stone columns

modified coefficient of radial consolidation and can be expressed as

$$c_r' = c_r\left(1 + n_s \frac{1}{N^2 - 1}\right) \tag{2}$$

where c_r is the coefficient of radial consolidation; n_s is the steady-stress concentration ratio as the consolidation is completed and can be expressed as

$$n_s = \xi \frac{E_c}{E_s} \tag{3}$$

where E_c and E_s are the modulus of elasticity of stone column material and soft soil, respectively; E_c / E_s is called as modular ratio; ξ is the Poisson's ratio factor and can be expressed as

$$\xi = \frac{(1+\mu_s)(1-2\mu_s)(1-\mu_c)}{(1+\mu_c)(1-2\mu_c)(1-\mu_s)} \tag{4}$$

where μ_c and μ_s are the Poisson's ratio of stone column material and soft soil, respectively. In the present analysis, both the Poisson's ratios are taken as same (0.3) as adopted by Balaam and Booker (1981). Thus, the material parameters those influence the rate of consolidation for stone column-improved ground are coefficient of radial consolidation, modulus of elasticity of stone column material and soft soil.

3. Load carrying capacity of the stone column

The load carrying capacity of the stone column is determined by the recommendation as per IS code (IS 15284 (Part 1) 2003). According to IS 15284-I (2003), the limiting axial stress in the stone column (q_{ult}) considering that the foundation soil is at failure when stressed horizontally due to bulging of the column can be written as

$$q_{ult} = Kp_{col}\sigma_{rL} \tag{5}$$

where $Kp_{col} = \frac{(1+\sin\phi_c)}{(1-\sin\phi_c)}$ and σ_{rL} is the limiting radial stress can be expressed as

$$\sigma_{rL} = \left(4c_u + \sigma_{r0}\right) \tag{6}$$

where c_u is the undrained cohesion of the surrounding clay, σ_{r0} is the initial effective radial stress equal to $K_0\sigma_{vo}$, where K_0 is the average coefficient of lateral earth pressure for clay equal to 0.6 or alternatively, as determined from the relationship K_0= (l-sinϕ), where ϕ is the effective angle of internal friction of surrounding soil (in the present study, K_0 is taken as 0.6), σ_{v0} is the average initial effective vertical stress considering an average bulge depth as 2 times diameter of the column that is $\sigma_{vo} = 2\gamma d_c$, γ is the effective unit weight of soft soil within the influence zone, ϕ_c is the angle of internal friction of the stone column material. Thus, Eq. (5) can be written as

$$q_{ult} = \frac{(1+\sin\phi_c)}{(1-\sin\phi_c)}\left(4c_u + 2K_0\gamma d_c\right) \tag{7}$$

Considering a factor of safety of 2, the safe load on column alone (Q_1) can be expressed as

$$Q_1 = \frac{q_{ult}\frac{\pi}{4}d_c^2}{2} \tag{8}$$

Taking into consideration the surcharge effect, the increase in mean radial stress ($\Delta\sigma_{r0}$) is given as

$$\Delta\sigma_{r0} = \frac{q_{Safe}}{3}(1+2K_0) \tag{9}$$

where q_{Safe} is the safe bearing pressure of the soil with a factor of safety 2.5 can be written as

$$q_{Safe} = c_u N_c / 2.5 \tag{10}$$

where N_c is the bearing capacity factor.

The increase in ultimate cavity expansion stress = $\Delta\sigma_{r0}F_q$, where F_q = Vesic's dimensionless cylindrical cavity expansion factor and is equal to 1 when $\phi = 0$. The increase in yield stress of the column = $Kp_{col}\Delta\sigma_{r0}$. Considering a factor of safety of 2, the increase in safe load of the stone column can be expressed as

$$Q_2 = \frac{Kp_{col}\Delta\sigma_{r0}\left(\frac{\pi}{4}\right)d_c^2}{2} \tag{11}$$

Considering the bearing support provided by the intervening soil, the safe load taken by the intervening soil is

$$Q_3 = q_{Safe} A_g \tag{12}$$

where A_g is the area of the intervening soil and can be expressed as:

$A_g = 0.866S^2 - (\pi/4)\, d_c^2$ for triangular arrangement

$= S^2 - (\pi/4)\, d_c^2$ for square arrangement

Therefore the overall safe load on each column and its surrounding soil is given as

$$Q = \frac{q_{ult} \frac{\pi}{4} d_c^2}{2} + \frac{Kp_{col} \Delta\sigma_{r0} \left(\frac{\pi}{4}\right) d_c^2}{2} + q_{Safe} A_g \tag{13}$$

4. Development of design charts

The design charts are developed according to the procedure proposed by Zhou *et al*. (1999) for the PVD-improved ground. However, in the study of Zhou *et al*. (1999) only consolidation has been considered for the design. In the present study, the design charts are developed for stone column-improved ground by considering both consolidation and bearing capacity of the improved ground.

The aim of the design of stone column-improved ground is to choose appropriate spacing between stone column (S) or diameter of the stone column (d_c) such that the target degree of consolidation $U_s(t_s)$ can be achieved at a specific time period t_s. The design is also done such that the stone column can carry the target safe load Q_s. However, the target degree of consolidation and/or safe load may or may not be achieved due to the uncertainty in the properties of the soft soil and stone column materials. Thus, probability based design charts are developed to incorporate the effect of the uncertainty in the properties of soil and column material.

If P_{s1} is the probability to achieve target degree of consolidation U_s at a specific time t_s and P_{s2} is the probability to carry target safe load Q_s, then P_{s1} and P_{s2} can be expressed as (Zhou *et al*. 1999)

$$P_{s1} = P_{s1}\left(U(t_s) \geq U_s(t_s)\right) \quad \text{or} \quad 1 - P_{s1} = P_{s1}\left(U(t_s) \leq U_s(t_s)\right) \tag{14}$$

$$P_{s2} = P_{s2}\left(Q \geq Q_s\right) \quad \text{or} \quad 1 - P_{s2} = P_{s2}\left(Q \leq Q_s\right) \tag{15}$$

Several researchers (e.g., Hong and Shang 1998, Zhou *et al*. 1999, Alonso and Jimenez 2011) suggested that c_r is the most important uncertain parameter that affects degree of consolidation. Thus, for consolidation purpose first it is considered that c_r is the only random variable to evaluate Eq. (14). In the bearing capacity of the stone column, the undrained shear strength or undrained cohesion of soft soil plays most significant role. The angle of internal friction of the soft soil is taken as zero. Thus, for bearing capacity purpose first it is considered that c_u is the only random variable to evaluate Eq. (15). It is observed that $U(t)$ and Q are constantly increasing function of c_r and c_u, respectively. Thus, Eqs. (14) and (15) can be written as (Zhou *et al*. 1999)

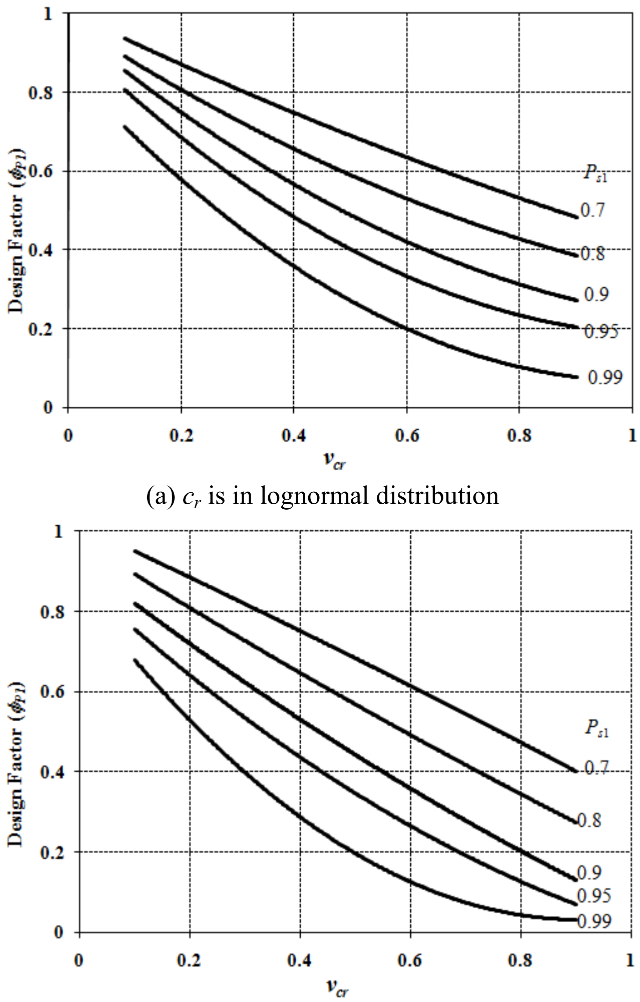
$$1 - P_{s1} = P_{s1}\left(U(t_s) \leq U_s(t_s)\right) = P_{s1}(c_r \leq c_{rp}) \tag{16}$$

$$1 - P_{s2} = P_{s2}\left(Q \leq Q_s\right) = P_{s2}(c_u \leq c_{up}) \tag{17}$$

where c_{rp} is the specified value of c_r that should be used for probabilistic design to achieve the target degree of consolidation at a specific time t_s with probability P_{s1}. Similarly, c_{up} is the specified value of c_u that should be used for probabilistic design to carry the target safe load with probability P_{s2}. Putting $c_{rp} = \phi_{P1} m_{cr}$ and $c_{up} = \phi_{P2} m_{cu}$ Eqs. (16) and (17) can be written as

$$1 - P_{s1} = P_{s1}\left(\frac{c_r}{m_{cr}} \le \phi_{P1}\right) \tag{18}$$

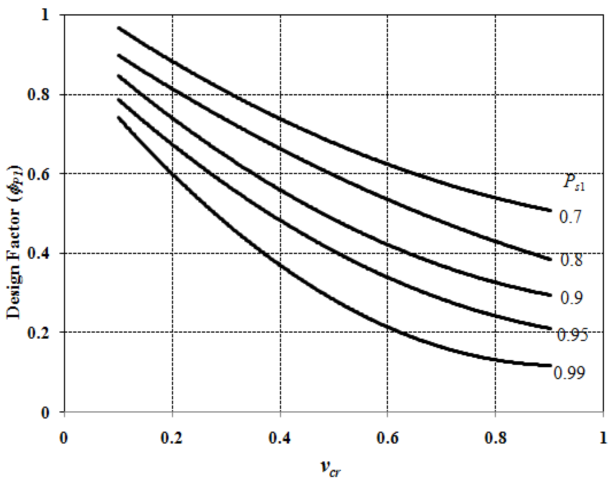
$$1 - P_{s2} = P_{s2}\left(\frac{c_u}{m_{cu}} \le \phi_{P2}\right) \tag{19}$$



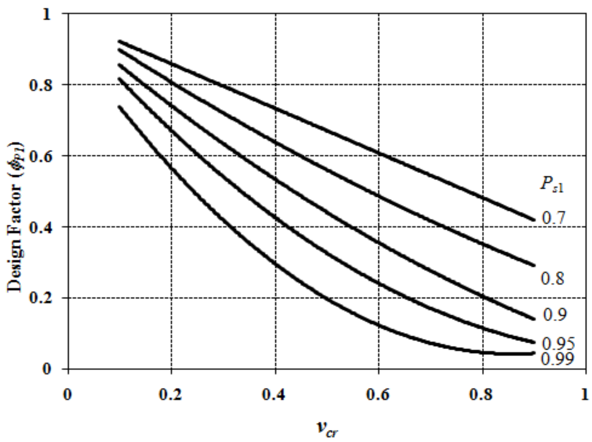
(a) c_r is in lognormal distribution

(b) c_r is in gamma distribution

Fig. 2 Design factor ϕ_{P1} for triangular arrangement where m_{cr} and m_{cu} are the mean (nominal) value of uncertain variables c_r and c_u, respectively. Thus, if c_{rp} and /or c_{up} are used to determine the stone column diameter or spacing then target degree of consolidation and/or safe load carrying capacity can be achieved with probability P_{s1} (for consolidation) and P_{s2} (for bearing capacity). The ϕ_{P1} and ϕ_{P2} are the design factors. Chang (1985) suggested that either lognormal or gamma distribution may be used for coefficient of vertical consolidation (Zhou *et al*. 1999). Griffiths *et al*. (2009) adopted lognormal distribution for undrained shear strength or undrained cohesion c_u. In the present study, lognormal and gamma distributions are selected for both c_r and c_u. For a given P_{s1}, coefficient of variation (COV) of the uncertain variable c_r (v_{cr}) and type of probability distribution of c_r, ϕ_{p1} can be evaluated iteratively from Eqs. (18) and (1) as suggested by Zhou *et al*. (1999). Similarly, for a given P_{s2}, coefficient of variation (COV) of the uncertain variable c_u (v_{cu}) and type of probability distribution of c_u, ϕ_{p2} can be evaluated iteratively from Eqs. (19) and (13).



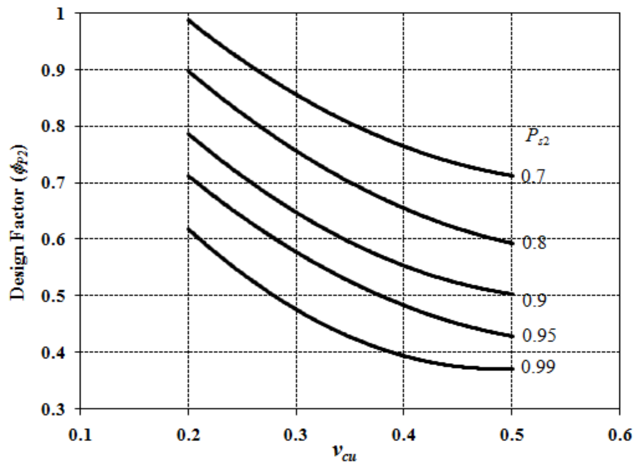
(a) c_r is in lognormal distribution



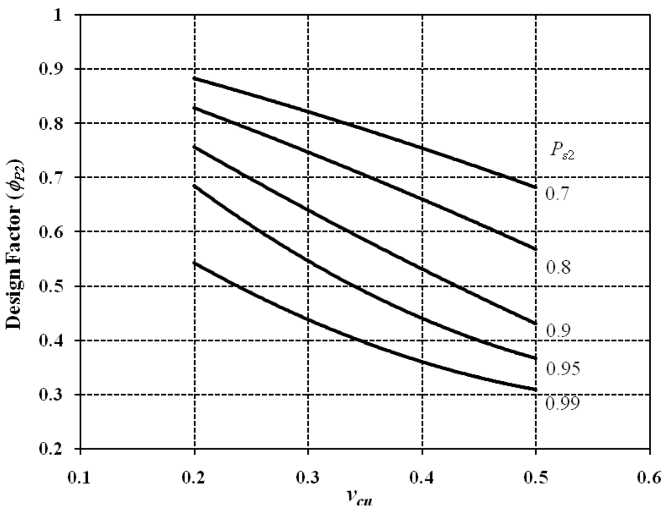
(b) c_r is in gamma distribution

Fig. 3 Design factor ϕ_{P1} for square arrangement

Figs. 2 and 3 show the charts for determining ϕ_{p1} value for triangular and square arrangement of stone columns for a range of P_{s1} and v_{cr} values, respectively. Figs. 4 and 5 show the charts for determining ϕ_{p2} value for square and triangular arrangement of stone columns for a range of P_{s2} and v_{cu} values. To prepare the design charts, the range of modular ratio is taken as 5 to 50 (Deb *et al*. 2007), diameter ratio is taken as 2 to 6 (Mitchell 1981), friction angle of stone column material 35° to 40° (Mitchell 1981). In the design charts for determining ϕ_{p1} and ϕ_{p2} value, both lognormal and gamma distribution of c_r and c_u are considered. If the proper distribution of c_r and c_u is unknown, the average value of the design factor obtained from both the distributions can be taken for design purpose. The charts corresponding to design factor for different cases almost reveal a similar trend. It can be seen that the design factors for a lower value of probability and lower value of coefficient of variation (COV) of the uncertain variables are higher as compared to the higher

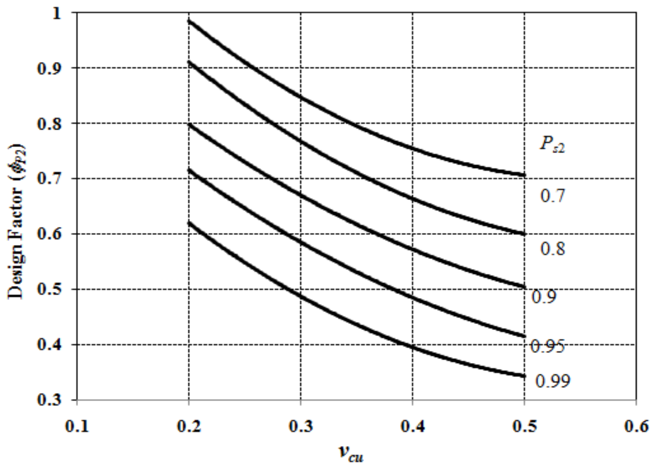


(a) c_u is in lognormal distribution

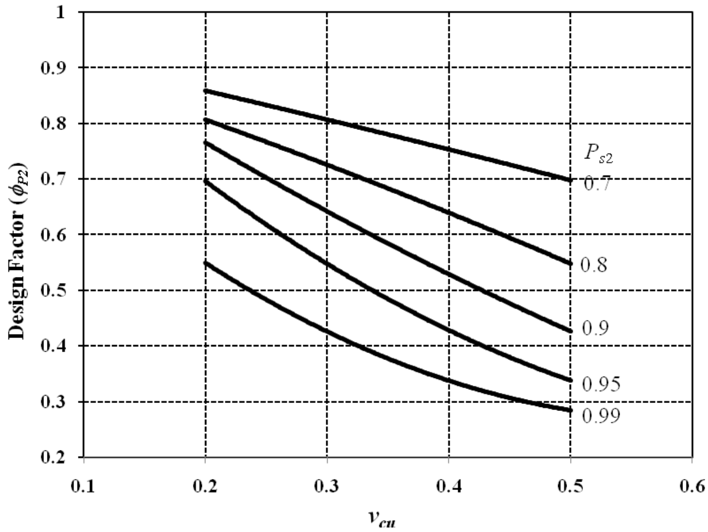


(b) c_u is in gamma distribution

Fig. 4 Design factor ϕ_{P2} for triangular arrangement



(a) c_u is in lognormal distribution



(b) c_u is in gamma distribution

Fig. 5 Design factor ϕ_{P2} for square arrangement

value of probability and higher value of coefficient of variation (COV) of the uncertain variables for the same case as expected. Thus, lower value of c_r and c_u should be used if uncertainty of the random variables increases and/or probability to achieve a target degree of consolidation or/and probability to carry a target safe load is high.

From the design charts it is observed that for lognormal distribution, the design factor value (ϕ_{P1}) is slightly higher in case of square stone column arrangement as compared to the triangular arrangement for higher P_{s1} value (P_{s1} = 0.99). For gamma distribution, the design factor ϕ_{P1} is higher in case of triangular stone column arrangement as compared to the square arrangement within lower range of v_{cr} (0.1 – 0.6) for lower range of P_{s1} (P_{s1} = 0.7 – 0.8). It is further observed that for gamma distribution, the design factor value (ϕ_{P2}) is slightly higher in case of triangular stone column arrangement as compared to the square arrangement within lower range of v_{cu} (0.1 – 0.2) for lower range of P_{s2} (P_{s2} = 0.7 – 0.8). In the field, mainly square and triangular stone column arrangements are selected. Thus, for design of stone column-improved ground, appropriate design charts are to be used according to the selected column arrangement.

The design charts are developed by considering c_r and c_u are only two random variables. However, there are others soil parameters like angle of internal friction of stone column materials and modular ratio of soft soil and stone column material may also be considered as random variables. Thus, further investigations are carried by considering c_u, ϕ_c, γ, c_r, E_c and E_s as random variables. To check the usefulness of the design charts, the probability obtained from the developed design charts to achieve target degree of consolidation and/or to carry target safe load is compared to the probability obtained by considering uncertainties of the other material properties involved in the design as suggested by Zhou *et al.* (1999). Table 1 shows the chosen design parameters and their mean and COV values to check the usefulness of the design charts. Lognormal distribution is considered for ϕ_c, E_c and E_s. Normal distribution is considered for γ. In case of c_r and c_u either lognormal or gamma distribution is considered. A triangular arrangement of the stone columns is considered. Safe design load is taken as 250 kN as design load of 200 kN to 300 kN per column is typical for columns in soft to medium stiff clays (Mitchell 1981). Based on these values and for a particular target probability 0.8 or 0.9 or 0.95, design factor (ϕ_{p1}) is determined from Fig. 2(a) for lognormal distribution and from Fig. 2(b) for gamma distribution. Similarly, for a particular target probability 0.8 or 0.9 or 0.95, design factor (ϕ_{p2}) is determined from Fig. 4(a) for lognormal distribution and from Fig. 4(b) for gamma distribution. From the design factors, design value of c_r and c_u is determined. According to the design values, required spacing is determined. After determining all the deterministic design parameters, probabilistic analysis is conducted by Monte-Carlo simulation to determine the probability to achieve target degree of consolidation and/or safe load where uncertainties are considered for other design parameters in addition to c_r and c_u. Table 2 shows the probability of achieving target degree of consolidation and target safe load by considering all random variables. From the results it is shown that the obtained probability by considering uncertainties for other design parameters in addition to c_r and c_u is near to the probability obtained from the design charts. Thus, ignoring the uncertainty

Table 1 Design Parameters and their mean and COV values to check the usefulness of the design charts

Parameter	Mean	COV
d_c	0.7 m	0
t_s	0.5, 0.75, 1.0 year	0
$U_s(t_s)$	0.90	0
c_u	20 kN/m^2	0.3*
ϕ_c	35°	0.1$
γ	15 kN/m^3	0.1*
c_r	2 m^2/yr	0.5#
E_c	30000 kN/m^2	0.3$
E_s	2000 kN/m^2	0.3$

* Baecher and Christian 2003; $Alonso and Jimenez 2011; #Zhou *et al.* 1999

Table 2 Probability of achieving target degree of consolidation and target safe load by considering all random variables

Distribution of c_r	Time (year)	Distribution of c_u	P_{s1}			P_{s2}		
			0.8	0.9	0.95	0.8	0.9	0.95
Lognormal	0.5	-	0.801	0.886	0.943	-	-	-
	0.75	-	0.802	0.892	0.944	-	-	-
	1.0	-	0.803	0.892	0.947	-	-	-
Gamma	0.5	-	0.793	0.892	0.940	-	-	-
	0.75	-	0.794	0.897	0.942	-	-	-
	1.0	-	0.795	0.897	0.942	-	-	-
-	-	Lognormal	-	-	-	0.795	0.905	0.950
-	-	Gamma	-	-	-	0.787	0.893	0.954

in the variables other than c_r and c_u is adequate. It is also shown that the simplified method is very convenient as the obtained design factors are independent of time (Zhou *et al*. 1999).

5. Design procedure

First select the type of arrangement of the stone column. Choose the mean or nominal value of the all the parameters involved in the design. It is to be noted that mean value of either diameter of the stone column or spacing between the stone columns has to be chosen. If the mean value of the spacing is chosen then required diameter will be determined from the design and vice versa. Find the COV and probability distribution type of c_r and c_u. For a given probability to achieve target degree of consolidation (P_{s1}) and/or safe load (P_{s2}), determine design factors from the respective design charts. If actual distribution of c_r and c_u is unknown then the average value of the design factor obtained from the lognormal and gamma distribution can be considered. From the design factors, determine the design value of c_r and c_u. Once the design value of c_r and c_u are known, determine the required spacing or diameter of stone column to achieve a target consolidation (from Eq. (1)) and/or a target safe load (from Eq. (13)). If design is done either for consolidation or load carrying capacity then obtained spacing or diameter will be chosen as design spacing or diameter in respective cases. However, if design is done for both consolidation and load carrying capacity then higher diameter obtained from both the cases will be chosen as design diameter or lower spacing obtained from both the cases will be chosen as design spacing. This is to be noted that the diameter and spacing of the stone columns usually be in the range of 0.6 to 1.0 m and 1.5 to 3.5 m, respectively (Mitchell 1981).

In the present design methodology, unit cell approach is considered during the calculation of load carrying capacity and consolidation of improved ground. Similar charts can also be produced by considering stone column group from the available equation. However, group effect of the stone column is not considered in the present study as most of the recommended design cases load carrying capacity and consolidation of stone column-improved ground are designed based on unit cell approach by considering one single column and its surrounding soft soil. Although it is observed that behaviour of stone column near the centre and edge of a structure is not same. After selecting the diameter or spacing of the stone column based on the proposed design charts, settlement of the improved ground can be determined as per IS-Code recommendation (or any other available method) and should be checked with the permissible settlement of the proposed structure. The required length of the stone column can be determined from the soil condition and as per the design requirements. In such case uncertainty in the design parameter (compressibility of soft soil) can also be considered. The design charts can be used for either consolidation and/or load carrying capacity of improved ground depending upon the design requirements.

6. Design example

For the design example, the basic design parameters are chosen as (considering the load carrying capacity of the stone column): c_u = 20 kN/m^2, S = 2.5 m, Q_{safe} = 250 kN, ϕ_c = 35°, γ = 15 kN/m^3. For the site interest, the COV of c_u is taken as 0.3. The probability (P_{s2}) to carry the target safe load is taken as 90%. Lognormal distribution is considered for c_u. However, both lognormal and gamma distribution can also be considered for c_u. In such case, average value of ϕ_{p2} can be considered. A triangular arrangement of the stone columns is considered. The corresponding design factor (ϕ_{p2}) is determined as 0.65 [from Fig. 4(a)] and 0.34 [from Fig. 2(b)] for lognormal and gamma distribution, respectively. An average value of [(0.65 + 0.64) / 2] = 0.645 is being considered. Thus, the probabilistic design value of c_u = 20 × 0.645 = 12.9 kN/m^2. Using Eq. (13) one can get the required diameter of stone column is equal to 0.96m (for probabilistic design). Similarly, taking the deterministic design value of c_u = 20 kN/m^2 and using Eq. (13) one can get the required diameter of stone column for probabilistic design is equal to 0.44 m.

Considering the rate of consolidation of stone column, the basic design parameters are chosen as: E_c = 60000 kPa, S = 2.5 m, c_r = 2 m^2/year. Target degree of consolidation is taken as 90%. E_s = 300 c_u (Bowles 1996) (or the measured E_s value can also be used directly). Thus, E_s = 300 × 13.2 = 3960 kPa for probabilistic design and E_s = 300 × 20 = 6000 kPa for deterministic design. However, the measured value of E_s can also be used directly. The target degree of consolidation after 6 months is taken as 90%. For the site interest, the COV of c_r is taken as 0.7. The probability (P_{s1}) to achieve the target degree of consolidation after 6 months is taken as 90%. The distribution of c_r is unknown. Thus, the average value of the design factor is taken during the design. The corresponding design factor (ϕ_{p1}) is determined as 0.36 [from Fig. 2(a)] and 0.30 [from Fig. 2(b)] for lognormal and gamma distribution, respectively. Since the exact distribution is unknown, an average value of [(0.30 + 0.36) / 2] = 0.33 is being considered. Thus, the probabilistic design value of c_r = 2 × 0.33 = 0.66 m^2/year. Using Eq. (1) one can get the required diameter of stone column is equal to 0.76m (for probabilistic design). Similarly, taking the deterministic design value of c_r = 2 m^2/year and using Eq. (1) one can get the required diameter of stone column for probabilistic design is equal to 0.6 m. Thus, the adopted value of diameter of the stone column for deterministic and probabilistic design approach is 0.6 m and 0.96 m, respectively by considering both load carrying capacity and consolidation into the design.

Comparing all the results based on both load carrying capacity and consolidation it is appropriate to take into consideration a probabilistic based design as it leads to a safer value (i.e., having a specified minimal risk) of required safe load along with a desired target degree of consolidation over a time. In the present study, only radial consolidation is considered as for stone column-improved ground for a particular time period the soil consolidation due to vertical drainage is much less than the consolidation occurred due to radial drainage. However, to calculate the overall degree of consolidation accurately, the part of the degree of consolidation contributed by the vertical drainage need to be computed and separated first from the overall (target) degree of consolidation to get the portion of the degree of consolidation contributed by the radial drainage. The design now can be performed only with the part of the degree of consolidation need to be achieved via radial drainage. It is also assumed that uncertainty of the soil parameters in each unit cell is also identical. The variation included in the analysis has been centered around a representative value which is based on the entire region to be improved.

7. Conclusions

In the present paper, probability-based design charts are presented for stone column-improved ground. Charts are presented by considering both load carrying capacity of the stone column as well as consolidation of the improved ground. The uncertainty in undrained cohesion (c_u) and coefficient of radial consolidation (c_r) of the soft soil is considered in the design charts. Simple design procedure with the illustration using design examples is presented for probabilistic design which also can be used for deterministic design if needed. The usefulness of the charts is checked by considering the uncertainties of the all design variables, therefore can be used by the practitioners with confidence. However, the design method presented in this paper cannot explicitly consider the spatial variability of c_u and c_r and the uncertainty in the model used to predict the consolidation and load carrying capacity.

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