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Nonlinear consolidation of soft clays subjected to cyclic loading - Part I: theory

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Abstract. In this paper, utilizing void ratio-effective stress and void ratio-permeability relationships, a system of two nonlinear partial differential equations is derived to predict the consolidation characteristics of normally consolidated (NC) and overconsolidated (OC) soft clays subjected to cyclic loading. A developed feature of the coefficient of consolidation containing two key parameters is emerged from the differential equations. Effect of these parameters on the consolidation characteristics of soft clays is analytically discussed. It is shown that the ratios between the slopes of e–logσ' and e–log k lines in the NC and OC states play a major role in the consolidation process. In the companion paper, the critical assumptions made in the analytical discussion are experimentally verified and a numerical study is carried out in order to examine the proposed theory.

Keywords: nonlinear; consolidation; cyclic loading; permeability; compressibility; soft clays.

1. Introduction

Every structure on the earth is founded on/in geotechnical materials such as soil and rock. If the produced settlement of the surrounding soil is not kept at tolerable limits, the desired use of the structure would be impaired and the design life of the structure may reduce. Fine-grained soils settle and rebound periodically when subjected to long-period cyclic loading of silos and reservoirs or short-period cyclic loading of offshore structures and tall buildings. Also, in highly populated semi-arid regions, consecutive oscillation of aquifer water level due to seasonal precipitation and over-exploitation of groundwater periodically changes the effective stresses in the soil layers and imposes cyclic loading resulting in an environmental disaster, called "Land Subsidence" (Yazdani *et al.* 2010, Ouria and Toufigh 2010).

The time-dependent process of volume changes in soil, as water is squeezed out of or returned back to the voids, is governed by complex interactions between effective and total stresses, pore pressure, seepage, compressibility and stress history. This process is referred to as "consolidation under cyclic loading". The most popular theory predicting the consolidation process is the celebrated Terzaghi's consolidation theory. Its publication with its clear implication of the concept of effective stress and pore-water pressure is often considered to mark the beginning of modern soil mechanics

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(Terzaghi *et al*. 1996). In the derivation of this theory, it was tacitly assumed that the permeability and compressibility of soil as well as loading intensity during time are constant. This is not usually the case; while the soil consolidates, void spaces, and hence, permeability and compressibility are reduced. It concludes that the coefficient of consolidation is not constant. Over the past five decades, a number of researchers have studied the complicated phenomenon of the soft soil consolidation.

Alonso and Krizek (1974) and Rahal and Vuez (1998) considered an elastic behavior for soft soils to analyze their consolidation under time-dependent loading. Subsequently, it was reported in some papers that assuming an elastic behavior for soft soil results in considerable inaccuracies in consolidation evaluation in many cases. Therefore, some different consolidation theories were developed to include variations in loading rate, boundary conditions, coefficient of permeability, and coefficient of compressibility (Schiffman 1958, Davis and Raymond 1965, Battaglio *et al*. 2003 and 2005, Zhuang *et al*. 2005). Baligh and Levadoux (1978), Favaretti and Mazzucato (1994) and Toufigh and Ouria (2009) introduced some simple prediction methods for consolidation of elastic and inelastic soils under time-dependent loading. In addition, consolidation of stratified or non-homogeneous soils under cyclic loading was analyzed by Zhu and Yin (1999) and Xie *et al*. (2005). Fox *et al*. (2003) and Cai *et al*. (2007) examined large strain and finite-strain consolidations under cyclic loading. A collection of analytical investigations as well as experimental and field tests on the consolidation under cyclic loading is found in Xie *et al*. (2006), Conte and Troncone (2006, 2007), and Yıldırım and Ersan (2007). Most of these studies assume that there is a constant ratio between the soil coefficient of consolidation in normal and over consolidation states. It is not usually the case since soil is a stress-dependent material and its characteristics change with changes in effective stress. On the other hand, the time factor application to determine the consolidation characteristics of a soil subjected to cyclic loading is problematic due to continuous changes in the soil properties and loading condition, although it has been widely used in the aforementioned researches. Therefore, developing a different approach to consider the real changes in the compressibility and permeability of soil during consolidation is advantageous.

Abbasi *et al*. (2007) presented a nonlinear partial differential equation for consolidation of NC clays. This equation considers the variation of compressibility and permeability using the idealized linear relationship between void ratio, effective stress and permeability. In this paper, the equation is extended to derive another differential equation to examine the consolidation behavior of OC clays (Yazdani 2008). The equations take into account stress time history but disregard the self-weight and creep effects. Also, by making a direct connection between settlement and time, the equations eliminate the limitations associated with the coefficient of consolidation, namely its variations and the empirical nature of the existing methods to determine it.

Section 2 describes the general behavior of inelastic clays under cyclic loading. The nonlinear partial differential equations and their theoretical bases are derived in Section 3. Section 4 is allocated to an analytical discussion on the variation of the coefficient of consolidation under cyclic loading. The companion paper reports the results of an experimental program and a numerical study carried out to verify the proposed theory.

2. Behavior of inelastic clays under cyclic loading

When load is applied on a soft clay layer, the effective stress will increase and the soft clay will consolidate. However, due to inelastic nature of the clay, the compression is not fully reversed upon

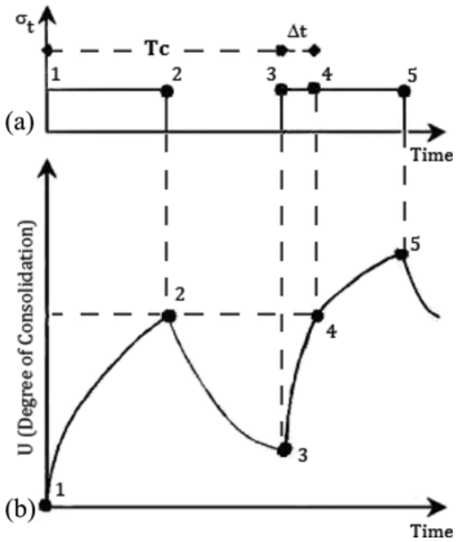
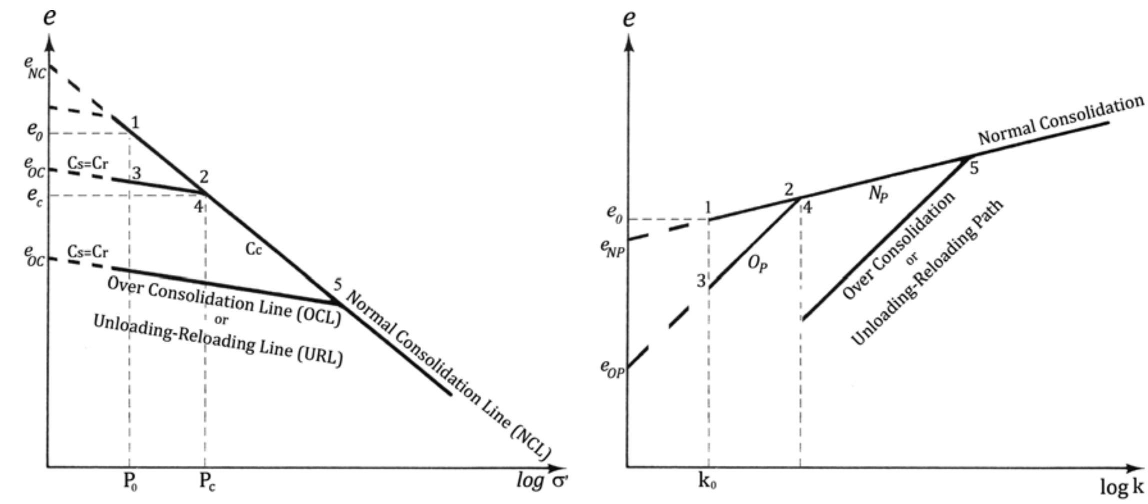


Fig. 1 Inelastic behavior of soft clay: (a) cyclic loading and (b) degree of consolidation

unloading. Inelastic behavior of the soft clay subjected to a cyclic loading in a limited stress range can be studied using an idealized bilinear model, shown in Figs. 1 and 2. The degree of consolidation of the inelastic clay under a rectangular cyclic loading, Fig. 1(a), is shown in Fig. 1(b). Also, variation of compressibility and permeability of the clay under cyclic loading is shown in Figs. 2(a) and 2(b), respectively.

The coefficients of compressibility and permeability of inelastic clays change during the loading and unloading half cycles. Consequently, the coefficient of consolidation, C_v, which is a function of these parameters, also changes in each loading cycle. Over the first loading half cycle soil is normally consolidated and behaves according to the route {1–2} while during the first unloading



(a) (b)

Fig. 2 Idealized variation of void ratio with (a) effective stress; (b) permeability cycle soil is overconsolidated and its characteristics change along with the route {2–3}, as shown in Figs. 1 and 2. After the first full cycle is completed, the soil properties change in consistent with the route {3−4−5} for the next loading half cycle. It should be stated here that, for simplicity, the slopes of e–logσ' in unloading and reloading steps are considered the same ($C_s = C_r$). At any time during cyclic consolidation, an element of clay at some elevation is said to be normally consolidated when the (incremental vertical) effective stress, σ', is equal to the maximum past pressure, pre-consolidation pressure (P_c), to which this element has been subjected since the beginning of cyclic loading. On the other hand, the element is said to be overconsolidated when $\sigma' < P_c$ (Baligh and Levadoux 1978). Then, position of point 4 is the same as the preconsolidation pressure (P_c) application point, which is imposed in the previous cycle. The preconsolidation pressure increases as the number of cycles is increased until a steady state is reached in which the soil layer remains OC. During unloading, soil rebounds in the upper layer near the pervious boundary while the lower part or the middle section of soil layer remains compressed depending on drainage conditions. Consequently, it is necessary to distinguish between the compressing part and rebounding part of soil during analysis so that the compressibility can be modeled appropriately. Therefore, developing a new model taking into account the cyclic loading condition as well as the variation of soil compressibility and permeability is advantageous.

3. Theoretical basis of the proposed model

3.1 Fundamental equation

The fundamental equation to derive the governing partial differential equations is the well-known Terzaghi's consolidation theory which can be written as (Terzaghi *et al.* 1996)

$$\frac{1}{1+e_0}\frac{\partial e}{\partial t} = -\frac{k}{\gamma_w}\frac{\partial^2 \sigma'}{\partial z^2} \tag{1}$$

where u is the excess pore-water pressure, k is the coefficient of permeability, γ_w is the unit weight of water, σ' is the effective stress, z is the downward coordinate through the soil layer, e_0 is the initial void ratio, and e is the void ratio at time t. The piece-wise linearization method is used in the companion paper to keep the assumptions behind Eq. (1) valid.

3.2 Void ratio-effective stress and void ratio-permeability relationships

The linear relationship between void ratio and effective stress in a semi-logarithmic scale has already been revealed. According to Fig. 2(a), for an NC clay this relationship can be expressed as

$$e = e_{NC} - C_c \log\sigma' \tag{2}$$

where C_c is the slope of the straight line and known as the compression index, e is the void ratio at the effective stress σ', and e_{NC}, which is the line's intercept, is the void ratio at unit effective stress and hence is a constant value for a given soil and is a function of the unit used for effective stress. Also, for an OC clay

$$e = e_{OC} - C_s \log\sigma' \tag{3}$$

where C_s is the slope of the straight line and known as the swell index and for the sake of simplicity it is considered equal to the recompression index, C_r, the slope of reloading line in e–logσ' space. The definition of e_{OC} is similar to that of e_{NC} except that depending on the pre-consolidation pressure, P_c, its value varies during the consolidation process. On the other hand, according to Fig. 2(b), for an NC clay a similar linear relationship can be presented between void ratio and permeability as (Davis and Raymond 1965, Mesri and Rokhsar 1974, Lekha *et al*. 2003, Abbasi *et al*. 2007)

$$e = e_{NP} + N_P \log k \tag{4}$$

where e_{NP} and N_P are the intercept and slope of the straight line, respectively. The intercept, e_{NP}, is the void ratio at unit coefficient of permeability ($k = 1$) and is constant for a given soil but varies with the unit used for k. In the OC state

$$e = e_{OP} + O_P \log k \tag{5}$$

where e_{OP} and O_P are the intercept and slope of the straight line, respectively. Again, depending on the preconsolidation pressure, P_c, e_{OP} varies during the cyclic consolidation process. For better recall, these parameters are tagged as letters “N, “O, “C and “P referring to the NC state, the OC state, Compressibility, and Permeability, respectively. It is worthy to mention that the well-known Kozeny-Carman formula is not appropriate for clayey soils due to its different assumptions such as the absence of electrochemical reactions between soil particles and water (Carrier 2003).

By replacing the left-side of Eq. (2) with the right-side of Eq. (4) with some arrangements an inverse relationship between effective stress and permeability is obtained as

$$k_{NC} = 10^{(e_{NC}-e_{NP})/N_P}(\sigma')^{-C_c/N_P} \tag{6}$$

Doing similar operations on Eqs. (3) and (5) leads to

$$k_{OC} = 10^{(e_{OC}-e_{OP})/O_P}(\sigma')^{-C_s/O_P} \tag{7}$$

It should be stated that the units of the involved parameters must be selected appropriately.

3.3 Mathematical models for soil consolidation

Calculating the first derivative of Eq. (2) with respect to time

$$\frac{\partial e}{\partial t} = \frac{-2.303 C_c}{\sigma'}\frac{\partial \sigma'}{\partial t} \tag{8}$$

and introducing Eqs. (6) and (8) into Eq. (1) with some rearrangements leads to

$$\frac{\partial \sigma'}{\partial t} = \frac{2.303(1+e_0)}{C_c \gamma_w} 10^{\frac{e_{NC}-e_{NP}}{N_P}} (\sigma')^{\left(1-\frac{C_c}{N_P}\right)} \frac{\partial^2 \sigma'}{\partial z^2} \tag{9}$$

By defining

$$\alpha_{NC} = 1 - \frac{C_c}{N_P} \tag{10}$$

$$\beta_{BC} = \frac{2.303(1+e_0)}{C_c\gamma_w}10^{\frac{e_{NC}-e_{NP}}{N_P}} \tag{11}$$

Eq. (9) is reduced to

$$\frac{\partial\sigma'}{\partial t} = \beta_{NC}(\sigma')^{\alpha_{NC}}\frac{\partial^2\sigma'}{\partial z^2} \tag{12}$$

Eq. (12) is a nonlinear partial differential equation which governs variations of effective stress during the nonlinear one-dimensional consolidation of an NC soil layer while assuming linear relationships between e–logσ' and e–log k. Coefficients α and β are called *nonlinearity factor* and *basic coefficient of consolidation*, respectively (Abbasi *et al.* 2007). Eq. (12) is similar to the Terzaghi's consolidation theory except that C_v has a generalized form of

$$C_{vNC} = \beta_{NC}(\sigma')^{\alpha_{NC}} \tag{13}$$

Eq. (13) shows that the coefficient of consolidation is not constant during consolidation and changes with the effective stress. For many soils C_c/N_P often lies within the limits of 0.5 to 1 (Mesri and Rokhsar 1974, Terzaghi *et al*. 1996, Abbasi *et al*. 2007). Hence, the nonlinearity factor, α, is often between zero to unity. When $\alpha = 0$ ($C_c/N_P = 1$), C_v will be constant and equal to β and Eq. (12) will be degenerated to the Terzaghi's consolidation equation (Rahal and Vuez 1998, Xie *et al*. 2006). For the OC state, a similar approach can be used to derive an analogous governing equation as

$$\frac{\partial\sigma'}{\partial t} = \beta_{OC}(\sigma')^{\alpha_{OC}}\frac{\partial^2\sigma'}{\partial z^2} \tag{14}$$

in which

$$\alpha_{OC} = 1 - \frac{C_s}{O_P} \tag{15}$$

$$\beta_{OC} = \frac{2.303(1+e_0)}{C_s\gamma_w}10^{\frac{e_O-e_{OP}}{O_P}} \tag{16}$$

$$C_{vOC} = \beta_{OC}(\sigma')^{\alpha_{OC}} \tag{17}$$

Using the effective stress principle, Eqs. (12) and (14) can be rewritten in terms of excess pore-water pressure, u, as

$$\frac{\partial(\sigma_t-u)}{\partial t} = \beta_{NC}(\sigma_t-u)^{\alpha_{NC}}\frac{\partial^2(\sigma_t-u)}{\partial z^2} \tag{18}$$

$$\frac{\partial(\sigma_t-u)}{\partial t} = \beta_{OC}(\sigma_t-u)^{\alpha_{OC}}\frac{\partial^2(\sigma_t-u)}{\partial z^2} \tag{19}$$

Eqs. (18) and (19) show the influence of a time-dependent loading on the consolidation process. It is also seen that the type and rate of loading is able to affect the coefficient of consolidation of a soil.

A very important difference between Eqs. (12) and (14) is that during the consolidation under cyclic loading, β_{NC} is constant but β_{OC} changes with changes in void ratio at unit effective stress, e_{OP}. There is a specific governing equation for each of the NC or OC states. Also, there is no closed form solution to determine the time at which the consolidation state of a point within the soil mass changes from NC to OC or vice versa, point 4 in Figs. 1 and 2. Therefore, there cannot exist an analytical solution for the nonlinear consolidation under cyclic loading. However, the variation of the obtained coefficients of consolidation is analytically studied.

4. Analytical study on the variation of the coefficient of consolidation under cyclic loading

In order to simplify the problem and also to express the study based on the conventional consolidation tests and the available information in the literature, some parameters are defined as follows

$$\lambda = \frac{C_c}{C_s} \tag{20}$$

$$\eta = \frac{C_c}{N_P} \tag{21}$$

$$\xi = \frac{N_P}{O_P} \tag{22}$$

$$CCR = \frac{C_{vNC}}{C_{vOC}} \tag{23}$$

$$CR = \frac{(m_v)_{OC}}{(m_v)_{NC}} \tag{24}$$

where $m_v = k/(\gamma_w C_v)$ is the modulus of volume compressibility. CCR and CR stand for the Coefficients of Consolidation Ratio and Compressibility Ratio, respectively. Substituting Eqs. (13) and (17) in Eq. (23) gives

$$CCR = \frac{\beta_{NC}}{\beta_{OC}}(\sigma_t - u)^{\alpha_{NC} - \alpha_{OC}} \tag{25}$$

Defining two new parameters as

$$A = \alpha_{NC} - \alpha_{OC} = \eta\left(\frac{\xi}{\lambda} - 1\right) \tag{26}$$

$$B = \frac{\beta_{NC}}{\beta_{OC}} = \frac{\dfrac{2.303(1+e_0)}{C_s\gamma_w}10^{\frac{e_{NC}-e_{NP}}{N_P}}}{\dfrac{2.303(1+e_0)}{C_s\gamma_w}10^{\frac{e_{OC}-e_{OP}}{O_P}}} = \frac{C_s}{C_c}10^{\left(\frac{e_{NC}-e_{NP}}{N_P} - \frac{e_{OC}-e_{OP}}{O_P}\right)} = \frac{10^{\left(\frac{e_{NC}-e_{NP}}{N_P} - \frac{e_{OC}-e_{OP}}{O_P}\right)}}{\lambda} \tag{27}$$

reduces Eq. (25) to

$$CCR = B(\sigma_t - u)^A \tag{28}$$

which is useful to investigate the variation of *CCR* during cyclic loading. If the ratio between the slopes of the NC and OC lines in e–logσ' and e–log k are the same, $A = 0$, then parameter B will independently govern the consolidation progress. Hence it is useful to investigate the variation of B during cyclic loading.

Based on the laboratory tests described in the companion paper, the slopes of e–log k in the NC and OC states are usually the same ($N_P = O_P$). It means that e–log k has usually a unique line during cyclic loading. Therefore, the slope and intercept of the line can be used to simplify the analysis. Defining two new parameters as

$$P = N_P = O_P \tag{29}$$

$$e_P = e_{NP} = e_{OP} \tag{30}$$

sets $\xi = 1$ and changes Eqs. (21), (26), and (27), respectively, to

$$\eta = \frac{C_c}{P} \tag{31}$$

$$A = \eta\left(\frac{1}{\lambda} - 1\right) \tag{32}$$

$$B = \frac{\beta_{NC}}{\beta_{OC}} = \frac{10^{\left(\frac{e_{NC} - e_{OC}}{P}\right)}}{\lambda} \tag{33}$$

Using the intersection point of the NC and OC lines in e–logσ', occurring at a point with coordinates (P_c, e_c), we have

$$e_c = e_{NC} - C_c \log P_c = e_{OC} - C_s \log P_c \tag{34}$$

Rearranging Eq. (34) leads to

$$e_{NC} - e_{OC} = C_c\left(1 - \frac{1}{\lambda}\right)\log P_c \tag{35}$$

Dividing Eq. (35) by P gives

$$\frac{e_{NC} - e_{OC}}{P} = \eta\left(1 - \frac{1}{\lambda}\right)\log P_c \tag{36}$$

Substituting Eq. (36) into Eq. (33) results in

$$B = \frac{1}{\lambda}P_c^{\eta\left(1 - \frac{1}{\lambda}\right)} \tag{37}$$

The exponent of P_c in the right-hand-side of Eq. (37) is always positive. Therefore, as the consolidation proceeds, B becomes bigger and the basic coefficient of consolidation of an OC clay

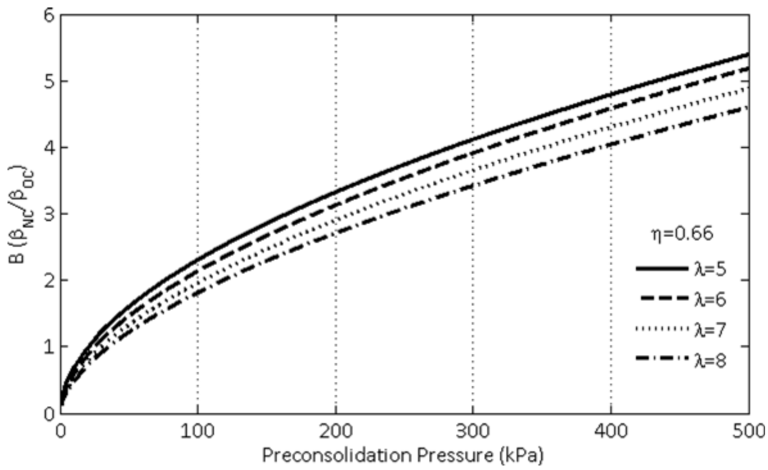


Fig. 3 The effect of preconsolidation pressure on B for different λ

recedes from that in the NC state. The difference depends on the preconsolidation pressure; the more the pressure at which the clay is unloaded, the more the changes in the coefficient of consolidation. Figure 3 shows the effect of preconsolidation pressure on B. It, also, shows that this effect is slightly more substantial for the soils having smaller values of λ. For elastic soils, $\lambda = 1$, Eq. (37) reduces to unity which shows that the basic coefficient of consolidation is constant and is not affected by the preconsolidation pressure.

It was mentioned in Section 3.2 that $\eta = C_c/N_P$ often lies within 0.5 to 1. Fig. 4 shows that for $\eta > 1$ the difference between the basic coefficients of consolidation in the NC and OC states is very large. It should be stated that during unloading-reloading phase the preconsolidation pressure and therefore the value of B remain constant.

Introducing the right-hand-side of Eqs. (26) and (27) into Eq. (28) gives

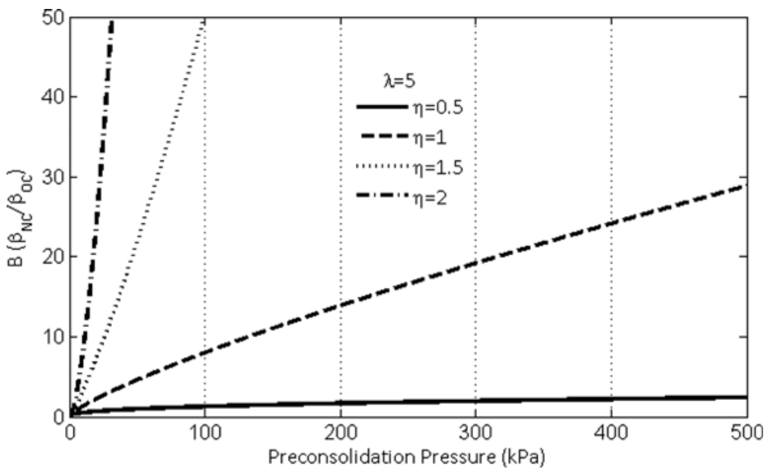


Fig. 4 The effect of preconsolidation pressure on B for different η

$$CCR = \frac{10^{\left(\frac{e_{NC}-e_{NP}}{N_P}-\frac{e_{OC}-e_{OP}}{O_P}\right)}}{\lambda}(\sigma')^{\eta\left(\frac{\xi}{\lambda}-1\right)} \tag{38}$$

Also, substituting Eqs. (6) and (7) in Eq. (24) results in

$$CR = \frac{k_{OC}/(\gamma_w C_{vOC})}{k_{NC}/(\gamma_w C_{vNC})} = CCR\frac{k_{OC}}{k_{NC}} = CCR \times 10^{\left(\frac{e_{OC}-e_{OP}}{O_P}-\frac{e_{NC}-e_{NP}}{N_P}\right)}(\sigma')^{\eta\left(\frac{\xi}{\lambda}-1\right)} = \frac{C_s}{C_c} = \frac{1}{\lambda} \tag{39}$$

Eq. (39) is experimentally validated in the companion paper. The modulus of volume compressibility, m_v, is usually related to the modulus of elasticity of a soil. In its simplest form, this parameter may be assumed to be the inverse of the modulus of elasticity. Having said that, Eq. (39) shows that the modulus of elasticity periodically changes between two values in the NC and OC states and the ratio of the values is λ.

Again, assuming a unique line in $e - \log k$ (Fig. 2(b)) reduces Eq. (39) to

$$CCR = \frac{1}{\lambda}P_c^{\eta\left(1-\frac{1}{\lambda}\right)}(\sigma')^{\eta\left(1-\frac{1}{\lambda}\right)} \tag{40}$$

or,

$$CCR = CR(OCR)^{\eta(1-CR)} \tag{41}$$

where OCR is the overconsolidation ratio, the ratio of the past maximum effective stress of the soil to its current effective stress. Baligh and Levadoux (1978) and Toufigh and Ouria (2009) assumed constant values for CCR and CR to analytically study the consolidation under cyclic loading. Eq. (39) confirms a constant value for CR but Eq. (41), as demonstrated in Fig. (5), shows that this assumption can lead to a significant error in solution, especially for heavily overconsolidated clays. When the clay is normally consolidated, $OCR = 1$, CCR remains constant. It means that unloading

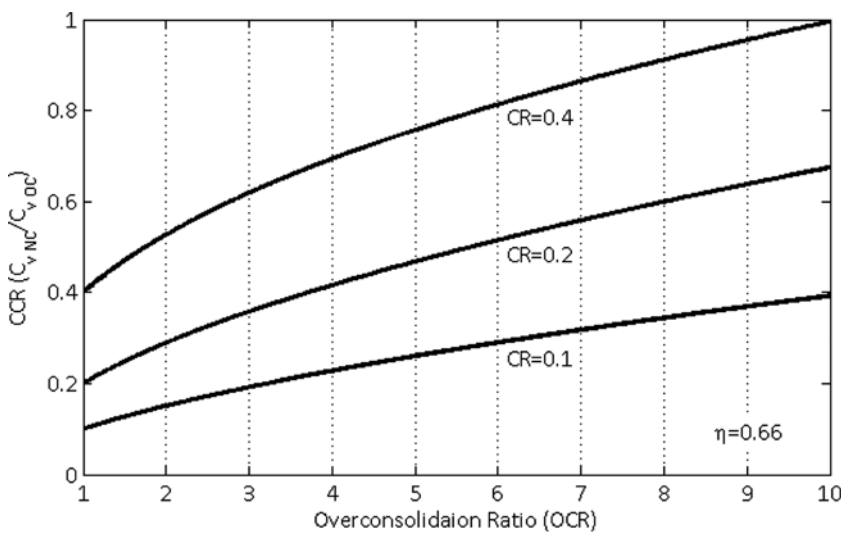


Fig. 5 The effect of overconsolidation ratio on the coefficient of consolidation

has the same effect on the coefficient of consolidation at every stress level for an NC clay. However, further unloading leads to a greater *OCR* resulting in more difference between the coefficients of consolidation in the NC and OC states at the same stress level.

5. Conclusions

Using void ratio-effective stress and void ratio-permeability relationships, two one-dimensional nonlinear partial differential equations were derived to predict the consolidation characteristics of normally consolidated and over consolidated soft clays. The variation of the coefficient of consolidation under cyclic loading was analytically discussed. The following remarks can be concluded:

(1) Under cyclic loading, effective stress periodically changes resulting in changes in compressibility, permeability, and the coefficient of consolidation of soil.
(2) Two parameters, namely nonlinearity factor and basic coefficient of consolidation, along with the effective stress govern the consolidation characteristics of soil under cyclic loading.
(3) Absence of a closed form solution to determine the time at which the state of a soil layer changes from NC to OC or vice versa hinders a comprehensive analytical study on the nonlinear consolidation under cyclic loading.
(4) Besides the preconsolidation pressure, the ratios between slopes of $e - \log\sigma'$ and $e - \log k$ lines in the NC and OC states significantly influence the consolidation progress.
(5) Assuming a constant ratio between the values of the coefficient of consolidation in NC and OC states, especially for heavily overconsolidated clays, can produce considerable errors in analyzing consolidation under cyclic loading.

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Notation

The following symbols are used in this paper:

A	= nonlinearity factors difference
B	= ratio of basic coefficients of consolidation in the NC and OC states
CCR	= coefficient of consolidation ratio
CR	= compressibility ratio
C_c	= compression index
C_r	= reloading index
C_s	= swell index

C_v = coefficient of consolidation $[L^2/T]$
C_{vNC} = coefficient of consolidation in the NC state $[L^2/T]$
C_{vOC} = coefficient of consolidation in the OC state $[L^2/T]$
e = void ratio
e_c = void ratio associated with preconsolidation pressure
e_0 = initial void ratio
e_{NC} = void ratio at unit effective stress in the NC state
e_{NP} = void ratio at unit permeability in the NC state
e_{OC} = void ratio at unit effective stress in the OC state
e_{OP} = void ratio at unit permeability in the OC state
k = permeability of soil $[L/T]$
k_{NC} = permeability of soil in the NC state $[L/T]$
k_{OC} = permeability of soil in the OC state $[L/T]$
m_v = unit weight of water $[LT^2/M]$
N_P = $e - \log k$ slope in the NC state
O_P = $e - \log k$ slope in the OC state
P = $e - \log k$ slope when $N_P = O_P$
P_c = preconsolidation pressure $[M/LT^2]$
t = time $[T]$
u = excess pore water pressure $[M/LT^2]$
z = vertical coordinate $[L]$
α_{NC} = nonlinearity factor in the NC state
α_{OC} = nonlinearity factor in the OC state
β_{NC} = basic coefficient of consolidation in the NC state
β_{OC} = basic coefficient of consolidation in the OC state
γ_w = unit weight of water $[M/L^2T^2]$
η = ratio of compression index to $e - \log k$ slope in the NC state
λ = compression to swell indices ratio
ξ = $e - \log k$ slopes ratio
σ' = vertical effective stress $[M/LT^2]$
σ = vertical total stress $[M/LT^2]$