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Free strain analysis of the performance of vertical drains for soft soil improvement

Sudip Basack[1a] and Sanjay Nimbalkar[*2]

[1]*Centre for Geomechanics and Railway Engineering, School of Civil, Mining and Environmental Engineering, University of Wollongong NSW 2522, Australia*
[2]*School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology, University of Technology Sydney NSW 2007, Australia*

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Abstract. Improvement of soft clay deposit by preloading with vertical drains is one of the most popular techniques followed worldwide. These drains accelerate the rate of consolidation by shortening the drainage path. Although the analytical and numerical solutions available are mostly based on equal strain hypothesis, the adoption of free strain analysis is more realistic because of the flexible nature of the imposed surcharge loading, especially for the embankment loading used for transport infrastructure. In this paper, a numerical model has been developed based on free strain hypothesis for understanding the behaviour of soft ground improvement by vertical drain with preloading. The unit cell analogy is used and the effect of smear has been incorporated. The model has been validated by comparing with available field test results and thereafter, a hypothetical case study is done using the available field data for soft clay deposit existing in the eastern part of Australia and important conclusions are drawn therefrom.

Keywords: finite difference method; foundation settlement; soft soil; vertical drain

1. Introduction

Reducing long-term settlement of infrastructure and providing cost-effective foundations with sufficient load-bearing capacities are national priorities for infrastructural development in most countries. Many estuarine plains and coastal belts in Australia consist of soft alluvial and marine clay deposits up to significant depths, having very low bearing capacity, low permeability and high compressibility (Fatahi *et al*. 2012, Indraratna *et al*. 2013a, 2014). The soft soil foundations cause excessive settlement and can initiate undrained failure of infrastructure if proper ground improvement is not carried out (Nimbalkar *et al*. 2012, Nimbalkar and Indraratna 2016). In order to improve the strength and stiffness of clayey soils, prefabricated vertical drains (PVDs), combined with preloading, are often used in practice (Holtz 1987, Bergado *et al*. 1993).

PVDs are installed in rectangular or triangular patterns with spacing varying from about 1.0 m to 3.0 m (Holtz 1987). The installation of PVDs creates a disturbed zone, which is also known as

*Corresponding author, Ph.D., E-mail: Sanjay.Nimbalkar@uts.edu.au
[a]Ph.D., E-mail: basackdrs@hotmail.com

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the smear zone, around the PVD in which the strength and the lateral coefficient of hydraulic conductivity decrease, leading to further delays in the consolidation process (Basu *et al*. 2007, Chai and Xu 2015). Predictions of improved properties of soft soil as well as the extent of the smear zone are extremely important for predicting the bearing capacity and settlement of the soft soil. The post construction behaviour of foundation reinforced with artificial inclusions depends on the improved properties of the soft soil, especially for settlement prediction (Yildiz and Uysal 2015, Ellouze *et al*. 2017).

Most of the available analytical and numerical methods in the relevant area (e.g., Barron 1948, Yoshikuni and Nakanodo 1974, Hansbo, 1981, Indraratna *et al.* 1994, 2013b, Leo 2004, Walker and Indraratna 2006, Yu *et al*. 2007, Mirjalili *et al*. 2011, Fatahi *et al*. 2013, Nimbalkar and Indraratna 2014, Basack *et al*. 2015, 2016a, Cheng and Wang 2016, Basack and Nimbalkar 2017, Ellouze *et al*. 2017) are based upon ‘unit cell’ analogy assuming ‘equal strain’ hypothesis, which is specifically applicable whenever the surcharge load applied on the ground surface is of rigid nature. In case of embankment loading for transport infrastructure however, the flexible nature of applied surcharge loading is most likely to induce an equal distribution of surface load resulting in an uneven settlement at the surface (Indraratna and Nimbalkar 2013, Nimbalkar and Indraratna 2016). The latter assumption, termed as ‘free strain’ approach (Barron 1948), has been incorporated in the present model.

The work reported herein is aimed towards developing a numerical model (finite difference technique) to analyse the response of soft soil improved by vertical drains considering free strain approach. The unit cell analogy is used and the effect of smear has been incorporated in the model. The model developed has been validated by comparison of the numerical results with the field test data. Thereafter, a hypothetical case study is done using the available field data for soft clay deposit existing in the eastern part of Australia and important conclusions are drawn therefrom.

2. Mathematical formulations

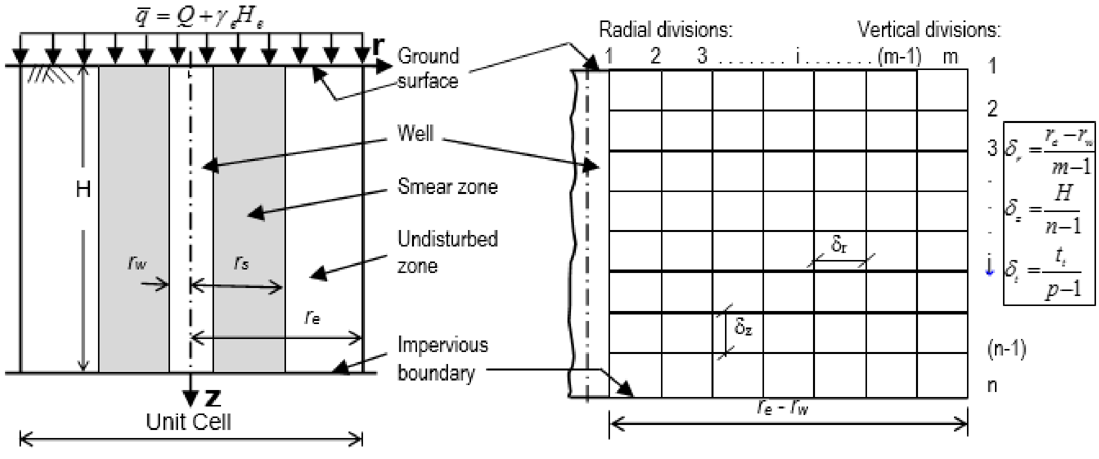
The soft clay layer of thickness H has been assumed to overlay on an impervious rigid boundary, and is improved by a group of vertical drains, each having a radius of r_w, extended to the bottom of the clay layer. The unit cell approach that conveniently represents the true response of the soft ground with vertical drains (Barron 1948) is considered for the current analysis. The radius of influence r_e of the unit cell may be calculated following the method described by Leo (2004). Due to the self-weight of the embankment, steady and uniform load intensity has been imposed on the ground surface, apart from the instantaneous surcharge load applied on its top. The average load intensity on the ground surface may therefore be written as

$$\overline{q} = Q + \gamma_e H_e \tag{1}$$

where, Q is uniform surcharge load intensity on the embankment fill and γ_e and H_e are the unit weight and maximum height of the embankment, respectively.

The analysis has been carried out based on the assumptions that the flow of water through the flow of water through the soil is purely horizontal (radially inward toward the drains) obeying Darcy’s law, the wells are free draining materials, the soil is fully saturated and the water is incompressible. Vertical flow of water, if any, is neglected.

The cross section of the entire zone of the unit cell is divided into three distinct zones (Fig. 1(a)), viz., the well zone, the column zone and the outer undisturbed soil zone. As reported by



(a) (b)

Fig. 1 The unit cell, (a) longitudinal section and (b) discretization for numerical analysis

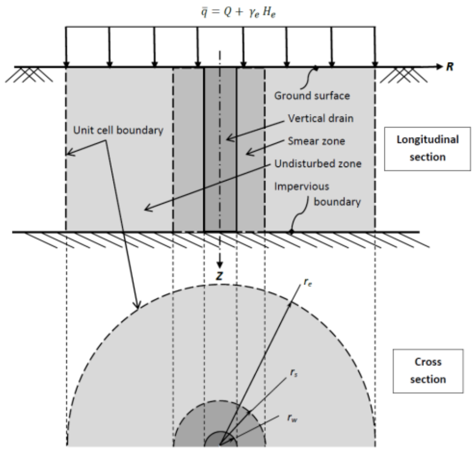


Fig. 2 Sections of the unit cell

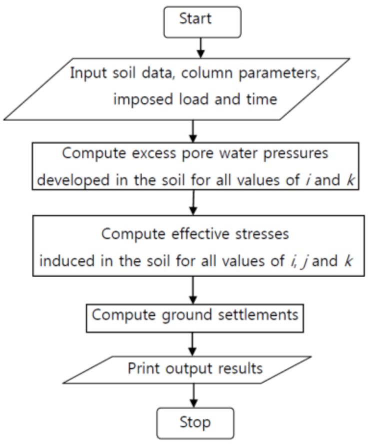


Fig. 3 Flow-chart for the computer program

Walker and Indraratna (2006), the ratio of r_s/r_c usually varies between 2 to 3 and the permeability of soil in the smear zone decreases in a parabolic pattern with radial distance. More information on the model can be found in Fig. 2.

As shown in Fig. 1(b), the soil mass within the unit cell has been divided both radially as well as vertically respectively into $(m-1)$ and $(n-1)$ numbers of equal divisions, such that each of such divisions may be expressed respectively as $\delta_r=(r_e-r_c)/(m-1)$ and $\delta_z=H/(n-1)$. The total time interval of computation t_t is divided into $(p-1)$ number of equal divisions, i.e., $\delta_t=t_t/(p-1)$. Here, m, n and p are positive integers greater than unity. Initially, the excess pore water pressures and the effective stresses developed at each separator at the corresponding time are computed, followed by determination of the other time-dependant variables such as the degree of consolidation.

Following the radial consolidation theory of Barron (1948), the variation of the excess pore water pressure in the soil in the space-time reference frame can be expressed as

$$\left[\frac{\partial}{\partial t} - c_{vr}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\right]u_{rt} = 0 \tag{2}$$

where, u_{rt} is the excess pore water pressure, r and t are the radial and time coordinates respectively, c_{vr} $[= k_h/(m_v\gamma_w)]$ is coefficient of radial consolidation of the soil, γ_w is the unit weight of water and k_h and m_v are the horizontal permeability and the coefficient of volume compressibility of the soil respectively.

Expressing the Eq. (2) above in finite difference form, the following expression is obtained

$$\frac{u_{i,k+1}-u_{i,k-1}}{2\delta_t} - c_{vr}\left[\frac{u_{i+1,k}-2u_{i,k}+u_{i-1,k}}{\delta_r^2} + \frac{1}{r_w+(i-1)\delta_r}\frac{u_{i+1,k}-u_{i-1,k}}{2\delta_r}\right] = 0 \tag{3}$$

In the model, the following boundary conditions are applied: (i) The initial pore water pressure in the soil is equal to the applied load intensity, (ii) No excess pore water pressure is developed at the soil-well interface during the process of consolidation, i.e., $u_{rt}=0$ at $r=r_w$, and (iii) Absence of any hydraulic gradient at the external boundary of the unit cell, i.e., $\frac{\partial u_{rt}}{\partial t} = \mathbf{0}$ at $r=r_e$.

Using Eq. (3) and the above boundary conditions, the following matrix equation has been obtained

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,mp} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2,mp} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3,mp} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{mp,1} & a_{mp,2} & a_{mp,3} & \cdots & a_{mp,mp} \end{bmatrix} \cdot \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ \cdots \\ U_{mp} \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ \cdots \\ b_{mp} \end{Bmatrix} \tag{4}$$

The above Equation can be written using matrix notation as

$$[A].\{U\}=\{b\} \tag{5}$$

where, $[A]$ is the coefficient matrix of the order of mp x mp, $\{b\}$is the augment vector of order of $mp\times 1$, and $\{U\}$is the unknown excess pore water pressure vector of the order of $mp\times 1$. A typical element U_i of the vector $\{U\}$ is given by

$$U_i = \{u_{i1}, u_{i2}, u_{i3}, \ldots.., u_{i,p}\}^T \tag{6}$$

Solving the Eq. (5) above, the unknown nodal pore water pressure relevant to the vector $\{U\}$ can be evaluated, and hence the average values of the excess pore water pressure and the degree of consolidation are obtained, using the following correlations

$$u_t^{av} = \frac{2}{r_e^2 - r_w^2} \sum_{i=1}^{m} [r_w + (i-1)\delta_r] u_{it} \tag{7}$$

$$U_t^c = \left(1 - \frac{u_t^{av}}{u_0^{av}}\right) x\ 100\% \tag{8}$$

where, u_t^{av} and U_t^c are the average values of the excess pore water pressure and the degree of consolidation at time t, respectively.

The displacement of a point *(r, z)* within the soil mass of the unit cell at time t is given by

$$\rho_{rzt} = -m_v \int_H^z \int_0^t \frac{\partial u_{rt}}{\partial t}\, dt\, dz \tag{9}$$

The Eq. (9) above can be simplified as

$$\rho_{rzt} = m_v (H - z)(u_{r0} - u_{rt}) \tag{10}$$

The average ground settlement is given by

$$\rho_t^{av} = \frac{2}{r_e^2 - r_w^2} \sum_{i=1}^{m} [r_w + (i-1)\delta_r] \rho_{r,0,t} \tag{11}$$

Similarly, the effective stress developed in the soil mass at any point *(r, z, t)* in the space-time coordinate may be expressed by (Khan *et al*. 2010)

$$\sigma'(r,z,t) = \gamma' z + \bar{q} - u(r,t) \tag{12}$$

where, γ' is the effective unit weight of the soil.

The computations have been carried out by a user-friendly program written in FORTRAN 99, the flowchart of which is given in Fig. 3. A sensitivity check has been carried out by varying the total number of nodes *m*, *n* and *p*. It has been observed that a reasonable optimization has been achieved by choosing the number of nodes as: *m*=30, *n*=100 and *p*=30. With this, a reasonable compromise was observed between the accuracy and the computational time. Hence in this paper, these optimal values have been taken for computations.

3. Free strain versus equal strain

When the imposed loading on the ground surface is of rigid nature (for example: rigid footing or raft), it is expected that the ground settlement assumes identical values at all the points while the vertical stress distribution becomes uneven. Such a condition is termed as '*equal strain*' condition. On the other hand, for flexible type of imposed loading (for example: embankments for transport corridors) essentially produce an uneven ground settlement with more-or-less even vertical stress distribution. Such a condition is termed as '*free strain*' condition (Das 2008, Indraratna *et al.* 2013b, Basack *et al*. 2016b). It is well established that difference in analytical/numerical results obtained from the two approaches are small (<10% in most cases) enough (Zhu and Yin 2004, Indraratna *et al.* 2008). However, mathematical rigor demands that in case of loading conditions relevant to transport infrastructure, free strain approach closely represents the field based observations (Basack *et al.* 2017).

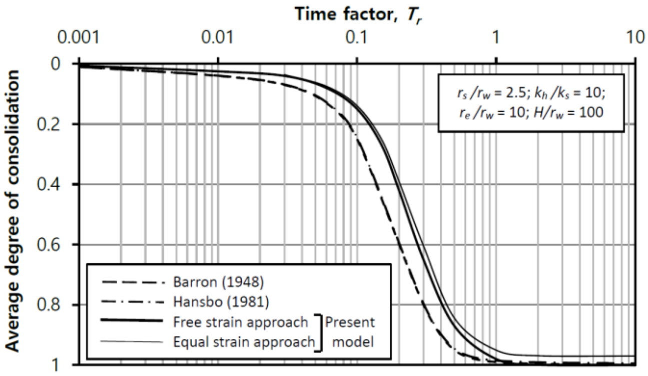
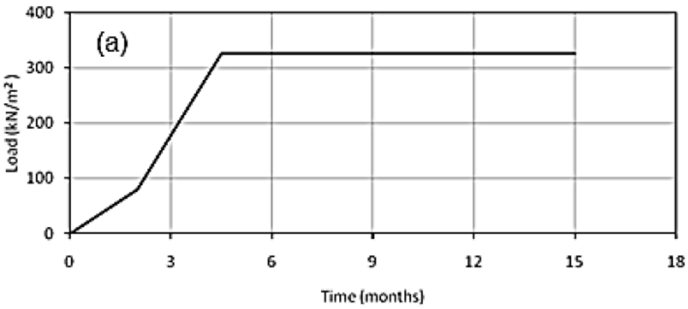
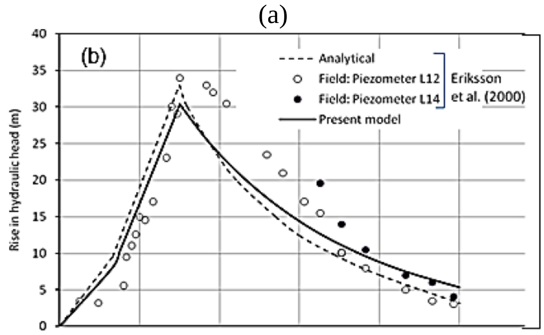


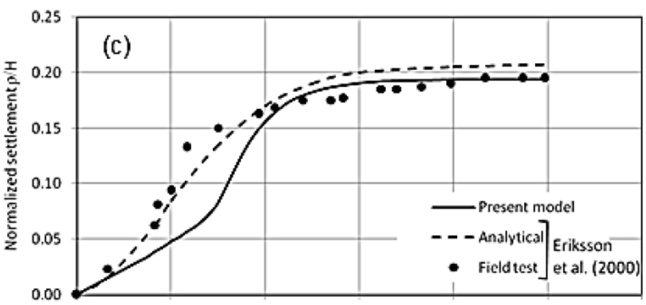
Fig. 4 Comparison of the rate of consolidation computed by present model with the existing analytical solutions



(a)



(b)



(c)

Fig. 5 Comparison of the computed results with the available field data and analytical results of Eriksson *et al*. (2000), (a) The applied ramp loading, (b) Rise in hydraulic head with time and (c) Average ground settlement versus time

4. Validation

First of all, the comparison of the average degree of consolidation by radial drainage only has been made with the existing models of Barron (1948), Hansbo (1981) and Wang (2010). The variations of average degree of consolidation with time factor T_r are presented in Fig. 4 for both the free strain and equal strain approaches. It may be observed that the numerical results obtained using the present model are in reasonably fair agreement with the other solutions. The slight overestimation in the free strain methodology can be justified by the assumption which leads to unequal strains on the soft ground surface resulting in faster rate of dissipation of excess pore water pressure compared to the other models having equal strain assumption.

Eriksson *et al.* (2000) carried out a field based study on a major ground improvement project at Arlanda airport, Stockholm by vertical drainage in combination with preloading (ramp loading, as shown in Fig. 5. While computing the excess pore water pressure developed at each load steps, Eriksson *et al.* (2000) assumed no dissipation to take place during the placement operation, which was reasonably justified by the dynamic disturbance effects produced during filling. The observed field test and analytical results has been compared with those obtained by using the present model. As reported, the input parameters used in the analysis are r_w=0.033 m, r_e=0.4725 m, r_s=0.075 m, H=4.5 m, k_h/k_s=3 and $m_v=2\times10^{-6}$ m^2/N.

The plots of the normalized average ground settlement and the rise in hydraulic head with time are shown in Fig. 5. The average deviations of the numerical results for the ground settlement using the present model with the field data and analytical results of Eriksson *et al.* (2000) were found to be about 9-18%. For the time zone $6<t<15$, t being the time in months, the curve obtained using the present model are well in agreement with the field data compared to the analytical curve of Erikson *et al.* (2000), although a notable deviation is observed in the zone of $3<t<6$. As regard to the rise in hydraulic head, a reasonably good agreement is observed for c_{vr}=0.45 m^2/year, although a notable deviation with a faster rate of the dissipation of excess pore pressure is found in case of the analytical results.

5. Case study

The numerical model developed has been utilized to study the behaviour of soft ground improved by vertical drains using the field data for eastern Australia. The Pacific Highway linking Sydney and Brisbane was constructed to reduce the high traffic congestion in Ballina, New South Wales, Australia (Fig. 6). This bypass route has to cross a floodplain consisting of highly compressible and saturated marine clay deposit. A system of vacuum assisted surcharge load with PVDs (34 mm diameter circular PVD at 1.0 m spacing in a square pattern) was adopted to improve the deeper clay layers (Indraratna 2009). A soft silty layer of clay approximately 10 m thick was underlain by moderately stiff, silty layer clay located 10-30 m deep, which was in turn underlain by firm clay. The groundwater was almost at the ground surface. A case study is carried out using the model, with the following input parameters: r_w=0.17 m, r_e=0.5 m, r_s=0.43 m, H=10 m, $k_h=1\times10^{-9}$ m/s, k_h/k_s=3 and $m_v=1\times10^{-6}$ kPa^{-1} and γ_s=15 N/m^3, q=80 kPa.

The time pattern of variation of ground settlement is shown in Fig. 7. The field measurement taken by settlement plate readings are compared with numerical results obtained by current model. For comparison, the data obtained by the equal strain approach are also plotted, which imply that the present model closely resembles the field observations. The agreement is generally good with an average variation below 10%, majority of the predicted values being on the lower side. This can be justified with the fact that in the current model, only the radial consolidation has been captured whereas in reality, a minor vertical component of pore water flow always exists in the field which increases the consolidation and resulting ground settlement (Han and Ye 2001).

The variation of the normalised pore water pressure in the space-time frame has been portrayed in Fig. 8. As observed, the excess pore pressure increases with radial distance, the slopes of the curves being gradually diminishing to zero at the boundary of the unit cell. In the advent of time, the excess pore pressure asymptotically decreases, the average pressure being dropping to as low as 15% of its initial value after 720 days. The normalized ground settlement has been found to increase gradually with radial distance, having a maximum value of 8% at the interface (Fig. 9(a)).

(a) (b)

Fig. 6 Ballina, New South Wales, (a) Geographical location and (b) View of the site

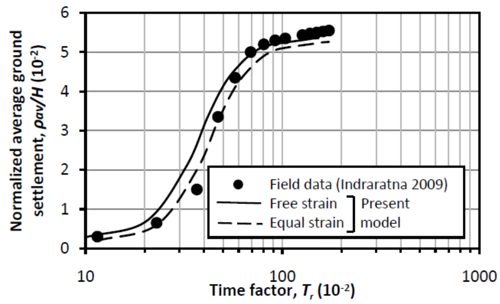
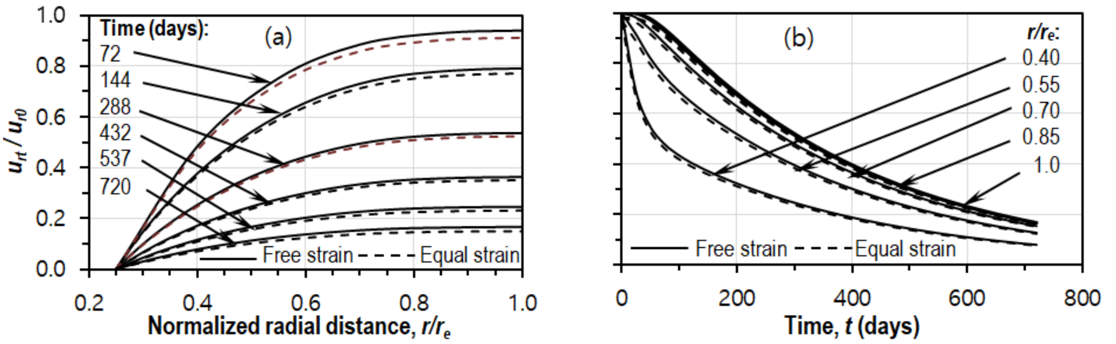
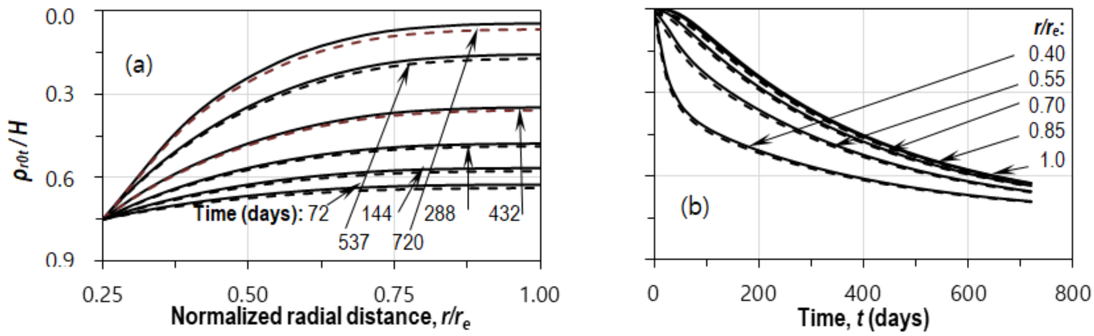


Fig. 7 Time pattern of variation of average ground settlement: comparison of numerical results with field measurement after Indraratna (2009)



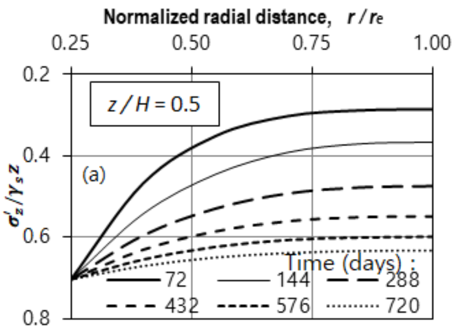
(a) (b)

Fig. 8 Variation of normalized average excess pore water pressure with: (a) radial distance and (b) time

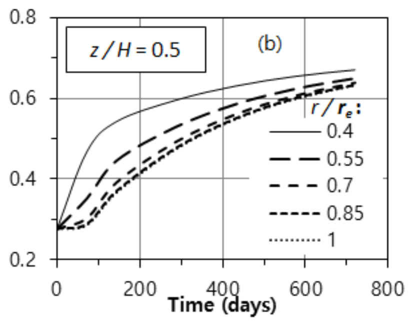


(a) (b)

Fig. 9 Variation of normalized ground settlement with: (a) radial distance and (b) time



(a)



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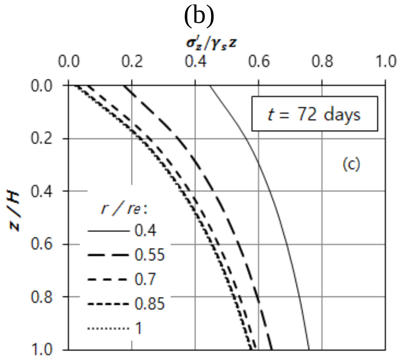

(c)

Fig. 10 Variation of effective stress within the soil mass with: (a) radial distance, (b) time and (c) depth

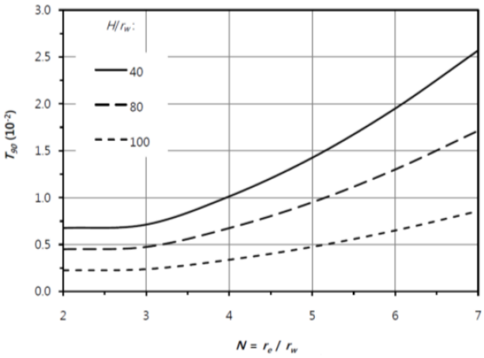

Fig. 11 Influence of installation geometry

At the cell boundary, the settlement has been observed to vary from 0.6% after 72 days to 7% at the end of 720 days. The slope of the ground gradually diminishes to zero at the cell boundary. The ground settlement was observed to increase asymptotically with time (Fig. 9(b)).

The effective stress induced in the soil is observed to decrease asymptotically with radial distance while increase with time, the normalized value approaching to unity (Figs. 10(a) and 10(b)). As consolidation proceeds, the slopes of the curves progressively diminish. With depth, the effective stress increases with decreasing slope, the variation is fairly hyperbolic (Fig. 10(c)).

The influence of installation geometry on the consolidation characteristics has been studied by varying the normalized drain spacing $N=r_e/r_w$ and radius H/r_w. The variation of the time factor T_{90} (normalized time to attain 90% degree of consolidation) with installation geometry has been plotted in Fig. 11. As observed, as the drain installation geometry varies in the range of $2\leq N\leq 7$ and $40\leq H/r_w\leq 100$, the value of T_{90} varies between 0.0024-0.026. With increase in N, the parameter T_{90} has been observed to increase in a curvilinear manner with ascending slope. With the increasing H/r_w , the value of T_{90} decreases. This can be justified by the fact that increase in drain spacing lengthens the drainage path for radial consolidation thereby retarding the consolidation. On the contrary, increase in drain radius (i.e., decrease in H/r_w) accelerates the consolidation.

For comparative study, the curves relevant to equal strain approach have also been plotted in Figs. 8 and 9. It is observed that the average deviation between the two approaches varies in the range of 5-10%, with the free strain approach being on the higher side. It is also observed from Fig. 7 that the free strain approach provides results closer to field observations, in case of embankment loading. This can be justified by the fact that the flexible nature of embankment loading in the Ballina field trial is properly simulated by the free strain approach.

6. Limitations of analysis

Although the numerical model yields quite promising results close to the field data, it has few inherent limitations as well, which are discussed below:

(i) The pore water flow is assumed to be purely horizontal, while the vertical component has been neglected. The latter is significant for partially penetrated drains.

(ii) The assumption that the base of the unit cell is rigid and impervious is true for stiff clay overlying the soft clay deposit. The numerical model should not be used when a pervious sand layer exists below the soft clay which may affect the chosen boundary condition.

7. Conclusions

In this study, a numerical solution based on unit cell theory is proposed considering the characteristics of soft soil improvement by vertical drains. The free strain hypothesis is adopted in the current numerical analysis and the smearing effect is taken into account. The comparison of the numerical results with the available field data and analytical results of Errikson *et al.* (2000) indicates reasonably good agreement which justifies the validity of the assumptions considered in the model. Considering a hypothetical case study on the soft soil deposit existing in Ballina, New South Wales, Australia, some important parametric studies are made. The study indicates that the pore water pressure within the unit cell increases with radial distance while decreases with time, with gradually diminishing slopes. Understandably, the opposite observation is found in case of ground settlement and effective stress in the soil. The increase in the effective stress with depth has found to be hyperbolic with decreasing slope. The drain installation geometry initiates pronounced influence on the consolidation.

For comparison, the data obtained from equal strain hypothesis have been studied as well. It was found that in case of flexible loading condition, for example transport infrastructure, the free strain approach yielded numerical results which are close to the field observations. For rigid type of loading (for example: raft footing), on the other hand, the equal strain assumption seems to be more valid.

Acknowledgments

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CC

Notations

H=*Thickness of clay layer*
H/r_w=*Normalized radius*
H_e=*maximum height of the embankment*
k_h=*Horizontal permeability of the soil*
m, n, p=*integer constants*
m_v=*Coefficient of volume compressibility of the soil*
N=*Normalized drain spacing* $(=r_e/r_w)$
q=*Surcharge pressure acting on embankment fill*
Q=*Uniform surcharge load intensity on embankment fill*
r=*Radial distance*
r_W=*Radius of the vertical drain*
r_e=*Radius of influence of unit cell*
r_s=*Radius of outer undisturbed soil zone*
r_w=*Radius of vertical drain*
t=*Time*
T_{90}=*Time factor (normalized time to attain 90% degree of consolidation)*
u=*Excess pore water pressure in soil*
γ'=*effective unit weight of soil*
γ_e=*unit weight of soil*
γ_w=*Unit weight of water*
δ_r=*Radial deformation of elemental slice*
δ_z=*Vertical deformation of elemental slice*
$\{b\}$=*augment vector of order* $np\times 1$
$\bar{q}$ =*average load intensity*
$\{u\}$=*nodal pore water pressure relevant to the vector of order* $np\times 1$
$[A]$=*the coefficient matrix of order* $np\times np$