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# Roof failure of shallow tunnel based on simplified stochastic medium theory

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**Abstract.** The failure mechanism of tunnel roof is investigated with upper bound theorem of limit analysis. The stochastic settlement and nonlinear failure criterion are considered in the present analysis. For the collapse of tunnel roof, the surface settlement is estimated by the simplified stochastic medium theory. The failure curve expressions of collapse blocks in homogeneous and in layered soils are derived, and the effects of material parameters on the potential range of failure mechanisms are discussed. The results show that the material parameters of initial cohesion, nonlinear coefficient and unit weight have significant influences on the potential range of collapse block in homogeneous media. The proportion of collapse block increases as the initial cohesion increases, while decreases as the nonlinear coefficient and the unit weight increase. The ground surface settlement increases with the tunnel radius increasing, while the possible collapse proportion decreases with increase of the tunnel radius. In layered stratum, the study is investigated to analyze the effects of material parameters of different layered media on the proportion of possible collapse block.

**Keywords:** nonlinear failure; roof collapse; stochastic medium theory; settlement; layered soils

## 1. Introduction

Chen (1975) expounded the principle of limit analysis method in detail, and the method was widely used to study the stability of tunnel. Leca and Dormieux (1990) calculated the upper bound of supporting force of shallow tunnel face under the active and passive failure mechanism. Atkinson and Potts (1977) built a wedge failure mechanism of tunnel in cohesionless soil to study the stability of tunnel, and calculated the upper bound and lower bound of supporting pressure respectively using the principle of limit analysis. Davis *et al.* (1980) obtained the upper bound solution and lower bound solution of stability coefficient of tunnel using limit analysis by constructing a failure mechanism consisting of multiple rigid blocks of different geometry. In order to research the failure mechanism of a deep rectangular cavity in Hoek-Brown rock medium, Li and Yang (2018) constructed a collapse mechanism of tunnel roof, and analyzed the influence of seepage forces on the failure mechanism. As for the failure mechanism under the limit state, a kind of parabolic failure mechanism was established, according to the analysis of shied and bias tunnels.

However, the influences of the ground surface settlement and the layered character of soil on tunnels are not taken into account in above mentioned work. In this paper, the collapse mechanisms of shallow tunnel considering stochastic settlement in homogeneous soil and in layered soils are discussed with the upper bound theorem of limit analysis.

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## 2. Upper bound theorem and simplified stochastic medium theory

### 2.1 Upper bound theorem

The limit analysis upper bound theorem considers that the load calculated through equating the internal energy dissipation to external work rate is no less than the actual limit load if velocity boundary condition and compatibility condition of strain are satisfied (Chen 1975, Khezri *et al.* 2016, Lee 2016). Upper bound theorem can be expressed as

$$\int_V \sigma_{ij}\dot{\varepsilon}_{ij}\mathrm{d}V \geq \int_S T_i v_i \mathrm{d}S + \int_V F_i v_i \mathrm{d}V \tag{1}$$

in which $\sigma_{ij}$ is the stress tensor, $\dot{\varepsilon}_{ij}$ is the strain rate, $T_i$ is the limit load on the boundary surface, $S$ is the length of velocity discontinuity, $F_i$ is the body force of the mechanism such as the force caused by self-weight, $V$ is the volume of the plastic zone, and $v_i$ is the velocity along the velocity discontinuity surface.

The limit analysis theory is adopted to make analysis about the moment of limit state when the tunnel or slope collapses (Xu *et al.* 2018a). At this moment, the limit state is between two scenarios described above. Any further change for ground surface settlement, no matter how small, will cause the tunnel roof collapse. The assumption in this work is also adopted by other scholars and their works are published already.

From the standpoint of physical modelling experiments, Wu and Lee (2003) carried out a series of centrifuge model


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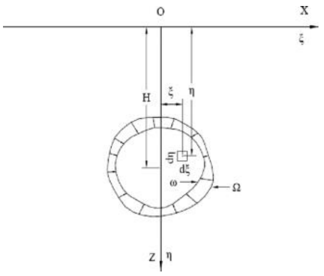



Fig. 1 Ground loss of single tunnel excavation

tests of unlined single and parallel tunnels in the plane strain condition to investigate the ground movement and the collapse mechanisms induced by tunneling. Based on this work and a large number of measured fields data, Osman (2010) put forward a simple plastic failure mechanism. Within this failure mode (in limit state), the settlement does not change any more, on the basis of work, Li and Yang (2018) used reliability method to introduce a new failure mechanism when the tunnel roof failures the settlement does not change.

### *2.2 Simplified stochastic medium theory*

The stochastic medium theory was proposed by Litwiniszyn (1975) and extended to predict the ground surface movement caused by tunneling and mining. According to the theory, the movement of stratum equals the sum of movement of stochastic medium elements induced by tunnel excavation. As shown in Fig. 1, assuming that every element collapse completely in the whole excavation zone Ω, the ground surface settlement can be obtained by applying superposition principle, which can be written as

$$W_{\Omega}(x)=\iint_{\Omega}\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \qquad (2)$$

The detail of derivation of settlement equation is in Appendix 1. Supposing that Ω is the initial cross section of tunnel, $\omega$ is the cross section after shrinkage and the radial convergence is $\Delta A=\Omega-\omega$, the ground surface settlement can be written as

$$W(x)=W_{\Omega}(x)-W_{\omega}(x)=\iint_{\Omega-\omega}\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \quad (3)$$

where $W(x)$ is the ground surface settlement and $\beta$ is the influence angle of ground settlement. Then, the movement of ground surface can be obtained when the parameters $\beta$ and $\Delta A$ re determined. Based on the theory, the ground surface settlement and transformation can be predicted considering the construction factors and formation conditions. The horizontal displacement of ground surface $U(x)$, the differential surface settlement $T(x)$, the horizontal deformation $E(x)$ and the curvature of surface settlement profile $K(x)$ can also be obtained as follows

$$U(x)=\iint_{\Omega-\omega}\frac{\left(x-\xi\right)\tan\beta}{\eta^2}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \quad (4)$$

$$\begin{aligned}T(x)&=\frac{\mathrm{d}W(x)}{\mathrm{d}x}\\&=\iint_{\Omega-\omega}\frac{-2\pi\tan^3\beta}{\eta^3}\left(x-\xi\right)\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta\end{aligned} \qquad (5)$$

$$\begin{aligned}E(x)&=\frac{\mathrm{d}U(x)}{\mathrm{d}x}\\&=\iint_{\Omega-\omega}\frac{\tan\beta}{\eta^2}\left[1-\frac{2\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta\end{aligned} \qquad (6)$$

$$\begin{aligned}K(x)&=\frac{\mathrm{d}^2W(x)}{\mathrm{d}x^2}\\&=\iint_{\Omega-\omega}\frac{2\pi\tan^3\beta}{\eta^3}\left[\frac{2\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2-1\right]\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta\end{aligned} \qquad (7)$$

From the equations mentioned above, it can be found obviously that the ground surface settlement, horizontal displacement, the differential surface settlement, horizontal deformation and the curvature of surface settlement profile can be obtained with the given boundary conditions. However, these functions cannot be integrated and numerical solution can only be got using calculation software. In order to obtain these values easily, the stochastic medium theory is simplified, and the simplified ground surface settlement functions can be expressed as follows. The total ground surface settlement for the uniform convergence displacement mode can be written as

$$W(x)\approx\frac{2\pi R\Delta A\tan\beta}{Z_0}\exp\left[-\frac{\pi\tan^2\beta}{Z_0^2}x^2\right] \qquad (8)$$

The total ground surface settlement for the non-uniform convergence displacement mode is expressed as

$$W(x)\approx\frac{\pi Rg\tan\beta}{Z_0}\exp\left[-\frac{\pi\tan^2\beta}{Z_0^2}x^2\right] \qquad (9)$$

where $g$ is the gap parameter, and the condition $g=2\Delta A$ should be satisfied. $Z_0$ is the distance between ground surface and the center of tunnel. It can also be noticed that the ground surface settlement for the uniform convergence displacement mode is the same as the settlement for the non-uniform convergence displacement mode.

## 3. Tunnel roof collapse with settlement

### *3.1 Roof collapse in homogeneous soil*

A large number of tests and theoretical studies have shown that geotechnical materials almost obey nonlinear failure criterion while the linear failure criterion is just a special case (Anyaegbunam 2015, Mohammadi and Tavakoli 2015, Xu *et al.* 2018b, Yang 2017, Yang *et al.* 2017, Yang and Yao 2018). Due to the limitation that the non-linear Hoek-Brown criterion is bias towards hard rock (Hoek and Brown 1997), the employment of the Power-law yield function would become the most suitable choice in this work (Yang 2018). In order to study the failure mechanism of tunnel roof, the Power-Law failure criterion is used in this paper. The Power-Law failure criterion of

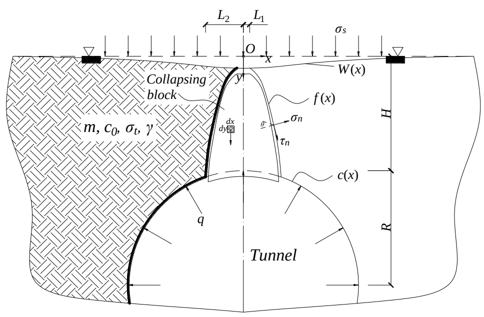



Fig. 2 Collapse of shallow tunnel under ground surface settlement in homogeneous soil

geotechnical material can be expressed as

$$\tau_n = c_0\left(1+\sigma_n/\sigma_t\right)^{1/m} \tag{10}$$

in which $c_0$ is initial cohesion, $\sigma_t$ is axial tensile stress, $m$ is nonlinear coefficient and these parameters can be determined by tests. The linear Mohr-Coulomb criterion could also be converted from Power-Law yield function if let nonlinear coefficient $m$=1.

The surface settlement mode put forward in this work is based on stochastic medium theory which assumes that every elemental excavation which occurs settlement is located at any point within tunnel region in any different soil layers. So the formulation describing settlement should not be affected by layered soils conditions. Furthermore the assumption that the layered soil condition does not affect the ground surface settlement is widely adopted by other scholars and their work is published.

Ou *et al.* (1993) studied the characteristics of ground surface settlement during excavation in layered sandy and clayey deposits in different fields, and proposed an empirical formula (without being effected by layered soils conditions) to predict the ground surface settlement profile in plane-strain conditions based on a lot of field observation data. In the work of Lee and Xiao (2001), a new transfer function is presented for analysis of the behavior of pile groups in multilayered soils. And the conclusion that settlement is not affected by layered soils could be concluded. Moormann (2004) found that the horizontal displacements of the retaining walls, the effect of a groundwater drawdown as well as of unloading have influences on the measured settlements result from different deformation mechanisms based on an extensive database of more than 530 current international case. Yang and Zhang (2017, 2018) put forward a new failure mechanism of shallow tunnel roof with considering surface settlement in layered soils. They assumed that the surface settlement obeying Guassian curve is not affected by layered soils conditions (Li and Yang 2017).

The failure mechanism of shallow tunnel is different from deep tunnel due to its failure mode extending to the ground surface (Fahimifar *et al.* 2015, Han and Liu 2016, Li *et al.* 2017). The failure mechanism of shallow tunnel in homogeneous soil considering stochastic settlement is established using upper bound theorem and variation method based on Power-Law failure criterion in this work, as shown in Fig. 2. The function $W(x)$ is the surface subsidence curve which is obtained using simplified stochastic medium theory, $f(x)$ is the expression of collapse block, $H$ is the distance between ground surface and tunnel roof and $R$ is tunnel radius. $L_1$ and $L_2$ are the half-width on the ground surface and the tunnel roof respectively. Assuming that the uniform convergence displacement mode is satisfied in this work, the expression of ground surface settlement curve is written as

$$W(x) = \frac{2\pi R\Delta A\tan\beta}{H+R}\exp\left[-\frac{\pi\tan^2\beta}{(H+R)^2}x^2\right] \tag{11}$$

in which $\Delta A$ is the radial convergence and $\beta$ is the influence angle of ground settlement. According to flow rule, the normal stress on the detaching surface can be derived as follows

$$\sigma_n = -\sigma_t + \sigma_t\left(\frac{m\sigma_t}{c_0}\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} \tag{12}$$

The derivation process of normal stress and energy dissipation density is illustrated in detail in Appendix 2. Based on the previous work (Fraldi and Guarracino 2009, Fraldi and Guarracino 2011, Zhang *et al.* 2010), the energy dissipation density of any point on the detaching surface results

$$\dot{D}_l = \sigma_n\dot{\varepsilon}_n + \tau_n\dot{\gamma}_n = \frac{v}{w}\left[1+f'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_t-(1-m)\sigma_t\left(\frac{c_0}{m\sigma_t}\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}}\right] \tag{13}$$

By considering the falling block to be symmetrical with respect to $y$-axis, the total energy dissipation can be obtained by integrating the unit energy dissipation along the right half velocity discontinuity. The total energy dissipation of right half collapse block results

$$\begin{aligned} D &= \int_{L_1}^{L_2}\dot{D}_l v\mathrm{d}s = \int_{L_1}^{L_2}\dot{D}_l v\cdot w\sqrt{1+f'(x)^2}\,\mathrm{d}x \\ &= \int_{L_1}^{L_2}\left[\sigma_t-(1-m)\sigma_t\left(c_0/m\sigma_t\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}}\right]v\mathrm{d}x \end{aligned} \tag{14}$$

The work rate generated by the weight of soil mass of the falling block can be expressed as

$$P_\gamma = v\int_0^{L_2}\gamma\cdot c(x)\mathrm{d}x - v\int_0^{L_1}\gamma\cdot W(x)\mathrm{d}x - v\int_{L_1}^{L_2}\gamma\cdot f(x)\mathrm{d}x \tag{15}$$

$\gamma$ being the weight per unit volume of soil mass and the function $c(x)$ describing the tunnel profile. The function of the tunnel profile can be written as

$$c(x) = (H+R) - \sqrt{R^2-x^2} \tag{16}$$

The power of the supporting force $q$ on the tunnel can be expressed as

$$P_q = -qR\arcsin\frac{L_2}{R}v \tag{17}$$

The work rate produced by the load on the ground surface is

$$P_{\sigma_s} = \sigma_s L_1 v \tag{18}$$

According to upper bound theorem of limit analysis, the upper bound solution closed to actual solution is the extremum one among all the feasible solutions determined by optimization calculation. Therefore, an objective function should be constructed in accordance with internal energy dissipation and external work rate

$$\begin{aligned}\zeta &= D - P_\gamma - P_q - P_{\sigma_s} \\ &= \int_{L_1}^{L_2} v\Lambda\left[f(x), f'(x), x\right]\mathrm{d}x - v\int_0^{L_2}\gamma\cdot c(x)\mathrm{d}x + v\int_0^{L_1}\gamma\cdot W(x)\mathrm{d}x + qR\arcsin\frac{L_2}{R}v - \sigma_s L_1 v\end{aligned} \tag{19}$$

in which the expression of $\Lambda\left[f(x), f'(x), x\right]$ can be written as follows

$$\Lambda\left[f(x), f'(x), x\right] = \sigma_t - (1-m)\sigma_t\left(c_0/m\sigma_t\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} + \gamma f(x) \tag{20}$$

It is obviously found from Eq. (19) that the extremum of the objective function $\xi$ is completely determined by the function $\Lambda\left[f(x), f'(x), x\right]$. Then, the problem transforms into a classical question of the calculus of variations, i.e., to find the extreme value of fonctionelle $\Lambda\left[f(x), f'(x), x\right]$ so as to obtain the upper bound solution of collapse mechanism. According to variation principle, the Euler-Lagrange equation for the function $\Lambda\left[f(x), f'(x), x\right]$ results

$$\frac{\partial\Lambda}{\partial x} - \frac{\partial}{\partial x}\left[\frac{\partial\Lambda}{\partial f'(x)}\right] = 0 \tag{21}$$

By substituting Eq. (20) into Eq. (21), a homogeneous second-order differential equation with constant coefficients can be obtained as follows

$$\gamma - \frac{m}{m-1}\sigma_t\left(\frac{c_0}{m\sigma_t}\right)^{\frac{m}{m-1}} f'(x)^{\frac{2-m}{m-1}} f''(x) = 0 \tag{22}$$

The function of detaching surface can be got through integrating the Eq. (22). The expression of collapse mechanism of tunnel can be written as

$$f(x) = k\left(x - \frac{a_0}{\gamma}\right)^m + a_1, \quad k = \sigma_t c_0^{-m}\gamma^{m-1} \tag{23}$$

$a_0$ and $a_1$ being two constants to be determined by boundary conditions. A geometrical relationship can be found from Fig. 2, that is

$$f(x = L_1) = W(x = L_1) \tag{24}$$

As there is no distribution of shear stress on the ground surface, the following equation is obtained

$$\tau_{xy}(x = L_1, y = W(L_1)) = 0 \tag{25}$$

Then, $a_0 = \gamma L_1$ and $a_1 = W(L_1)$. The final expression of falling block of shallow tunnel can be written as

$$f(x) = k\left(x - L_1\right)^m + W(L_1) \tag{26}$$

Hence, the concrete expression of function $\xi$ can be expressed through calculation. According to the upper bound theorem, the upper bound solution can be obtained through equating the energy dissipation to the external power, results

$$\begin{aligned}\zeta &= [\sigma_t + \gamma W(L_1)](L_2 - L_1) + \frac{m}{m+1}\sigma_t\left(\frac{\gamma}{c_0}\right)^m (L_2 - L_1)^{m+1} \\ &\quad - \gamma\left[(H+R)L_2 - \frac{L_2}{2}\sqrt{R^2 - L_2^2} - \frac{R^2}{2}\arcsin\frac{L_2}{R}\right] + \gamma\int_0^{L_1} W(x)\mathrm{d}x + qR\arcsin\frac{L_2}{R} - \sigma_s L_1 = 0\end{aligned} \tag{27}$$

It can be found that the concrete failure mechanism of tunnel roof can be obtained as long as the half-width on the ground surface $L_1$ and half-width on the tunnel roof $L_2$. However, the values of $L_1$ and $L_2$ cannot be got from Eq. (27). According to Fig. 2, a relationship between $L_1$ and $L_2$ can be obtained as follows

$$f(L_2) = c(L_2) \Leftrightarrow k\left(L_2 - L_1\right)^m + W(L_1) = (H+R) - \sqrt{R^2 - L_2^2} \tag{28}$$

By combining Eq. (27) with Eq. (28), a system of nonlinear equations about $L_1$ and $L_2$ can be obtained and the values of $L_1$ and $L_2$ can also be obtained. It is noticed that in Eq. (27) cannot be integrated for the simplest form. Then, coding the computer program, the numerical solution of this integral can be obtained and the values of $L_1$ and $L_2$ can be obtained afterwards. The concrete expression of failure mechanism of shallow tunnel subjected to surface settlement can be affirmed as long as the values of $L_1$ and $L_2$ are ensured and substituted into Eq. (26), and the shape of collapse block can be drawn afterwards.

It is also worthy noticing that the roof of collapse block extends to the ground surface exactly when $L_1$=0 is satisfied. Then, the critical height $H_{cr}$ can be obtained by combing Eq. (27) with Eq. (28). And this failure mechanism of shallow tunnel is considered valid when the condition $H \le H_{cr}$ is satisfied.

### *3.2 Roof collapse in layered soils*

In practical engineering, the soil mass is frequently not homogeneous, so the collapse mechanism of shallow tunnel in layered soils is established in this section. As shown in Fig. 3, it is assumed that the failure mechanism is symmetrical with respect to $y$-axis and the curve of falling block is smooth. The function $W_1(x)$ is the surface subsidence curve which is obtained simplified stochastic medium theory and the curve of falling block is made up of two functions $f_1(x)$ and $f_2(x)$. $h_1$ is the height between ground surface and the layered position and $h_2$ is the height between the layered position and the top of tunnel. $L_3$, $L_4$ and $L_5$ are the half-width of the falling block on the ground surface, the layered position and the tunnel roof

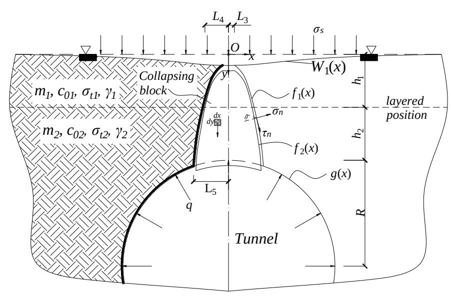



Fig. 3 Collapse of shallow tunnel under ground surface settlement in layered soils

respectively. The 1 and 2 in the subscript of soil's parameters $c_0$, $\sigma_t$, $m$ and $\gamma$ represent the upper soil and lower soil respectively.

The expression of ground surface settlement curve shown in Fig. 3 is expressed as follows

$$W_1(x)=\frac{2\pi R\Delta A\tan\beta}{h_1+h_2+R}\exp\left[-\frac{\pi\tan^2\beta}{\left(h_1+h_2+R\right)^2}x^2\right] \tag{29}$$

Through calculating, the dissipation densities of the internal forces on the detaching surface in two layers result respectively

$$\dot{D}_{i1}=\sigma_{n1}\dot{\varepsilon}_{n1}+\tau_{n1}\dot{\gamma}_{n1}=\frac{v}{w}\left[1+f_1'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_{t1}-(1-m_1)\sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{m_1-1}}f_1'(x)^{\frac{m_1}{m_1-1}}\right] \tag{30}$$

$$\dot{D}_{i2}=\sigma_{n2}\dot{\varepsilon}_{n2}+\tau_{n2}\dot{\gamma}_{n2}=\frac{v}{w}\left[1+f_2'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_{t2}-(1-m_2)\sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{m_2-1}}f_2'(x)^{\frac{m_2}{m_2-1}}\right] \tag{31}$$

Thus the total energy dissipation can be obtain by integrating the $\dot{D}_{i1}$ and $\dot{D}_{i2}$ along the velocity discontinuity

$$\begin{aligned}D=&\int_{L_3}^{L_4}[\sigma_{t1}-(1-m_1)\sigma_{t1}(c_{01}/m_1\sigma_{t1})^{\frac{m_1}{m_1-1}}f_1'(x)^{\frac{m_1}{m_1-1}}]v\mathrm{d}x\\&+\int_{L_4}^{L_5}[\sigma_{t2}-(1-m_2)\sigma_{t2}(c_{02}/m_2\sigma_{t2})^{\frac{m_2}{m_2-1}}f_2'(x)^{\frac{m_2}{m_2-1}}]v\mathrm{d}x\end{aligned} \tag{32}$$

For the tunnel in layered soils, the work rate produced by self-weight is expressed as

$$P_\gamma=v\int_0^{L_5}\gamma_2 g(x)\mathrm{d}x-v\int_{L_3}^{L_4}\gamma_1 f_1(x)\mathrm{d}x-v\int_{L_4}^{L_5}\gamma_2 f_2(x)\mathrm{d}x-v\int_0^{L_3}\gamma_1 W_1(x)\mathrm{d}x+(\gamma_1-\gamma_2)h_1L_4 \tag{33}$$

where the expression of circular tunnel is written as follows

$$g(x)=\left(h_1+h_2+R\right)-\sqrt{R^2-x^2} \tag{34}$$

The powers of the supporting force on the tunnel and the load on the ground surface result respectively

$$P_q=-qR\arcsin\frac{L_5}{R}v \tag{35}$$

$$P_{\sigma_s}=\sigma_s L_3 v \tag{36}$$

In order to obtain the upper bound solution, an objective function should be established using the energy dissipation and power of external forces, results

$$\begin{aligned}\psi&=D-P_\gamma-P_q-P_{\sigma_s}\\&=\psi_1+\psi_2-v\int_0^{L_5}\gamma_2 g(x)\mathrm{d}x+v\int_0^{L_3}\gamma_1W_1(x)\mathrm{d}x-(\gamma_1-\gamma_2)h_1L_4+qR\arcsin\frac{L_5}{R}v-\sigma_sL_3v\end{aligned} \tag{37}$$

where

$$\begin{aligned}\psi_1&=\int_{L_3}^{L_4}v\Lambda_1\left[f_1(x),f_1'(x),x\right]\mathrm{d}x\\&=\int_{L_3}^{L_4}v\left[\sigma_{t1}-(1-m_1)\sigma_{t1}(c_{01}/m_1\sigma_{t1})^{\frac{m_1}{m_1-1}}f_1'(x)^{\frac{m_1}{m_1-1}}+\gamma_1f_1(x)\right]\mathrm{d}x\end{aligned} \tag{38}$$

$$\begin{aligned}\psi_2&=\int_{L_4}^{L_5}v\Lambda_2\left[f_2(x),f_2'(x),x\right]\mathrm{d}x\\&=\int_{L_4}^{L_5}v\left[\sigma_{t2}-(1-m_2)\sigma_{t2}(c_{02}/m_2\sigma_{t2})^{\frac{m_2}{m_2-1}}f_2'(x)^{\frac{m_2}{m_2-1}}+\gamma_2f_2(x)\right]\mathrm{d}x\end{aligned} \tag{39}$$

The problem also transforms into a typical calculus of variations, i.e., to find two functions, $y=f_1(x)$ and $y=f_2(x)$ when the extremum of objective function $\psi$ is obtained. It can be found from Eq. (37) that the extremum of $\psi$ is determined completely by functions $\psi_1$ and $\psi_2$. Thus, it is assumed that the extremum of objective function $\psi$ can be obtained when the extreme values of two functions $\psi_1$ and $\psi_2$ are obtained simultaneously. According to variation principle, the Euler-Lagrange equations for the functions $\Lambda_1\left[f_1(x),f_1'(x),x\right]$ and $\Lambda_2\left[f_2(x),f_2'(x),x\right]$ result

$$\frac{\partial\Lambda_1}{\partial f_1(x)}-\frac{\partial}{\partial x}\left[\frac{\partial\Lambda_1}{\partial f_1'(x)}\right]=0\Leftrightarrow\gamma_1-\frac{m_1}{m_1-1}\sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{m_1-1}}f_1'(x)^{\frac{2-m_1}{m_1-1}}f_1''(x)=0 \tag{40}$$

$$\frac{\partial\Lambda_2}{\partial f_2(x)}-\frac{\partial}{\partial x}\left[\frac{\partial\Lambda_2}{\partial f_2'(x)}\right]=0\Leftrightarrow\gamma_2-\frac{m_2}{m_2-1}\sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{m_2-1}}f_2'(x)^{\frac{2-m_2}{m_2-1}}f_2''(x)=0 \tag{41}$$

The expressions of the detaching curve $f_1(x)$ and $f_2(x)$ can be derived by integral calculation

$$f_1(x)=k_1\left(x-\frac{a_2}{\gamma_1}\right)^{m_1}+a_3,\quad k_1=\frac{\sigma_{t1}}{\gamma_1}\left(\frac{\gamma_1}{c_{01}}\right)^{m_1} \tag{42}$$

$$f_2(x)=k_2\left(x-\frac{a_4}{\gamma_2}\right)^{m_2}+a_5,\quad k_2=\frac{\sigma_{t2}}{\gamma_2}\left(\frac{\gamma_2}{c_{02}}\right)^{m_2} \tag{43}$$

$a_2$, $a_3$, $a_4$ and $a_5$ being constants which can be determined by boundary conditions. A geometrical relationship can be found from Fig. 3, that is

$$f_1(x=L_3)=W_1(x=L_3) \tag{44}$$

As there is also no distribution of shear stress on the ground surface, the following equation is obtained

$$\tau_{xy}(x=L_3,y=W_1(L_3))=0 \tag{45}$$

The expressions of $a_2$ and $a_3$ can be determined, $a_2=\gamma_1L_3$ and $a_3=W_1(L_3)$. Thus, the concrete expression of falling block in upper soil can be written as follows

$$f_1(x)=k_1\left(x-L_3\right)^{m_1}+W_1(L_3) \tag{46}$$

When the curve of collapse block is smooth, the equations, $f_1'(x=L_4)=f_2'(x=L_4)$ is satisfied. Another geometrical relationship $f_2(x=L_4)=h_1$ can also be found from Fig. 3. Thus, the expressions of $a_4$ and $a_5$ can be obtained, $a_4=\gamma_2(L_4-Z)$ and $a_5=h_1-k_2Z^{m_2}$ in which

$$Z=\left(\frac{m_1k_1}{m_2k_2}\right)^{\frac{1}{m_2-1}}\left(L_4-L_3\right)^{\frac{m_1-1}{m_2-1}} \tag{47}$$

The concrete function of collapse block in lower soil results

$$f_2(x)=k_2\left(x-L_4+Z\right)^{m_2}+(h_1-k_2Z^{m_2}) \tag{48}$$

The concrete expression of objective function $\psi$ can be obtained through integral. To get the upper bound solution, an equation can be obtained by equating the internal energy dissipation to the external work rate, in other words equating the function $\psi$ to zero, that is

$$\begin{aligned}\xi &= \left[\sigma_{t1} + \gamma_1 W_1(L_3)\right](L_4 - L_3) + \frac{m_1}{m_1+1}\sigma_{t1}\left(\frac{\gamma_1}{c_{01}}\right)^{m_1}(L_4 - L_3)^{m_1+1} \\ &+ \left[\sigma_{t2} + \gamma_2(h_1 - k_2 Z^{m_2})\right](L_5 - L_4) + \frac{m_2}{m_2+1}\sigma_{t2}\left(\frac{\gamma_2}{c_{02}}\right)^{m_2}\left[(L_5 - L_4 + Z)^{m_2+1} - Z^{m_2+1}\right] \\ &- \gamma_2\left[(h_1 + h_2 + R)L_5 - \frac{L_5}{2}\sqrt{R^2 - L_5^2} - \frac{R^2}{2}\arcsin\frac{L_5}{R}\right] - (\gamma_1 - \gamma_2)h_1 L_4 \\ &+ \gamma_1\int_0^{L_3} W_1(x)\mathrm{d}x + qR\arcsin\frac{L_5}{R} - \sigma_s L_3 = 0\end{aligned} \quad (49)$$

It can also be found that the failure mechanism of shallow tunnel can be determined as long as the values of $L_3$, $L_4$ and $L_5$. According to Fig. 3, two geometrical conditions are still left, which can be expressed as

$$k_2\left(L_5 - L_4 + Z\right)^{m_2} + (h_1 - k_2 Z^{m_2}) = \left(h_1 + h_2 + R\right) - \sqrt{R^2 - L_5^2} \quad (50)$$

$$k_1\left(L_4 - L_3\right)^{m_1} + W_1(L_3) = h_1 \quad (51)$$

Combining Eqs. (49)-(51), the values of $L_3$, $L_4$ and $L_5$ can be solved by using software. The integral $\int_0^{L_3} W_1(x)\mathrm{d}x$ in Eq. (49) cannot also be integrated for the simplest form, so the numerical solution of this integral is also calculated, and is substituted into the system of equation. Based on $L_3$, $L_4$ and $L_5$, the final forms of detaching curves $f_1(x)$ and $f_2(x)$ are obtained, and the shape of failure surface is drawn by Eq. (46) and Eq. (48).

## 4. Numerical results and discussions

### *4.1 Influence of parameters on collapse mechanism in homogeneous soil*

It can be noticed through Eq. (11) or Eq. (29) that the surface settlement curve is just related to the tunnel radius, buried depth, section convergence value and the influence angle of ground settlement when the movement of ground surface is estimated using simplified stochastic medium theory. Fig. 4 illustrates the settlement curve $W(x)$ (or $W_1(x)$) in the scope of 15 m on each side of tunnel central line corresponding to $R$=5 m, $H$=5 m (or $h_1$+$h_2$=5 m), $\Delta A$=20 mm and tan$\beta$=0.9. In other word, the values of surface settlement will not be impact by the parameters $c_0$, $\sigma_t$, $m$ and $\gamma$.

#### *4.1.1 Influence of initial cohesion*

In order to investigate the influence of initial cohesion $c_0$ on potential collapse block, Fig. 5 illustrates the effects of the initial cohesion $c_0$ on the range of falling block corresponding to $\sigma_t$=30 kPa, $m$=1.3, $\sigma_s$=10 kPa, $R$=5 m, $H$=5 m, $q$=50 kPa, $\gamma$=22 kN/m$^3$, $\Delta A$=20 mm and tan$\beta$=0.9 with initial cohesion $c_0$ varying from 6 kPa to 14 kPa. It can be seen from Fig. 5 that the initial cohesion $c_0$ has a significant influence on the failure mechanism of tunnel roof in homogeneous soil. When the other parameters remain constant, the potential collapse range of circular tunnel increases with the increase of initial cohesion $c_0$.

#### *4.1.2 Influence of nonlinear coefficient*

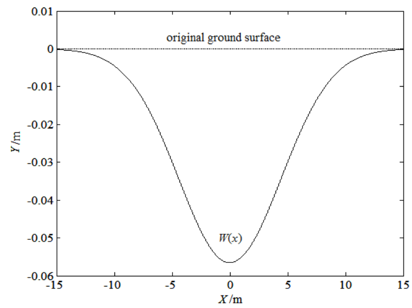



Fig. 4 Ground surface settlement curves with constant R, H, $\Delta A$ and tan$\beta$

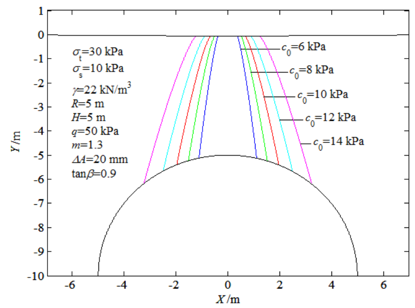



Fig. 5 Effects of initial cohesion $c_0$ on failure mechanisms

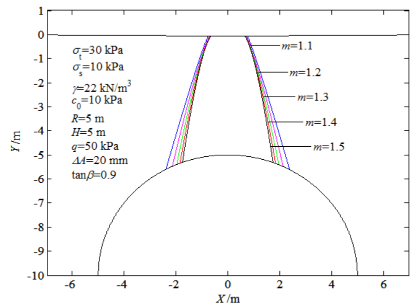



Fig. 6 Effects of nonlinear coefficient $m$ on failure mechanisms

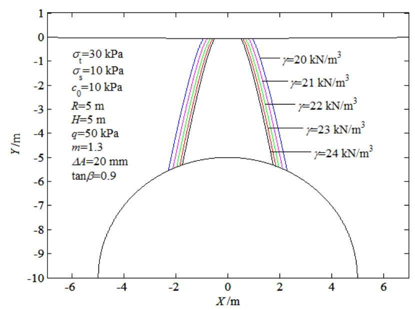
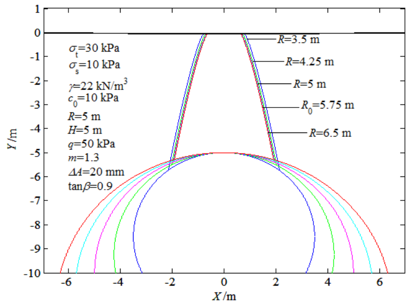
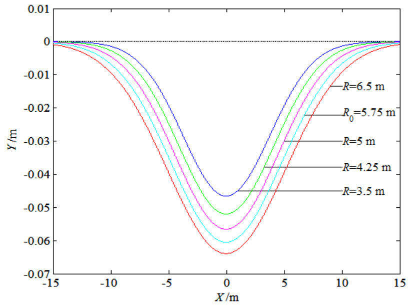



Fig. 7 Effects of unit weight $\gamma$ on failure mechanisms

To investigate how the collapse mechanism of shallow tunnel roof in homogeneous soil is influenced by nonlinear coefficient $m$, the failure surfaces of shallow tunnel corresponding to $\sigma_t$=30 kPa, $c_0$=10 kPa, $\sigma_s$=10 kPa, $R$=5 m, $H$=5 m, $q$=50 kPa, $m$=1.3, $\Delta A$=20 mm and tan$\beta$=0.9 with nonlinear coefficient $m$ varying from 1.1 to 1.5 are plotted in Fig. 6. It can be observed that the proportion of potential collapse block decreases with the nonlinear coefficient $m$ increasing when other parameters remain constant.

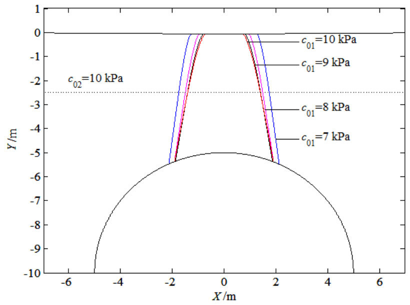



(a) Potential collapse blocks with varying $R$

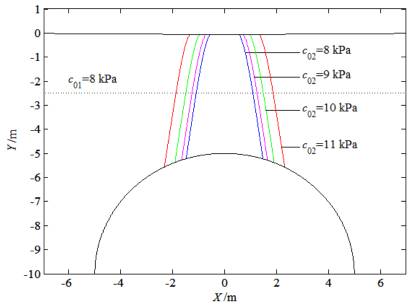



(b) Ground surface settlement curves

Fig. 8 Effects of tunnel radius $R$ on failure mechanisms




(a) $c_{02}$=10 kPa

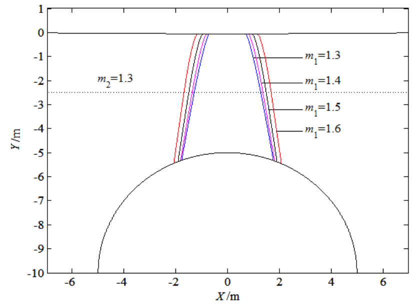



(b) $c_{01}$=8 kPa

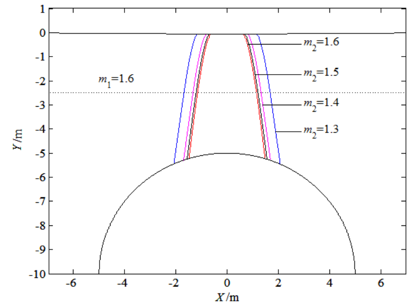
Fig. 9 Effects of initial cohesions on failure mechanisms

*4.1.3 Influence of unit weight*

Fig. 7 represents the influence of unit weight $\gamma$ of soil mass on the collapse mechanism corresponding to $\sigma_t$=30 kPa, $c_0$=10 kPa, $\sigma_s$=10 kPa, $R$=5 m, $H$=5 m, $q$=50 kPa, $\gamma$=22 kN/m$^3$, $\Delta A$=20 mm and tan$\beta$=0.9 with unit weight $\gamma$ varying from 20 kN/m$^3$ to 24 kN/m$^3$. It can be seen from Fig. 7 that the proportion of collapse block will decrease with the unit weight of the soil increase. From the perspective of energy analysis, this means that in order to set the equation $\zeta$=0 to be satisfied, the size of collapse block will be large when the values of unit weights is low.




(a) $m_2$=1.3




(b) $m_1$=1.5

Fig. 10 Effects of nonlinear coefficients on failure mechanisms

*4.1.4 Influence of tunnel radius*

The failure surfaces corresponding to $\sigma_t$=30 kPa, $c_0$=10 kPa, $\sigma_s$=10 kPa, $\gamma$=22 kN/m$^3$, $H$=5 m, $q$=50 kPa, $m$=1.3, $\Delta A$=20 mm and tan$\beta$=0.9 with tunnel radius $R$ varying from 3.5 m to 6.5 m are plotted so as to analyze the influence of tunnel radius $R$, as shown in Fig. 8(a). Fig. 8(b) illustrates the settlement curves with tunnel radius changing. It can be observed that the values of ground settlement increase with the radius increasing. However, the potential collapse block decreases with the tunnel radius $R$ increasing when other parameters remain constant.

*4.2 Influence of parameters on collapse mechanism in layered soils*

In order to study the influence of initial cohesions on the failure mechanism, the detaching surfaces corresponding to $\sigma_{t1}$=$\sigma_{t2}$=30 kPa, $m_1$=1.5, $m_2$=1.3, $\gamma_1$=21 kN/m$^3$, $\gamma_2$=23 kN/m$^3$, $\sigma_s$=10 kPa, $q$=50 kPa, $h_1$=$h_2$=2.5 m, $\Delta A$=20 mm and tan$\beta$=0.9 are plotted in Fig. 9 with varying $c_{01}$ and $c_{02}$. It can be observed through Fig. 9 that the possible collapse block decreases with $c_{01}$ and the potential falling block increases with only $c_{02}$ increasing.

In order to investigate how nonlinear coefficients influence the failure mechanism of shallow tunnel, the falling blocks corresponding to $\sigma_{t1}$=$\sigma_{t2}$=30 kPa, $c_{01}$=8 kPa, $c_{02}$=10 kPa, $\gamma_1$=21 kN/m$^3$, $\gamma_2$=23 kN/m$^3$, $\sigma_s$=10 kPa, $q$=50 kPa, $h_1$=$h_2$=2.5 m, $\Delta A$=20 mm and tan$\beta$=0.9 are plotted in Fig. 10 with varying $m_1$ and $m_2$. It can be seen from Fig.10 that the possible collapse block increases with $m_1$ increasing or $m_2$ decreasing when other parameters remain constant.

**5. Conclusions**

Considering the influence of the ground surface settlement, the failure mechanism of shallow tunnel is proposed with nonlinear failure criterion and limit analysis method. The influence of the ground surface settlement is considered using simplified stochastic medium theory. The expressions of falling block in homogeneous media and in layered media are derived with variation principle. Through calculation analysis, it is found that the initial cohesion, nonlinear coefficient and unit weight have an important effect on collapse scope.

As to the tunnel in homogeneous soil, the potential collapse block increases with the increase of initial cohesion, and decreases with the increase of nonlinear coefficient and unit weight. With the tunnel radius increasing, the ground surface settlement increases but the possible falling block decreases.

As to the tunnel in layered stratum, the possible failure mode of shallow tunnel is made up of two equations $y=f_1(x)$ and $y=f_2(x)$. The failure mechanism of shallow tunnel is obtained with boundary conditions and limit analysis method. Through calculation analysis, the initial cohesions and nonlinear coefficients have significant influence on the possible range of collapse block. The potential collapse block decreases with the initial cohesion $c_{01}$ and nonlinear coefficient $m_2$ increasing. The possible collapse block increases with the initial cohesion $c_{02}$ and nonlinear coefficient $m_1$ increasing.

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## Appendix 1: Derivation of the settlement equation

It is assumed that an underground excavation can be divided into infinitesimal excavation can be divided into infinitesimal excavation elements and that the ground movement induced by the excavation equals the sum of the movement due to each elemental excavation. The elemental excavation has dimensions of by $d\xi$ by $d\zeta$ by $d\eta$. According to Liu (1993), based on the stochastic medium theory of Litwiniszyn (1957), the ground surface settlement at a point $(x,y)$, $w_e(x,y)$, due to an elemental excavation can be expressed below

$$w_e(x,\ y)=\frac{1}{r^2(z)}\exp\left[-\frac{\pi}{r^2(z)}\left(x^2+y^2\right)\right]\mathrm{d}\xi\mathrm{d}\zeta\mathrm{d}\eta \tag{52}$$

in which $r(z)$= radius of influence zone=z/tan$\beta$. $\beta$=angle of influence zone of ground settlement.

For a long excavation with the excavation axis in $y$ direction, the ground settlement profile normal to $y$ axis, $w_e(x)$, can be obtained by integrating $d\zeta$ along axis $\zeta$. The resulting equation is as follows

$$w_e(x)=\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{53}$$

In the analysis of horizontal displacement, the soil is assumed to be incompressible. This requires that the sum of normal strains in $x$, $y$, and $z$ directions must be equal to zero. Meanwhile, for a long excavation with a plane strain condition, the strain in $y$ direction equals zero. Thus

$$\frac{\partial u_e(x)}{\partial x}+\frac{\partial w_e(x)}{\partial z}=0 \tag{54}$$

in which, $u_e(x)$ and $w_e(x)$ are displacements in $x$ and $z$ directions, respectively. Solving for $u_e(x)$ with a boundary condition of $u_e(x)=0$ at $x\to 8$ yields the following equation

$$u_e(x)=\frac{x\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{55}$$

Both Eqs. (54) and (55) are the basic formulae for developing the equations for ground surface movement due to tunnelling.

The preceding formulation is applicable to tunnels with different shapes of cross section. As before, the tunnel is assumed to be composed of infinitesimal elements of excavation, each having dimensions of $d\xi$ by $d\eta$. The elemental excavation is located at $(\xi,\eta)$ within the tunnel region $\Omega$. Based on Eq. (2), the ground surface settlement, $w(x)$, due to tunnel excavation can be expressed below

$$w(x)=\iint_{\Omega}\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{56}$$

Ground surface movements depend on the nature and extent of convergence over the cross section of the working. The ground settlement induced by the convergence from $\Omega$ to $\omega$ can be determined from the difference in settlement due to region $\Omega$ and that due to region $\omega$ as shown below

$$w(x)=w_{\Omega}(x)-w_{\omega}(w)=\iint_{\Omega-\omega}\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{57}$$

The tunnel cross section has a maximum height of $2B$, and a maximum width of $2A$. Its center is located at $H$ below the ground surface. Assume that the amount of radial convergence from $\Omega$ to $\omega$ equals $\Delta A$ the ground settlement, $w(x)$, can be expressed as follows

$$w(x)=\int_a^b\int_c^d\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta-\int_e^f\int_g^h\frac{\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{58}$$

in which

$$a=H-B,\ b=H+B,\ c=-A\sqrt{1-\left(\frac{H-\eta}{B}\right)^2},$$
$$d=-ce=H-\left(B-\Delta A\right),\ f=H+\left(B-\Delta A\right),$$
$$g=-\left(A-\Delta A\right)\sqrt{1-\left(\frac{H-\eta}{B-\Delta A}\right)^2},\ h=-g$$

For a circular tunnel, $A$=$B$, with a radial convergence of$\Delta A$, the ground settlement, $w(x)$, can be computed using Eq. (58) together with the following limits of integration

$$a=H-A,\ b=H+A,\ c=-\sqrt{A^2-\left(H-\eta\right)^2},$$
$$d=-ce=H-\left(A-\Delta A\right),\ f=H+\left(A-\Delta A\right),$$
$$g=-\sqrt{\left(A-\Delta A\right)^2-\left(H-\eta\right)^2},\ h=-g$$

By following the same principle and reasoning, the horizontal displacement of ground surface, $u(x)$, due to convergence from $\Omega$ to $w$ can be expressed as

$$u(x)=u_{\Omega}(x)-u_{\omega}(w)=\iint_{\Omega-\omega}\frac{\left(x-\xi\right)\tan\beta}{\eta}\exp\left[-\frac{\pi\tan^2\beta}{\eta^2}\left(x-\xi\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta \tag{59}$$

For the determination of ground surface settlement and deformation, a computer program was developed by Yang *et al.* (2017) to solve the differential equations. The program adopted the Gaussian numerical integration method. This program is used to perform further analysis presented below.

## Appendix 2: Derivation of the normal stress and internal energy dissipation

In the appendix, the derivation of the dissipation density of a random point on the failure surface, Eq. (13), is shown in detail. By assuming the plastic potential, $\zeta$, the plastic strain rate can be written as follows

$$\dot{\varepsilon}_n = \lambda\frac{\partial\zeta}{\partial\sigma} = -\lambda\frac{c_0}{m\sigma_t}\left(1+\sigma_n/\sigma_t\right)^{\frac{1-m}{m}} \qquad \dot{\gamma}_n = \lambda\frac{\partial\zeta}{\partial\tau} = \lambda \tag{60}$$

where $\lambda$ is a scalar parameter, $\dot{\varepsilon}_n$ is normal plastic strain rate and $\dot{\gamma}_n$ is shear plastic strain rate. The plastic strain rate components can be written in the form

$$\dot{\varepsilon}_n = -\frac{v}{w}\left[1+f'(x)^2\right]^{-\frac{1}{2}} \qquad \dot{\gamma}_n = \frac{v}{w}f'(x)\left[1+f'(x)^2\right]^{-\frac{1}{2}} \tag{61}$$

where $f(x)$ is the function of velocity discontinuity surface and $f'(x)$ is the first derivative of $f(x)$. $v$ is the velocity of the failure block. A dot denotes differentiation with respect to time and a prime with respect to $x$, i.e., $v=\partial u/\partial t$, $f'(x)=\partial f(x)/\partial x$.

In order to enforce compatibility, from Eqs.(60) and (61) it follows

$$\lambda = \frac{v}{w}f'(x)\left[1+f'(x)^2\right]^{-\frac{1}{2}} \tag{62}$$

Based on Eq. (62) and considering the equal of $\dot{\varepsilon}_n$, the normal component of stress can be written as

$$\sigma_n = -\sigma_t + \sigma_t\left(\frac{m\sigma_t}{c_0}\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} \tag{63}$$

Substituting Eq. (63) into Eq. (10), the shear stress can be written as

$$\tau_n = -c_0\left(\frac{m\sigma_t}{c_0}\right)^{\frac{1}{1-m}} f'(x)^{\frac{1}{m-1}} \tag{64}$$

So that, by virtue of the Greenberg minimum principle, the effective collapse mechanism can be found by minimizing the total dissipation, the dissipation density of the internal forces on the detaching surface, $\dot{D}_i$, results

$$\dot{D}_i = \sigma_n\dot{\varepsilon}_n + \tau_n\dot{\gamma}_n = \frac{v}{w}\left[1+f'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_t - (1-m)\sigma_t\left(\frac{c_0}{m\sigma_t}\right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}}\right] \tag{65}$$

It can be concluded the normal stress of any point on the velocity discontinuity both in upper and lower soil layers are

$$\sigma_{n1} = -\sigma_{t1} + \sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}} f_1'(x)^{\frac{m_1}{(m_1-1)}} \qquad \sigma_{n2} = -\sigma_{t2} + \sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}} f_2'(x)^{\frac{m_2}{(m_2-1)}} \tag{66}$$

During the process of the impending collapse, the dissipation densities of the internal forces on the detaching surface, $\dot{D}_{i1}$ and $\dot{D}_{i2}$, are

$$\dot{D}_{i1} = \sigma_{n1}\dot{\varepsilon}_{n1} + \tau_{n1}\dot{\gamma}_{n1} = \frac{v}{w}\left[1+f_1'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_{t1} - (1-m_1)\sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{m_1-1}} f_1'(x)^{\frac{m_1}{m_1-1}}\right]$$
$$\dot{D}_{i2} = \sigma_{n2}\dot{\varepsilon}_{n2} + \tau_{n2}\dot{\gamma}_{n2} = \frac{v}{w}\left[1+f_2'(x)^2\right]^{-\frac{1}{2}}\left[\sigma_{t2} - (1-m_2)\sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{m_2-1}} f_2'(x)^{\frac{m_2}{m_2-1}}\right] \tag{67}$$

The 1 and 2 in the subscript of soil's parameters $c_0$, $\sigma_t$, $m$ and $\gamma$ represent the upper soil and lower soil respectively.