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Stochastic design charts for bearing capacity of strip footings

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Abstract. Traditional design methods of bearing capacity of shallow foundations are deterministic in the sense that they do not explicitly consider the inherent uncertainty associated with the factors affecting bearing capacity. To account for such uncertainty, available deterministic methods rather employ a fixed global factor of safety that may lead to inappropriate bearing capacity predictions. An alternative stochastic approach is essential to provide a more rational estimation of bearing capacity. In this paper, the likely distribution of predicted bearing capacity of strip footings subjected to vertical loads is obtained using a stochastic approach based on the Monte Carlo simulation. The approach accounts for the uncertainty associated with the soil shear strength parameters: cohesion, c, and friction angle, ϕ, and the cross correlation between c and ϕ. A set of stochastic design charts that assure target reliability levels of 90% and 95%, are developed for routine use by practitioners. The charts negate the need for a factor of safety and provide a more reliable indication of what the actual bearing capacity might be.

Keywords: stochastic analysis; Monte Carlo; bearing capacity; shallow foundations; strip footings.

1. Introduction

Predicting bearing capacity of shallow foundations is a common practice in geotechnical engineering and an accurate estimation of its value is essential for a safe and reliable design. If bearing capacity is over-estimated, soil will fail, resulting in serious consequences. If, on the other hand, bearing capacity is under-estimated, undue costs will incur. Bearing capacity prediction of shallow foundations, as in many geotechnical engineering problems, is often affected by a considerable level of uncertainty. Such uncertainty may produce an unreliable estimation of the magnitude of bearing capacity, while reliable bearing capacity prediction is essential for design purposes. Uncertainties affecting most geotechnical engineering problem, including bearing capacity of foundations, is generally caused by one or more of the following two categories (Baecher and Christian 2003): (*i*) natural variability and (*ii*) knowledge uncertainty. Natural variability is due to the natural spatial variability of soil from one point to another, which is caused by variations in the mineral composition and characteristics of soil strata during soil formation. Knowledge uncertainty, on the other hand, can be divided into model uncertainty and parameter uncertainty.

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Model uncertainty is caused by the inability of chosen mathematical models to mimic reality. This is usually due to the simplified nature of models that are used to describe soil behaviour, which are generally based on a number of assumptions that provide uncertainty (Frey 1998). Model uncertainty is usually difficult to measure physically (Juang *et al.* 1991) and in most instances, the model used to describe a certain phenomenon is assumed to be a perfect predictor (i.e. has minimal or no prediction error). However, if sufficient measured and predicted data are available and assuming that the measured data are error free, the uncertainty associated with the prediction method used can then be quantified and assumed to correspond to model uncertainty.

Parameter uncertainty is due to inaccuracy in assessing soil properties as a result of a limited soil sampling and testing data. It is also due to inadequacy of interpreting the subsurface geology due to the measurements error, data handling and transcription errors, inconsistency of data and inadequate representation of data sampling due to time and space limitations (Baecher and Christian 2003). Parameter uncertainty can also be due to the discrepancies between the in-situ implementation of structure and what appears in construction drawings.

Traditional deterministic methods for modelling bearing capacity of shallow foundations do not explicitly consider the abovementioned uncertainties in their analysis and simulation, and rather simplify the problem by assuming a fixed global factor of safety to account for such uncertainties. This factor of safety is in reality a “factor of ignorance” as it is usually derived from past experience and does not necessary reflect the inherent uncertainty associated with each of the uncertainties mentioned above. One way to include such uncertainties in the analysis and design of shallow foundations is to use reliability and stochastic analyses. These probabilistic methods are currently possible due to the significant improvement of the computational ability of recent computers and enhancement of existing knowledge in relation to the statistical properties of soil properties (Phoon and Kulhawy 1999). Applications of reliability and stochastic analyses to bearing capacity of shallow foundations have been investigated by many researchers (e.g. Abdel Massih *et al.* 2008, Basheer and Najjar 1998, Cherubini 1990, 2000, Easa 1992, Fenton and Griffiths 2003, Griffiths *et al*. 2002, Popescu *et al.* 2005, Przewlocki 2005, Vessia *et al.* 2009). However, most of these applications have limited use by practitioners as they require a great deal of probability knowledge and significant computer resources. In this paper, stochastic analysis using the Monte Carlo simulation is applied to deterministic modelling of bearing capacity of strip footings and a set of stochastic design charts suitable for routine use by practitioners are developed. In the sections that follow, the proposed stochastic approach and the procedure used for developing the stochastic design charts are explained and presented.

2. Deterministic bearing capacity of strip footings

In order to obtain stochastic solutions for bearing capacity of shallow foundations, a deterministic model shall first be selected. In the present work, the commonly used model proposed by Terzaghi (1943) is selected in which the ultimate bearing capacity of strip footings can be calculated as follows (Terzaghi 1943)

$$q_u = cN_c + qN_q + 0.5\gamma BN_\gamma \tag{1}$$

where: q_u is the ultimate bearing capacity; c is the soil cohesion; γ is the soil unit weight; B is the footing breadth; q is the overburden pressure (i.e. the soil unit weight × depth of foundation, D);

and N_c, N_q and N_γ are the bearing capacity factors. The bearing capacity factors rely solely on the soil friction angle, ϕ, and are estimated as follows (Terzaghi 1943)

$$N_q = \frac{[e^{(0.75\pi - \phi/2)\tan\phi}]^2}{2\cos^2(45^\circ + \phi/2)} \tag{2}$$

$$N_c = (N_q - 1)\cot\phi \tag{3}$$

$$N_\gamma = \frac{1}{2}\left(\frac{k_{p\gamma}}{\cos^2\phi} - 1\right)\tan\phi \tag{4}$$

where: $\pi = 3.14$ and $k_{p\gamma}$ is the passive earth pressure coefficient that relies on ϕ. From values of $k_{p\gamma}$ corresponding to ϕ given by Terzaghi (1943), the following matching empirical equations can be proposed for $k_{p\gamma}$

$$k_{p\gamma} = 10.49e^{0.0363\phi} \quad (R^2 = 0.98, \text{ for } \phi = 0.0 - 15^\circ) \tag{5}$$

$$k_{p\gamma} = 5.82e^{0.074\phi} \quad (R^2 = 0.99, \text{ for } \phi = 15^\circ - 35^\circ) \tag{6}$$

$$k_{p\gamma} = 0.364e^{0.1516\phi} \quad (R^2 = 0.98, \text{ for } \phi = 35^\circ - 50^\circ) \tag{7}$$

3. Stochastic bearing capacity of strip footings

In the present work, the stochastic analysis of bearing capacity of strip footings is conducted by applying the Monte Carlo simulation and considering the parameter uncertainty in relation to the input variables affecting the bearing capacity obtained from Eq. (1). Detailed description of the Monte Carlo simulation can be found in many publications (e.g. Hammersley and Handscomb 1964, Rubinstein 1981). As mentioned earlier, natural variability and knowledge uncertainty (i.e. model uncertainty + parameter uncertainty) are the two main sources of uncertainty affecting most of the geotechnical engineering problems. It should be noted that the natural variability is beyond the scope of this paper and Terzaghi's model used to estimate the bearing capacity of strip footings is assumed to be a perfect predictor (i.e. has no model uncertainty). Consequently, the stochastic approach used in the current study accounts mainly for parameter uncertainty. For an individual case of bearing capacity prediction, the procedure used to obtain stochastic solution that incorporates parameter uncertainty is as follows:

(1) For each of the bearing capacity input variables of Eq. (1) (i.e. c, ϕ, γ, B and D), a random value is generated in relation to parameter uncertainty from the input variable mean, coefficient of variation (COV), known or assumed probability distribution function and any correlation exists between that input variable and the other available input variables;

(2) Using the generated input values from Step (1) and assuming that Terzaghi's model of bearing capacity of strip footings is a perfect predictor, a deterministic value of bearing capacity is obtained;

(3) Steps 1 and 2 are repeated hundreds or thousands of times, as part of the Monte Carlo simulation, until certain acceptable convergence is met; and

(4) Finally, all the bearing capacities obtained are collated and used to determine the cumulative

distribution function (CDF) or to plot the cumulative probability distribution curve from which predictions associated with target reliability levels can be estimated.

Among the five input variables of Terzaghi's bearing capacity model, the cohesion, c, and friction angle, ϕ, are likely to include significant parameter uncertainty and thus are assumed to be random variables in the current study. The soil unit weight, γ, is assumed to be constant in as it contributes to parameter uncertainty of a lesser degree, as demonstrated by Lee *et al.* (1983). In addition, the footing breadth, B, and depth of foundation, D, are likely to provide marginal parameter uncertainty and are thus assumed to be deterministic for practical purposes.

In order to illustrate the stochastic procedure set out above, the following case study is investigated. A strip footing of breadth B = 2.0 m is founded at a depth D = 1.5 m below the ground surface, and the soil is clayey sand of unit weight $\gamma = 18$ kN/m^3. The statistical values for c and ϕ are selected as follows: μ_c (mean of cohesion) = 5 kPa, μ_ϕ (mean of friction angle) = 30^o, COV_c (coefficient of variation of cohesion) = 27%, COV_ϕ (coefficient of variation of friction angle) = 10% and $\rho_{c,\phi}$ (correlation coefficient between c and ϕ) = −0.6. The probability distribution functions for both c and ϕ are assumed to follow a lognormal distribution, as has been used in several geotechnical engineering applications. It should be noted that these statistical values are within the practical ranges cited in the literature. For example, the mean value of ϕ is typically between 20^o and 40^o (Abdel Massih *et al.* 2008), with COV ranging from 5% to 15% for sands and 12% to 56% for clays (Lee *et al.* 1983, Phoon and Kulhawy 1999). The COV for c varies between 10% to 70% (Cherubini 2000) with a recommended value of 30% can be used for practical purposes (Lee *et al.* 1983). For the coefficient of correlation between c and ϕ, the COV ranges between −0.24 and −0.7 (Lumb 1970, Wolff 1985, Yuceman *et al.* 1973) with a recommended value of −0.6 can be used in practice (Cherubini 2000).

The statistical data mentioned above are used to generate sample values of c and ϕ (Step 1) and the corresponding deterministic bearing capacity is calculated using Terzaghi's model (Step 2). As mentioned previously, Terzaghi's model is assumed to be a perfect predictor with no model uncertainty and consequently, parameter uncertainty associated with the soil shear strength properties c and ϕ is the only source of uncertainty. Steps 1 and 2 are repeated many times until a convergence criterion is achieved (Step 3). To determine whether convergence has been achieved, the statistics describing the distribution of the predicted bearing capacities are calculated at fixed numbers of simulations and compared with the same statistics at previous simulations. Convergence is deemed to have occurred if the change in the statistics describing the distribution of predicted bearing capacity is 1.5% or less. The predicted bearing capacities obtained from the many simulations conducted are used to plot the cumulative probability distribution curve from which bearing capacity predictions that assure target reliability levels are obtained (Step 4). It should be noted that the stochastic simulation described in Steps 1 to 4 are conducted with the aid of the PC-based software @Risk (Palisade 2000) and the results are shown in Fig. 1, which also includes the predicted deterministic value of bearing capacity. The predicted deterministic bearing capacity is obtained using Eq. (1) and is found to be equal to 1067 kPa. For target reliability levels of 90% and 95% (i.e. the reliability levels that usually needed for design purposes), the corresponding bearing capacities are estimated from the cumulative probability function (or Fig. 1) to be 730 kPa and 658 kPa, respectively. These values indicate equivalent factors of safety of 1067/730 = 1.5 and 1067/658 = 1.6, respectively. These results indicate that, for the case study considered, the factor of safety of 3 that usually used in the deterministic analysis is conservative. The results also demonstrate that

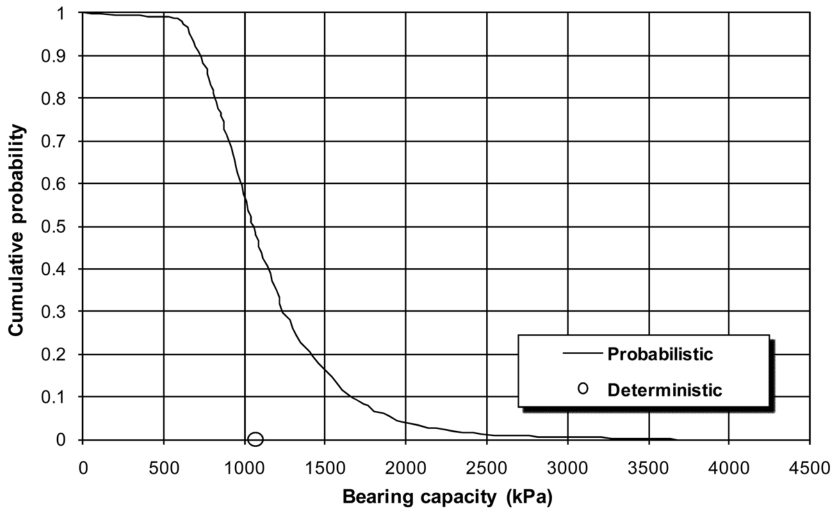

Fig. 1 Cumulative probability distribution incorporating parameter uncertainty for the case study considered

the uncertainty associated with c and ϕ can considerably affect the bearing capacity of strip footings and thus should not be neglected.

4. Stochastic bearing capacity design charts

The stochastic simulation applied to the case study described in Section 3 is used to develop a generic set of stochastic design charts for routine use in practice. The charts are expected to be a useful tool for practitioners from which, predicted bearing capacity corresponding to 90% and 95% reliability levels can be readily obtained. The charts are based on the practical recommended parameter uncertainty of COV_c of 30% and COV_ϕ of 20% and a coefficient of correlation between c and ϕ of −0.6 with lognormal distribution for both c and ϕ. The procedure that is used to develop the charts is as follows:

(1) A combination of input values for c, ϕ, γ, B and D are selected so as to be within the ranges that can be expected in practical applications, as given in Table 1;
(2) The stochastic approach, outlined previously, which incorporates parameter uncertainty of c and ϕ is applied and the corresponding CDF is obtained;

Table 1 Values of the input variables used for development of the stochastic design charts

Input variable	Values	Number of values
Cohesion, c (kPa)	0, 20, 40, 60, 80, 100	6
Friction angle, ϕ (degrees)	0, 10, 20, 30, 40	5
Soil unit weight, γ (kN/m^3)	16, 18, 20	3
Footing breadth, B (m)	0.5, 1, 2, 3	4
Depth of foundation, D (m)	0, 1.5, 3	3

(3) From the CDF, the bearing capacities corresponding to 90% and 95% reliability levels are determined; and

(4) Another combination of values of c, ϕ, γ, B and D are selected from Table 1 and Steps 2 to 3

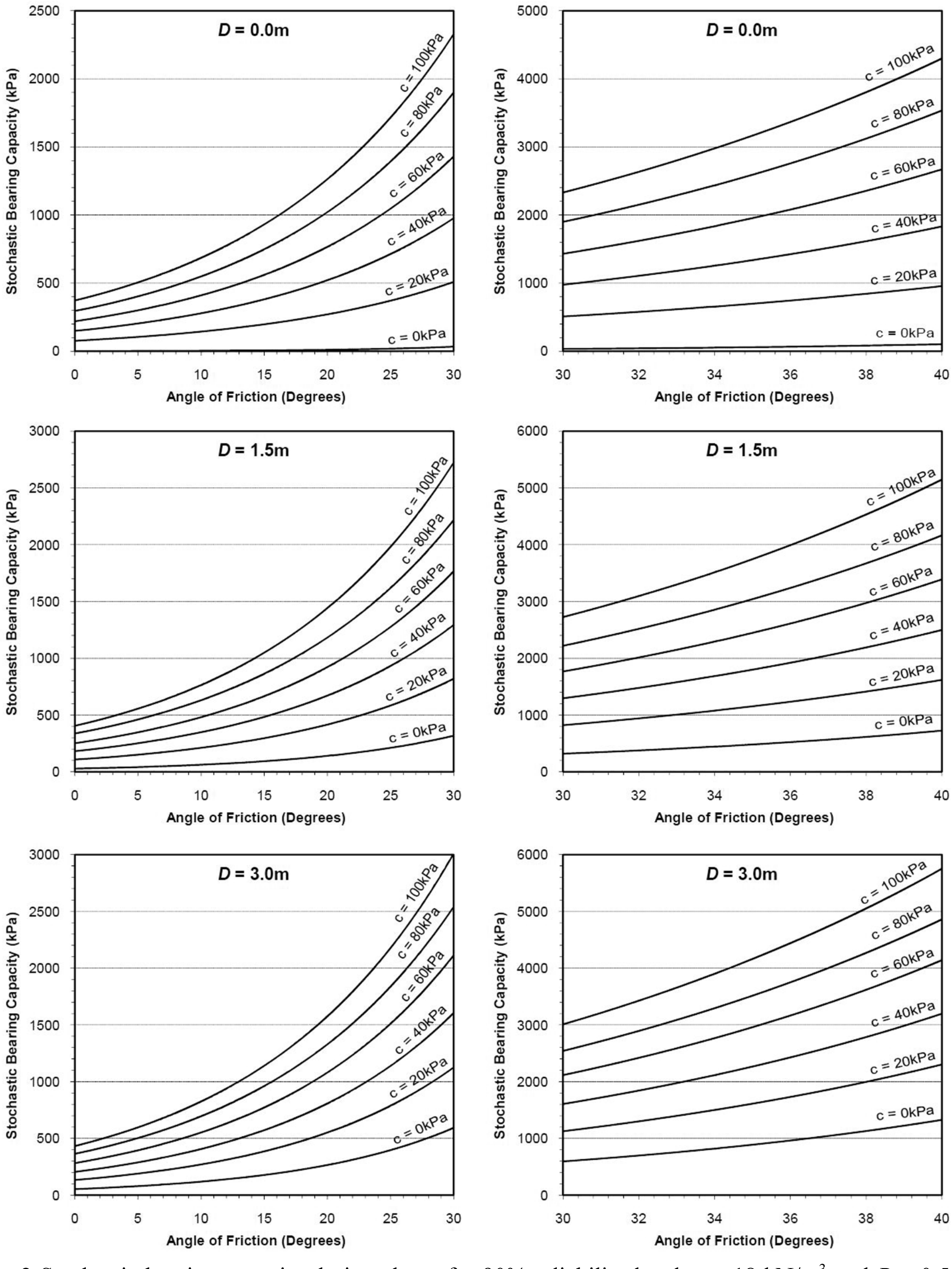

Fig. 2 Stochastic bearing capacity design charts for 90% reliability level, γ = 18 kN/m^3 and B = 0.5 m

are repeated until all possible combinations of values of c, ϕ, γ, B and D given in Table 1 are selected and their stochastic simulations are conducted. The results are used to develop the sto-

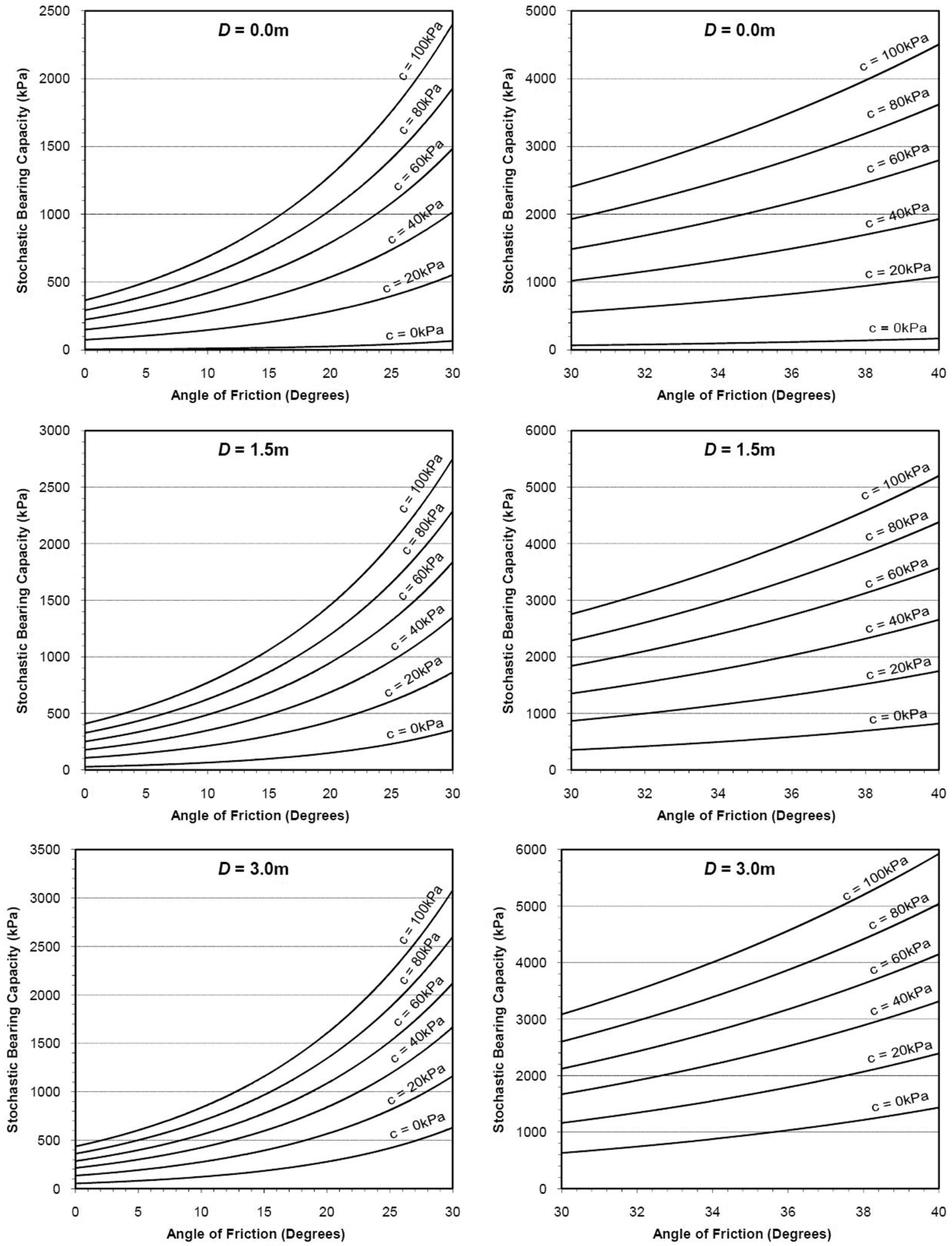

Fig. 3 Stochastic bearing capacity design charts for 90% reliability level, $\gamma = 18$ kN/m^3 and $B = 1.0$ m

chastic design charts shown in Figs. 2-9, corresponding to 90% and 95% reliability levels.

It should be noted that the charts in Figs. 2-9 are for soil unit weight $\gamma = 18$ kN/m^3, and for any

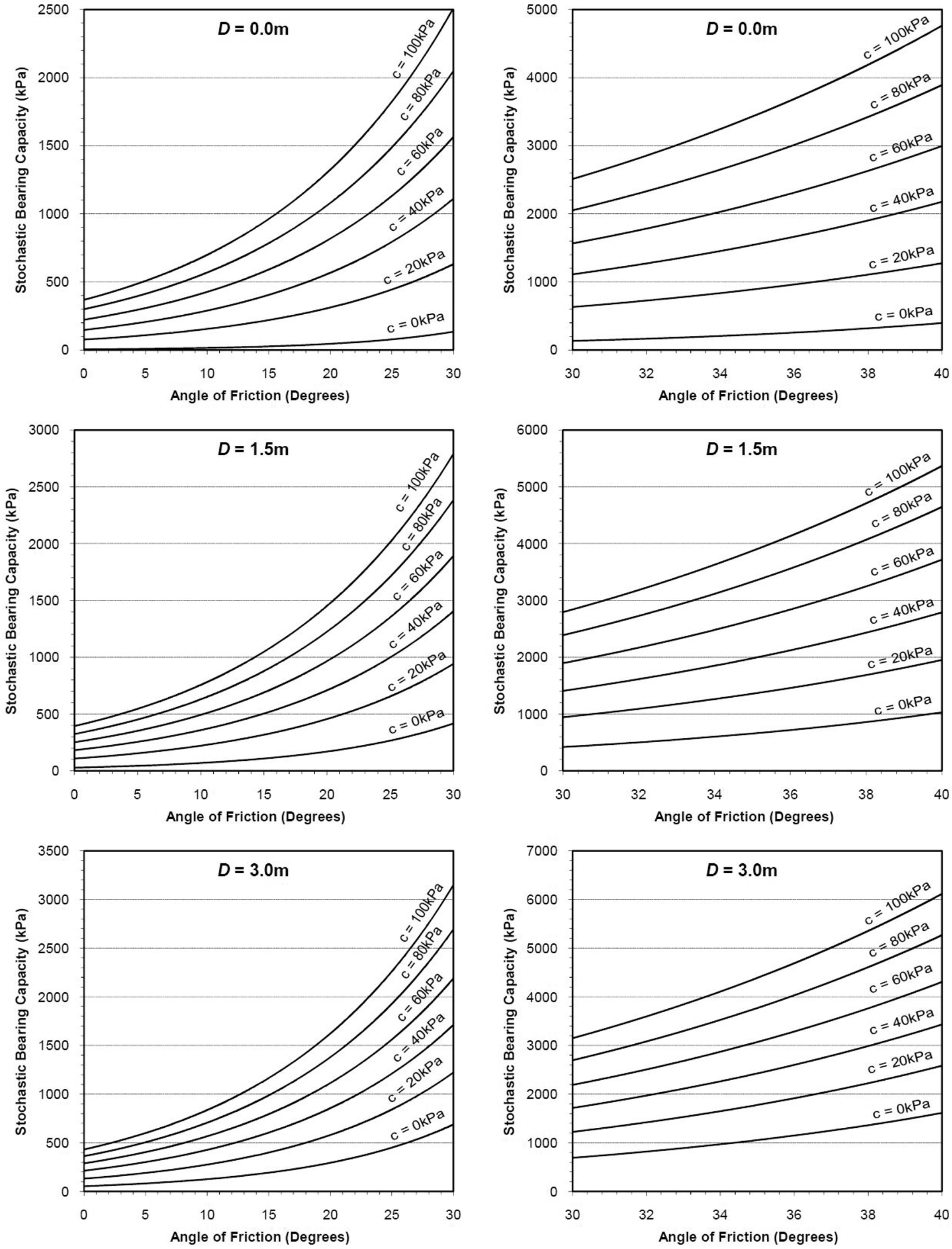

Fig. 4 Stochastic bearing capacity design charts for 90% reliability level, $\gamma = 18$ kN/m^3 and B = 2.0 m

other value of γ between the range given in Table 1, more simulations are carried out and a correction factor curve is developed, as shown in Fig. 10. By considering the number of values given in Table 1 for c, ϕ, γ, B and D, it can be derived that the number of stochastic simulations

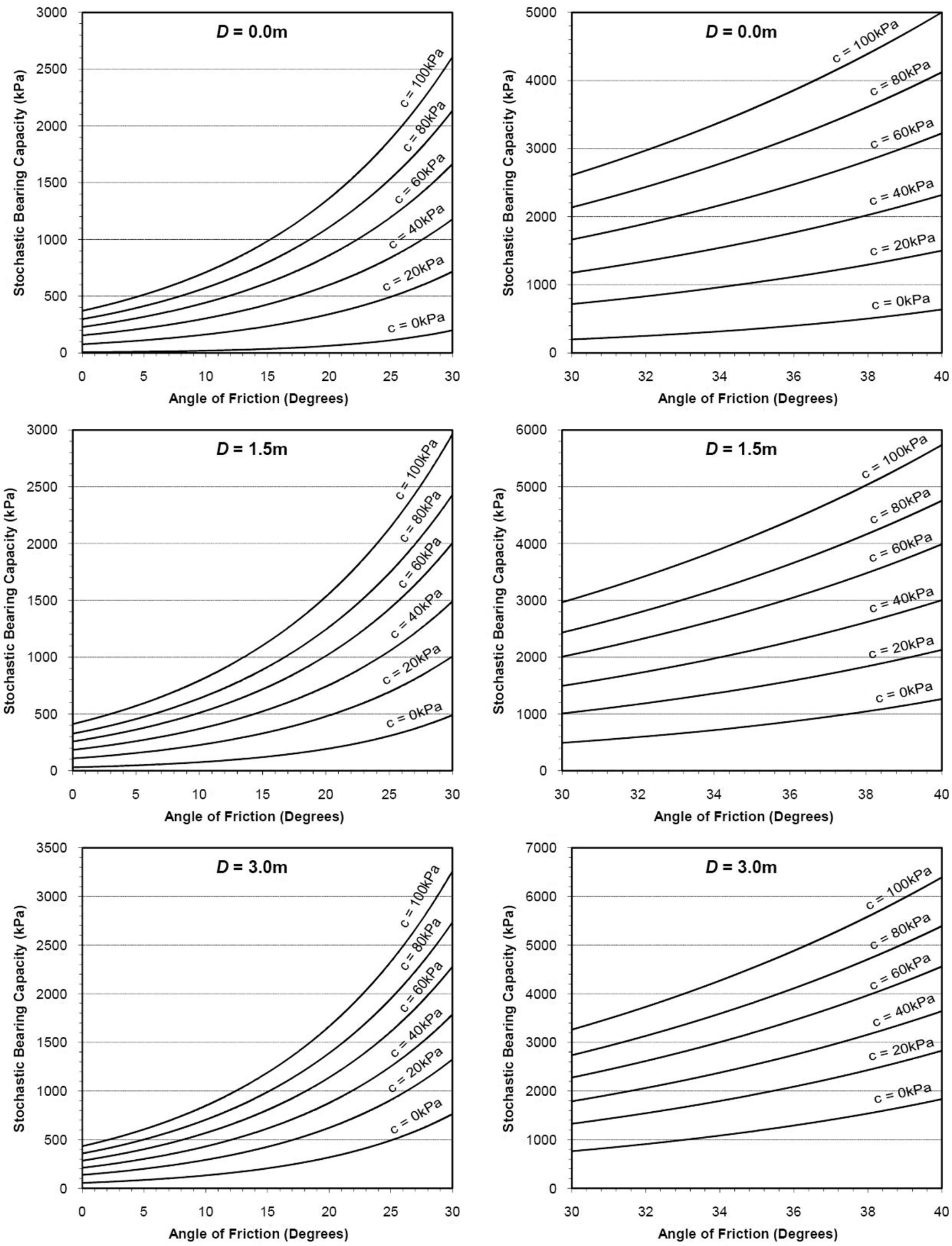

Fig. 5 Stochastic bearing capacity design charts for 90% reliability level, $\gamma = 18$ kN/m^3 and $B = 3.0$ m

conducted in order to develop the stochastic design charts and soil unit weight correction factor curve are: 6 × 5 × 3 × 4 × 3 = 1080. In order to illustrate the use of the design charts, the following numerical example is examined.

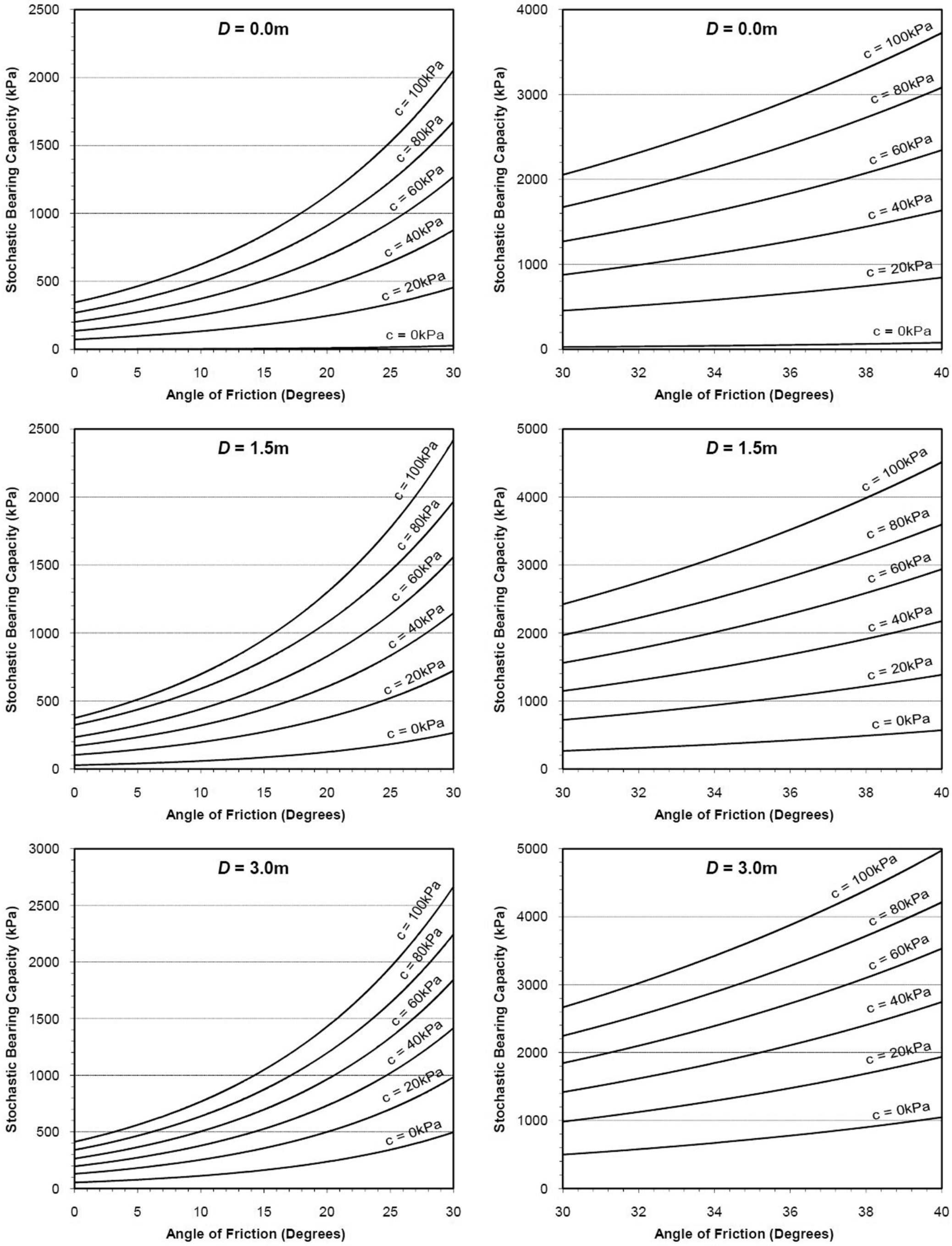

Fig. 6 Stochastic bearing capacity design charts for 95% reliability level, $\gamma = 18$ kN/m^3 and $B = 0.5$ m

4.1 Example

A strip footing of breadth B = 1.5 m is to be constructed at a depth D = 3 m below the ground

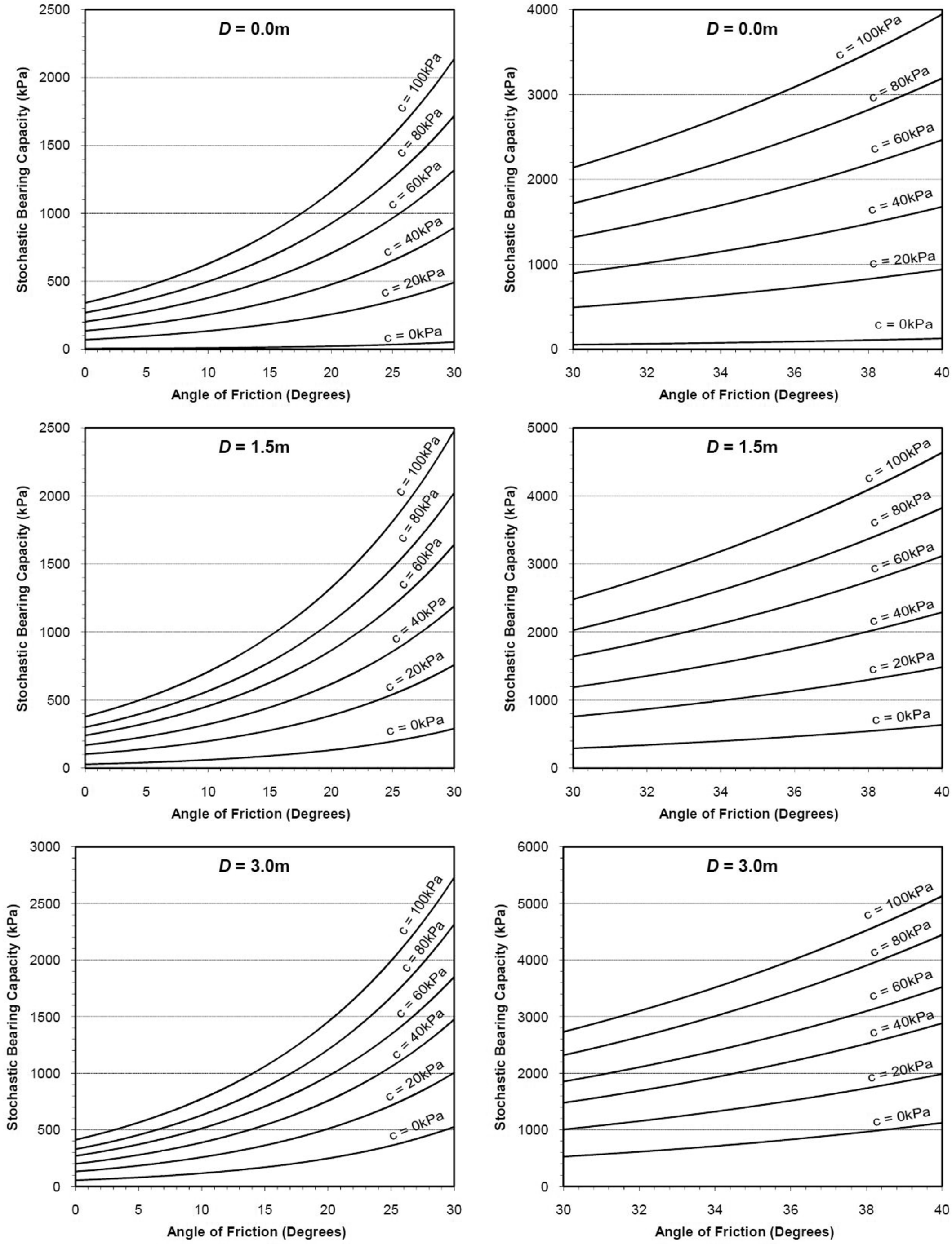

Fig. 7 Stochastic bearing capacity design charts for 95% reliability level, γ = 18 kN/m^3 and B = 1.0 m

surface in a soil that has the following properties: c = 20 kPa, ϕ = 35° and γ = 20 kN/m^3. It is required to find the bearing capacity corresponding to reliability level of 90%, and also estimate the equivalent FOS.

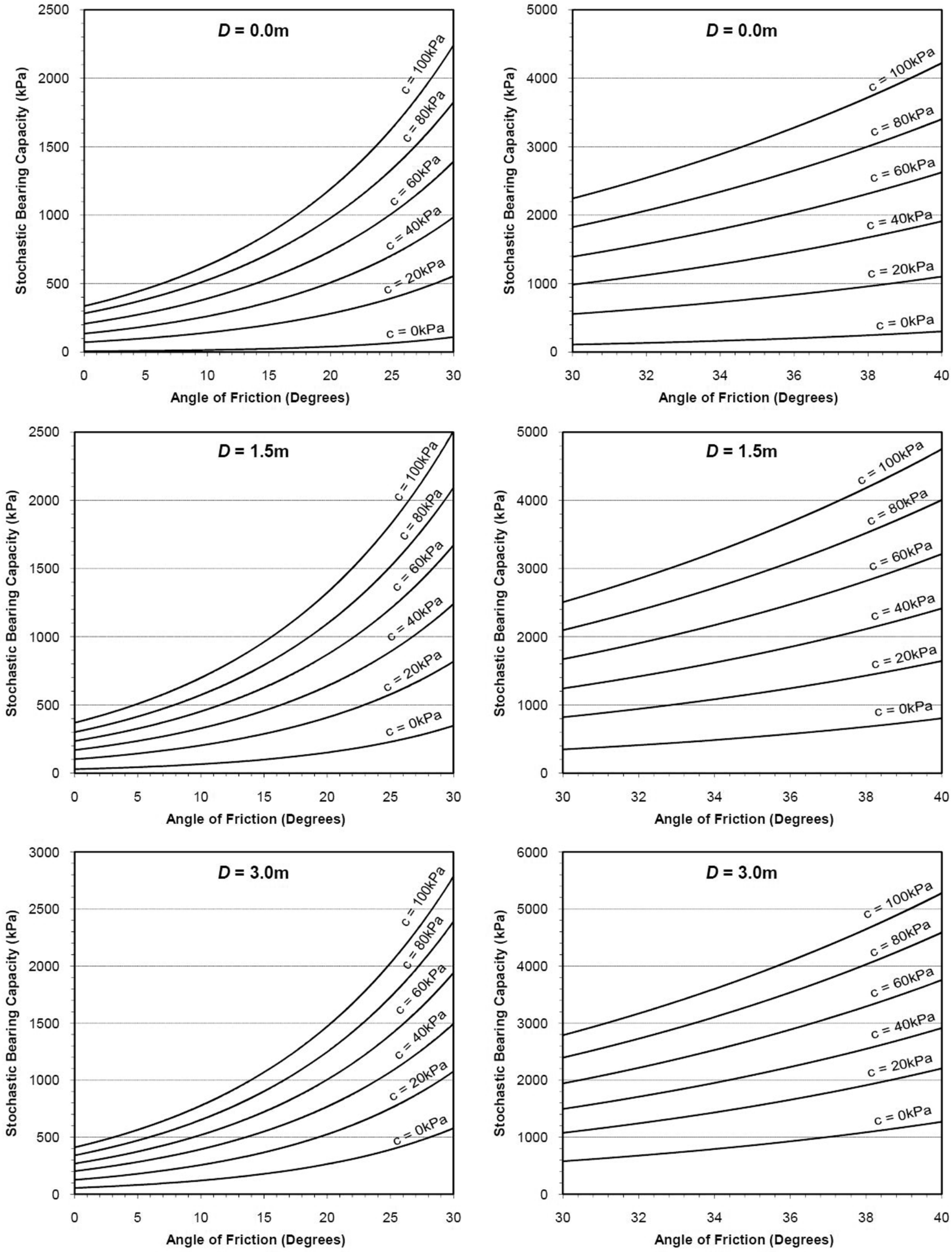

Fig. 8 Stochastic bearing capacity design charts for 95% reliability level, γ = 18 kN/m^3 and B = 2.0 m

4.2 Solution

For a reliability level of 90%, the charts in Figs. 2-5 should be used to obtain the bearing capacity

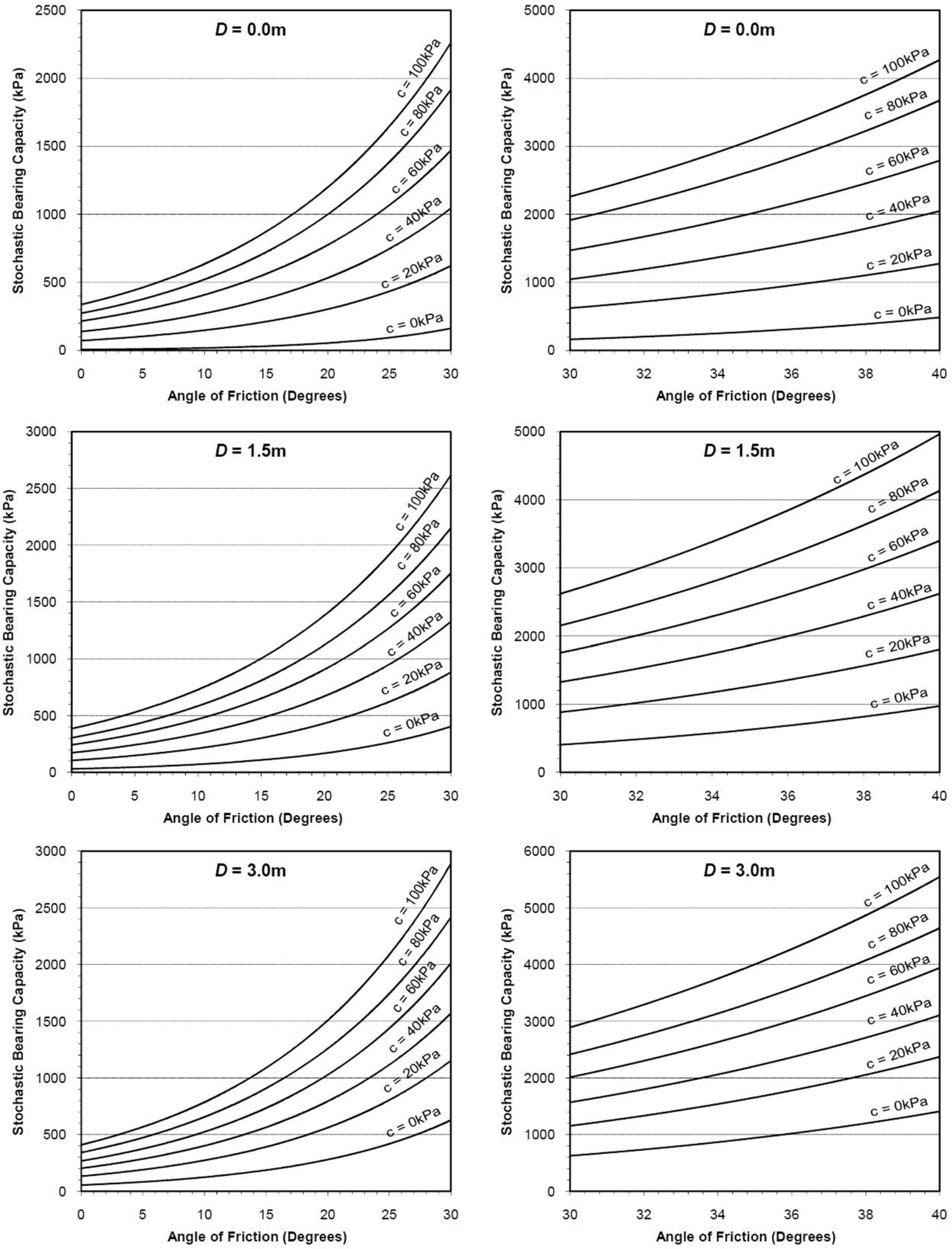

Fig. 9 Stochastic bearing capacity design charts for 95% reliability level, $\gamma = 18$ kN/m^3 and $B = 3.0$ m

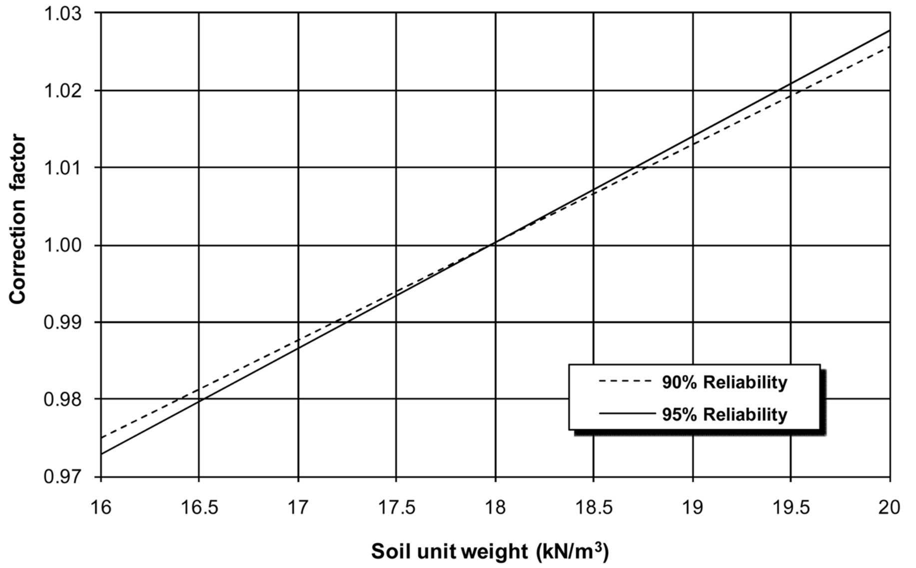

Fig. 10 Unit weight correction factors corresponding to 90% and 95% reliability levels

corresponding to γ = 18 kN/m^3, and Fig. 10 should then be used to correct obtained bearing capacity for γ = 20 kN/m^3. As there is no standard design chart developed for B = 1.5 m, linear interpolation between the values of bearing capacities obtained at B = 1 m and B = 2 m is made. Using Figs. 3 and 4, resulting in bearing capacities of 1665 kPa and 1870 kPa for B = 1 m and B = 2 m, respectively, are determined. The linear interpolation between these two bearing capacities lead to a bearing capacity of 1742 kPa for B = 1.5 m. This value should then be corrected using Fig. 10, which for γ = 20 kN/m^3 gives a correction factor of 1.026, leading to a final corrected bearing capacity of 1787 kPa. The deterministic bearing capacity is obtained using Eq. (1) and is found to be equal to 4211 kPa. For this case, the equivalent FOS = 4211/1787 = 2.4.

5. Conclusions

Stochastic approach that utilizes the Monte Carlo technique was used to obtain stochastic bearing capacity of strip footings from the commonly used deterministic Terzaghi's model. The proposed stochastic approach accounts for parameter uncertainty of soil cohesion and friction angle, and enables bearing capacity to be quantified in the form of a cumulative probability distribution function that provides bearing capacity predictions corresponding to certain reliability levels. A series of stochastic design charts that incorporate parameter uncertainty of coefficient of variation of 30% and 20% for soil cohesion and friction angle, respectively, were developed for routine use by practitioners, and a numerical example was given to illustrate the use of charts.

The results of the numerical example indicate that the suggested factor of safety of 3 that usually used by most deterministic models is conservative, as equivalent factors of safety of 1.5 and 1.6 for reliability levels of 90% and 95%, respectively, are obtained. These results indicate the importance of adopting stochastic analyses in favour of the factor of safety. It was also shown in this work that the developed stochastic design charts can be used to predict bearing capacity of strip footings for reliability levels equal to 90% and 95%. The charts are believed to be a useful tool that can be readily used by practitioners for design of strip footings.

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