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Optimization of ground response analysis using wavelet-based transfer function technique

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Abstract. One of the most advanced classes of techniques for ground response analysis is based on the use of Transfer Functions. They represent the ratio of Fourier spectrum of amplitude motion at the free surface to the corresponding spectrum of the bedrock motion and they are applied in frequency domain usually by FFT method. However, Fourier spectrum only shows the dominant frequency in each time step and is unable to represent all frequency contents in every time step and this drawback leads to inaccurate results. In this research, this process is optimized by decomposing the input motion into different frequency sub-bands using Wavelet Multi-level Decomposition. Each component is then processed with transfer Function relating to the corresponding component frequency. Taking inverse FFT from all components, the ground motion can be recovered by summing up the results. The nonlinear behavior is approximated using an iterative procedure with nonlinear soil properties. The results of this procedure show better accuracy with respect to field observations than does the Conventional method. The proposed method can also be applied to other engineering disciplines with similar procedure.

Keywords: ground response analysis; transfer functions; wavelet analysis; multi-level decomposition

1. Introduction

One of the most important issues in geotechnical earthquake engineering is the prediction of ground motions in soil layers which is carried out using many different techniques. These earthquake ground motions can be significantly affected by the local subsurface geology and morphology (Bazrafshan and Bagheripour 2011). An important class of techniques for ground response analysis is based on the use of transfer functions. They represent the ratio of various surface response parameters such as displacement, velocity and acceleration of a ground motion to those of an input motion at bedrock. Transfer functions are usually expressed in terms of mathematical functions incorporating geometrical and dynamic soil properties and define frequencies at which soil amplification or deamplification occurs. As can be seen in recent literatures (Assimaki and Li 2012, Hassani *et al.* 2011, Lee and Trifunac 2010, Obando *et al.* 2011), the proposed transfer functions are usually frequency dependent and hence they were

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formulated in frequency domain. A major drawback of these researches (Assimaki and Li 2012, Hassani *et al*. 2011, Lee and Trifunac 2010, Obando *et al*. 2011) is that they are essentially based on the use of transfer functions which in turn needs Fast Fourier Transform (FFT). Since the input motion data such as acceleration time histories are in time domain, Therefore, in order to use the transfer functions, the input motion has been transformed from time domain to frequency domain using FFT method. However, Fast Fourier Transform gives a unique representation of the signal in the frequency domain and provides information about frequencies which appear in the signal but not about the time instants that these frequencies are encountered. Further, Fourier spectrum only shows the dominant frequency in each time step and is unable to represent all frequency contents in every time step. The major drawback of FFT based methods is that FFT only tells whether a certain frequency component exists or not and this information is independent of where in time this component appears or repeats and this issue imposes computational errors in the final results.

This drawback has been removed by the recently developed time-frequency transform tool called wavelet transform. Wavelets are composed of a family of basis functions that are capable of describing a signal in a localized time and frequency (or scale) domain whereas the Fourier transform is only localized in frequency domain (Bazrafshan and Bagheripour 2012, Bagheripour *et al*. 2010). The Short-Time Fourier Transform (STFT) is also time and frequency localized, and although it is an improved version of Fourier Transform, but there are issues with the frequency time resolution whereas wavelets often give a better signal representation using multi-resolution analysis (Walnut 2001). The wavelet transform is a two-parameter transform. For time signals, the two domains of the wavelet transform are time t and scale a, which is related to the frequency ω.

If an earthquake accelerogram, e.g., EW component of Loma Prieta earthquake (Fig. 1), is transformed to frequency domain using FFT method, its Fourier spectrum will be represented as in Fig. 2. This spectrum is two dimensional and shows only the frequency contents of the input motion and has specific amplitude for each frequency. If the aforementioned input motion is analyzed by wavelet transform method, a three dimensional spectrum will be obtained as shown in Fig. 3. The wavelet spectrum shows both the frequency contents and the related times in which each frequency occurred. It can be observed that if a specific frequency occurs in different time steps throughout the earthquake duration, the wavelet transform will distinguish and will provide all of the time occurrences of that frequency. However, as shown in Fig. 2, Fourier transform was unable to represent the time information and it cannot show that which frequency is related to which time instants. Also, if a frequency occurs more than once in the input motion, the Fourier transform is not able to show those points of times and ignored them. Since the transfer functions are frequency dependent, therefore, for each frequency the corresponding magnitude for transfer function is unique. If the input motion is transformed to frequency domain by means of FFT method, the calculated transfer function can be applied only to one of the time steps in which the corresponding frequency occurs and hence other time steps will not be multiplied by the related transfer function. This issue may cause errors in the analysis results and as will be shown in this research, FFT method implementation may lead to unacceptable results.

In this paper, despite other approaches, the ground response analysis problem is solved by decomposing the input motion into different frequency sub-bands using Wavelet Multi-level Decomposition. The decomposition algorithm separates a signal into components at various scales corresponding to related frequencies. Each decomposed accelerogram has a dominant frequency which differs from other components (see Fig. 4). By taking the Fourier transform of the decomposed accelerograms, it can be observed that each spectrum has value near its dominant frequency and vanishes outside its frequency band as can be seen in Fig. 5.

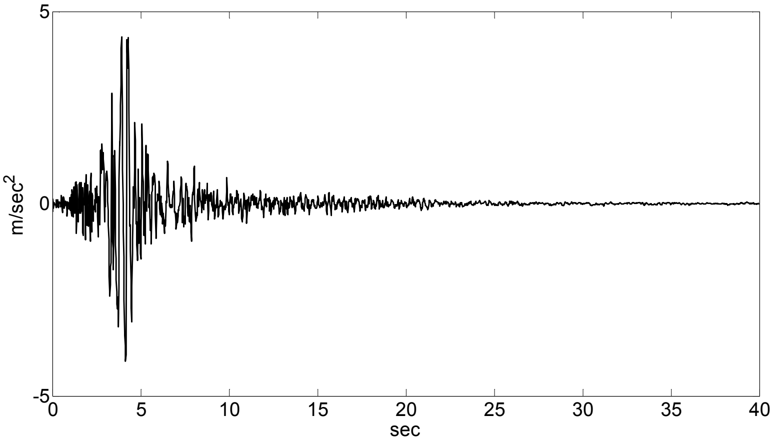


Fig. 1 EW component of Loma Prieta earthquake

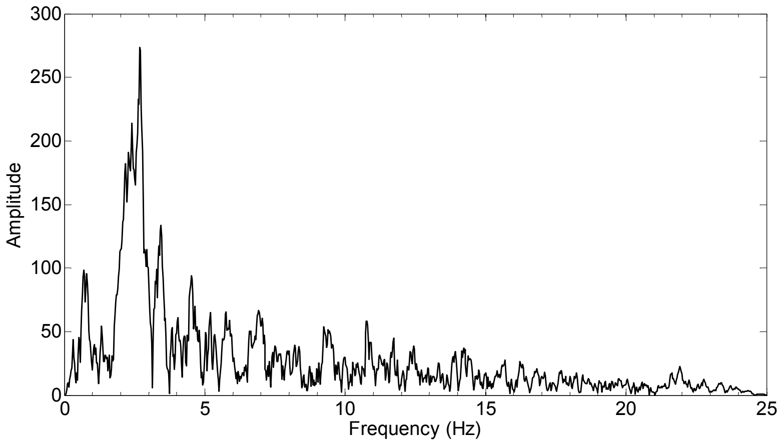


Fig. 2 Fourier spectrum of EW component of Loma Prieta earthquake

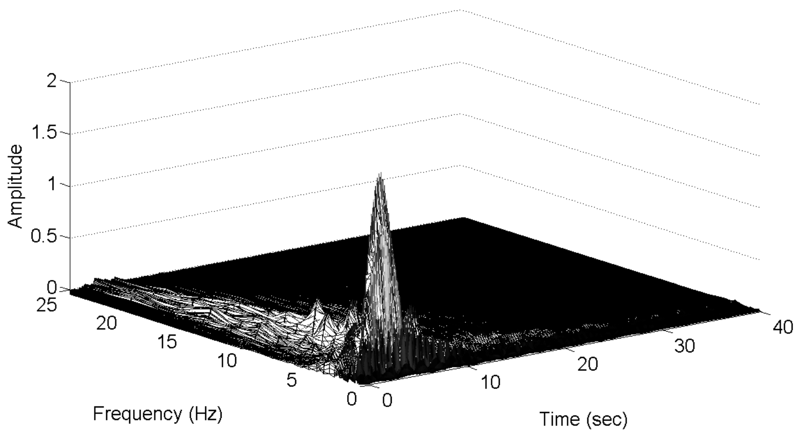


Fig. 3 3D wavelet spectrum of EW component of Loma Prieta earthquake

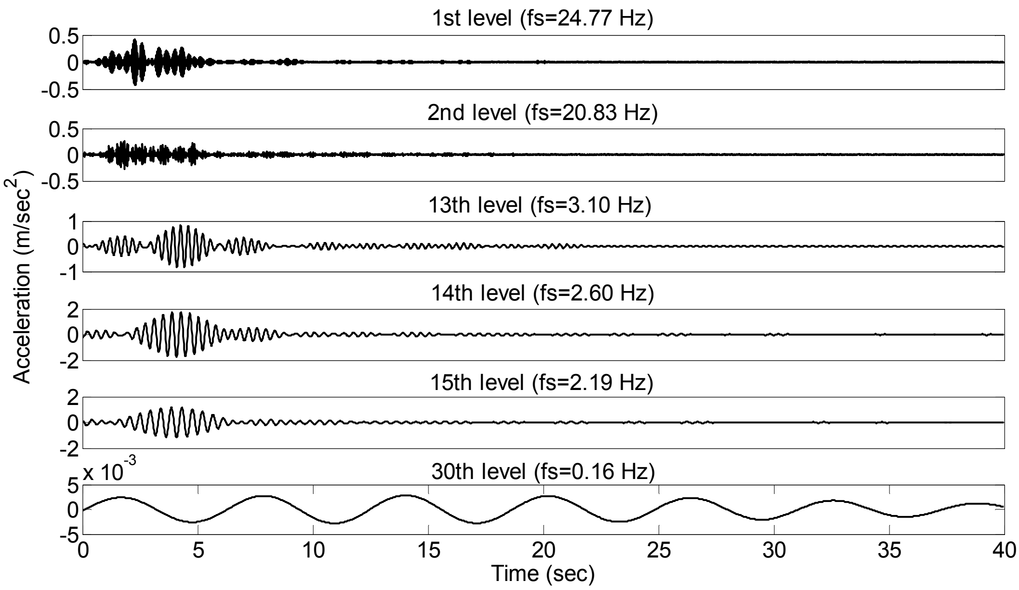


Fig. 4 Decomposed accelerograms of EW component of Loma Prieta earthquake

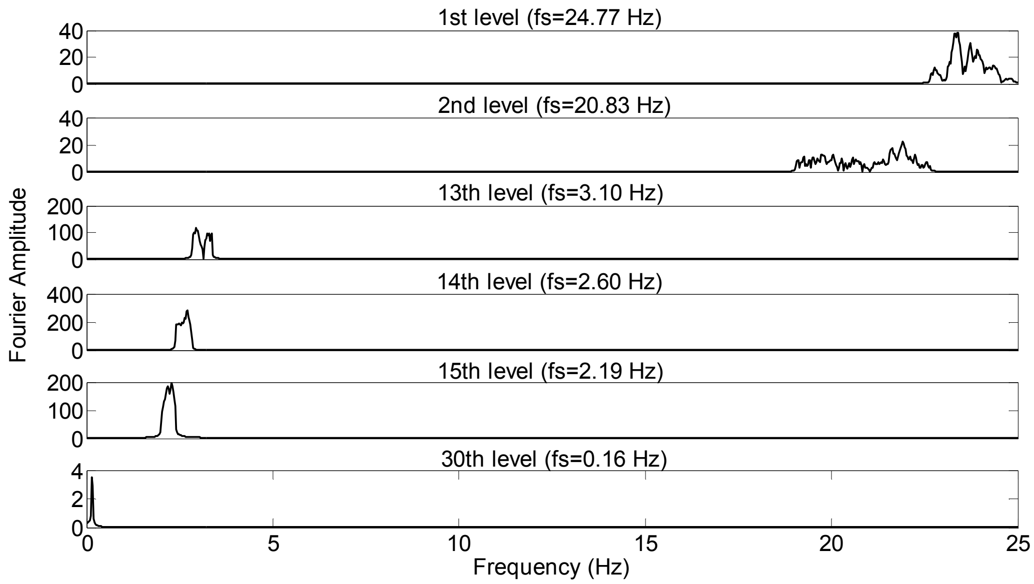


Fig. 5 Fourier transform of decomposed accelerograms of EW component of Loma Prieta earthquake

Fig. 5 shows that the input accelerogram is fully decomposed into non-overlapping frequency bands and therefore, each decomposed accelerogram can be processed individually and transformed into frequency domain using FFT method without causing errors (comparing to those obtained from conventional FFT methods) in the results. The transformed components are then multiplied by Transfer Functions relating to the corresponding component frequencies. Taking inverse FFT from all components, the motion at top of the desired layer can be recovered by summing up the multiplied components. The proposed method can be applied to all similar methods in which the input data is in time domain and the transfer or amplification functions are formulated in terms of frequency parameter. The output data can be calculated using the proposed procedure. The analysis results have been compared with recorded surface motions and with those obtained from FFT based method. It has been shown that the proposed method give proper results compared with recorded field observations than do the conventional method.

2. Formulation

For a SDOF system of a mass m, stiffness k and damping c which is subjected to a periodic loading $Q(t) = \sum_{n=0}^{N} q_n^* e^{i\omega_n t}$ where * denotes for complex value and N is the number of sample data, the equation of motion can be expressed as

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = Q(t) \tag{1}$$

And the complex Fourier coefficients, q_n^*, can be determined directly from $Q(t)$ as

$$q_n^* = \frac{1}{T_e} \int_0^{T_e} Q(t) e^{-i\omega_n t} dt \tag{2}$$

where T_e is the duration of loading. The response motion can be related to the loading by

$$u_n(t) = H(\omega_n) q_n^* e^{i\omega_n t} \tag{3}$$

In the above equation $H(\omega_n)$ is called Transfer Function. Substituting Eq. (3) into the equation of motion gives

$$-m\omega_n^2 H(\omega_n) q_n^* e^{i\omega_n t} + ic\omega_n H(\omega_n) q_n^* e^{i\omega_n t} + kH(\omega_n) q_n^* e^{i\omega_n t} = q_n^* e^{i\omega_n t} \tag{4}$$

Simplifying Eq. (4) yields

$$H(\omega_n) = \frac{1}{-m\omega_n^2 + ic\omega_n + k} \tag{5}$$

It is noted that the denominator also appears in the equation of motion in frequency domain

$$(-m\omega_n^2 + ic\omega_n + k) u_n(\omega_n) = q_n^* \tag{6}$$

For a soil deposit including some horizontal layers, where the n^{th} layer overlain the bedrock (Fig. 6), each layer was assumed to be homogeneous, isotropic, and was characterized by the thickness h, mass density ρ, shear modulus G, and damping ratio ζ, the transfer function, $H(\omega)$, becomes (Kramer 1996)

$$H_{ij}(\omega) = \frac{|u_i|}{|u_j|} = \frac{a_i(\omega) + b_i(\omega)}{a_j(\omega) + b_j(\omega)} \tag{7}$$

which relates the displacements in layer i to that of layer j. $a_i(\omega)$ and $b_i(\omega)$ in layer i are functions of h_i, ρ_i, G_i and ζ_i where their relations can be expressed as (Kramer 1996)

$$\begin{aligned} A_m &= a_m(\omega) A_1 \\ B_m &= B_m(\omega) B_1 \end{aligned} \tag{8}$$

Where A_m and B_m are given as

$$\begin{aligned} A_m &= \frac{1}{2} A_{m-1}(1+\alpha^*_{m-1}) e^{ik^*_{m-1}h_{m-1}} + \frac{1}{2} B_{m-1}(1-\alpha^*_{m-1}) e^{-ik^*_{m-1}h_{m-1}} \\ B_m &= \frac{1}{2} A_{m-1}(1-\alpha^*_{m-1}) e^{ik^*_{m-1}h_{m-1}} + \frac{1}{2} B_{m-1}(1+\alpha^*_{m-1}) e^{-ik^*_{m-1}h_{m-1}} \end{aligned} \tag{9}$$

Where α^*_{m-1} is the complex impedance ratio at the boundary between layers m and $m-1$

$$\alpha^*_{m-1} = \frac{k^*_{m-1} G^*_{m-1}}{k^*_m G^*_m} \tag{10}$$

Transfer function $H_{ij}(\omega)$ for each layer can be calculated using Eqs. (7)-(10). Since the acceleration time histories are in time domain, a transformation to frequency domain is necessary. Rather than using FFT method to transform the input motion from time domain into frequency domain, wavelet multi-level decomposition algorithm was used to decompose the input motion into sub-band frequency components. The chosen wavelet for this research is the modified Littlewood-Paley (LP) (Basu and Gupta 1998) wavelet which has an excellent localization in frequency domain. This wavelet is widely used in earthquake analytical methods and since it has non-overlapping frequency bands, it is suitable for wavelet multiresolution analysis (Chang and Shi 2010, Das and Gupta 2010, 2011). Its mathematical expression is given as (Basu and Gupta 1998)

$$\psi(t) = \frac{1}{\pi\sqrt{\sigma - 1}} \frac{\sin(\sigma\pi t) - \sin(\pi t)}{t} \tag{11}$$

where σ is a constant scalar which is used in discretizing scales. Basu and Gupta (1998) found $\sigma = 2^{1/4}$ is more suitable for earthquake motions.

In multiresolution analysis, a signal $S(t)$ is resolved into several signals at different levels as

$$A_j(t) = A_{j+1}(t) + D_{j+1}(t) \qquad j \in Z, j = 0, \ldots, n \tag{12}$$

where, j is the level number representing a particular range of frequency and it is related to the scale a by the relation

$$a = \sigma^j \quad \text{or} \quad j = \log(a)/\log(\sigma) \tag{13}$$

The idea of decomposition is illustrated in Fig. 7. A_j's are local averages or approximation functions at a particular level and D_j's are local differences or details functions (the so-called decomposed accelerograms). Decomposition is cut at a level where there is no appreciable information remained in the approximation signal and the entropy of the decomposed signal becomes negligible. The entropy chosen for this research is Shannon type entropy which is defined as (Misiti *et al*. 2011)

$$E = -\sum_{n=1}^{N} s_i^2 \log(s_i^2) \tag{14}$$

where N is the number of sample data and s_i is the magnitude of each signal sample. Therefore, the original signal can be reconstructed from the details signals as

$$S = A_0(t) = \sum_{j=1}^{n} D_j(t) \tag{15}$$

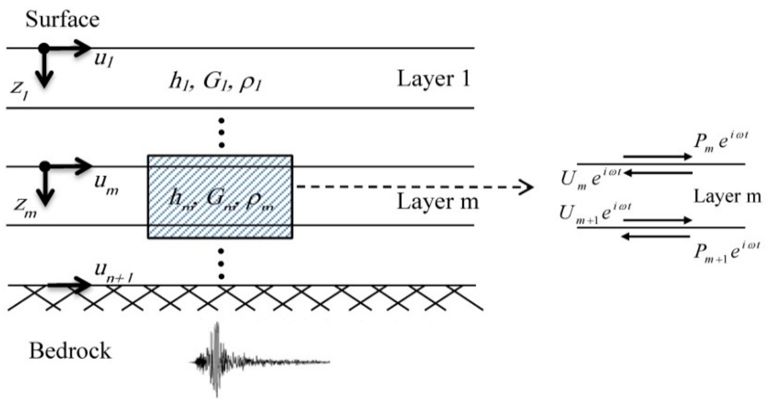


Fig. 6 One-dimensional soil-bedrock system subjected to SH waves

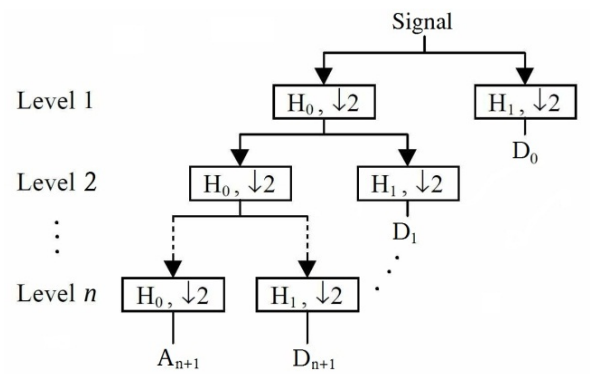


Fig. 7 Schematic of wavelet decomposition tree

In the above, the details functions $D_j(t)$ are expressed as

$$D_j(t) = \sum_{k=0}^{T_e} C_{j,k}\psi_{j,k} \tag{16}$$

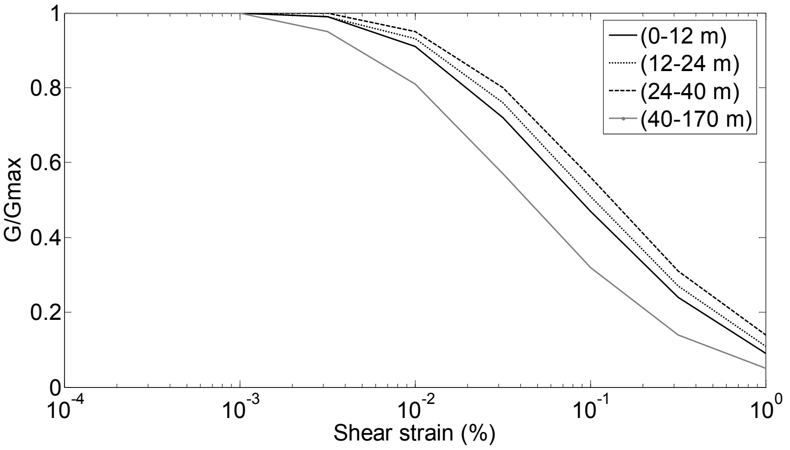
with $\psi_{j,k} = \sigma^{j/k}\psi(\sigma^j t - k)$. Parameter k represents an index on time scale, $\psi_{j,k}$ are the dilated and translated versions of LP wavelet while $C_{j,k}$ are corresponding wavelet coefficients (Basu and Gupta 1998).

3. Case studies

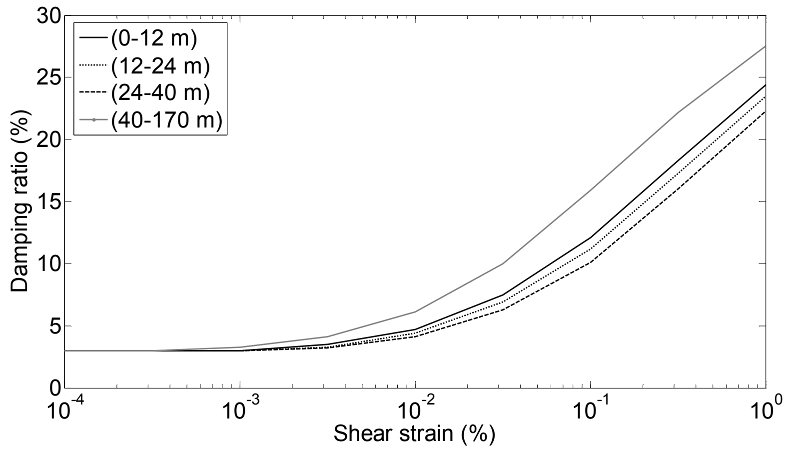
A reference site with its measured in-situ dynamic soil properties was chosen based on EPRI technical research (1993). The control motions, as mentioned by EPRI (1993) were taken from the reference rock site Gilroy 1, located about 2 km west of soil site Gilroy 2. Two earthquakes, different in magnitude and PGA, have been analyzed in this research which were both occurred near Gilroy 1 and 2 sites. Loma Prieta earthquake of 17 October 1989 with magnitude of 6.9 and PGA = 0.473 g from its EW component and Coyote Lake earthquake of 6 August 1979 with magnitude of 5.7 and PGA = 0.132 g from its SE component (EPRI 1993). The Gilroy 2 site is characterized by deep water table; therefore, pore pressure build-up and liquefaction were not of major concern, so the model was appropriate for this case study. The soil at Gilroy 2 is about 170 m deep and consists of sands and clays up to a depth of 40 m. Beyond 40 m is a deposit of gravel underlain by weathered bedrock at a depth of about 170 m. The characteristics of soil profile at Gilroy 2 are given in Table 1 (EPRI 1993). The nonlinear behavior of soil was modeled using an iterative nonlinear method, therefore, modulus reduction and damping characteristics of the soils beneath the Gilroy 2 recording station was obtained from an extensive laboratory testing program conducted by EPRI (1993), which is shown in Figs. 8(a) and 8(b).

Table 1 Gilroy 2 soil profile (EPRI 1993)

Layer	H (m)	V_s (m/sec)	ρ(kN/m^3) ρ (kN/m^3)
1	10.7	198	18.9
2	3.0	305	18.9
3	9.1	475	18.9
4	9.4	305	18.9
5	6.1	347	20.9
6	3.7	375	20.9
7	34.1	640	20.9
8	6.1	640	20.9
9	7.0	472	20.9
10	8.8	527	20.9
11	72.5	701	20.9
Bedrock	--	1189	22.6



(a)



(b)

Fig. 8 (a) Moduli ratio curve for soil at Gilroy 2 site; (b) Damping curve for soil at Gilroy 2 site (EPRI 1993)

4. Results

A computer program was developed in MATLAB environment based on the formulation presented by Eqs. (7)-(16). The program is capable of developing models for geometrical complexities and material diversities in soil layers and has no limitations in quantity. The analysis is performed from the bedrock layer towards the surface layer. In each layer, the calculated acceleration at the top of the underneath layer is decomposed into non-overlapping frequency bands using LP wavelet and then transformed into frequency domain. Then the transfer functions which are computed using the initial values of shear modulus and damping ratio, are multiplied with the transformed accelerations. By taking inverse Fourier transform from the multiplied values and summing up the results, the acceleration time history at top of the layer can be calculated. Therefore, shear strains for the layer can be obtained. In an iterative procedure, shear modulus and damping ratios are rationally updated for each time step, according to the calculated value of shear strain at the corresponding time and hence the method avoid overestimation or underestimation of the input acceleration. Then the transfer functions, in each time step, are corrected according to the modified values of shear modulus and damping ratio. The process continues until differences between the computed shear modulus values in two successive iterations tends to zero.

4.1 Analysis of Loma Prieta earthquake

The necessary number of levels for decomposition was chosen based on the entropy of the decomposed components which is fully discussed in (Misiti *et al*. 2011). Fig. 9 shows the entropy of the decomposed accelerograms of EW component of Loma Prieta earthquake. As can be seen in Fig. 9, the decomposition process has been continued until two successive components have negligible entropies and therefore, the input motion was decomposed into 30 levels based on this criterion. With this enough levels of decomposition, the reconstructed signal was fully matched with the original acceleration as well. Fig. 9 shows that 13^{th}, 14^{th} and 15^{th} decomposed components have the highest energy among the other components corresponding to frequencies of 3.10, 2.60 and 2.19 Hz respectively. These frequencies are in the bound of dominant frequency of the input motion and have the highest amplitudes of acceleration as well as Fourier spectrum shown in Figs. 4 and 5 respectively.

It can be concluded that, these components have more effects on magnitude of the surface motion than other decomposed components. Applying transfer functions for each soil layer from bedrock to ground surface into each of the decomposed accelerations, implementing iterative method for soil nonlinearity, leads to calculation of ground surface motion shown in Fig. 10. Fig. 10 also compares the calculated acceleration time histories at the surface for the proposed method with those given by conventional method and with field observations. It can be seen that the conventional method is unable to predict precise surface accelerations and the proposed and optimized method improves the results, provides better predictions when both are compared with recorded surface acceleration.

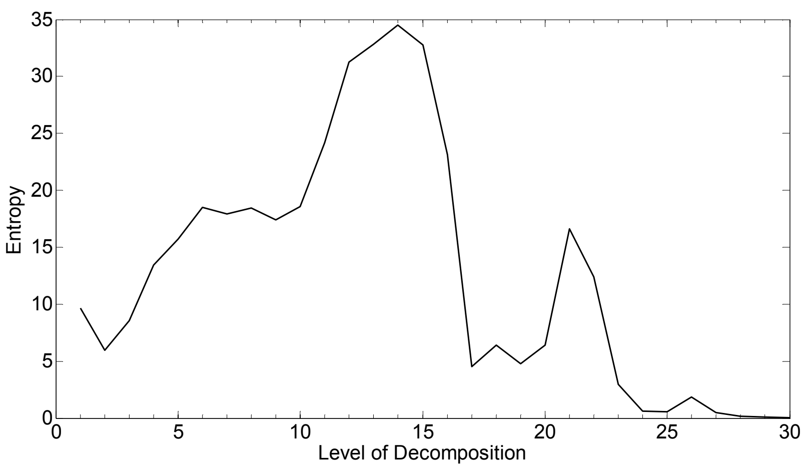


Fig. 9 Entropy of decomposed components in each level for EW component of Loma Prieta earthquake

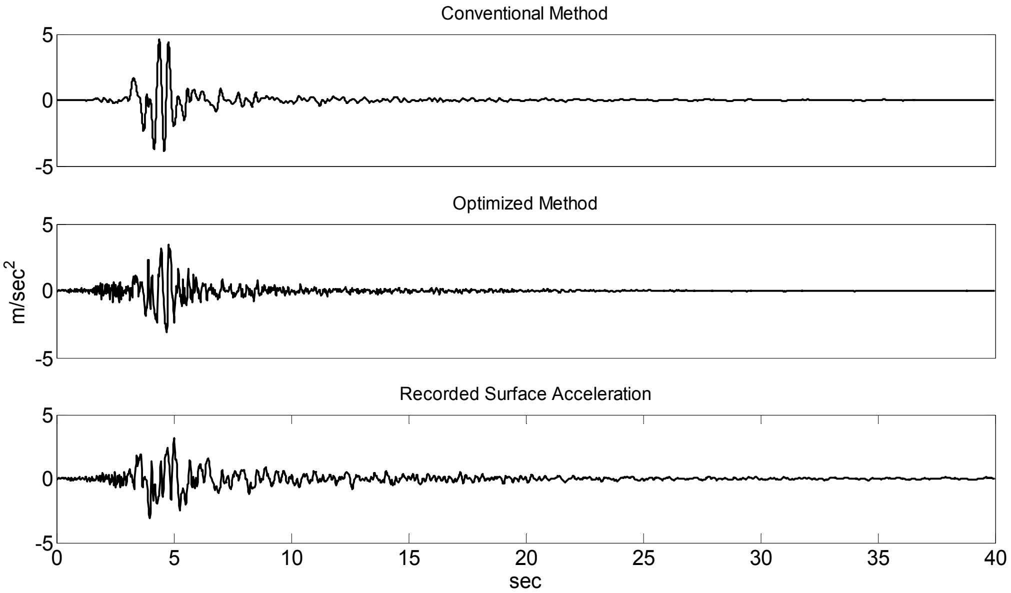


Fig. 10 Comparison of the proposed method with conventional and Recorded surface acceleration (Loma Prieta earthquake)

Inspecting Figs. 1 and 10 reveals that the EW component of Loma Prieta earthquake attenuated at the surface. Fig. 2 shows that higher amplitudes of motion concentrate in low frequency range (less than 5 Hz). Therefore, due to the dependency of shear modulus and damping ratio to frequency (Kausel and Assimaki 2002), the initial part of the motion (first 10 seconds), which has the main effect on ground response, would have smaller shear modulus than the one related to the shear strain level. This excessive decreasing of the shear modulus will lead to the deamplification of the input motion as it is observed in EW component of Loma Prieta earthquake.

4.2 Analysis of Coyote Lake earthquake

The analysis is also performed on SE component of Coyote Lake earthquake. Fig. 11 shows the acceleration time history of this earthquake recorded in Gilroy 1 rock site. Figs. 12 and 13 show the Fourier spectrum and wavelet spectrum of the input acceleration, respectively. Unlike the EW component of Loma Prieta earthquake, these spectrums show that the initial and effective part of the input motion has medium to high frequencies. Since this earthquake has a magnitude of 5.7 and PGA = 0.132 g, therefore it is weaker than the Loma Prieta earthquake and it is expected that less number of levels for decomposition is needed. Calculating the entropy of the decomposed accelerations reveals that 28 levels are enough for the decomposition analysis which is shown in Fig. 14. It is observed that the 10th level has the highest entropy which corresponds to the frequency of 5.2 Hz. This frequency is the dominant frequency of SE component of Coyote Lake and it can be recognized in Fourier spectrum and wavelet spectrum of the original acceleration. 28 decomposed accelerations are obtained after the decomposition process. The first, the Last and the two high energy components are shown in Fig. 15. It can be seen that the first and the last accelerograms, which have negligible entropies, have small amplitudes and the last component has

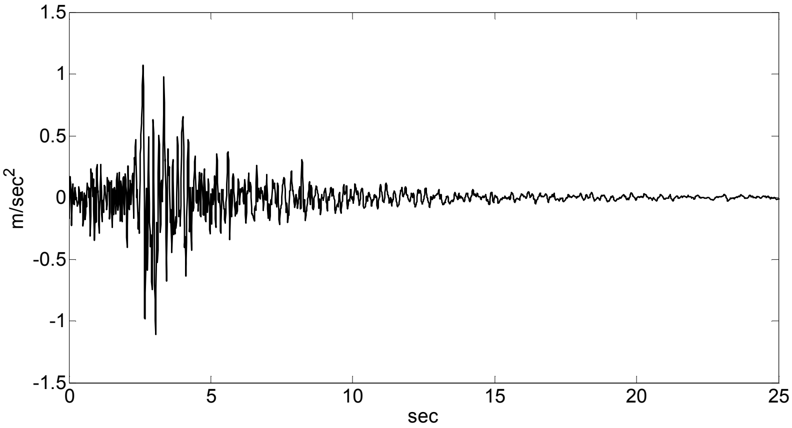


Fig. 11 SE component of Coyote Lake earthquake

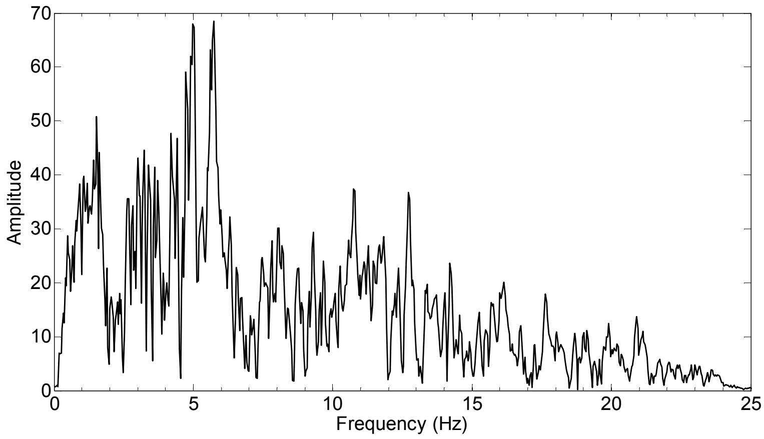


Fig. 12 Fourier spectrum of SE component of Coyote Lake earthquake

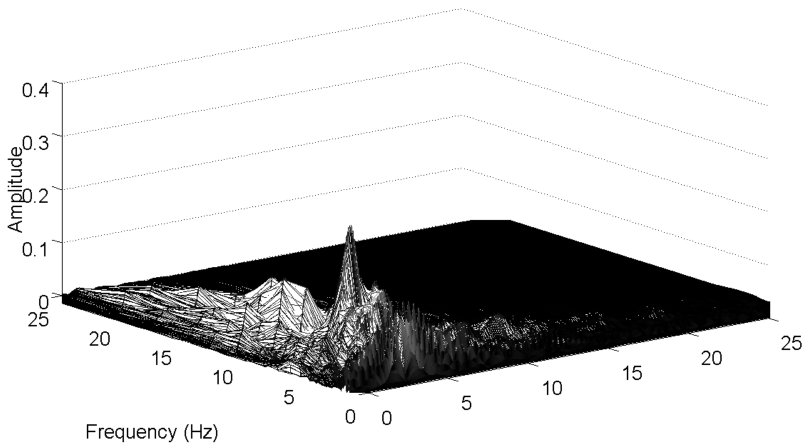


Fig. 13 3D wavelet spectrum of SE component of Coyote Lake earthquake

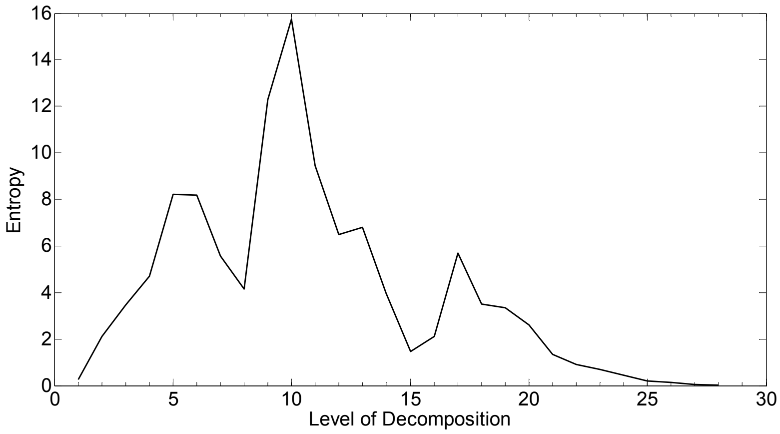


Fig. 14 Entropy of decomposed components in each level for SE component of Coyote Lake earthquake

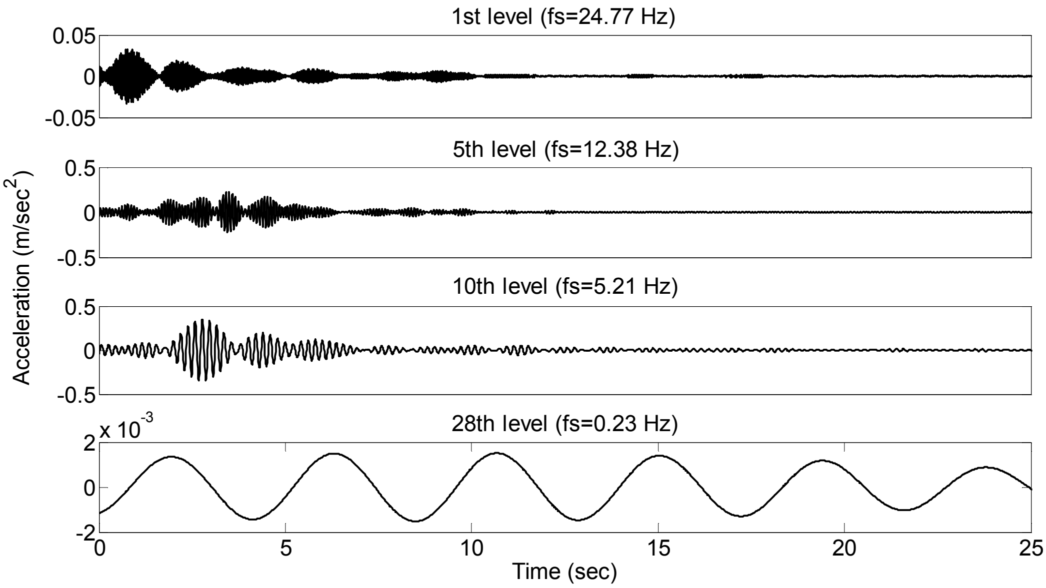


Fig. 15 Decomposed accelerograms of SE component of Coyote Lake earthquake

the form of damped free vibration. In Fig. 16, the Fourier spectrums of decomposed accelerograms are shown. Each component has a specific and non-overlapping frequency band and the calculated transfer functions in each layer can be multiplied to these transformed accelerograms to obtain the transformed acceleration for the next layer. Applying this recursive approach from bedrock to the surface layer, the surface acceleration can be obtained which is compared with the recorded field observation and those calculated from the conventional method in Fig. 17. It can be seen that, unlike the conventional method, the proposed procedure gives acceptable results and predict better acceleration than the FFT-based method. Comparisons of Figs. 11 and 17 shows that amplification occurs for Coyote lake earthquake which is the opposite case for Loma Prieta earthquake. The reason of this amplification is related to the frequency content of the input acceleration. As can be seen in Figs. 12 and 13, the initial and effective part of the input motion has medium to high

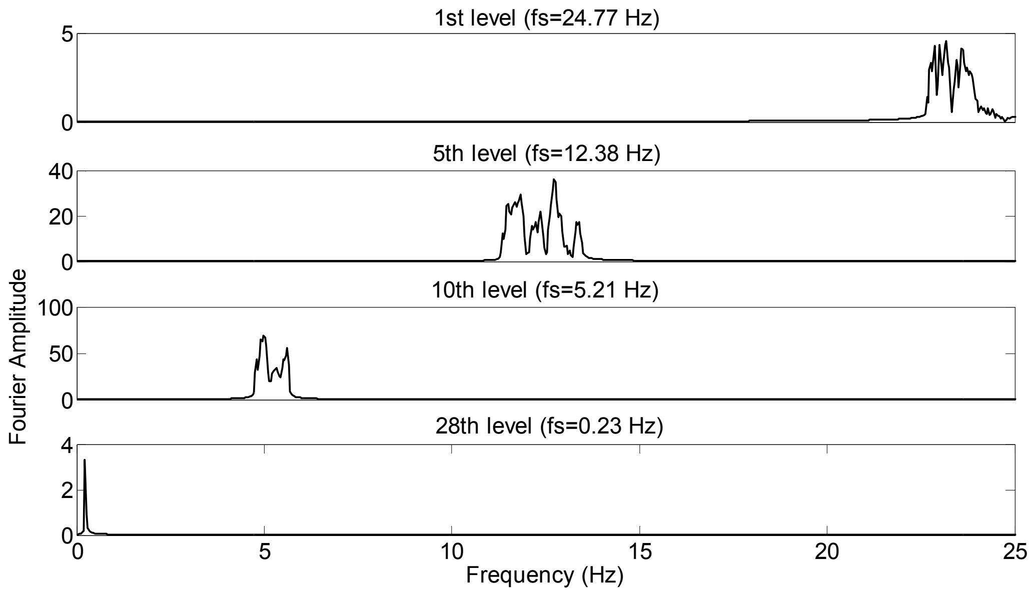


Fig. 16 Fourier transform of decomposed accelerograms of SE component of Coyote Lake earthquake

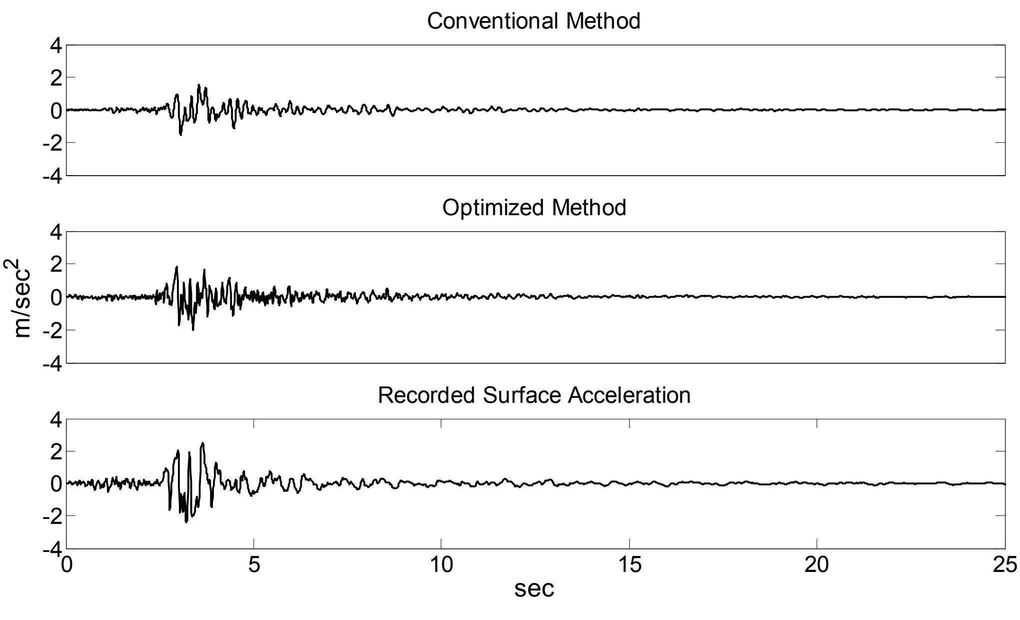


Fig. 17 Comparison of the proposed method with conventional and Recorded surface acceleration (Coyote Lake earthquake)

frequencies. Kausel and Assimaki (2002) studies show that shear modulus decreases and damping ratio increases just for low frequency range. Therefore, the frequency dependency of shear modulus and damping ratio does not affect or reduce the values of shear modulus in soil layers leading to amplification of input motion as seen in Fig. 17. It is observed that in a given site, both amplification and deamplification may develop which is dependent to the frequency content of the accelerations occurred in the site. These results confirm the importance of frequency analysis and the need for developing efficient methods for calculating the ground response analysis such as the one proposed in this research.

5. Conclusions

This paper presented a new method for optimizing the performance of transfer functions based methods. Since the transfer functions are formulated in terms of frequency, therefore, acceleration time histories were transformed into frequency domain usually by Fourier transform method. However, FFT method is unable to represent all frequencies in each time steps and resolve them in terms of time. This deficiency causes error in the analysis process and leads to spurious results. In the method developed in this research, unlike other approaches, the accelerations first decomposed into several components, which have non-overlapping frequency bands, using wavelet multi-level decomposition analysis. Since all frequency information are maintained in decomposed accelerations, they can be transformed into frequency domain and multiplied by transfer functions giving more accurate results for ground response analysis. A program in MATLAB environment is developed based on the proposed formulations. Using EPRI technical research, a reference site with its measured in-situ dynamic soil properties is chosen while two earthquakes, different in PGA and magnitude, occurred near the reference site, are chosen for investigations. Using the analysis concept presented here, acceleration time histories at the surface are obtained. As was discussed in the paper, the proposed method predicted more reasonable and acceptable results than the conventional method. It was observed that for a given site, both amplification and deamplification may occur depending on the nature of frequency content of incident earthquake motion, however, the conventional method was unable to predict this issue and the proposed method shows its efficiency in providing more accurate results.

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