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A complement to Hoek-Brown failure criterion for strength prediction in anisotropic rock

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Abstract. In this paper, a complement to the Hoek-Brown criterion is proposed in order to derive the strength of anisotropic rock from strength of the corresponding truly intact rock. The complement is a decay function, which unlike other modifications or suggestions made in the past, is multiplied to the function of the original Hoek-Brown failure criterion for intact rock. This results in a combined and extended form of the criterion which describes the strength of anisotropic rock as a varying fraction of the corresponding truly intact rock strength. Statistical procedures and in particular regression analyses were conducted into data obtained in experiments conducted in the current research program and those collected from the literature in order to define the Hoek-Brown's criterion complement. The complement function was best described by a simple polynomial including only three constants to be empirically evaluated. Further investigations also showed that these constants can be related to the other readily available parameters of rock material which further facilitate determining the constants. A great and prime advantage of the proposed complement is that it is mathematically simple including the least possible number of empirical constants which are easily estimated with minimum experimental effort. Moreover, proposed concept does not suggests any change to the original Hoek-Brown criterion itself or its constants and serves whenever anisotropy does exist in the rock. This further implies on the possibility of using any other failure criterion for intact rock in conjunction with the compliment to reach the strength of anisotropic rock.

Keywords: Hoek-Brown criterion; anisotropic rock; complement; strength prediction.

1. Introduction

Many rock types have naturally occurring and directional features such as bedding planes, foliation, or flow structures which introduce anisotropy in their strength and deformational properties. Most foliated metamorphic rocks such as schist, slates, gneisses and phylites contain a natural orientation in their flat/long minerals or a banding phenomenon which results in anisotropy in their mechanical properties. Stratified sedimentary rocks like sandstone, shale or sandstone-shale alteration often display anisotropic behaviour due to presence of bedding planes. Anisotropy can also be exhibited by igneous rocks having flow structures as may be observed in rhyolites. These anisotropies are

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often referred to as inherent anisotropy and the corresponding rocks are sometimes categorized as intact anisotropic rock.

Other directional features often present in rock include discontinuities in the form of a single or a set of parallel through-going joints as well as faults which are referred to as induced anisotropy. This paper primarily concerns intact anisotropic rock.

In general, important aspect of anisotropic rock is that it contains plane or parallel planes of weaknesses or discontinuities. Further, as far as the failure is controlled by slip on these planes, strength and deformational properties of such rock is dependant on the geometry and distribution properties, such as orientation and frequency, as well as on the shear strength, which in turn depends on level of stress, of the weakness planes.

In rock engineering problems, the design and analysis into anisotropic rock requires knowledge of rock material strength and shear resistance of weakness planes. It is, therefore, necessary to introduce failure criteria as basic tools for practicing engineers.

The strength criteria developed for prediction of rock strength, including anisotropic rock which is the prime concern in this paper, can be classically divided broadly into two categories: a) the theoretical criteria, b) empirical criteria. It is to be mentioned that in recent years, other intelligent concepts such as Neural Networks (NN) or Genetic Programming (GP) have also been introduced for evaluating the strength of rock (e.g. Asadi *et al.* 2010).

In the following a brief review is made on these criteria before new complement is introduced.

2. Theoretical background and related criteria

Earliest theoretical studies back to Jaeger (1960) and also to Jaeger and Cook (1979) who developed a relation to predict the strength of rock containing a single through-going (Fig. 1) or a set of parallel discontinuities. The relation developed stated that the strength was a function of the stress to cause slip on plane of weakness and of its angle (β) as follows

$$\sigma_{1(\beta)} = \sigma_3 + \frac{2(C_j + \sigma_3 \tan \phi_j)}{(1 - \tan \phi_j \tan \beta) \sin 2\beta} \tag{1}$$

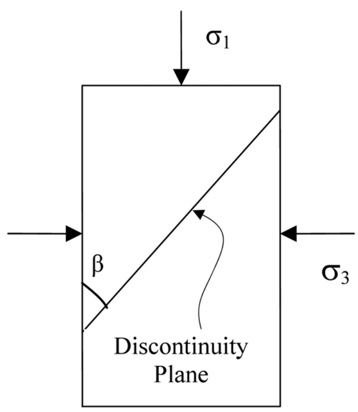

Fig. 1 Anisotropic rock with single plane of discontinuity studied by Jaeger (1960)

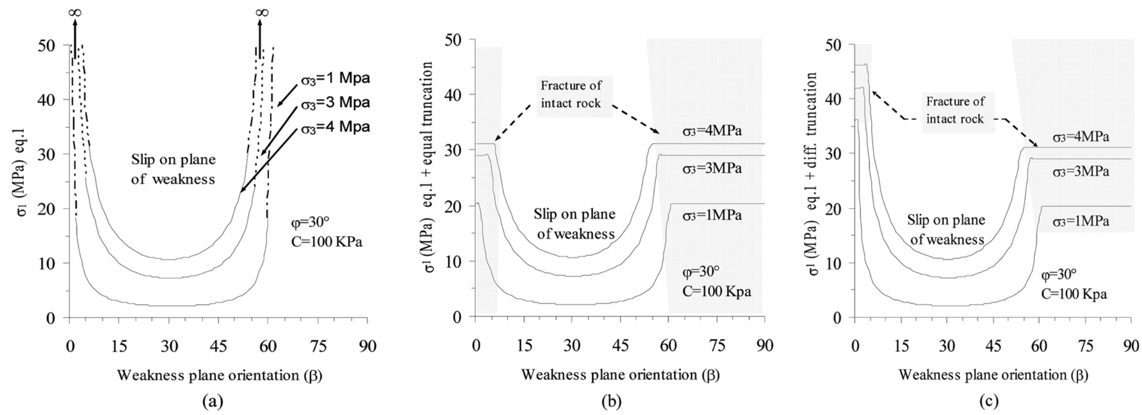

Fig. 2 Graphical presentation of Jaeger (1960) criterion: (a) original criterion with no truncation (b) equal truncation at shoulders (c) differential truncation at shoulders

where C_j and ϕ_j are Mohr-Coulomb's shear strength parameters of the weakness plane. σ_3 is the confining stress and $\sigma_{1(\beta)}$ is the major principal stresses at failure which defines the anisotropic strength of the rock. Plot of Eq. (1) is presented in Fig. 2(a) which can now be referred to as the anisotropic strength curve.

Despite the concrete theoretical basis and the fact that it is perhaps the only well known theoretical criterion, Eq. (1) suffers serious drawbacks; A) the strength of rock tends to infinity at boundaries when the orientation of plane of weakness becomes flat ($\beta \rightarrow 90^o$) or very steep ($\beta \rightarrow 0^o$) as predicted by Eq. (1). Moreover, when ($\tan\beta$) $\rightarrow$ 1/($\tan(\phi)$), the strength again tends to infinity as shown in Fig. 2(a). Jaeger and Cook (1979) discuss that in such cases, failure may be controlled by fracture through intact rock as shown by shaded zones in Figs. 2(b) and 2(c). However, the strength of discontinuous rock at a specific confining pressure can not exceed the strength of the corresponding truly intact rock and it is necessary to truncate the strength curve of Jaeger criterion as it over-predicts the anisotropic rock strength. This is often referred to as extended Jaeger (1960) criterion and is graphically shown in Fig. 2(b) as two horizontal lines truncating the plot of Eq. (1). These lines do not differentiates between the strength of rock having very steep ($\beta \rightarrow 0^o$) or very flat ($\beta \rightarrow 90^o$) plane of weaknesses as opposed to what has been observed in some experiments. Borecki and Kwasnieski 1981, Kwasnieski 1993, and Sheorey 1997 proposed differentiating the strength of rock having flat or highly steep plane of weakness and hence use of truncating lines leveled differently to the left and to the right of the curve ,depending on the experimental results, as graphically shown in Fig. 2(c).

Hoek and Brown (1980a, b) evaluated Jaeger and Cook (1979) criterion and noticed that it is almost incapable of describing the strength of intact anisotropic rocks. Duveau and Shao (1998) used a nonlinear failure criterion for discontinuity's shear strength and extended Jaeger and Cook (1979) but it suffered difficulty in determining numerous experimental constants. An alternative approach for the theoretical prediction of strength anisotropy was also introduced by Tien and Kuo (2001) in which two different failure modes were combined. The well-known Mohr-Coulomb criterion was considered for slip failure mode on plane of weakness, similar to the Jaeger and Cook (1979) approach, whereas the principle of maximum axial strain energy was presumed to govern failure (or brittle fracture) of intact rock. A major drawback also for this criterion was that it involved mathematically two separate functions incapable of continuous describing failure in both slip and non-slip modes.

3. Empirical prediction of strength anisotropy

The difficulties experienced with strength prediction by theoretical criteria, as explained briefly above, have led to the development of empirical criteria for anisotropic rock. Jaeger (1960) generalized the Mohr-Coulomb failure criterion to account for the situation where the inherent shear strength (i.e. Cohesion) of rock varied with the orientation of plane of weakness, β, and proposed the following relationship:

$$C = c_1 - c_2 \cos 2(\beta - \beta_{\min}) \tag{2}$$

where c_1 and c_2 are empirical constants and $\beta_{\min}$ is the specific angle of plane of weakness at which minimum strength for anisotropic rock is attained and is often presumed to follow the well known relation of $\beta \approx 45 - \phi/2$ where ϕ is the friction angle of the material.

Mclamore and Gray (1967) suggested similar mathematical expressions for shear strength parameters (ϕ_j and C_j) as that of Jaeger (1960), i.e. Eq. (2), and used them in well known Mohr-Coloumb failure criterion. Donath (1964) and Sing *et al.* (1989) proposed the following relation for strength of anisotropic rock having plane of weakness oriented at an angle (β) (Fig. 1)

$$\sigma_{c(\beta)} = A - B[\cos 2(\beta_{\min} - \beta)] \tag{3}$$

Constants A and B may be determined by a set of unconfined compression tests and predictions may be made for various values of β. Bagheripour and Mostyn (1996) considered variation of strength for anisotropic rock and conducted compression tests on the specimens of anisotropic model rock. They proposed the following relation for strength prediction in anisotropic rock as

$$\sigma_{1(\beta)} = A\ \sigma_{1i} \tag{4}$$

where $\sigma_{1(\beta)}$ and σ_{1i} are the compressive strength of anisotropic and the corresponding truly intact rock respectively. Parameter A is, in fact, a reduction factor, or in fact a decay function, relating these two strengths. In their studies, Bagheripour and Mostyn (1996) found that the function A could be further defined as

$$A = 1 - Bf(\beta) \qquad \text{where} \quad B = b_o + b_1\ \sigma_3/\sigma_{ci}$$
$$\text{and} \tag{5}$$
$$f(\beta) = [\cos 3/2(\beta_{\min} - \beta)]^C \qquad \text{where} \quad C = 1 + c_1\ \sigma_3/\sigma_{ci}$$

Parameters b_o, b_1, c_1 were all empirical constants determined statistically through regression analysis into anisotropic rock strength data. For unconfined compressive strength prediction, parameter C was set to unity and also B becomes a constant equal to b_o which resulted in a more simple relation.

Ramamurthy and Arora (1994) conducted extensive experimental study on the strength of anisotropic rock and introduced an empirical failure criterion for anisotropic rock and explained that the shapes of the strength curves for anisotropic rock could be broadly divided into two main groups namely: a) U shape and b) shoulder shape such that for weak rock having parallel joints or joint sets the U shape strength curve was observed whereas, the shoulder shape prevailed in rocks with well defined bedding or joint planes in strong rocks. In the following Hoek-Brown failure criterion, which is the matter of investigation in this study, for both isotropic and anisotropic conditions is reviewed.

3.1 Hoek-Brown failure criterion

The classical Griffith theory for brittle failure of rock and the empirical curve fitting concept were used by Hoek and Brown (1980b) and Hoek (1983) to reach an empirical failure criterion for intact and rock mass. Modifications have been suggested or introduced in the past into the criterion mainly based on the accumulation of experimental evidence and various rock strength data (Hoek *et al.* 1992, Hoek 1994, Hoek and Brown 1997, Hoek *et al.* 2002). Over the years, it has gained increasing popularity with practicing engineers and researchers.

The latest version of the criterion known as generalized Hoek-Brown criterion (Hoek *et al.* 2002) takes the following form

$$\sigma_1 = \sigma_3 + \sigma_{ci}(m_b\sigma_3/\sigma_{ci} + s)^{\alpha} \tag{6}$$

where σ_1 and σ_3 are respectively the major and minor principal stresses at failure, σ_{ci} is the uniaxial compressive strength of intact portion of the rock mass, while m_b and s are rock mass constants. In a special case where $\alpha = 0.5$ and constant m_b is set to m (i.e. $m = m_b$), the criterion returns to its original form

$$\sigma_1 = \sigma_3 + (m\sigma_3\sigma_{ci} + s\sigma_{ci}^2)^{1/2} \tag{7}$$

In which constants s and m define structural pattern, or quality, of rock mass and rock type respectively. In the modified Hoek-Brown criterion (Hoek *et al.* 1992), parameter m_b accounts for broken rock and is defined by the ratio m_b/m_i where m_i is the material constants for intact rock.

For the original and modified Hoek-Brown criterion, different attempts have been made either by Hoek and Brown or by other researchers to evaluate m or m_b, and also s, from some classification schemes such as *RMR* or *GSI*.

Benz *et al.* (2008) proposed an extended version of the Hoek-Brown criterion with intrinsic material factorization. The original Hoek-Brown criterion was additionally enhanced by adopting the Spatial Mobilized Plane (SMP) concept which accounts for the experimentally proven influence of intermediate principal stress on failure which is disregarded in original Hoek-Brown criterion.

Hoek-Brown failure criterion is applied to a variety of the engineering problems stability of slopes or underground excavations. Other applications have also been introduced. For example, it was used to evaluate the ultimate bearing capacity of rock mass overlain by surface footing by Merifield *et al.* (2006). Rigorous bounds on the ultimate bearing capacity are obtained by employing finite element analysis in conjunction with the upper and lower bound limit theories of the classical plasticity. Bagheripour and Hakimipour (2009) discussed the efficiency of the Hoek-Brown failure criterion for predicting rock mass strength applied to dam foundations and tunnels. Also a new model based on Hoek-Brown failure criterion was also proposed by Wu and Zhou (2010) to predict the strength of circular and square columns confined by Fiber reinforced Polymers (FRP). A relatively large number of test data were used to evaluate the model and comparison was made between the test and the model results which showed the accuracy of the model proposed.

3.2 Modified Hoek-Brown criterion for anisotropic rock

A basic assumption involved in original and modified versions of Hoek-Brown criterion was that the intact or the rock mass behaves isotropically and hence parameters m and s are evaluated for such rock. For anisotropic rock Hoek and Brown (1980a) primarily introduced special modification for constants m and s as follows

$$\frac{m_j}{m_i} = 1 - A\ \exp\left[-\left(\frac{\beta - \xi_m}{A_2 + \beta A_3}\right)^4\right] \quad \text{where} \quad A = m_i - \frac{m_{min}}{m_i} \tag{8}$$

$$s_j = 1 - P\ \exp\left[-\left(\frac{\beta - \xi_s}{P_2 + \beta P_3}\right)^4\right] \quad \text{where} \quad P = 1 - s_{min} \tag{9}$$

where m_j and s_j are Hoek-Brown constants for jointed rock particularly defined for anisotropic rock and m_i is the intact material constant. The latter can be obtained by testing rocks loaded normal to the plane of weakness. ξ_m and ξ_s are the specific angle of weakness plane, β, at which m and s are respectively minimum. A_2, A_3, P_2, P_3 are additional empirical parameters which are determined experimentally. Also m_{min} and s_{min} are constants specifically related to the case when rock strength is minimum. The strength variation for Martinsburg slate was investigated by Hoek and Brown (1980a) using Eqs. (8)-(9) which is depicted in Fig. 3.

Fig. 4 shows the schemes of rock samples concerning types of anisotropies that may be exhibited by rock and values of parameters attributed to each type. Since parameter s has been originally introduced by Hoek and Brown (1980a, b) to quantify the structural pattern of rock, this parameter is set to unity for intact anisotropic rock (Fig. 4(a)) whereas for rock masses shown in Figs. 4b and 4c the parameter s attains values less than unity. Fig. 5 also shows strength anisotropy curve in principal stress plane, i.e. $\sigma_1 - \sigma_3$, which helps visualizing the impact of plane of weakness orientation on strength of anisotropic rock.

More recently, Colak and Unlu (2004) investigated parameter m_i in Hoek-Brown criterion and observed how it was affected by rock anisotropy. They noticed that the m_i values for same generic rock types varied within a range. They discussed that the broadness of the range may be attributed to the strength anisotropy. Therefore, they attempted to quantify this parameter based on normalization, determination, and biasing the parameter m_i for any given angle of plane weakness ($m_{i(\beta)}$) to a

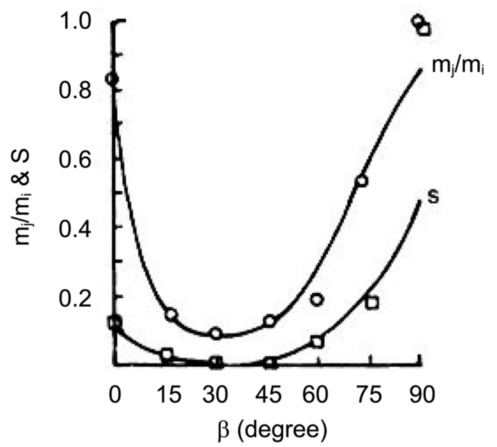

Fig. 3 Variation of s and m_j/m_i vs β for anisotropic rock (adopted from Hoek and Brown 1980a)

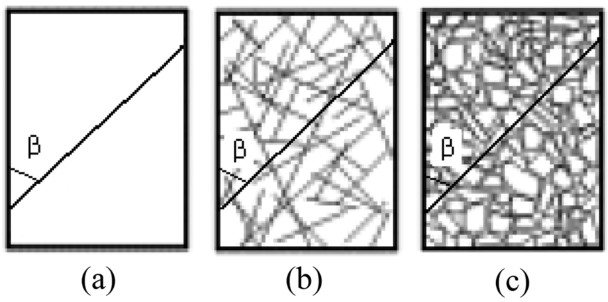

Fig. 4 Samples of anisotropic rock: (a) with intact pieces ($s = 1$); (b) and (c) with crushed pieces ($0 < s < 1$)

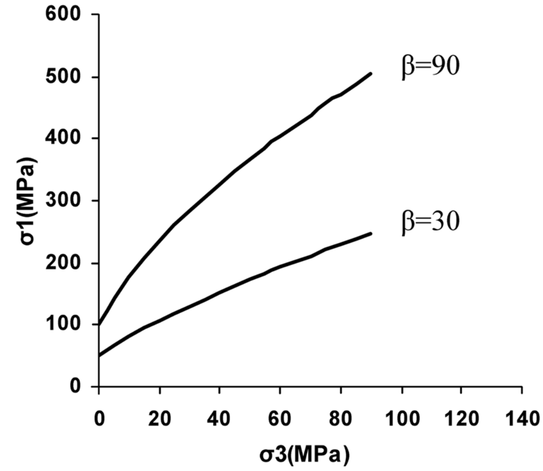

Fig. 5 Strength anisotropy presentation in principal stresses plane for a sample anisotropic rock

reference value ($m_{i(90)}$). Colak and Unlu (2004) adopted the following modified form of Hoek-Brown criterion for statistical and regression analysis into their experimental data obtained in compression tests to evaluate the values of $\sigma_{ci(\beta)}$ and also $m_{i(\beta)}$

$$\sigma_{1(\beta)} = \sigma_3 + (\sigma_{ci(\beta)} m_{i(\beta)} \sigma_3 + \sigma_{ci(\beta)}^2)^{1/2} \tag{10}$$

The ratio of $m_{i(\beta)}/m_{i(90)}$ obtained was further investigated by Colak and Unlu (2004) and they found that it was in good agreement with a relation earlier proposed by Hoek and Brown (1980a)

$$m_{i(\beta)}/m_{i(90)} = 1 - A\ \exp\{-[(\beta - B)/(C + D\beta)]^4\} \tag{11}$$

In which parameters A, C, D are all empirical constants while B is the value of β corresponding to $m_{i(\beta)}$ at its minimum value. Also $m_{i(90)}$ was assumed as the reference value being equal to m_i for intact rock. They discussed that the ratio $m_{i(\beta)}/m_{i(90)}$ could be correlated well with the degree of anisotropy of uniaxial strength (R_C). The modified Hoek-Brown criterion defined by Colak and Unlu (2004) is limited and applicable to only rock types they experienced, i.e. sandstone, siltstone, claystone, and that it should be further investigated for other anisotropic rock types.

A new parameter, namely K_β, was more recently introduced into the Hoek-Brown failure criterion by Saroglou and Tsiambaos (2008) which also took the degree of rock strength anisotropy (R_C) into account. Further, the reduction of uniaxial strength of intact rock due to existence of weakness planes was also considered as an important factor, denoted by $\sigma_{c\beta}$, and was incorporated into the criterion. The modified Hoek-Brown criterion suggested by Saroglou and Tsiambaos (2008) took the following form

$$\sigma_1 = \sigma_3 + (k_\beta m_i \sigma_{c\beta} \sigma_3 + \sigma_{c\beta}^2)^{1/2} \tag{12}$$

In which m_i is constant of intact rock material. As can be seen from the above modified criterion, parameter s (when Eq. (12) is compared with Eq. (7)) is assumed fixed and is set to unity (i.e. $s = 1$) in a similar manner to that suggested for the original Hoek-Brown criterion (Hoek and Brown 1980b, Hoek 1983). They also investigated the correlation of the variation of K_β, with the degree of anisotropy.

It can be concluded from the discussion given in the proceeding section that parameter m played an important and key role in modifications introduced into the Hoek-Brown criterion to reach strength of anisotropic rock. Considering the fact that dimensionless parameter m depends on inter-particle friction and the degree of particle interlocking, careful selection of this parameter is necessary. Moreover, further redefinition and ramifications introduced for this parameter may cause confusion and inconvenience in practical use of modified Hoek-Brown criterion for anisotropic rock.

3.2.1 Complement instead of modification

An alternative and better solution appears to exist for prediction of anisotropic strengths which is based on the study conducted here suggesting the use of a complement to the Hoek-Brown criterion. Introducing a special reduction factor, or in fact a decay function, multiplied to the original form of Hoek-Brown criterion for truly intact rock, results in strength of anisotropic rock. The decay function should be carefully defined to account for relative reduction of strength due to existence of weakness planes. In fact, such an idea was examined by Bagherpour and Mostyn (1996) for a soft sedimentary model rock through Eqs. (4)-(5). The concept appeared promising and remained appealing for further statistical and experimental investigations. Current study was conducted to revise and optimize this concept while further empirical data became available from the current experimental program and also from the literature.

4. Strength and degree of anisotropy

The strength variation observed in uniaxial or triaxial compression of rock with respect to the weakness plane's orientation (β) is often defined as strength anisotropy (Bagheripour and Asadi 2006, Bagheripour and Pashnesaz 2006, Saroglou and Tsiambaos 2008). In addition, the degree of anisotropy is also quantified by parameter "R_C" through the following relation

$$R_C = \sigma_{ci(90)}/\sigma_{ci(min)} \tag{13}$$

which is simply the ratio of the maximum to minimum strength attainable in anisotropic rock. The maximum strength ($\sigma_{ci(90)}$) is obtained in samples having flat plane of weakness and may rationally be assumed equal to strength of corresponding truly intact rock (i.e. σ_{ci}). While $\sigma_{ci(min)}$ exhibits by rock having steep plane of weakness corresponding to (β_{min}). β_{min} is often observed to vary in a range of 30°-45° and agrees experimentally with the well-known relation of $\beta_{min} \approx 45\text{-}\phi/2$ in geotechnical engineering.

Rammamurthy (1993) classified strength anisotropy based on R_C values evaluated for various rocks. Despite the fact that the degree of anisotropy defined by R_C is essentially based on uniaxial compressive strength of rock, however, reports on the strength anisotropy in confined compression state have shown that the degree of anisotropy for a specific rock is not constant. As reported by some researchers (Brown *et al.* 1977, Nasseri *et al.* 2003) even at low level of confinements, R_C still remains dependant on the confining pressure. Other investigators (e.g. Donath 1964) suggest that the strength anisotropy may not remain dominant and may even vanishes (e.g. Mostyn and Bagheripour 1995, Bagheripour and Mostyn 1996) at sufficiently high normal stress levels. Using a newly introduced experimental criterion for discontinuous rock, Bagheripour and Mostyn (1996) estimated a specific level of confining pressure about which the jointed weak sandstone ceased to behave as anisotropic rock. This specific level of confining pressure $(\sigma_3)_o$ was evaluated in terms of the uniaxial compressive strength of the corresponding intact rock as $(\sigma_3)_o = 0.58\sigma_{ci}$ which was also in well agreement with the relative value reported by Rammamurthy (1994). It is understood from the experimental evidence that the effect of confining pressure on degree of anisotropy is significant on the jointed anisotropic rock. For intact anisotropic rock, however, less experimental supporting evidence exists and hence in this study, this effect is not considered to be of prime concern.

More recent studies regarding the strength anisotropy in rock backs to Dehler and Labuz (2007) who studied the elastic response of anisotropic sandstone with P-wave velocities normal and parallel to the beddings. They also carried out triaxial compression and extension tests using cyclic loading normal a parallel to the beddings. They observed that with axial stress parallel to the bedding, the friction angle was more than that with axial stress normal to the bedding. This anomalous behaviour was attributed to the strength anisotropy of the sandstone.

It will be seen, in the following sections, that the decay function introduced here indirectly incorporates R_C (defined by Eq. (13)) through its empirical constants since the constants observed to be well correlated to parameter R_C and is also comparable to that proposed earlier by Bagheripour *et al.* (2003).

5. Proposed criterion

The following relation is mathematically proposed to generally predict the strength of anisotropic rock

$$\sigma_{1(anis)} = F_{(\beta)}\sigma_{1(intact)} \tag{14}$$

where $\sigma_{1(anis)}$ defines the strength of intact anisotropic rock while $F_{(\beta)}$ and $\sigma_{1(intact)}$ refer to the decay function and to the strength of truly intact rock respectively. If $\sigma_{1(intact)}$ is defined by original Hoek-Brown for intact rock ($\sigma_{1(H\text{-}B)}$), then

$$\sigma_{1(anis)} = F_{(\beta)}\sigma_{1(H\text{-}B)} \tag{15}$$

If $\sigma_{1(H\text{-}B)}$ is represented by Eq. (7) with the parameters $s = 1$ and $m = m_i$, then the relation shown by Eq. (14) is stated by the following relation

$$\sigma_{1(anis)} = F_{(\beta)}[\sigma_3 + (m_i\sigma_{ci}\sigma_3 + \sigma_{ci}^2)^{1/2}] \tag{16}$$

$F_{(\beta)}$ is hereafter referred to as a complement to the Hoek-Brown original failure criterion. As mentioned earlier, the complement acts as a decay function (or reduction factor) which varies between its boundary values. In particular, if it is set to unity, the original Hoek-Brown failure criterion is retrieved (i.e. Eq. (16) is reduced to Eq. (7)) and is applied to the intact rock.

Extensive course of investigations was undertaken in order to evaluate the effectiveness and workability of trial functions initially defined for $F_{(\beta)}$ and to arrive in a final and optimum shape of the decay function. A simple polynomial was found as best fit to the experimental data describing the rate of reduction in anisotropic rock strength relative to the strength of corresponding truly intact rock. The complement was finally defined by the following relation

$$F_{(\beta)} = \beta_D^2(A - B\beta_D) + \mathrm{C} \quad \text{where} \quad \beta_D = \beta - \beta_{mi} \tag{17}$$

In the above equations, A, B, and C are the regression constants to be further investigated and discussed later in this paper. β_D is defined as "deviatoric angle" which is the difference between orientation of plane of weakness exists in rock (β) and the specific orientation of plane of weakness at which anisotropic rock strength attains its minimum value (β_{min}). Eq. (17) is substituted into Eq. (16) to arrive in a combined form of Hoek-Brown failure criterion for anisotropic rock

$$\sigma_{1(anis)} = [\beta_D^2(A - B\beta_D) + C][\sigma_3 + (m_i\sigma_{ci}\sigma_3 + \sigma_{ci}^2)^{1/2}] \quad (18)$$

A prime and great advantage of such a combined form of criterion is that (unlike other modified Hoek-Brown criterion introduced for anisotropic rock discussed in preceding sections) the original form of the Hoek-Brown failure criterion for truly intact rock remains intact and its corresponding rock parameters are unchanged. Interested readers may also note that any other failure criterion for truly intact rock may be considered instead of Hoek-Brown criterion ($\sigma_{1(H\text{-}B)}$). This is because the complement keeps its general functionality as a multiplier to any relation defining the strength of intact rock. However, based on the popularity of the Hoek-Brown criterion in practical problems, preference is given to this criterion in current study.

In order to validate the proposed combined criterion, experimental data are required. In the following, experimental program carried out in the current study is explained first. The results obtained in this program associated with those of interest found in literature were used to develop a relatively good data set for statistical calculations including regression analysis into Eq. (18). Discussions on the results obtained, validity of the combined criterion, efficiency and dependency of the empirical constants are given next.

6. Experimental program

The experimental program involved developing model sedimentary intact and anisotropic rock followed by various investigations into the strength prediction for intact and anisotropic jointed rock. The gap in the available and required data into anisotropic rock was filled by data obtained in testing model rock. The artificial jointing/bedding process associated with the description of the main constituents and also admixtures used for the model rock material are similar to those reported by Bagheripour and Mostyn (1996). The rock itself developed was tested at various stress conditions including uniaxial and triaxial compression (triaxial apparatus/cell is shown in Fig. 6) and tensile

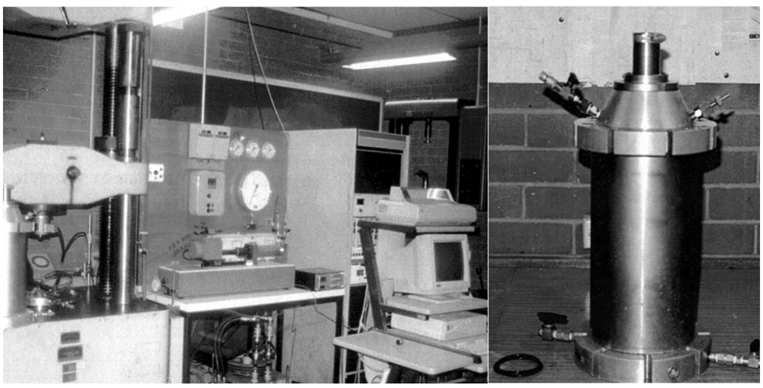

Fig. 6 Triaxial testing apparatus/cell used for the current experiments

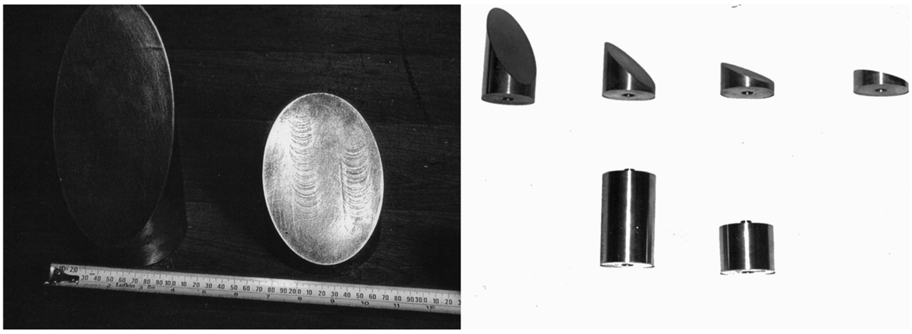

Fig. 7 Typical inclined steel rammers used for development of weakness planes in model rock

tests which provided a very good model of a prototype sedimentary rock, satisfied a broad range of similarity conditions, and well agreed with the results earlier reported by Mostyn and Bagheripour (1995).

Special consideration was given to the development of intact and anisotropic samples using inclined and also flat rammers (shown in Fig. 7) held in the casting moulds. Fig. 8 shows these casting moulds which included split cylinders used to keep the mixture compacted by rammer. Both 50 mm and 100 mm diameter samples were developed, however, it was observed in the experimental program that using 100 mm mould to develop larger diameter samples would result in the much less segregation in the truly intact rock samples and also in the intact portion of the anisotropic rock samples developed. This led also to a better consistency and less unfavorable discrepancies in the results which would be observable even in those of natural rocks and often caused difficulties in analysis and interpretation of the results. Fig. 9 shows some of rock samples of the model anisotropic rock failed in triaxial tests.

It is interesting to note that by controlling the compaction pressures as well as the constituents of

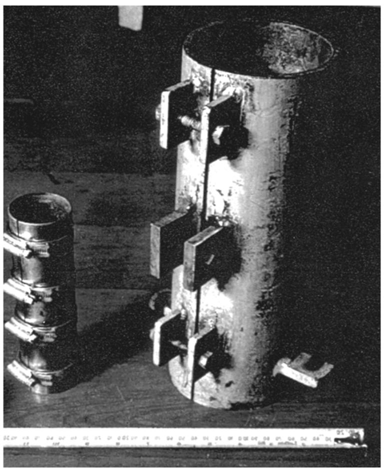

Fig. 8 Split cylinders used to mould and compact the materials for the development of rock and their weakness planes.

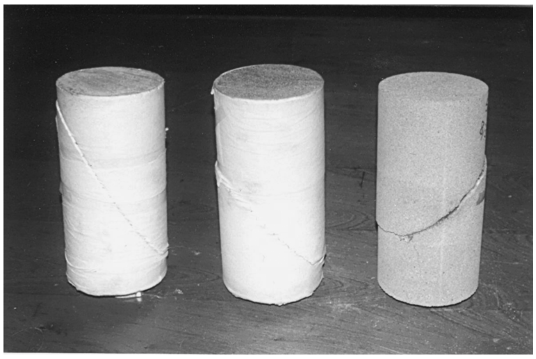

Fig. 9 Some anisotropic model rock samples with single plane of weakness failed in triaxial tests

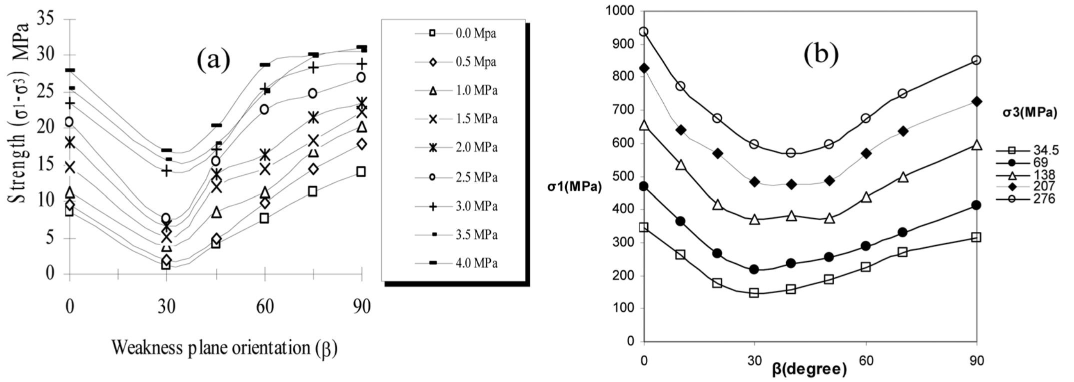

Fig. 10 Strength anisotropy in (a): model rock developed in this study (Bagheripour and Mostyn 1995 and 1996) and (b): Martingsburg slate (Mclamore and Gray 1967)

the raw materials, used for development of model rocks, and especially the percentage of cementing agents it would be possible to develop a wide range of rocks having various strength characteristics. After careful curing, compression tests including uniaxial and triaxial tests were conducted on specimens. This broad range of conditions for model rock developed helped obtaining desired and unavailable samples for which the data set suffered gaps. The description of the whole process is lengthy and is beyond the scope of this paper, however, interested readers may refer to other published works by authors (e.g. Bagheripour and Mostyn 1996, Mostyn and Baghgeripour 1995).

Typical results obtained are presented in Fig. 10(a) showing the strength anisotropy curve for a set of model sedimentary rock (weak sandstone) developed and tested at relatively low confining pressures. For the sake of initial comparison, the strength anisotropy curve observed by Mclamor and Gray (1967) on Martinsburg slate is also shown in Fig. 10(b). This indicates that anisotropic model rock developed also satisfied the similarity conditions to natural anisotropic sedimentary rock.

7. Validation of the proposed combined criterion

Table 1 presents a summary of the most important strength characteristics of various rock types investigated in this study. As mentioned before, these included results of the tests conducted in current experimental program as well as those collected from the literature. As can be seen from this table, various rocks investigated cover a relatively broad range of rock type and strength characteristics. Test results related to five types of model rock developed in the current experimental study are indexed by 10A-10E. As mentioned earlier, these results are believed to fill the gaps in available strength data and to control the natural rock strength data.

Parameters such as $\sigma_{ci(90)}$ and $m_{i(90)}$ were also quoted/reported in Table 1 which were considered as good indices of strength for the corresponding truly intact rock. Further, the ratio $\sigma_{ci(90)}$, to $\sigma_{ci(min)}$ defines the anisotropy degree described before in Sec. 4 which was given attention in this study. Nonlinear regressions were carried out in order to evaluate how best the proposed combined criterion fits experimental data. In Figs. 11-18, variations in strength predicted by Eq. (18) are shown for some of the various anisotropic rocks, characterized in Table 1, associated with the corresponding experimental data. Presentation of the same graphs for all other rock types studied here is lengthy

Table 1 Properties of various rock types investigated in current study

Rock	No.	$\sigma_{ci(90)}$ (MPa)	$\sigma_{ci(\min)}$ (MPa)	$m_{i(90)}$	Reasercher
Martinsburg Slate	1	155	18	14.08	Donath (1964)
Austin Slate	2	250	100	5.72	Mclamor and Gray (1967)
Green River Shale I	3	208	140	6.99	Mclamor and Gray (1967)
Green River Shale II	4	106	80	4.90	Mclamor and Gray (1967)
Limestone	5	56.6	28	6.21	Horino and Ellickson (1970)
Blue Penrhyn Slate	6	206.6	38.2	6.23	Attewell and Sandford (1974)
Tournemire Shale	7	45	18	4.33	Niandou *et al.* (1997)
Gneiss	8	252.8	215	28.55	Horino and Ellickson (1970)
Model Rock	9	14	1.09	14.47	Bagheripour and Mostyn (1996)
	10A	83	9.24	16	
	10B	48.5	6.9	18	
Model Rock	10C	34	8	13.5	Bagheripour and Pashnesaz (Current study)
	10D	60	5	19	
	10E	74	7	21	
Artificial Interlayered Rock	11	35.21	13.68	4.73	Tien and Tsao (2000)
Gneiss(A)	12	60.6	28.2	27.83	Saroglou and Tsiambaos (2008)
Gneiss(B)	13	73.45	22.4	27.69	Saroglou and Tsiambaos (2008)
Athens Schist	14	67.2	51.6	9.42	Saroglou and Tsiambaos (2008)
Marble	15	84.7	71	10.38	Saroglou and Tsiambaos (2008)

and would be beyond the limit of the paper. However, they can be presented in an appendix or a separate report. The coefficients of correlation (R^2) obtained in regression analyses are listed in Table 2 for all rock types studied. It can be seen from this table that these coefficients obtained, except in one case, are all well above 90% indicating a relatively good agreement between experimental anisotropic strength data and prediction made by Eq. (18). This is also deduced easily from the graphs of Figs. 11-18 that the criterion well aggress with the experimental data. The general trend observed for the criterion suggests that it easily captures the strength variation aspects of experimental

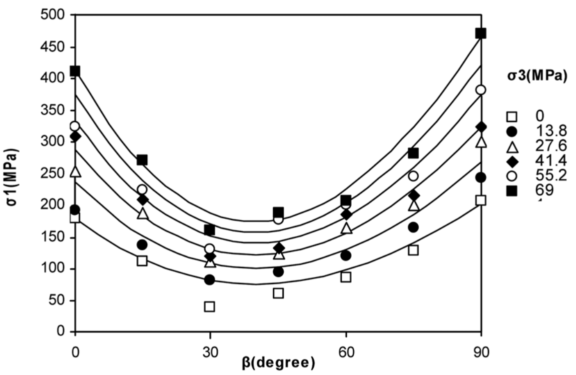

Fig. 11 Strength anisotropy observed in Slate of Penrhyn (Attwell and Sanford 1974) and prediction by Eq. (18)

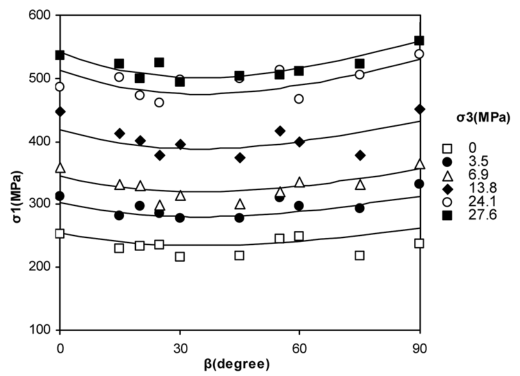

Fig. 12 Strength anisotropy observed in Gneiss (Horino and Ellickson 1970) and prediction by Eq. (18)

Table 2 Constants of the complement for rock types investigated in current study

Rock	No.	A	B	C	R_C	R^2 (%)	Reasercher
Martinsburg Slate	1	1.1443	0.4158	0.3105	8.61	97.04	Donath (1964)
Austin Slate	2	0.8470	0.5188	0.5748	2.50	95.10	Mclamor and Gray (1967)
Green River Shale I	3	0.2479	0.1891	0.9092	1.49	98.90	Mclamor and Gray (1967)
Green River Shale II	4	0.4861	0.3698	0.8285	1.33	99.01	Mclamor and Gray (1967)
Limestone	5	0.5365	0.4307	0.8605	2.02	93.40	Horino and Ellickson (1970)
Blue Penrhyn Slate	6	0.9322	0.1454	0.3646	5.41	95.70	Attewell and Sandford (1974)
Tournemire Shale	7	0.6356	0.4077	0.7568	2.50	96.10	Niandou *et al.* (1997)
Gneiss	8	0.1676	0.0541	0.9283	1.18	97.98	Horino and Ellickson (1970)
Model rock	9	1.6052	1.0061	0.4186	10.33	97.80	Bagheripour and Mostyn (1996)
	10A	1.3	0.55	0.2	9	96.5	
	10B	1.25	0.6	0.23	7	97.1	
Model rock	10C	0.95	0.72	0.3	4.25	92.3	Bagheripour and Pashnesaz (current study)
	10D	1.4	0.75	0.12	12	94.2	
	10E	1.5	0.82	0.2	10.5	91.3	
Artificial Interlayered Rock	11	0.6591	0.6010	0.7200	2.57	89.40	Tien and Tsao (2000)
Gneiss (A)	12	1.2616	0.7872	0.5249	2.15	98.18	Saroglou and Tsiambaos (2008)
Gneiss (B)	13	0.8475	0.3004	0.4170	3.28	98.84	Saroglou and Tsiambaos [25]
Athens Schist	14	0.4561	0.3182	0.8723	1.30	96.33	Saroglou and Tsiambaos (2008)
Marble	15	0.4192	0.2988	0.9014	1.19	98.40	Saroglou and Tsiambaos (2008)

data and covers both "U" and "shoulder" type anisotropies exhibited by various rock types.

Four other parameters are also listed in Table 2 including regression constants A, B, C used in Eq. (18) associated with anisotropy ratio (R_C) for all rock types investigated. The constants (A, B, C) are to be further studied for their inter-correlations with other rock strength parameters. This is carried

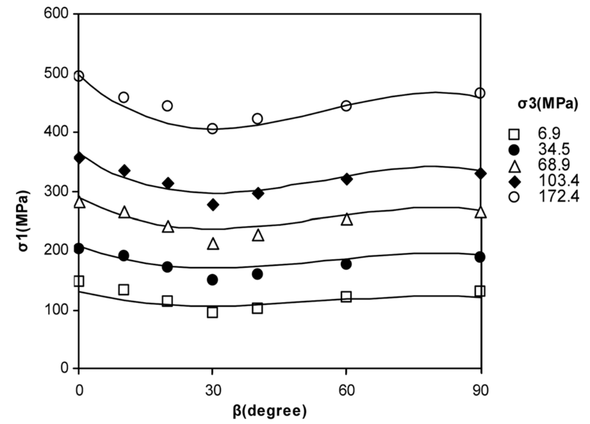

Fig. 13 Strength anisotropy observed in Green River Shale *II* (McLamore and Gray 1967) and prediction by Eq. (18)

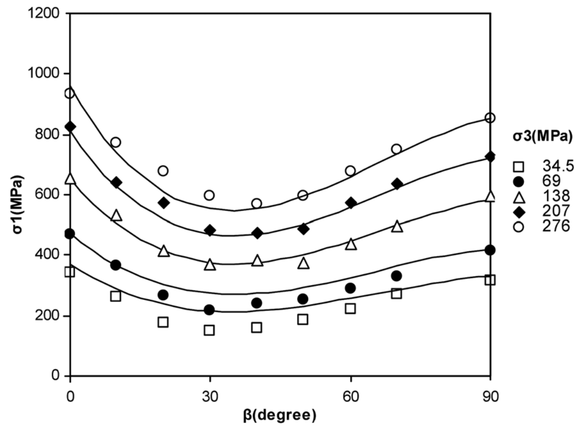

Fig. 14 Strength anisotropy observed in Austin Slate (Mclamore and Gray 1967) and prediction by Eq. (18)

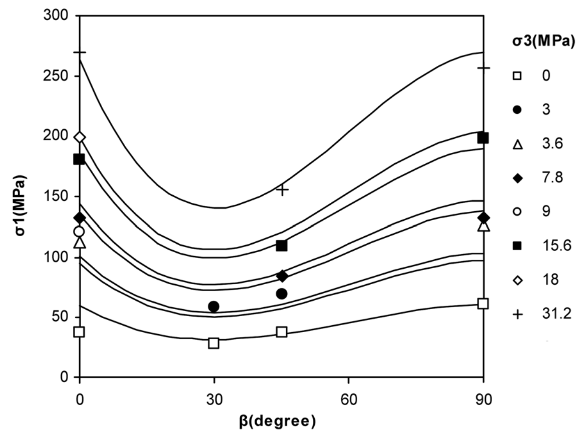

Fig. 15 Strength anisotropy observed in Gneiss (A) (Saroglou and Tsiambaos 2008) and prediction by Eq. (18)

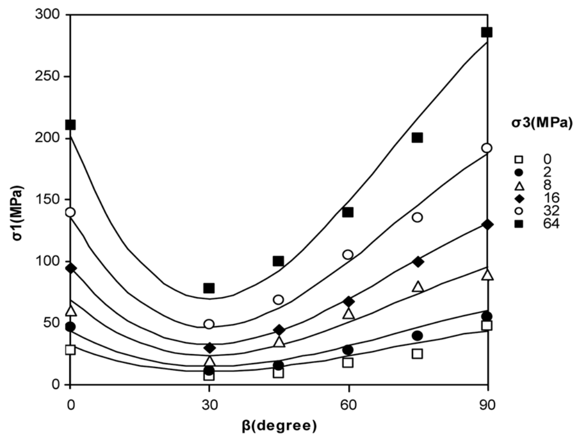

Fig. 16 Strength anisotropy observed in model rock (current study) and prediction by Eq. (18)

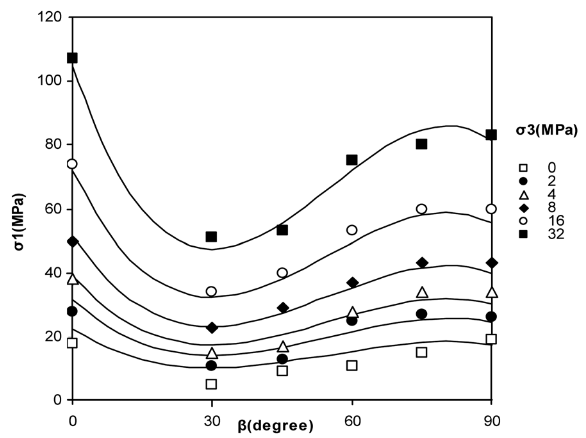

Fig. 17 Strength anisotropy observed in model rock (current study) and prediction by Eq. (18)

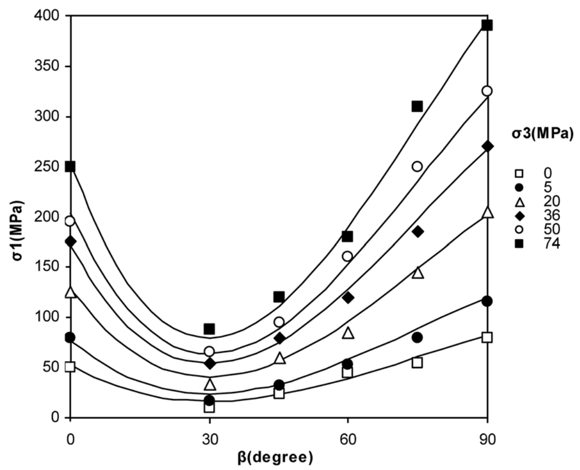

Fig. 18 Strength anisotropy observed in model rock (current study) and prediction by Eq. (18)

and discussed in following sections.

An obvious aspect of the combined criterion developed in this study is that for any given confining pressure (σ_3) the function $\sigma_{1(H\text{-}B)}$ reduces to a constant and strength variation for all values of β is established similar to those presented in Figs. 11-18. On the other hand, for any given β, the function $F_{(\beta)}$ holds as a constant and strength variation in principal stress planes ($\sigma_1 - \sigma_3$) is defined similar to that schematically shown in Fig. 5. This is carried for some rock types studied here and the results are shown in Figs. 19-24. It is generally deduced, from this alternative presentation of strength variation provided by these figures, that the proximity or farness of the two strength envelopes in $\sigma_1 - \sigma_3$ space is an indication of the degree of strength anisotropy. Therefore, the closer two strength curves are in $\sigma_1 - \sigma_3$ plane to each other, the lesser is the degree of anisotropy and vice versa.

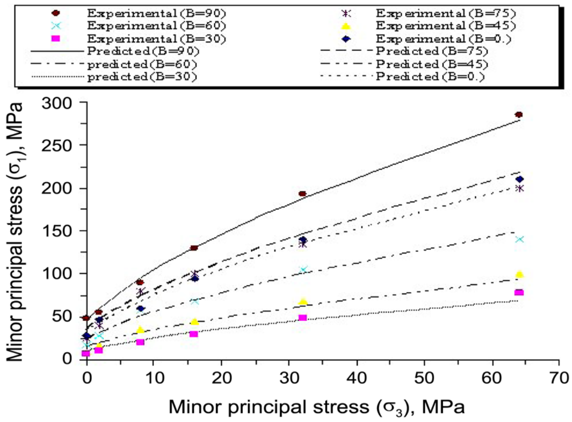

Fig. 19 Variation of strength in model rock (type 10B developed in this study) vs. confing pressure for given angle of plane of weakness (β)

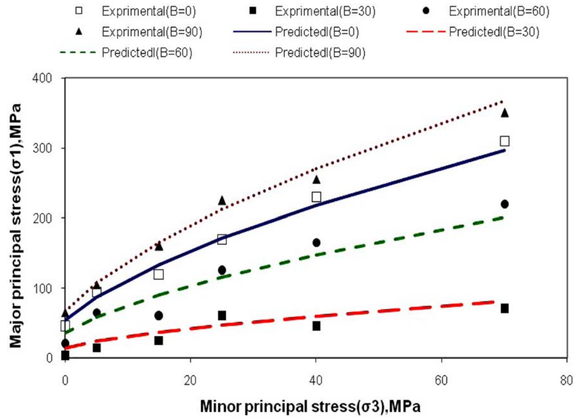

Fig. 20 Variation of strength in model rock (type 10E developed in this study) vs. confing pressure for given angle of plane of weakness (β)

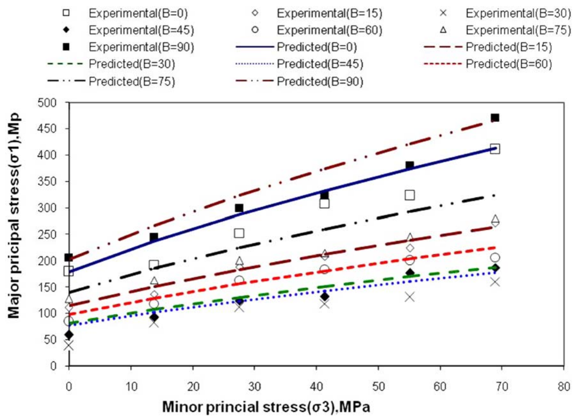

Fig. 21 Variation of strength depicted in principal stress plane ($\sigma_1 - \sigma_3$) for various β observed in Slate of Penrhyn (Attwell and Sanford 1974) and prediction by Eq. (18)

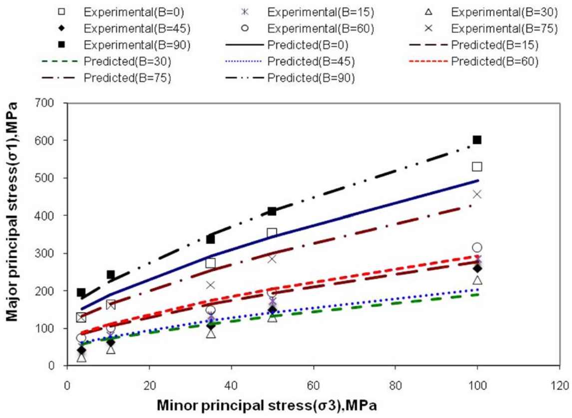

Fig. 22 Variation of strength depicted in principal stress plane ($\sigma_1 - \sigma_3$) for various β observed in Martinsburg Slate (Donath 1964) and prediction by Eq. (18)

8. Discussion on the criterion's constants

8.1 Parameter A

It is desirable to describe the parameters/constants of any function defining a failure criterion in such a way that they are easily interpreted based on their deterministic role in that function. Further, it is also preferred to relate these constants to the most readily available rock strength parameters (whenever a relation does exist) so that they are easily evaluated with minimum computational or experimental efforts.

For various rock types studied here, parameter A was found related well to the anisotropy ratio, i.e. R_C, defined by Eq. (13). Hence the relation between parameter A and R_C was investigated using

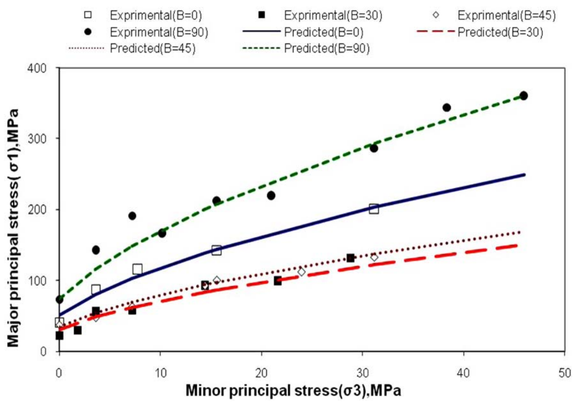

Fig. 23 Variation of strength depicted in principal stress plane ($\sigma_1 - \sigma_3$) for various β observed in Gneiss (B) (Saroglou and Tsiambaos 2008) and prediction by Eq. (18)

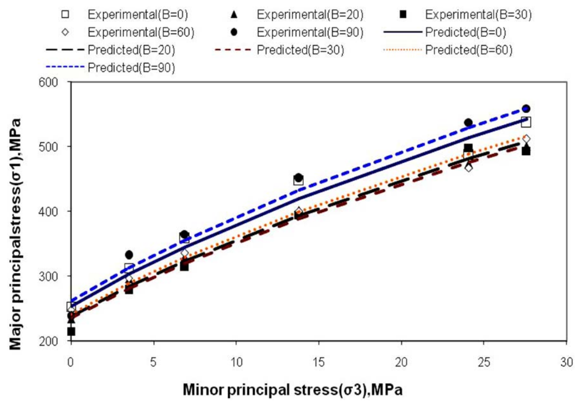

Fig. 24 Variation of strength depicted in principal stress plane ($\sigma_1 - \sigma_3$) for various β observed in Limestone (Horino and Ellickson 1970) and prediction by Eq. (18)

statistical analysis. Regression analysis led to the following relation

$$A = 1.15 \log(R_C) + 0.23 \tag{19}$$

The coefficient of correlation obtained was about 92%. Fig. 25 shows plot of Eq. (19) associated with experimental values of R_C listed in Table 2. It can be seen from Fig. 24 that as the anisotropy ratio increases, parameter A also increase. As the logarithmic plot implies, however, the rate of increase in parameter A is reduced as R_C increases.

8.2 Parameter B

As mentioned earlier in this paper (Sec. 3), the general shape of anisotropy strength curve is controlled by some factors such as the strength of intact rock and that of the weakness plane. A very good explanation for this may be quoted from Ramamurthy and Arora (1994) which describes the shape of strength anisotropy curve depending on the properties of the weakness plane itself and its surrounding rock. As mentioned before, they found that for weak rocks having parallel joints or joint sets, U shape strength curve was observed whereas, the shoulder shape prevailed in rocks with well defined bedding or joint planes in strong rocks. A study was conducted, as part of the current research program, to compare the shapes of anisotropic strength curves shown in Figs. 11-24 and to find which parameter/variable included in the complements' function might generally control the shape these curves.

Referring to Fig. 1, from Mohr's stress transformation principles, one can calculate shear stress on the weakness plane as

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin\,[2(90-\beta)] \tag{20}$$

In a state of stress corresponding to $\sigma_{ci(min)}$, the minimum shear strength on plane of weakness can be found using Eq. (20) which results in the following relation

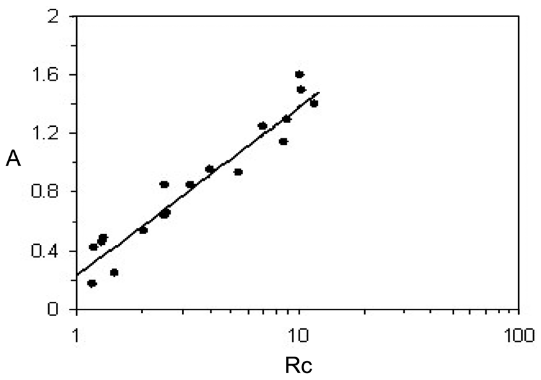

Fig. 25 Variation of parameter A against anisotropy degree (R_C)

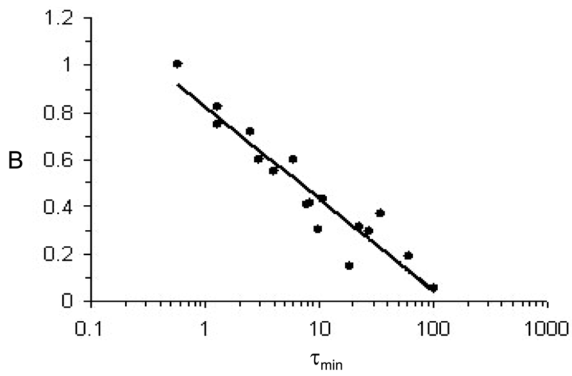

Fig. 26 Variation of parameter B against τ_{min}

$$\tau_{min} = \frac{\sigma_{ci(min)}}{2}\sin(2\beta_{min}) = \frac{\sigma_{ci(90)}}{2R_C}\sin(2\beta_{min}) = \frac{\sigma_{ci}}{2R_C}\sin(2\beta_{min}) \tag{21}$$

When regressed parameter B for all types of rock listed in Table 2 are plotted against the corresponding values of τ_{min}, calculated by Eq. (21), a declining and consistent trend as that shown in Fig. 26 is observed. Regression analysis led to a relation between parameter B and τ_{min} as follows

$$B = -0.39\log(\tau_{min}) + 0.82 \tag{22}$$

This relation obtained implies that parameter B is inversely dependant on τ_{min} (or in fact on log (τ_{min})) as is also deduced from Fig. 26. In practice, Eq. (21) is used to evaluate τ_{min} for a given β_{min} and from σ_{ci} as well as its corresponding R_C value. The calculated value of τ_{min} can be subsequently used to evaluate parameter B using Eq. (22).

Further discussion on parameter B may now be conducted regarding the general shape of strength anisotropy curve. U shape and shoulder type anisotropic strength curves can be distinguished for various rocks studied depending on their corresponding τ_{min} values and hence parameters B. It appears that the variation of B is relatively consistent with the definition of the types of anisotropy strength curves defined by Ramamurthy and Arora (1994) quoted in Sec. 3. Therefore, at this stage, parameter B is suggested to be used for differentiating U shape or shoulder type anisotropies. In this study, parameter B can be referred to as "*Shouldering Number* or *SN*" which varies approximately between zero and one (see also Table 2). For low shear strength on bedding planes, *SN* approaches unity (U shape) while for high shear strength on bedding planes it tends to zero (Shoulder shape). However, a more concrete statement in this regard should be made based on further experimental investigations.

8.3 Parameter C

If Eq. (18) is applied to condition of $\beta = \beta_{min}$ and to the state of unconfined compression stress ($\sigma_3 = 0$) then the minimum anisotropic strength is attained

$$\sigma_1 = \sigma_{ci(min)} = C\ \sigma_{ci} \tag{23}$$

If σ_{ci} is rationally approximated to $\sigma_{ci(90)}$, (i.e. $\sigma_{ci} = \sigma_{ci(90)}$) and Eq. (13) is applied, one can reach a simple relation which will be further used to derive parameter C as a function of anisotropy ratio

$$R_C = \sigma_{ci(90)}/\sigma_{ci(min)} = \sigma_{ci}/C.\sigma_{ci} = 1/C \tag{24}$$

Eq. (24) simply suggests that the parameter C is the inverse of anisotropy degree defined by Eq. (13).

9. Recommended procedure for practical problems

In order to apply the concept presented here in practical rock engineering problems, a step by step procedure for determination of variation of strength anisotropy is suggested as follows:

1. The strength of truly intact rock at given confining stress σ_3 (i.e. $\sigma_{1(intact)}$ defined in Eq. (14)) is evaluated at a given confining pressure (σ_3) using the original Hoek-Brown criterion. In fact, m_i and σ_{ci} are needed to define and characterize the strength of truly intact portion of the anisotropic rock. σ_{ci} and m_i can be approximated to $\sigma_{ci(90)}$ and $m_{i(90)}$ respectively if these two parameters are available from anisotropic rock samples tested.

2. Unconfined compressive strength of intact anisotropic rock at $\beta = \beta_{min} \approx 45-\phi/2$ (i.e. $\sigma_{ci(min)}$) is determined which is regarded as the minimum possible strength attainable in anisotropic rock. The degree of anisotropy (R_C) can be calculated using values of $\sigma_{ci(90)}$ and $\sigma_{ci(min)}$ assigned to Eq. (13). Alternatively R_C can be estimated using available $m_{i(min)}$ and $m_{i(90)}$ in accordance to suggestions made by Colak and Unlu (2004) discussed in some detail in Secs. 3 and 4 of this paper. Thereafter, empirical constant A can be evaluated using directly Eq. (19) or be deduced from graph of Fig. 25.

3. Having determined R_C and σ_{ci} (or alternatively $\sigma_{ci(90)}$), τ_{min} is calculated using Eq. (21). It is subsequently used to calculate parameter B directly from Eq. (22) or to deduce graphically from Fig. 26.

4. As stated by Eq. (23), the inverse of the degree of anisotropy is equal to the parameter C. Therefore, calculation of parameter C is straight forward with regard to this equation.

5. Prediction of anisotropic strength can be made using parameters/constants defined in preceding steps and through Eq. (18). At any given confining pressure (σ_3) the function $\sigma_{1(H\text{-}B)}$ in Eq. (18) reduces to a constant and strength variation for all values of β is determined. Using the complement, these results in graphs similar to those presented in Figs. 11-18. On the other hand, for any given β, the function $F_{(\beta)}$ holds constant and strength variation in principal stress planes ($\sigma_1 - \sigma_3$) is defined similar to those shown in Figs. 5 and 19-24.

It can be concluded from the discussion given above that for the model presented here, parameter $\sigma_{ci(min)}$ is the only additional parameter required to evaluate the strength variation for anisotropic rock from that of the corresponding truly intact rock which is characterized by Hoek-Brown parameters (i.e. σ_{ci} and m_i). If R_C value is readily available, there is no need to define this parameter since it can be obtained through the procedure discussed in Secs. 4 and 9 of this paper.

10. Conclusions

In this paper, a complement to the original Hoek-Brown criterion for intact rock was proposed in order to derive the strength of anisotropic rock. The complement was, in fact, a reduction factor or a decay function acting as a multiplier to the original Hoek-Brown criterion. This results in a combined and extended form of the Hoek-Brown criterion which describes the strength variation for anisotropic rock as a varying fraction of the corresponding truly intact rock. If the decay function is set to unity, the original Hoek-Brown criterion is retrieved.

In the current study, it was found that the complements' empirical constants are all related to the degree of anisotropy of the corresponding rock (R_C). The latter, in turn, can be easily calculated through other well known and readily available rock strength parameters such as $\sigma_{ci(min)}$ and $\sigma_{ci(90)}$. Therefore, a very few number of empirical constants, unlike other criteria, were required in the complement that can be determined with minimum experimental effort.

Experimental data obtained through the current research program and those gathered from the interested literature were used to examine and validate the proposed complement and hence the resulting combined Hoek-Brown criterion for anisotropic rock. A step by step procedure was suggested at the end of the paper in order to evaluate the complements constants from the relations introduced. Hence prediction of strength anisotropy would precede straight forward with minimum required experimental data. Another great and prime advantage of the proposed complement is that, because of its general functionality, it can not only be applied to the Hoek-Brown criterion, but may also to any other failure criterion describing the intact rock strength.

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