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In-house calibration of pressure transducers and effect of material thickness

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Abstract. Pressure transducers are increasingly used within soil mass or at soil-structure interface for appraisal of stresses acting at point of installation. Calibration of pressure transducers provides a unique relationship between applied pressure and voltage or strain sensed by transducer during various loading conditions and is crucial for proper interpretation of results obtained from pressure transducers. In the present study an in-house calibration device is used to calibrate pressure transducers and the study is divided into two parts: 1) demonstration of developed calibration device for fluid and in-soil calibration of pressure transducers; 2) effect of soil layer thickness on the earth pressure cell (EPC) output. Results obtained from the present study revealed successful performance of the developed calibration device, and significant effect of sand layer thickness on the calibration results. The optimum sand layer thickness is obtained as 1.5 times the diameter of EPC.

Keywords: in-house calibration device; stress measurement; in-soil calibration; particulate material; sand layer thickness

1. Introduction

Stress evaluation within a soil-mass or at soil-structure interface is always a matter of investigation because of its importance in the fields of geotechnical, structural, mechanical and allied fields. As the rheology of soil is a complicated function of material type, stress history, shear and normal stress levels, boundary conditions and many environmental effects the stress registered by the earth pressure cell (EPC) will not be the same as the stress which have existed at that point, if the EPC was not present.

In order to obtain proper information about stress at the point of installation, proper correlation between applied pressure and pressure sensed by transducer is obtained by calibration. Calibration of pressure transducer is done either by applying fluid pressure or in-soil pressure. Calibration using fluid is done to check 1) instrument's physical condition 2) response to applied pressure and 3) return to zero after removal of applied pressure. In-soil calibration using soil is done to check 1) hysteretic behavior upon loading and unloading 2) variation of calibration factor with soil condition and 4) variation of calibration factor with stress history.

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2. Literature review

2.1 Fluid calibration

Various approaches have been adopted in the past for laboratory fluid calibration of pressure transducers. Redshaw (1954) and Ramirez *et al.* (2010) calibrated pressure transducers by subjecting them to external oil pressure by means of dead weight calibrator. The other approaches are I) use of centrifugal technique to calibrate pressure transducers (Pang 1986, Take 1997, Chen and Randolph 2006), II) use of application of fluid pressure to calibrate pressure transducer (Frydman and Keissar 1987, Clayton and Bica 1993, Labuz and Theroux 2005, Rusinek *et al.* 2009, Ramirez *et al.* 2010).

2.2 In-soil calibration

In the past, calibration of EPC within the soil was carried out using modified triaxial test set up/ Rowe cell set up (Clayton and Bica 1993, Chen and Randolph 2006) and direct shear box set up (Madabhushi and Khokher 2010). Clayton and Bica (1993) used a Rowe cell to calibrate the EPC by maintaining a ratio of diameter of EPC diaphragm to the height of sand as 0.57. Labuz and Theroux (2005) also developed a calibration device similar to a Rowe cell for calibration of EPC. Madabhushi and Khokher (2010) calibrated EPC in a direct shear box, and applied loading-unloading cycles using standard weights. However, the conventional direct shear apparatus (60 mm \times 60 mm) restricts calibration of EPC, whose diaphragm diameter is less than 20 mm, in order to avoid boundary effects. Further, arching effect may influence the calibration results, as for in-soil calibration, EPC should be fixed flushed with the surrounding mounting body so as to avoid arching between sand and diaphragm of the EPC.

The devices discussed above were EPC dimension specific, and cannot be used for calibrating EPC of varying specifications. These devices may not be suitable to calibration of EPC using critical soil height, due to their dimensional restrictions. Also, the devices developed by previous researchers could either be used to calibrate the EPC in fluid or in-soil.

Data interpretation using standard calibration curve, obtained using fluid calibration, leads to measurement errors when EPC is used in soil for stress measurement. The alternative but more complex procedure of in-soil calibration of EPC may reduce this error considerably as a more realistic calibration curve can be obtained (Selig 1980, Weiler and Kulhawy 1982, Selig 1989). The reliable measurement of stresses in soil is still difficult to achieve due to strong dependency of measurement on relationship between EPC and soil stiffness (Hadala 1967, Hvorslev 1976, Dunnicliff 1988). Various factors affecting EPC output during in-soil calibration were broadly classified into inclusion effects, EPC-soil interaction, placement effects, environmental influence and dynamic response (Weiler and Kulhawy 1982, Dunnicliff 1988). The literature is full of examples illustrating the effect of placement method, soil density/stiffness, geometry, grain size, loading history and soil type on the calibration of EPC. Askegaard (1994) suggested to test EPC under as varied conditions as possible to get an estimate of the accuracy obtainable when the EPC are used in practice in unknown material and loading histories.

Previous studies on the effect of soil thickness on EPC output (Terzaghi 1943, McNulty 1965, Mason 1965, Ingram 1968) revealed that at soil thickness about 2 D_{EPC} (where D_{EPC} is diameter of EPC) no additional decrease in the sensitivity should be expected. This thickness is called critical soil thickness above which no further arching will develop if the peak soil strength is mobilized

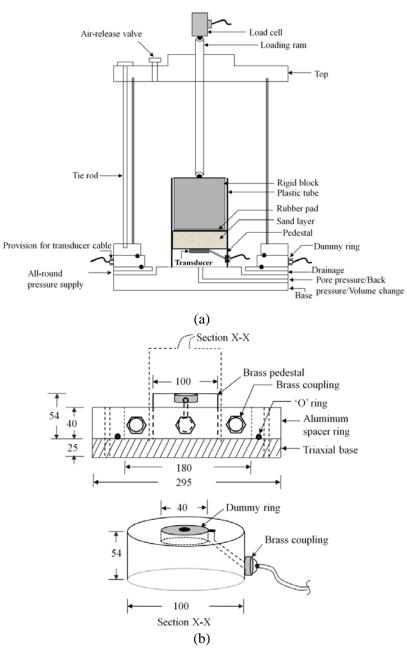


Fig. 1 Details of in-house calibration device by modification in triaxial set up (a) modified triaxial apparatus (b) details of modifications (Dave and Dasaka 2012b) (all dimensions are in mm)

and would render accurate pressure. However, soil thickness less than D_{EPC} were not recommended as the EPC output was somewhat erratic. Labuz and Theroux (2005) used EPC of 55 mm diameter and soil thickness of 12.5 and 25 mm for calibration and observed a decrease in EPC sensitivity with increase in soil height and noted that the critical thickness of soil depends on the deflection of diaphragm of the EPC and it can be less than D_{EPC} .

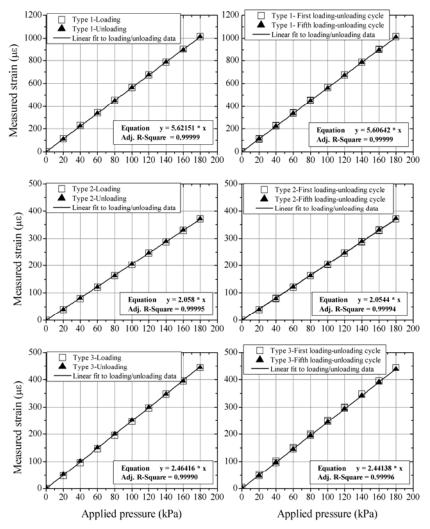


Fig. 2 Performance of Type 1, Type 2 and Type 3 transducers (a) under first loading-unloading cycle; (b) under first & fifth loading-unloading cycle

3. Need for the present study

Based on the above reviewed literature, it is worthy to develop a device for both fluid calibration of pressure transducers and in-soil calibration of EPC of different diameters. In case of in-soil calibration, critical soil thickness (i.e. soil thickness where no arching develops and sidewall friction is minimal) differs with stiffness and diameter of EPC diaphragm. Further, it may differ with density and particle size distribution of soil. Hence, the developed device should be capable of using with a wide range of soil thicknesses. The developed calibration device should be economically viable, having simple working mechanism, reasonably accurate, easy to adopt, and can be fabricated by incorporating limited modifications to the existing triaxial cell.

Present study is divided into two parts: 1) description and demonstration of calibration device

developed, and 2) study of effect of thickness of sand layer to calibrate the EPC using modified triaxial set up and large calibration chamber. In the above set ups, the ratio of thickness of sand layer and diameter of EPC is restricted to 2.5 and 15, respectively.

4. Details of calibration device

A calibration device was developed by modifying a conventional triaxial apparatus suitable for testing 100 mm diameter soil specimen, as shown in Fig. 1(a). The modifications consist of 1) a dummy aluminum spacer ring with brass couplings 2) a brass pedestal with a set of replaceable dummy rings (Fig. 1(b)). The combination of dummy ring and pedestal was planned to allow transducers of various diameters to fix flushed on the pedestal. Efforts were made to prevent water entry through the space around pressure transducer by applying silicon rubber gel around the cables of transducers. Further details about the developed device can be seen in Dave and Dasaka (2012b).

5. Calibration procedure

5.1 Fluid calibration using the developed device

The calibration device was fully filled with de-aired water, and pressure was applied by an air-water bladder cylinder, using compressed air with an accuracy of 0.1 kPa. Three different transducers namely pore pressure transducer (Druck–PDCR 81) hereafter referred to as Type 1, medium size EPC (Haris make) of diameter 40 mm (Type 2) and miniature EPC (TML make) of diameter 6.5 mm (Type 3), were used for verifying the accuracy of developed calibration device. A detailed description of all three transducers is presented in Table 1. The fluid pressure in the triaxial cell was increased to a maximum of 180 kPa, in increments of 20 kPa. The transducer data was recorded in terms of the output strain from the transducer at each step of loading as well as unloading. Readings corresponding to each loading/unloading were obtained once the fluid pressure had stabilized typically in 30 seconds. Total 5 loading/unloading pressure cycles were

pressure had stabilized, typically in 30 seconds. Total 5 loading/unloading pressure cycles were performed for calibration test on each transducer. The relationship between applied pressure and measured strain for all three transducers is shown in Fig. 2. Measurement non-linearity was also obtained for all three transducers.

	Table1 Detailed specification of transducers used for calibration (Dave and Dasaka 2012b)
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Terminology Pressure range	$0-15 \text{ kg/cm}^2$	$0-2 \text{ kg/cm}^2$	$0-2 \text{ kg/cm}^2$
Sensitivity	1.146 mV/V/bar	1.420 mV/V at FS	$+946 \ \mu V/V$
Non-linearity & hysteresis	0.4% of full scale	0.5 % of full scale	0.5 % of full scale
Thermal sensitivity shift	±0.2%/°C	0.1 % of full scale/°C	1%/°C
Dimensions	$5mm \Phi \times 10mm$	$40mm \ \Phi \times 10mm$	6.5 mm $\Phi \times 1$ mm

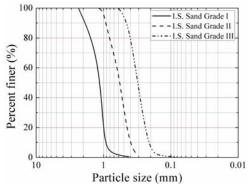


Fig. 3 Particle size distributions for materials used in the present study

5.2 In-soil performance of EPC

In-soil performance of Type 2 transducer was observed using three grades of sand (Indian Standard sand, commercially known as Ennore sand of Grade I, Grade II and Grade III). The particle size distribution curves of sand are shown in Fig. 3. The modified triaxial setup with additional rigid plastic tube of 2.5 mm thickness and 100 mm internal diameter (ID) placed tightly on the brass pedestal was used with Type 2 transducer fixed flush on the pedestal. Considering D_{EPC} (for Type 2 transducer $D_{EPC} = 40$ mm) as reference, sand layer thicknesses of $0.25D_{EPC}$, $0.5D_{EPC}$, D_{EPC} , $1.5D_{EPC}$ and $2.5D_{EPC}$ were placed above the EPC to obtain the effect of sand layer thickness on the relation between applied pressure and measured strain. Greased polyethylene sheets of 60 µm thickness were pasted to inner surface of plastic tube in order to reduce side wall friction. A rigid wooden block overlying a thick rubber pad was placed on sand layer for transfer and uniform distribution of pressure between load cell of triaxial apparatus and EPC as shown in Fig. 1.

Sand pouring method was adopted to obtain a unit weight of 16 kN/m³, which was maintained constant for all the tests. Incremental displacements were applied manually to the triaxial base, thereby monitoring applied stress on wooden block through load cell and induced strains in the EPC. Measured strains for all three grades of sand and thicknesses for first loading cycle up to 50 kPa pressure were taken into consideration. Results obtained from the in-soil calibration were compared with that of fluid calibration as shown in Fig. 4.

5.3 Performance studies of EPC using large calibration chamber

Performance of EPC was also studied using a calibration chamber of internal dimensions 1.2 m $\times 0.31 \text{ m} \times 0.7 \text{ m}$ height, which was further used to conduct model experiments on retaining wall. The three sides of chamber were made up of 12 mm stainless steel plates and Perspex sheet of 25 mm was placed on the other side. All sides of tank were applied with 10 cm wide greased polyethylene sheets of 60 µm thickness, overlapping one another to reduce surface friction between sand particles and sides of tank. Sand filling in the tank was restricted to 0.9 m $\times 0.31 \text{ m} \times 0.6 \text{ m}$ by providing a vertical support of 20 mm thick wooden ply of same width as that of tank, which was further supported by spacer blocks as shown in Fig. 5. A stainless steel plate of 15 mm thickness having multiple recesses of dimensions same as that of EPCs to fix them flush was

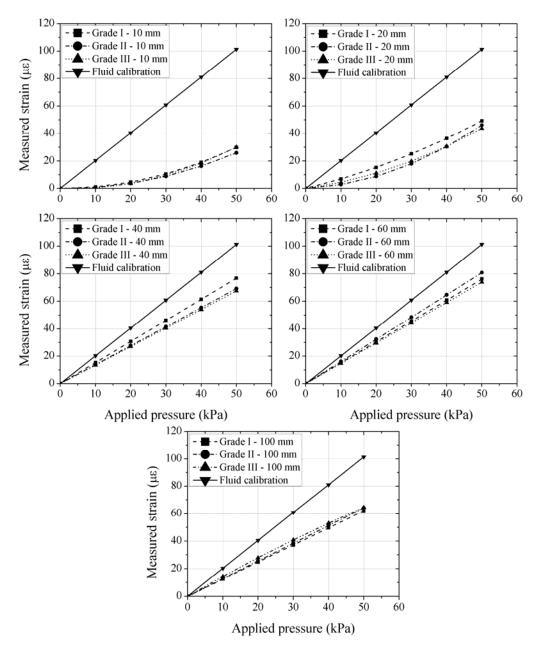
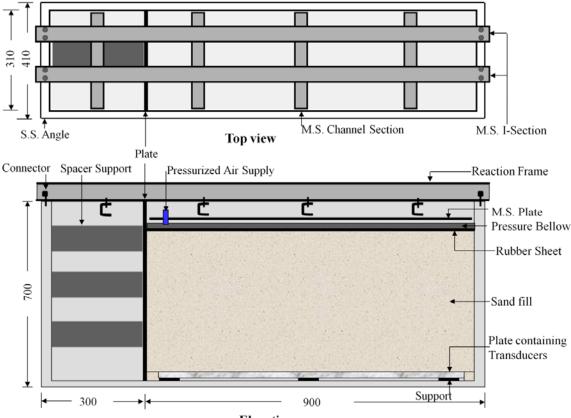


Fig. 4 Effect of particle size on EPC performance using: (a) 10 mm sand thickness; (b) 20 mm sand thickness; (c) 40 mm sand thickness; (d) 60 mm sand thickness and (e) 100 mm sand thickness

placed at the bottom of the tank. Total 7 EPCs (Type 2 and Type 3) were fixed at pre-defined locations on the steel plate. The EPC cables were safeguarded by wrapping them with a sheet of geotextile around it. Thin latex rubber sheet (0.2 mm thick) were pasted on the EPC surfaces based on suggestion of manufacturer, as shown in Fig. 6. After placing steel plate at the bottom, sand bed

of 60 cm height and 16 kN/m³ unit weight was placed using a traveling pluviator of the type developed by Dave and Dasaka (2012a), which consists of an orifice and diffuser system. To apply uniformly distributed surcharge on the backfill, a rubber bellow was placed over an 8 mm thick rubber sheet overlying the backfill. A steel plate of 10 mm thickness was placed on the rubber bellow such that when rubber bellow was inflated with compressed air, the plate moves upwards to come in contact with the reaction frame, which was rigidly connected to the tank, thereby allows transferring pressure to the sand fill.

Pressure was applied in 10 kPa increments and next increment was applied only after steady readings were achieved under the application of previous increment (typically about 1-2 min) in the pressure range of 0 to 50 kPa. Using Grade II sand, three sets of tests were performed to see reliability of EPC output data. Little scatter was observed between applied pressure and measured strain and a linear regression curve fitting was done to obtain relationship between applied pressure and measured strain for all EPCs as shown in Fig. 7. The large calibration chamber test results were compared with results of fluid calibration and EPC performance for various thicknesses and grades of sand were shown in Fig. 8. Further, output of EPC in large calibration chamber for all three grades of sand was compared with fluid calibration as shown in Fig. 9.



Elevation

Fig. 5 Details of experimental set up for large scale performance studies of pressure cell (all dimensions are in mm)

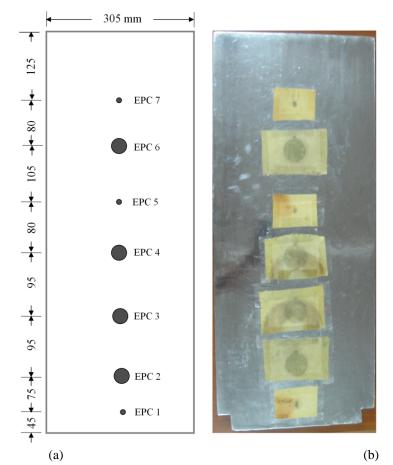


Fig. 6 Location and identification of pressure cells fixed on steel plate (a) schematic diagram (b) pictorial view

6. Results and discussion

6.1 Fluid calibration

Calibration performance of three different transducers viz. a pore-pressure transducer, medium and miniature size EPC were studied using universal calibration device developed for this purpose. Calibration performances of these transducers in the range of 0-180 kPa for first loading-unloading cycle and fifth loading-unloading cycles are presented in Fig. 2. Results of first loading-unloading cycle indicated maximum non-linearity of 0.6% of FS, 0.31% of FS and 2.64% of FS for each of these transducers, respectively. Further, maximum non-linearity of 0.79% of FS, 0.4% of FS and 2.67% of FS were observed for five consecutive loading-unloading cycles for each of these transducers, respectively.

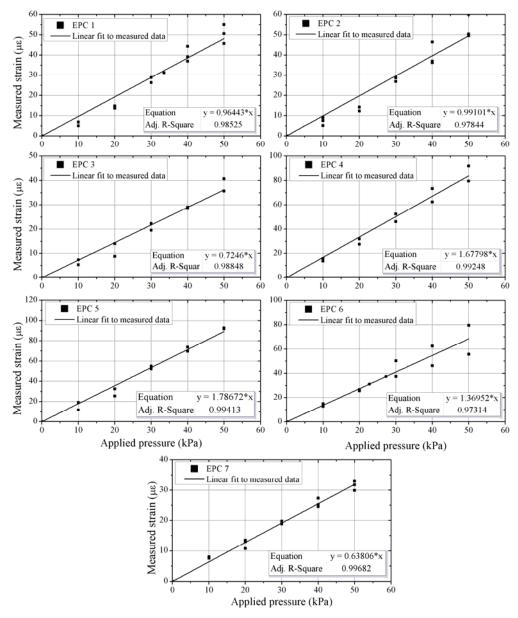


Fig. 7 In-soil performance of EPC 1 - EPC 7 at large scale

6.2 Effect of sand thickness

Effect of sand layer thickness on calibration of EPCs was studied by considering sand thicknesses of 10 mm (0.25 D_{EPC}), 20 mm (0.5 D_{EPC}), 40 mm (D_{EPC}), 60 mm (1.5 D_{EPC}), 100 mm (2.5 D_{EPC}) and 600 mm (15 D_{EPC}). Three different grades of sands were used during studies and test results are presented in Fig. 4.

The relation between applied pressure and measured strain is non-uniform for sand layer

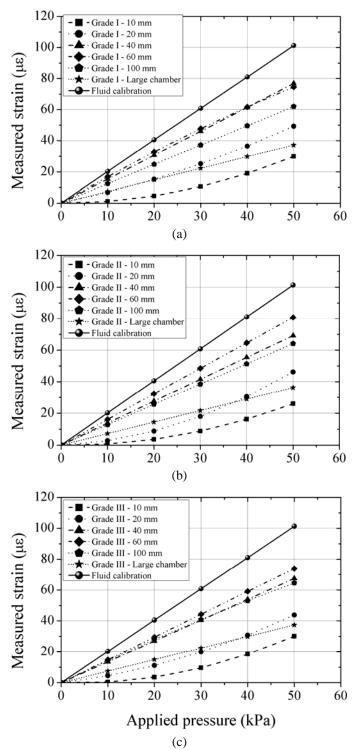


Fig. 8 Effect of sand bed thickness on performance of EPC: (a) Performance for sand of Grade I, (b) Performance for sand of Grade II and (c) Performance for sand of Grade III

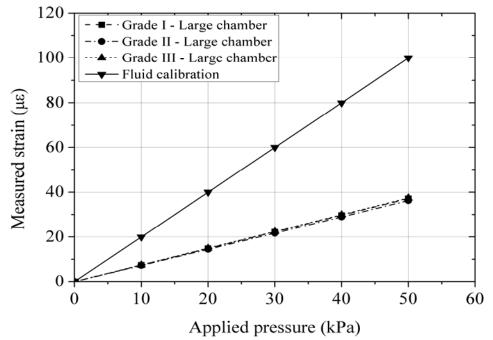


Fig. 9 Performance comparison for sand of Grade I, Grade II and Grade III with Fluid

thickness of 0.25 D_{EPC} and 0.5 D_{EPC} . As the pedestal body was more rigid than diaphragm of EPC, arching of sand might have caused non-uniform transfer of pressure, in line with observations by Ingram (1968). With increase in the sand layer thickness effect of arching would have been reduced, thus allowing uniform transfer of pressure from load cell to EPC, and thereby increasing output of EPC. Outputs using sand thicknesses of D_{EPC} and 1.5 D_{EPC} were almost similar and on higher side of that obtained for 0.25 D_{EPC} and 0.5 D_{EPC} , as shown in Fig. 4. However, with further increase in thickness of sand layer beyond 1.5 D_{EPC} , EPC output was reduced, which may be attributed to stress dissipation inside the soil causing only a portion of applied pressure to transfer to the diaphragm of the EPC. Also, the influence of surface friction, friction between the sand layer and the container holding the sand, may not be ruled out at higher thicknesses. To verify the observed trend of reduced strain with increased sand thickness, large calibration tests involving sand layer thickness of 15 D_{EPC} were also performed.

Performance of total seven EPC, viz. EPC 1 to EPC 7 (viz., 4 Nos. Type 2 and 3 Nos. Type 3 transducers), was observed using large calibration chamber tests. A set of three tests was performed using Grade II sand and one test each using Grade I and Grade III sand. Regression analysis of results obtained from the above tests indicated that a linear fit is good enough to represent the applied pressure-measured strain data. However, for the sake of brevity only results on Grade II sand were presented in Fig. 7.

Comparison of performance of EPCs in large calibration chamber with that using fluid calibration and modified triaxial in-soil calibration are presented in Fig. 8. As illustrated in figures, pressure sensed by EPC during fluid calibration was highest among all results. However, with increase in sand layer thickness, pressure sensed by EPC was reduced, and found lowest for large calibration chamber. Further, test results with sand layer thickness in the range of D_{EPC} to 2.5 D_{EPC}

showed sensed pressures in the range of 60-80% of that obtained from fluid calibration, in line with observations of previous researchers.

In-soil calibration experiments using three different grades of sand demonstrated significant effect of sand layer thickness on performance of EPC using modified triaxial setup. 0.25 D_{EPC} and 0.5 D_{EPC} thick sand layers produced nonlinear and erratic strains for various applied pressures. Sand thickness in the range of 1.5 D_{EPC} to 15 D_{EPC} sensed reduced strains with increased applied pressure. Hence it can be concluded that the optimum sand layer thickness is 1.5 D_{EPC} for D_{EPC} of 40 mm used in this study.

EPC 3 exhibited calibration factor of 2.054 in fluid calibration, whereas the corresponding calibration factors using in-soil calibration with sand layer thickness of 1.5 D_{EPC} were obtained as 1.522, 1.616 and 1.478 for sand of Grade I, II and III respectively.

From the results presented in Figs. 4 and 9 for Grade I, II and III sands, it is noted that the variation of measured strain is within +/-10%, in line with the observations of Labuz and Theroux (2005). The ratio of diaphragm of the EPC and maximum particle size (d/d_{max}) for the grades of sand used in the present study varies in the range of 18-66. This insignificant variation of measured strain with particle size matches with the observations of Muira *et al.* (2003), wherein it was concluded that the particle size effect on measured strain is quite minimal, if d/d_{max} is greater than 8.

Significant effect of chamber size on performance of EPC using relatively large chamber is presented in Fig. 8. From the studies, it can be understood that, one should be very cautious during interpretation of pressure obtained in model experiments involving sand using calibration factors obtained by fluid calibration. The above results highlighted the importance of EPC under conditions identical to its intended use.

7. Conclusions

The following are some of the salient conclusions drawn from the present study:

- In-house calibration device was developed by modifying existing triaxial device for fluid calibration and in-soil calibration of pressure transducers and results exhibited successful performance of developed calibration device. Hence the developed device can be used to check calibration of new pressure transducers and recalibrate used transducers.
- 2) From the calibration studies, it is noted that sand layer thickness of 0.25 D_{EPC} and 0.5 D_{EPC} exhibited nonlinear and erratic measured strain. Sand layer thickness more than 1.5 D_{EPC} sensed reduced strains with increased applied pressure. Hence it can be concluded that the optimum sand layer thickness is 1.5 D_{EPC} for D_{EPC} of 40 mm used in the present study.
- 3) Experiments with comparatively large calibration chamber sense significantly lower pressures compared to fluid calibration, which may be attributed to the stress dissipation within the soil.
- 4) No significant effect of particle size of sand on EPC output was observed during in-soil calibration of EPC, using modified triaxial set up and large calibration chamber.

In conclusion, calibration factors should be obtained by in-soil calibration of EPC, under the conditions similar to that prevail at the place of intended use for obtaining reliable results.

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