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A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates

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Abstract. This work presents a simple and refined *n*th-order shear deformation theory for mechanical and thermal buckling behaviors of functionally graded (FG) plates resting on elastic foundation. The proposed refined nth-order shear deformation theory has a new displacement field which includes undetermined integral terms and contains only four unknowns. Governing equations are obtained from the principle of minimum total potential energy. A Navier type analytical solution methodology is also presented for simply supported FG plates resting on elastic foundation which predicts accurate solution. The accuracy of the present model is checked by comparing the computed results with those obtained by classical plate theory (CPT), first-order shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). Moreover, results demonstrate that the proposed theory can achieve the same accuracy of the existing HSDTs which have more number of variables.

Keywords: buckling; functionally graded plate; elastic foundation; plate theory

1. Introduction

Functionally graded materials (FGMs) are inhomogeneous composites which have smooth and continuous distribution of material characteristics in space (Mahi *et al*. 2015, Arefi 2015a, b, Nguyen *et al*. 2015, Tagrara *et al*. 2015, Pradhan and Chakraverty 2015, Chen 2015, Arefi and Allam 2015, Bouguenina *et al*. 2015, Saidi *et al*. 2016, Celebi *et al*. 2016, Aizikovich *et al*. 2016, Rajabi *et al*. 2016, Tounsi *et al*. 2016, Ebrahimi and Shafiei 2016, Benferhat *et al*. 2016, Ait Atmane *et al*. 2017). With increasing applications, functionally graded (FG) plates are found in many structures and components. FG plates are mainly designed for applications under thermal environment (Bouafia *et al*. 2017, Bousahla *et al*. 2016, Beldjelili *et al*. 2016, El-Hassar *et al*. 2016, Akbaş 2015, Bouchafa *et al*. 2015, Attia *et al*. 2015, Zidi *et al*. 2014, Bouderba *et al*. 2013,

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Tounsi *et al*. 2013). They are often fabricated from a mixture of ceramics and metals to attain the significant requirement of material characteristics (Belkorissat *et al*. 2015). Buckling response of FG structures under different types of loading is important for practical applications and has received considerable interest.

Lanhe (2004) analytically investigated the thermal buckling problem of a FG plate with moderate thickness and simply supported boundary conditions based on the FSDT. Sohn and Kim (2008) dealt with the stabilities of FG panels subjected to combined thermal and aerodynamic loads. The FSDT was used to simulate supersonic aerodynamic loads acting on the panels. Matsunaga (2009) proposed a two-dimensional global HSDT for thermal stability of plates made of FGMs. He determined the critical buckling temperatures of a simply supported FG plate under uniformly and linearly distributed temperatures. Zhao *et al*. (2009) discussed the buckling behavior of FG plates under mechanical and thermal loads with arbitrary geometry, including plates that contain square and circular holes at the centre, via the element-free *kp*-Ritz method. Tung and Duc (2010) studied buckling of thick FG plates with initial geometrical imperfection under thermal loadings. By Galerkin procedure, the resulting equations were solved to obtain analytical solutions of critical buckling temperature difference. Bachir Bouiadjra *et al*. (2012) presented a four-variable refined plate theory for buckling analysis of FG plates subjected to thermal loads. Bourada *et al*. (2012) proposed a novel four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. Bachir Bouiadjra *et al*. (2013) studied the nonlinear response of FG plates under thermal loads using an efficient sinusoidal shear deformation theory. Kettaf *et al*. (2013) examined the thermal buckling response of FG sandwich plates by proposing a new hyperbolic displacement model.

Recently, investigations of FG plates resting on elastic foundations are identified as an interesting field. Duc and Tung (2011) studied the buckling and post-buckling responses of thick FG plates supported by elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Yaghoobi and Torabi (2013) presented an exact solution for thermal buckling of FG plates resting on elastic foundations with various boundary conditions. Ait Amar Meziane *et al*. (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Yaghoobi and Fereidoon (2014) presented a refined *n*th-order shear deformation theory for the mechanical and thermal buckling responses of FG plates resting on elastic foundation. Bakora and Tounsi (2015) studied the post-buckling of thick plates made of functionally graded material resting on elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Tebboune *et al*. (2015) investigated the thermal buckling of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Taibi *et al*. (2015) presented a simple shear deformation theory for thermo-mechanical behavior of FG sandwich plates on elastic foundations. Hadji *et al*. (2016a) analyzed FG beam using a new first-order shear deformation theory. Barati and Shahverdi (2016) presented a four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions. Ghorbanpour Arani *et al*. (2016) examined the dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory. Laoufi *et al*. (2016) studied the mechanical and hygrothermal behavior of FG plates using a hyperbolic shear deformation theory. Bouderba *et al*. (2016) investigated the thermal stability of FG sandwich plates using a simple shear deformation theory. Ghasemabadian and Kadkhodayan (2016) presented an investigation of buckling behavior of FG piezoelectric rectangular plates under open and closed circuit conditions. Barka *et al*. (2016) discussed the thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation. Abdelhak *et al*. (2016) examined the thermal buckling response of FG sandwich plates with clamped boundary conditions. Trinh *et al*. (2016) analyzed the post-buckling responses of elastoplastic FGM beams on nonlinear elastic foundation. Becheri *et al*. (2016) studied the buckling of symmetrically laminated plates using nth-order shear deformation theory with curvature effects. Ebrahimi and Jafari (2016) presented the thermo-mechanical vibration analysis of temperature- dependent porous FG beams based on Timoshenko beam theory. Chikh *et al*. (2016) analyzed the thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. Benahmed *et al*. (2017) presented a novel quasi-3D hyperbolic shear deformation theory for FG thick rectangular plates on elastic foundation.

The present work deals with the *n*th-order shear deformation theory (Xiang *et al*. 2011). Moreover, the present article mainly utilizes the ideas behind the new FSDT (Mantari and Granados 2015) that the authors include undetermined integral terms to model the warping effect of the shear deformation theories. In the present work, the authors combine this idea for developing the *n*th-order shear deformation theory with modified displacement field to its optimization. The present theory contains only four variables and four governing equations, but it satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factors. Governing equations are obtained from the principle of minimum total potential energy. Analytical solutions for mechanical and thermal buckling response of FG plates resting on elastic foundations are determined. Numerical examples are proposed to demonstrate the accuracy of the present theory.

2. Material properties of FG plate

In this work, material characteristics of a FG plate are assumed to be graded in accordance with the rule of mixtures as (Sugano 1990). Simple power law variation from pure metal at lower face ($z = -h / 2$) to pure ceramic at the upper face ($z = +h / 2$) in terms of the volume fractions of the constituents is assumed (Praveen and Reddy 1998, Bousahla *et al*. 2014, Al-Basyouni *et al*. 2015, Ait Yahia *et al*. 2015, Ait Atmane *et al*. 2015, Hadji *et al*. 2015, Hassaine Daouadji and Hadji 2015, Bounouara *et al*. 2016, Ahouel *et al*. 2016, Boukhari *et al*. 2016, Hadji *et al*. 2016b, Houari *et al*. 2016, Bellifa *et al*. 2016). The mechanical and thermal characteristics of FGMs are obtained from the volume fraction of the material constituents. We suppose that the material characteristics such as the Young's modulus (E), the thermal conductivity (K), coefficient of thermal expansion (α) and Poisson's ratio (ν) can be obtained by (Yaghoobi and Torabi 2013, Meksi *et al*. 2015)

$$E(z) = E_M + \left(E_C - E_M\right)\left(\frac{2z+h}{2h}\right)^k \tag{1a}$$

$$K(z) = K_M + \left(K_C - K_M\right)\left(\frac{2z+h}{2h}\right)^k \tag{1b}$$

$$\alpha(z) = \alpha_M + \left(\alpha_C - \alpha_M\right)\left(\frac{2z+h}{2h}\right)^k, \quad \nu(z) = \nu = \text{constant} \tag{1c}$$

where k is the power law exponent and subscripts M and C denote denote the metallic and ceramic components, respectively. The value of k equal to zero and infinity represents a fully ceramic and metal plate, respectively.

3. Novel refined *n*th-order plate theory

3.1 Kinematics and constitutive equations

In this work, further simplifying suppositions are adopted to the nth-order shear deformation theory so that the number of variables is reduced. The kinematic of the conventional nth-order shear deformation theory is given by (Xiang *et al.* 2011)

$$u(x,y,z)=u_0(x,y)+z\phi_x(x,y)-\frac{1}{n}\left(\frac{2}{h}\right)^{n-1}z^n\left(\phi_x(x,y)+\frac{\partial w_0(x,y)}{\partial x}\right) \tag{2a}$$

$$v(x,y,z)=v_0(x,y)+z\phi_y(x,y)-\frac{1}{n}\left(\frac{2}{h}\right)^{n-1}z^n\left(\phi_y(x,y)+\frac{\partial w_0(x,y)}{\partial y}\right) \tag{2b}$$

$$n=3,5,7,9,....$$

$$w(x,y,z)=w_0(x,y) \tag{2c}$$

where u_0, v_0, w_0, ϕ_x and ϕ_y are five unknown displacement functions of the mid-plane of the plate; and h is the thickness of the plate. By considering that $\phi_x=\int\theta(x,y)dx$ and $\phi_y=\int\theta(x,y)dy$, the displacement field of the new refined theory can be expressed in a simpler form as (Chikh *et al.* 2017, Bourada *et al.* 2016, Merdaci *et al.* 2016, Hebali *et al.* 2016, Meftah *et al.* 2017, Meksi *et al.* 2017)

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_0}{\partial x}+k_1\,f(z)\int\theta(x,y)\,dx \tag{3a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+k_2\,f(z)\int\theta(x,y)\,dy \tag{3b}$$

$$w(x,y,z)=w_0(x,y) \tag{3c}$$

with

$$f(z)=z-\frac{1}{n}\left(\frac{2}{h}\right)^{n-1}z^n \tag{4}$$

It can be observed that the displacement field in Eq. (3) uses only four unknowns (u_0, v_0, w_0 and θ). The nonzero strains associated with the kinematic in Eq. (3) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{6a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta \, dx + k_2 \dfrac{\partial}{\partial x}\int \theta \, dy \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1 \int \theta \, dy \\ k_2 \int \theta \, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{6b}$$

The integrals appearing in the above expressions shall be resolved by a Navier type solution and can be expressed as follows

$$\frac{\partial}{\partial y}\int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \qquad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \tag{7}$$

where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A' and B' are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \qquad B' = -\frac{1}{\beta^2}, \qquad k_1 = \alpha^2, \qquad k_2 = \beta^2 \tag{8}$$

where α and β are defined in expression (19).

It should be noted that unlike the FSDT, this theory does not require shear correction factors. Moreover, the constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha(z)T(z) \\ \varepsilon_y - \alpha(z)T(z) \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{9}$$

where $T(z)$ is the temperature difference with respect to the reference and C_{ij} (i, j = 1, 2, 4, 5, 6) is the elastic stiffness of the FG plate given by

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \qquad C_{12} = \frac{\nu E(z)}{1-\nu^2}, \qquad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10}$$

3.2 Stability equations

The equilibrium equations of FG plates resting on elastic foundation under thermo-mechanical loadings may be determined on the basis of the stationary potential energy (Reddy 1984). The equilibrium equations are deduced as

$$\begin{aligned}
&\delta\, u_0: \ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\delta\, v_0: \ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
&\delta\, w_0: \ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}-K_w w_0+K_s\nabla^2 w_0+N_x^0\frac{\partial^2 w_0}{\partial x^2}+2N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+N_y^0\frac{\partial^2 w_0}{\partial y^2}=0\\
&\delta\,\theta: \ -k_1 M_x^s-k_2 M_y^s-\left(k_1 A'+k_2 B'\right)\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+k_1 A'\frac{\partial S_{xz}^s}{\partial x}+k_2 B'\frac{\partial S_{yz}^s}{\partial y}=0
\end{aligned} \tag{11}$$

where K_w is the modulus of subgrade reaction (elastic coefficient of the foundation) and K_s is the shear moduli of the subgrade (shear layer foundation stiffness).

Using constitutive relations, the stress and moment resultants are expressed as

$$\left(N_i, M_i^b, M_i^s\right)=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz, \quad (i=x,y,xy) \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right)=\int_{-h/2}^{h/2} g\left(\tau_{xz},\tau_{yz}\right)dz \tag{12}$$

Upon substitution of Eq. (5) into Eq. (9) and the subsequent results into Eq. (12) the stress resultants are determined in the matrix form as

$$\begin{Bmatrix} N_x\\ N_y\\ N_{xy}\\ M_x^b\\ M_y^b\\ M_{xy}^b\\ M_x^s\\ M_y^s\\ M_{xy}^s \end{Bmatrix}=\begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0\\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0\\
0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s\\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0\\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0\\
0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s\\
B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0\\
B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0\\
0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s
\end{bmatrix}\begin{Bmatrix} \varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\\ k_x^b\\ k_y^b\\ k_{xy}^b\\ k_x^s\\ k_y^s\\ k_{xy}^s \end{Bmatrix}-\begin{Bmatrix} N_x^T\\ N_y^T\\ 0\\ M_x^{bT}\\ M_y^{bT}\\ 0\\ M_x^{sT}\\ M_y^{sT}\\ 0 \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} S_{yz}^s\\ S_{xz}^s \end{Bmatrix}=\begin{bmatrix} A_{44}^s & 0\\ 0 & A_{55}^s \end{bmatrix}\begin{Bmatrix} \gamma_{yz}^0\\ \gamma_{xz}^0 \end{Bmatrix} \tag{13b}$$

where stiffness components are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz, \tag{14a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{14b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{14c}$$

The stress and moment resultants, $N_x^T = N_y^T$; $M_x^{bT} = M_y^{bT}$ and $M_x^{sT} = M_y^{sT}$; to thermal loading are defined by

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_y^{bT} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-\nu}\alpha(z)T(z) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{15}$$

In order to obtain the stability equations and investigate the mechanical and thermal buckling response of the FG plate resting on elastic foundation, the adjacent equilibrium criterion is employed (Brush and Almroth 1975). By employing this approach, the governing stability equations are determined as

$$\begin{aligned} &\delta u_0 : \frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0 \\ &\delta v_0 : \frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0 \\ &\delta w_0 : \frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} - K_w w_0^1 + K_s \nabla^2 w_0^1 + N_x^0 \frac{\partial^2 w_0^1}{\partial x^2} + 2 N_{xy}^0 \frac{\partial^2 w_0^1}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0^1}{\partial y^2} = 0 \\ &\delta \theta : -k_1 M_x^{s1} - k_2 M_y^{s1} - \left(k_1 A' + k_2 B'\right)\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^{s1}}{\partial x} + k_2 B' \frac{\partial S_{yz}^{s1}}{\partial y} = 0 \end{aligned} \tag{16}$$

where N_x^0, N_{xy}^0 and N_y^0 are the pre-buckling forces. Eq. (16) can be written in terms of displacements $(u_0^1, v_0^1, w_0^1, \theta^1)$ by substituting for the stress resultants from Eq. (13). For FG plate resting on elastic foundation, the governing equations Eq. (16) take the form

$$\begin{aligned} &A_{11}\frac{\partial^2 u_0^1}{\partial x^2} + A_{12}\frac{\partial^2 v_0^1}{\partial x \partial y} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial y^2} + \frac{\partial^2 v_0^1}{\partial x \partial y}\right) - B_{11}\frac{\partial^3 w_0^1}{\partial x^3} - B_{12}\frac{\partial^3 w_0^1}{\partial x \partial y^2} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x \partial y^2} \\ &+ B_{11}^s A' k_1 \frac{\partial^3 \theta^1}{\partial x^3} B_{12}^s B' k_2 \frac{\partial^3 \theta^1}{\partial x \partial y^2} + B_{66}^s\left(A' k_1 + B' k_2\right)\frac{\partial^3 \theta^1}{\partial x \partial y^2} = 0 \end{aligned} \tag{17a}$$

$$A_{12}\frac{\partial^2 u_0^1}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0^1}{\partial y^2} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial x \partial y} + \frac{\partial^2 v_0^1}{\partial x^2}\right) - B_{12}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} - B_{22}\frac{\partial^3 w_0^1}{\partial y^3} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} \tag{17b}$$

$$+B_{12}^{s}A'k_1\frac{\partial^3\theta^1}{\partial x^2\partial y}B_{22}^{s}B'k_2\frac{\partial^3\theta^1}{\partial y^3}+B_{66}^{s}\left(A'k_1+B'k_2\right)\frac{\partial^3\theta^1}{\partial x^2\partial y}=0 \tag{17b}$$

$$\begin{aligned}&B_{11}\frac{\partial^3 u_0^1}{\partial x^3}+B_{12}\left(\frac{\partial^3 u_0^1}{\partial x\partial y^2}+\frac{\partial^3 v_0^1}{\partial x^2\partial y}\right)+B_{22}\frac{\partial^3 v_0^1}{\partial y^3}+2B_{66}\left(\frac{\partial^3 u_0^1}{\partial x\partial y^2}+\frac{\partial^3 v_0^1}{\partial x^2\partial y}\right)-D_{11}\frac{\partial^4 w_0^1}{\partial x^4}\\&-2D_{12}\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0^1}{\partial y^4}-4D_{66}\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}+D_{11}^{s}A'k_1\frac{\partial^4\theta^1}{\partial x^4}+D_{12}^{s}\left(A'k_1+B'k_2\right)\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\&+D_{22}^{s}B'k_2\frac{\partial^4\theta^1}{\partial y^4}+2D_{66}^{s}\left(A'k_1+B'k_2\right)\frac{\partial^4\theta^1}{\partial x^2\partial y^2}-K_w w_0^1+K_s\nabla^2 w_0^1\\&+N_x^0\frac{\partial^2 w_0^1}{\partial x^2}+N_y^0\frac{\partial^2 w_0^1}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w_0^1}{\partial x\partial y}=0\end{aligned} \tag{17c}$$

$$\begin{aligned}&-B_{11}^{s}A'k_1\frac{\partial^3 u_0^1}{\partial x^3}-B_{12}^{s}\left(A'k_1\frac{\partial^3 v_0^1}{\partial x^2\partial y}+B'k_2\frac{\partial^3 u_0^1}{\partial x\partial y^2}\right)-B_{22}^{s}B'k_2\frac{\partial^3 v_0^1}{\partial y^3}\\&-D_{66}\left(\left(A'k_1+B'k_2\right)\frac{\partial^3 u_0^1}{\partial x\partial y^2}+\left(A'k_1+B'k_2\right)\frac{\partial^3 v_0^1}{\partial x^2\partial y}\right)+D_{11}^{s}A'k_1\frac{\partial^4 w_0^1}{\partial x^4}+D_{12}^{s}\left(A'k_1+B'k_2\right)\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}\\&D_{22}^{s}B'k_2\frac{\partial^4 w_0^1}{\partial y^4}+2D_{66}^{s}\left(A'k_1+B'k_2\right)\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}-H_{11}^{s}\left(A'k_1\right)^2\frac{\partial^4\theta^1}{\partial x^4}-2H_{12}^{s}A'k_1B'k_2\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\&-H_{22}^{s}\left(B'k_2\right)^2\frac{\partial^4\theta^1}{\partial y^4}-H_{66}^{s}\left(A'k_1+B'k_2\right)^2\frac{\partial^4\theta^1}{\partial x^2\partial y^2}+A_{55}^{s}\left(A'k_1\right)^2\frac{\partial^2\theta^1}{\partial x^2}+A_{44}^{s}\left(B'k_2\right)^2\frac{\partial^2\theta^1}{\partial y^2}=0\end{aligned} \tag{17d}$$

4. Analytical solution

The Navier solution method is used to determine the analytical solutions for which the displacement functions are expressed as product of arbitrary parameters and known trigonometric functions to respect the governing equations and boundary conditions.

$$\begin{Bmatrix}u_0^1\\v_0^1\\w_0^1\\\theta^1\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\, x)\sin(\beta\, y)\\V_{mn}\sin(\alpha\, x)\cos(\beta\, y)\\W_{mn}\sin(\alpha\, x)\sin(\beta\, y)\\X_{mn}\sin(\alpha\, x)\sin(\beta\, y)\end{Bmatrix} \tag{18}$$

where U_{mn}, V_{mn}, W_{mn}, X_{mn} are arbitrary parameters to be determined. α and β are defined as

$$\alpha=m\pi/a \qquad \text{and} \qquad \beta=n\pi/b \tag{19}$$

Substituting Eq. (18) into Eq. (17), the closed-form solution of buckling load can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33}+N_x^0\alpha^2+N_y^0\beta^2 & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{20}$$

where

$$\begin{aligned} a_{11} &= -(A_{11}\alpha^2 + A_{66}\beta^2) \\ a_{12} &= -\alpha\,\beta\left(A_{12}+A_{66}\right) \\ a_{13} &= \alpha(B_{11}\alpha^2 + (B_{12}+2B_{66})\beta^2) \\ a_{14} &= -\alpha(B_{11}^s A'k_1\alpha^2 + B_{12}^s B'k_2\beta^2 + B_{66}^s\left(A'k_1+B'k_2\right)\beta^2) \\ a_{22} &= -\alpha^2 A_{66} - \beta^2 A_{22} \\ a_{23} &= \beta(B_{22}\beta^2 + (B_{12}+2B_{66})\alpha^2) \\ a_{24} &= -\beta\left(B_{22}^s B'k_2\beta^2 + \alpha^2\left(B_{12}^s A'k_1 + B_{66}^s\left(A'k_1+B'k_2\right)\right)\right) \\ a_{33} &= -\alpha^2\left(D_{11}\alpha^2 + \left(2D_{12}+4D_{66}\right)\beta^2\right) - D_{22}\beta^4 - K_w - K_s\left(\alpha^2+\beta^2\right)\alpha^2 \\ a_{34} &= D_{11}^s A'k_1\alpha^4 + D_{12}^s\left(A'k_1+B'k_2\right)\beta^2\alpha^2 + D_{22}^s B'k_2\beta^4 + 2D_{66}^s\left(A'k_1+B'k_2\right)\beta^2\alpha^2 \end{aligned} \tag{21}$$

By using the condensation technique to eliminate the axial displacements U_{mn} and V_{mn}, Eq. (20) can be rewritten as

$$\begin{bmatrix} \bar{a}_{33}+N_x^0\alpha^2+N_y^0\beta^2 & \bar{a}_{34} \\ \bar{a}_{43} & \bar{a}_{44} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{22}$$

where

$$\begin{aligned} \bar{a}_{33} &= a_{33} - \frac{a_{13}\left(a_{13}a_{22}-a_{12}a_{23}\right) - a_{23}\left(a_{11}a_{23}-a_{12}a_{13}\right)}{a_{11}a_{22}-a_{12}^2} \\ \bar{a}_{34} &= a_{34} - \frac{a_{14}\left(a_{13}a_{22}-a_{12}a_{23}\right) - a_{24}\left(a_{11}a_{23}-a_{12}a_{13}\right)}{a_{11}a_{22}-a_{12}^2} \\ \bar{a}_{43} &= a_{34} - \frac{a_{13}\left(a_{14}a_{22}-a_{12}a_{24}\right) - a_{23}\left(a_{11}a_{24}-a_{12}a_{14}\right)}{a_{11}a_{22}-a_{12}^2} \\ \bar{a}_{44} &= a_{44} - \frac{a_{14}\left(a_{14}a_{22}-a_{12}a_{24}\right) - a_{24}\left(a_{11}a_{24}-a_{12}a_{14}\right)}{a_{11}a_{22}-a_{12}^2} \end{aligned} \tag{23}$$

The system of homogeneous Eq. (22) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients (W_{mn}, X_{mn}) must equal zero

$$\begin{vmatrix} \bar{a}_{33}+N_x^0\alpha^2+N_y^0\beta^2 & \bar{a}_{34} \\ \bar{a}_{43} & \bar{a}_{44} \end{vmatrix} = 0 \tag{24}$$

The obtained equation may be solved for the buckling load. This gives the following relation for buckling load

$$N_x^0\alpha^2 + N_y^0\beta^2 = \frac{\bar{a}_{34}\bar{a}_{43} - \bar{a}_{33}\bar{a}_{44}}{\bar{a}_{44}} \tag{25}$$

4.1 Mechanical buckling

In this part, a simply supported rectangular plate resting on elastic foundation with length a and width b is considered by applying axial loading in two directions $N_x^0 = \chi_1 N_{cr}$, $N_y^0 = \chi_2 N_{cr}$, $N_{xy}^0 = 0$. By employing the Eq. (25), the following expression for mechanical buckling load is determined

$$N_{cr}(m,n) = \frac{1}{\chi_1\alpha^2 + \chi_2\beta^2}\frac{\bar{a}_{34}\bar{a}_{43} - \bar{a}_{33}\bar{a}_{44}}{\bar{a}_{44}} \tag{26}$$

The critical buckling load is the minimum value of the determinant for values of m and n.

4.2 Thermal buckling

In this case, a rectangular plate subjected to thermal loads is considered. To obtain the critical buckling temperature, the pre-buckling thermal loads should be determined. Hence, solving the membrane form of the equilibrium equations and by employing the technique proposed by Meyers and Hyer (1991), the pre-buckling load resultants of FG plate exposed to the temperature variation within the thickness are found to be (Yaghoobi and Torabi 2013)

$$N_x^0 = -\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\alpha(z)T(z)dz, \qquad N_y^0 = -\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\alpha(z)T(z)dz, \qquad N_{xy}^0 = 0 \tag{27}$$

In this article, to examine the effect of the considered type of temperature variation within the thickness on stability buckling response of FG plate resting on elastic foundation, three types of thermal loading within the plate thickness are taken.

4.2.1 Uniform temperature rise (*UTR*)

It is supposed that the initial uniform temperature of the FG plate is T_i, and the temperature is uniformly elevated to a final value T_f such that the plate buckles. Thus, the temperature change is (Yaghoobi and Torabi 2013)

$$T(z) = T_f - T_i = \Delta T \tag{28}$$

By employing the Eqs. (25), (27), and (28) the following equation for thermal buckling load is deduced

$$\Delta T(m,n) = \frac{1}{\alpha^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44} - \bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}}\frac{1}{\int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)}{1-\nu}dz} \tag{29}$$

4.2.2 Linear temperature distribution through the thickness (LTD)

The following linear temperature distribution within the thickness of the FG plate is considered (Yaghoobi and Torabi 2013)

$$T(z) = \frac{\Delta T}{h}\left(z + \frac{h}{2}\right) + T_M, \qquad \Delta T = T_C - T_M \tag{30}$$

Identically to the UTR procedure, the following expression for thermal buckling load is deduced

$$\Delta T(m,n) = \frac{1}{\displaystyle\int_{-h/2}^{h/2} \frac{E(z)\,\alpha(z)\left(\frac{z}{h}+\frac{1}{2}\right)}{1-\nu}\,dz}\left(\frac{1}{\alpha^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44}-\bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}} - \int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)T_M}{1-\nu}\,dz\right) \tag{31}$$

4.2.3 Non-linear temperature distribution through the thickness (NTD)

The temperature field considered to be uniform over the plate surface but varying across the thickness direction due to heat conduction. In this a case, the temperature variation across the thickness can be determined by solving the steady-state heat transfer equation as (Yaghoobi and Torabi 2013)

$$\frac{d}{dz}\left(K(z)\frac{dT(z)}{dz}\right) = 0, \qquad T\left(\frac{h}{2}\right) = T_C \qquad \text{and} \qquad T\left(-\frac{h}{2}\right) = T_M \tag{32}$$

The differential Eq. (32) can be easily solved by employing the polynomial series. Thus, the temperature variation within the plate thickness is determined as

$$T(z) = T_M + \left(\frac{1}{2}+\frac{z}{h}\right)\Delta T\,\frac{\sum_{i=0}^{\infty}\left(\frac{1}{k\,i+1}\left(\frac{(K_M-K_C)\left(\frac{1}{2}+\frac{z}{h}\right)^k}{K_M}\right)^i\right)}{\sum_{i=0}^{\infty}\left(\frac{1}{k\,i+1}\left(\frac{(K_M-K_C)}{K_M}\right)^i\right)}, \qquad \Delta T = T_C - T_M \tag{33}$$

Following the same procedure as indicated in Section 4.2.1, we can determine the ΔT. Needless to say that the lowest value among all of these ΔT for each m and n is known as the critical buckling temperature difference ΔT_{cr}.

5. Results and discussion

In this section, numerical examples are examined and discussed for checking the accuracy of the proposed theory in computing the mechanical and thermal buckling forces. Analytical solutions are determined by employing the Navier method for simply supported FG plates resting on Winkler-Pasternak elastic foundations. Critical buckling loads are determined and the comparison is carried out with the existing results. For numerical results, an Al/Al_2O_3 plate made of aluminum (as metal) and alumina (as ceramic) is assumed. The Young's modulus, thermal conductivity and coefficient of thermal expansion are E_M = 70 GPa, $\alpha_M = 23\times10^{-6}/°C$, K_M = 204 W/mK and those of alumina are E_C = 380 GPa, $\alpha_C = 7.4\times10^{-6}/°C$, K_C = 10.4 W/mK, respectively. The Poisson's ratio of the plate is considered to be constant within the thickness and equal to 0.3 (Duc and Tung 2011, Yaghoobi and Torabi 2013). For convenience, the following nondimensional quantities are employed in presenting the numerical results in tabular form

$$\overline{K}_w = \frac{K_w a^4}{D_C}, \quad \overline{K}_s = \frac{K_s a^2}{D_C}, \quad \hat{N} = \frac{N_{cr} b^2}{D_C}, \quad \overline{N} = \frac{N_{cr} a^2}{E_M h^3}, \quad \widetilde{N} = \frac{N_{cr} a^2}{\pi^2 D_C},$$

$$D_C = \frac{E_C h^3}{12(1-\nu^2)}, \quad D_M = \frac{E_M h^3}{12(1-\nu^2)}$$

5.1 Comparisons for mechanical stability

Example 1: The dimensionless critical buckling forces $\widetilde{N}$ of simply supported thin homogeneous square plate without or resting on elastic foundations are given in Table 1. The obtained results are compared with those given by Lam *et al*. (2000) based on CPT, Akhavan *et al*. (2009) and Sobhy (2013) based on FSDT and Yaghoobi and Fereidoon (2014) based on RPT. It is mentioned that the solutions of Lam *et al*. (2000) are deduced via the Green's function. Good agreement is observed between the proposed theory and the published ones. Also, Table 2 demonstrates the dimensionless buckling loads $\hat{N}$ of simply supported homogeneous plate under in-plane compression ($\chi_1 = -1$, $\chi_2 = 0$). The computed results based on the proposed model are compared with those of Akhavan *et al*. (2009) and Sobhy (2013) based on FSDT, and Yaghoobi and Fereidoon (2014) based on RPT. Excellent agreement can be seen for different values of foundation coefficients, $\overline{K}_w$ and $\overline{K}_s$, aspect ratio a/b, and thickness ratio h/a.

Example 2: Tables 3-5 list the dimensionless critical buckling forces $\overline{N}$ of simply supported Al/Al_2O_3 square plate for different values of gradient index k and foundation coefficients $\overline{K}_w$ and

Table 1 Comparison of non-dimensional critical buckling load $\widetilde{N}$ of a simply supported thin homogeneous square plate resting on elastic foundations ($a/h = 1000$, $\chi_1 = -1$, $\chi_2 = 0$)

Theory	$(\overline{K}_w, \overline{K}_s)$			
	(0,0)	(0,100)	(100,0)	(100,100)
CPT (Lam *et al*. 2000)	4.00000	18.92[a]	5.027	19.17[a]
FSDT (Akhavan *et al*. 2009)	3.99998	18.9151[a]	5.02658	19.1717[a]
FSDT (Sobhy 2013)	3.99998	18.91506[a]	5.02658	19.17171[a]
(Yaghoobi and Fereidoon 2014) $n = 3$	3.99990	18.91400[a]	5.02650	19.17200
Present $n = 3$	3.99999	18.91513[a]	5.02659	19.17178[a]
Present $n = 5$	3.99999	18.91514[a]	5.02659	19.17179[a]
Present $n = 6$	3.99999	18.91514[a]	5.02659	19.17179[a]
Present $n = 7$	3.99999	18.91514[a]	5.02659	19.17179[a]

[a] Mode for plate is $(m, n) = (2, 1)$

Table 2 Comparison of non-dimensional critical buckling load $\hat{N}$ of a simply supported homogeneous plate resting on elastic foundations ($\chi_1 = -1$, $\chi_2 = 0$)

a/b	$(\overline{K}_w, \overline{K}_s)$	Theory	a/h			
			5	10	100	1000
0.5	(0,0)	FSDT (*)	54.3207	59.6629	61.6641	61.6848
		FSDT (**)	54.0859	59.5887	61.6633	61.6848
		RPT (***) n = 3	54.0737	59.5856	61.6633	61.6848
		Present n = 3	54.0737	59.5856	61.6633	61.6848
		Present n = 5	54.2031	59.6265	61.6637	61.6848
		Present n = 9	54.4901	59.7136	61.6647	61.6848
	(100,10)	FSDT (*)	144.6952	150.1910	152.1930	152.2130
		FSDT (**)	144.6140	150.1170	152.1920	152.2130
		RPT (***) n = 3	144.6022	150.1141	152.1918	152.2133
		Present n = 3	144.6022	150.1141	152.1918	152.2133
		Present n = 5	144.7315	150.1549	152.1922	152.2133
		Present n = 9	145.0185	150.2421	152.1931	152.2133
	(1000,100)	FSDT (*)	643.5000[b]	686.1710[a]	704.3860[a]	704.5890[a]
		FSDT (**)	641.380[b]	685.567[a]	704.378[a]	704.589[a]
		RPT (***) n = 3	640.9782 [b]	685.5369 [a]	704.3775 [a]	704.5888 [a]
		Present n = 3	640.9782 [b]	685.5369 [a]	704.3775 [a]	704.5888 [a]
		Present n = 5	642.1623 [b]	685.8694 [a]	704.3819 [a]	704.5888 [a]
		Present n = 9	646.0620 [b]	686.6014 [a]	704.3910 [a]	704.5889 [a]
1	(0,0)	FSDT (*)	32.4414	37.4477	39.457	39.4782
		FSDT (**)	32.2398	37.3753	39.4562	39.4782
		RPT (***) n = 3	32.2276	37.3721	39.4562	39.4782
		Present n = 3	32.2276	37.3721	39.4562	39.4782
		Present n = 5	32.3387	37.4120	39.4566	39.4782
		Present n = 9	32.5932	37.4977	39.4576	39.4782
	(100,10)	FSDT (*)	55.0289[a]	67.5798	69.5891	69.6103
		FSDT (**)	54.6116[a]	67.5074	69.5883	69.6103
		RPT (***) n = 3	54.5692 [a]	67.5042	69.5883	69.6103
		Present n = 3	54.5692 [a]	67.5042	69.5883	69.6103
		Present n = 5	54.7998 [a]	67.5441	69.5887	69.6103
		Present n = 9	55.4045 [a]	67.6299	69.5897	69.6103
	(1000,100)	FSDT (*)	174.9760[b]	204.6510 [a]	211.9610 [a]	212.0140 [a]
		FSDT (**)	174.391[b]	204.416 [a]	211.928 [a]	212.014 [a]
		RPT (***) n = 3	174.2676 [b]	204.4040 [a]	211.9285 [a]	212.0145 [a]
		Present n = 3	174.2676 [b]	204.4040 [a]	211.9285 [a]	212.0145 [a]
		Present n = 5	174.5952 [b]	204.5334 [a]	211.9302 [a]	212.0145 [a]
		Present n = 9	175.7258 [b]	204.8204 [a]	211.9340 [a]	212.0145 [a]

Table 2 Continued

a / b	$(\overline{K}_w, \overline{K}_s)$	Theory	a / h			
			5	10	100	1000
2	(0,0)	FSDT (*)	19.2255 [b]	32.4414 [a]	39.3930 [a]	39.4776 [a]
		FSDT (**)	19.0400 [b]	32.2398 [a]	39.3897 [a]	39.4775 [a]
		RPT (***) n = 3	18.9794 [b]	32.2276 [a]	39.3896 [a]	39.4775 [a]
		Present n = 3	18.9794 [b]	32.2276 [a]	39.3896 [a]	39.4775 [a]
		Present n = 5	19.0850 [b]	32.3387 [a]	39.3914 [a]	39.4775 [a]
		Present n = 9	19.5382 [b]	32.5932 [a]	39.3952 [a]	39.4776 [a]
	(100,10)	FSDT (*)	22.7476 [c]	37.5182 [b]	45.0262 [a]	45.1108 [a]
		FSDT (**)	22.6778 [c]	37.8581 [b]	45.0229 [a]	45.1108 [a]
		RPT (***) n = 3	22.5785 [c]	37.8358 [b]	45.0228 [a]	45.1108 [a]
		Present n = 3	22.5785 [c]	37.8358 [b]	45.0228 [a]	45.1108 [a]
		Present n = 5	22.6248 [c]	37.9967 [b]	45.0246 [a]	45.1108 [a]
		Present n = 9	23.1299 [c]	38.3858 [b]	45.0284 [a]	45.1108 [a]
	(1000,100)	FSDT (*)	–	72.8290 [c]	85.0953 [b]	85.2563 [b]
		FSDT (**)	52.2276 [d]	72.4117 [c]	85.0889 [b]	85.2562 [b]
		RPT (***) n = 3	50.0214 [d]	72.3694 [c]	85.0887 [b]	85.2562 [b]
		Present n = 3	50.0214 [d]	72.3694 [c]	85.0887 [b]	85.2562 [b]
		Present n = 5	49.9604 [d]	72.5999 [c]	85.0921 [b]	85.2562 [b]
		Present n = 9	50.4899 [d]	73.2046 [c]	85.0994 [b]	85.2563 [b]

(*) (Akhavan *et al*. 2009); (**) (Sobhy 2013); (** *) (Yaghoobi and Fereidoon 2014)
[a] Mode for plate is (m, n) = (2, 1); [b] Mode for plate is (m, n) = (3, 1);
[c] Mode for plate is (m, n) = (4, 1); [d] Mode for plate is (m, n) = (5, 1)

Table 3 Comparison of non-dimensional critical buckling load $\overline{N}$ of a simply supported FG plate resting on elastic foundations (a / b = 1, a / h = 1, $\chi_1 = -1$, $\chi_2 = 0$)

$(\overline{K}_w, \overline{K}_s)$	Theory	k					
		0	0.5	1	2	5	10
(0,0)	TSDT (*)	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
	Present n = 3	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
	Present n = 5	18.5983	12.1337	9.3476	7.2744	6.0593	5.4700
	Present n = 7	18.6224	12.1468	9.3578	7.2855	6.0780	5.4869
	Present n = 9	18.6409	12.1569	9.3657	7.2938	6.0911	5.4999
(100,10)	TSDT (*)	21.3379	14.8823	12.0985	10.0224	8.7947	8.2122
	Present n = 3	21.3379	14.8823	12.0985	10.0224	8.7947	8.2122
	Present n = 5	21.3577	14.8930	12.1069	10.0337	8.8187	8.2294
	Present n = 7	21.3817	14.9062	12.1172	10.0448	8.8373	8.2463
	Present n = 9	21.4003	14.9163	12.1251	10.0532	8.8504	8.2592

Table 3 Continued

$(\overline{K}_w, \overline{K}_s)$	Theory	k					
		0	0.5	1	2	5	10
(1000,100)	TSDT (*)	40.6477[a]	31.4605[a]	27.4319[a]	24.3470[a]	22.3602[a]	21.4516[a]
	Present n = 3	40.6477[a]	31.4605[a]	27.4319[a]	24.3470[a]	22.3602[a]	21.4516[a]
	Present n = 5	40.7120[a]	31.4960[a]	27.4600[a]	24.3845[a]	22.4373[a]	21.5053[a]
	Present n = 7	40.7922[a]	31.5405[a]	27.4950[a]	24.4222[a]	22.4982[a]	21.5593[a]
	Present n = 9	40.8547[a]	31.5750[a]	27.5222[a]	24.4506[a]	22.5415[a]	21.6010[a]

(*) (Thai and Kim 2013); [a] Mode for plate is (m, n) = (2, 1)

$\overline{K}_s$. The computed dimensionless critical buckling loads are compared with those given by Thai and Kim (2013). It should be noted that the results given by Thai and Kim (2013) were based on TSDT with five independent variables. In these tables, three different loading cases are assumed, and six arbitrary values of the gradient index k are considered. Three combinations of foundation coefficients, $\overline{K}_w$ and $\overline{K}_s$ are also considered. It can be observed that the results computed using the proposed model are in excellent agreement with those reported by Thai and Kim (2013) for all loading types, power law index, and foundation coefficients. It should be signaled that in this example the dimensionless foundation coefficients, $\overline{K}_w$ and $\overline{K}_s$ are $K_w a^4/D_M$ and $K_s a^4/D_M$ respectively.

Table 4 Comparison of non-dimensional critical buckling load $\overline{N}$ of a simply supported FG plate resting on elastic foundations (a / b = 1, a / h = 1, $\chi_1 = 0$, $\chi_2 = -1$)

$(\overline{K}_w, \overline{K}_s)$	Theory	k					
		0	0.5	1	2	5	10
(0,0)	TSDT (*)	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
	Present n = 3	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
	Present n = 5	18.5983	12.1337	9.3476	7.2744	6.0593	5.4700
	Present n = 7	18.6224	12.1468	9.3578	7.2855	6.0780	5.4869
	Present n = 9	18.6409	12.1569	9.3657	7.2938	6.0911	5.4999
(100,10)	TSDT (*)	21.3379	14.8823	12.0985	10.0224	8.7947	8.2122
	Present n = 3	21.3379	14.8823	12.0985	10.0224	8.7947	8.2122
	Present n = 5	21.3577	14.8930	12.1069	10.0337	8.8187	8.2294
	Present n = 7	21.3817	14.9062	12.1172	10.0448	8.8373	8.2463
	Present n = 9	21.4003	14.9163	12.1251	10.0532	8.8504	8.2592
(1000,100)	TSDT (*)	40.6477[e]	31.4605[e]	27.4319[e]	24.3470[e]	22.3602[e]	21.4516[e]
	Present n = 3	40.6477[e]	31.4605[e]	27.4319[e]	24.3470[e]	22.3602[e]	21.4516[e]
	Present n = 5	40.7120[e]	31.4960[e]	27.4600[e]	24.3845[e]	22.4373[e]	21.5053[e]
	Present n = 7	40.7922[e]	31.5405[e]	27.4950[e]	24.4222[e]	22.4982[e]	21.5593[e]
	Present n = 9	40.8547[e]	31.5750[e]	27.5222[e]	24.4506[e]	22.5415[e]	21.6010[e]

(*) (Thai and Kim 2013); [a] Mode for plate is (m, n) = (1, 2)

Table 5 Comparison of non-dimensional critical buckling load $\overline{N}$ of a simply supported FG plate resting on elastic foundations ($a / b = 1$, $a / h = 1$, $\chi_1 = 1$, $\chi_2 = -1$)

$(\overline{K}_w, \overline{K}_s)$	Method	k = 0	0.5	1	2	5	10
(0,0)	TSDT (*)	9.2893	6.0615	4.6695	3.6315	3.0177	2.7264
	Present n = 3	9.2893	6.0615	4.6696	3.6315	3.0177	2.7264
	Present n = 5	9.2992	6.0668	4.6738	3.6372	3.0297	2.7350
	Present n = 7	9.3112	6.0734	4.6789	3.6427	3.0390	2.7435
	Present n = 9	9.3205	6.0785	4.6829	3.6469	3.0455	2.7499
(100,10)	TSDT (*)	10.6689	7.4411	6.0492	5.0112	4.3973	4.1061
	Present n = 3	10.6689	7.4411	6.0492	5.0112	4.3973	4.1061
	Present n = 5	10.6788	7.4465	6.0535	5.0169	4.4093	4.1147
	Present n = 7	10.6909	7.4531	6.0586	5.0224	4.4187	4.1231
	Present n = 9	10.7001	7.4581	6.0625	5.0266	4.4252	4.1296
(1000,100)	TSDT (*)	23.0860	19.8582	18.4663	17.4283	16.8144	16.5232
	Present n = 3	23.0860	19.8582	18.4663	17.4283	16.8144	16.5232
	Present n = 5	23.0959	19.8636	18.4705	17.4339	16.8264	16.5318
	Present n = 7	23.1079	19.8702	18.4757	17.4395	16.8357	16.5402
	Present n = 9	23.1172	19.8752	18.4796	17.4437	16.8423	16.5467

5.2 Comparisons for thermal stability

Example 3: In order to check the thermal stability solutions determined in this work, the critical buckling temperature difference, ΔT_{cr}, for FG plates resting on Winkler-Pasternak elastic foundations for the UTR, LTD and NTD are presented in Tables 6-8, respectively. It can be deduced from Tables 6–8 that there is a very good agreement between the present theory (with four variables) and other higher-order plate theories (with five variables). The significant differences

Table 6 Comparison of critical buckling temperature difference $\Delta T_{cr} \times 10^{-3}$ of square FG plate resting on elastic foundation under UTR

k	Theory	$(\overline{K}_w, \overline{K}_s) = (0,0)$			$(\overline{K}_w, \overline{K}_s) = (10,0)$			$(\overline{K}_w, \overline{K}_s) = (10,10)$		
		$a/h = 5$	$a/h = 10$	$a/h = 20$	$a/h = 5$	$a/h = 10$	$a/h = 20$	$a/h = 5$	$a/h = 10$	$a/h = 20$
0	CPT (a)	6.83964	1.70991	0.42748	7.01519	1.7538	0.43845	10.48019	2.62005	0.65501
	FSDT (b)	5.58069	1.61862	0.42153	5.75623	1.66251	0.43251	9.22123	2.52876	0.64907
	HSDT (a)	5.58344	1.61868	0.42154	5.75899	1.66257	0.43251	9.22398	2.52882	0.64907
	TPT (a)	5.58556	1.61882	0.42154	5.76109	1.6627	0.43252	9.2261	2.52896	0.64908
	RPT (c) n = 3	5.58344	1.61868	0.42154	5.75898	1.66257	0.43251	9.22398	2.52882	0.64907
	Present n = 3	5.58344	1.61868	0.42154	5.75898	1.66257	0.43251	9.22398	2.52882	0.64907
	Present n = 5	5.60269	1.62041	0.42165	5.77823	1.66429	0.43263	9.24323	2.53055	0.64919
	Present n = 9	5.64677	1.62412	0.42191	5.82231	1.66801	0.43288	9.28731	2.53426	0.64944

Table 6 Continued

k	Theory	$(\overline{K}_w,\overline{K}_s)=(0,0)$			$(\overline{K}_w,\overline{K}_s)=(10,0)$			$(\overline{K}_w,\overline{K}_s)=(10,10)$		
		$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$
1	CPT [(a)]	3.17751	0.79438	0.19859	3.34112	0.83528	0.20882	6.57068	1.64267	0.41067
	FSDT [(b)]	2.67039	0.75837	0.19626	2.834	0.79928	0.20649	6.06356	1.60667	0.40834
	HSDT [(a)]	2.67153	0.7584	0.19627	2.83515	0.7993	0.20649	6.0647	1.60669	0.40835
	TPT [(a)]	2.67241	0.75845	0.19627	2.83603	0.79935	0.20649	6.06558	1.60674	0.40834
	RPT [(c)] $n=3$	2.67153	0.75840	0.19627	2.83514	0.79930	0.20649	6.06470	1.60669	0.40834
	Present $n=3$	2.67153	0.75840	0.19627	2.83514	0.79930	0.20649	6.06470	1.60669	0.40834
	Present $n=5$	2.67951	0.75908	0.19631	2.84312	0.79999	0.20654	6.07268	1.60738	0.40839
	Present $n=9$	2.69776	0.76056	0.19641	2.86137	0.80146	0.20664	6.09094	1.60885	0.40848
5	CPT [(a)]	2.90629	0.72657	0.18164	3.13305	0.78326	0.19582	7.60938	1.90234	0.47559
	FSDT [(b)]	2.35948	0.68678	0.17905	2.58625	0.74347	0.19322	7.06257	1.86255	0.47299
	HSDT [(a)]	2.27501	0.67931	0.17854	2.50179	0.736	0.19271	6.9781	1.85508	0.47248
	TPT [(a)]	2.27131	0.67895	0.17851	2.49808	0.73564	0.19268	6.9744	1.85472	0.47245
	RPT [(c)] $n=3$	2.27501	0.67931	0.17854	2.50178	0.73600	0.19271	6.97810	1.85508	0.47248
	Present $n=3$	2.27501	0.67931	0.17854	2.50178	0.73600	0.19271	6.97810	1.85508	0.47248
	Present $n=5$	2.30466	0.68201	0.17872	2.53144	0.73871	0.19290	7.00775	1.85779	0.47267
	Present $n=9$	2.34556	0.68559	0.17897	2.57233	0.74228	0.19314	7.04865	1.86136	0.47291
10	CPT [(a)]	2.9877	0.74693	0.18673	3.24365	0.81091	0.20273	8.29575	2.07394	0.51848
	FSDT [(b)]	2.36822	0.70108	0.18373	2.62416	0.76507	0.19972	7.67626	2.02809	0.51548
	HSDT [(a)]	2.27678	0.69269	0.18314	2.53273	0.75668	0.19914	7.58483	2.0197	0.5149
	TPT [(a)]	2.27551	0.69254	0.18313	2.53146	0.75653	0.19913	7.58356	2.01955	0.51489
	RPT [(c)] $n=3$	2.27679	0.69269	0.18314	2.53273	0.75668	0.19914	7.58483	2.01970	0.51490
	Present $n=3$	2.27679	0.69269	0.18314	2.53273	0.75668	0.19914	7.58483	2.01970	0.51490
	Present $n=5$	2.29950	0.69488	0.18330	2.55544	0.75886	0.19929	7.60754	2.02189	0.51505
	Present $n=9$	2.34110	0.69867	0.18356	2.59704	0.76266	0.19956	7.64914	2.02568	0.51531

[(a)] (Zenkour and Sobhy 2011); [(b)] (Yaghoobi and Torabi 2013a); [(c)] (Yaghoobi and Fereidoon 2014)

Table 7 Comparison of critical buckling temperature difference $\Delta T_{cr} \times 10^{-3}$ of square FG plate resting on elastic foundation under LTD

k	Theory	$(\overline{K}_w,\overline{K}_s)=(0,0)$			$(\overline{K}_w,\overline{K}_s)=(10,0)$			$(\overline{K}_w,\overline{K}_s)=(10,10)$		
		$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$
0	CPT [(a)]	13.66929	3.40982	0.84496	14.02036	3.49759	0.8669	20.95037	5.23009	1.30002
	FSDT [(b)]	11.15138	3.22725	0.83307	11.50246	3.31502	0.85501	18.43246	5.04752	1.28814
	HSDT [(a)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	TPT [(a)]	11.16112	3.22764	0.83309	11.5122	3.31541	0.85503	18.4422	5.04791	1.28816
	RPT [(c)] $n=3$	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814

Table 7 Continued

k	Theory	$(\overline{K}_w,\overline{K}_s)$ = (0,0)			$(\overline{K}_w,\overline{K}_s)$ = (10,0)			$(\overline{K}_w,\overline{K}_s)$ = (10,10)		
		a/h = 5	a/h = 10	a/h = 20	a/h = 5	a/h = 10	a/h = 20	a/h = 5	a/h = 10	a/h = 20
0	Present n = 3	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	Present n = 5	11.19537	3.23082	0.83331	11.54645	3.31859	0.85525	18.47646	5.05109	1.28838
	Present n = 9	11.28354	3.23824	0.83381	11.63462	3.32601	0.85575	18.56463	5.05852	1.28888
1	CPT [(a)]	5.94993	1.48045	0.36308	6.25678	1.55716	0.38226	12.31372	3.0714	0.76082
	FSDT [(b)]	4.99885	1.41292	0.35871	5.3057	1.48964	0.37789	11.36263	3.00387	0.75645
	HSDT [(a)]	5.00099	1.41297	0.35871	5.30784	1.48968	0.37789	11.36477	3.00391	0.75645
	TPT [(a)]	5.00264	1.41307	0.35872	5.30948	1.48978	0.37789	11.36642	3.00402	0.75645
	RPT [(c)] n = 3	5.00099	1.41297	0.35871	5.30784	1.48968	0.37789	11.36477	3.00391	0.75645
	Present n = 3	5.00099	1.41297	0.35871	5.30784	1.48968	0.37789	11.36477	3.00391	0.75645
	Present n = 5	5.01595	1.41426	0.35880	5.32280	1.49097	0.37798	11.37974	3.00520	0.75654
	Present n = 9	5.05019	1.41702	0.35898	5.35703	1.49374	0.37816	11.41397	3.00797	0.75672
5	CPT [(a)]	4.99396	1.24204	0.30405	5.3843	1.33962	0.32845	13.08936	3.26588	0.81002
	FSDT [(b)]	4.05274	1.17354	0.29959	4.44308	1.27113	0.32399	12.14814	3.19739	0.80555
	HSDT [(a)]	3.90735	1.16069	0.29871	4.2977	1.25827	0.3231	12.00275	3.18453	0.80467
	TPT [(a)]	3.90098	1.16006	0.29866	4.29132	1.25765	0.32306	11.99637	3.18391	0.80462
	RPT [(c)] n = 3	3.90735	1.16069	0.29871	4.29770	1.25827	0.32310	12.00275	3.18453	0.80467
	Present n = 3	3.90735	1.16069	0.29871	4.29770	1.25827	0.32310	12.00275	3.18453	0.80467
	Present n = 5	3.95839	1.16534	0.29903	4.34873	1.26293	0.32342	12.05379	3.18919	0.80499
	Present n = 9	4.02878	1.17149	0.29945	4.41913	1.26907	0.32385	12.12418	3.19534	0.80541
10	CPT [(a)]	5.28555	1.31474	0.32204	5.7391	1.42813	0.35039	14.69174	3.66629	0.90993
	FSDT [(b)]	4.18778	1.2335	0.31672	4.64132	1.34688	0.34506	13.59396	3.58504	0.9046
	HSDT [(a)]	4.02576	1.21864	0.31568	4.4793	1.33203	0.34403	13.43194	3.57019	0.90357
	TPT [(a)]	4.0235	1.21837	0.31566	4.47705	1.33176	0.34401	13.42969	3.56992	0.90355
	RPT [(c)] n = 3	4.02576	1.21864	0.31568	4.47930	1.33203	0.34403	13.43194	3.57019	0.90357
	Present n = 3	4.02576	1.21864	0.31568	4.47930	1.33203	0.34403	13.43194	3.57019	0.90357
	Present n = 5	4.06600	1.22251	0.31596	4.51954	1.33589	0.34430	13.47218	3.57405	0.90384
	Present n = 9	4.13972	1.22923	0.31642	4.59326	1.34261	0.34477	13.54590	3.58077	0.90431

[(a)] (Zenkour and Sobhy 2011); [(b)] (Yaghoobi and Torabi 2013a); [(c)] (Yaghoobi and Fereidoon 2014)

deformation effect which is neglected by CPT. In addition, an excellent agreement is demonstrated between the present model and RPT for all values of k and a / h.

In Fig. 1, the influence of the power law index (k) on the dimensionless critical buckling forces $\overline{N}$ of a square FG plate resting on elastic foundations is examined. It can be deduced from this figure that the dimensionless critical buckling load initially diminishes, and then the variation of curves are not significant by increasing in the value of the power law index. Fig. 2 shows the variation of the ΔT_{cr} versus the variation of the a / h for all three types of thermal loads. From this figure, it can be observed that ΔT_{cr} is highest for NTD compare with two other thermal loads. Moreover, with increasing the plate side-to-thickness ratio, the ΔT_{cr} diminishes.

Table 8 Comparison of critical buckling temperature difference $\Delta T_{cr} \times 10^{-3}$ of square FG plate resting on elastic foundation under NTD

k	Theory	$(\overline{K}_w,\overline{K}_s)$ = (0,0)			$(\overline{K}_w,\overline{K}_s)$ = (10,0)			$(\overline{K}_w,\overline{K}_s)$ = (10,10)		
		$a/h = 5$	$a/h = 10$	$a/h = 20$	$a/h = 5$	$a/h = 10$	$a/h = 20$	$a/h = 5$	$a/h = 10$	$a/h = 20$
0	CPT [(a)]	13.66929	3.40982	0.84496	14.02036	3.49759	0.8669	20.95037	5.23009	1.30002
	FSDT [(b)]	11.15138	3.22725	0.83307	11.50246	3.31502	0.85501	18.43246	5.04752	1.28814
	HSDT [(a)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	TPT [(a)]	11.16112	3.22764	0.83309	11.5122	3.31541	0.85503	18.4422	5.04791	1.28816
	RPT [(c)] $n = 3$	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	Present $n = 3$	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	Present $n = 5$	11.19537	3.23082	0.83331	11.54645	3.31859	0.85525	18.47646	5.05109	1.28838
	Present $n = 9$	11.28354	3.23824	0.83381	11.63462	3.32601	0.85575	18.56463	5.05852	1.28888
1	CPT [(a)]	8.25905	2.055	0.50399	8.68499	2.16148	0.53061	17.09257	4.26338	1.05608
	FSDT [(b)]	6.93886	1.96127	0.49792	7.36479	2.06775	0.52454	15.77238	4.16965	1.05002
	HSDT [(a)]	6.94183	1.96133	0.49792	7.36777	2.06781	0.52455	15.77535	4.16971	1.05002
	TPT [(a)]	6.94412	1.96147	0.49793	7.37005	2.06796	0.52455	15.77763	4.16985	1.05003
	RPT [(c)] $n = 3$	6.94183	1.96133	0.49792	7.36777	2.06781	0.52455	15.77535	4.16971	1.05002
	Present $n = 3$	6.94183	1.96133	0.49792	7.36777	2.06781	0.52455	15.77535	4.16971	1.05002
	Present $n = 5$	6.96261	1.96312	0.49805	7.38854	2.06960	0.52467	15.79612	4.17150	1.05014
	Present $n = 9$	7.01012	1.96696	0.49830	7.43606	2.07344	0.52492	15.84364	4.17534	1.05040
5	CPT [(a)]	6.24563	1.55334	0.38026	6.73381	1.67538	0.41077	16.37004	4.08444	1.01304
	FSDT [(b)]	5.06851	1.46768	0.37468	5.55669	1.58972	0.40519	15.19291	3.99878	1.00745
	HSDT [(a)]	4.88668	1.4516	0.37357	5.37486	1.57364	0.40408	15.01109	3.9827	1.00635
	TPT [(a)]	4.87871	1.45082	0.37352	5.36688	1.57286	0.40403	15.00311	3.98192	1.00629
	RPT [(c)] $n = 3$	4.88668	1.45160	0.37357	5.37486	1.57364	0.40408	15.01109	3.98270	1.00635
	Present $n = 3$	4.88668	1.45160	0.37357	5.37486	1.57364	0.40408	15.01109	3.98270	1.00635
	Present $n = 5$	4.95051	1.45742	0.37398	5.43869	1.57946	0.40449	15.07492	3.98852	1.00675
	Present $n = 9$	5.03855	1.46511	0.37450	5.52672	1.58715	0.40501	15.16295	3.99621	1.00728
10	CPT [(a)]	6.10899	1.51957	0.37221	6.6332	1.65062	0.40497	16.98057	4.23746	1.05169
	FSDT [(b)]	4.8402	1.42567	0.36606	5.3644	1.55672	0.39882	15.71178	4.14356	1.04553
	HSDT [(a)]	4.65293	1.40849	0.36486	5.17714	1.53954	0.39763	15.52451	4.12639	1.04434
	TPT [(a)]	4.65033	1.40818	0.36484	5.17453	1.53923	0.3976	15.52191	4.12608	1.04432
	RPT [(c)] $n = 3$	4.65293	1.40849	0.36486	5.17714	1.53954	0.39763	15.52451	4.12639	1.04434
	Present $n = 3$	4.65293	1.40849	0.36486	5.17714	1.53954	0.39763	15.52451	4.12639	1.04434
	Present $n = 5$	4.69944	1.41296	0.36518	5.22365	1.54401	0.39794	15.57102	4.13086	1.04465
	Present $n = 9$	4.78465	1.42073	0.36572	5.30885	1.55178	0.39848	15.65623	4.13862	1.04519

[(a)] (Zenkour and Sobhy 2011); [(b)] (Yaghoobi and Torabi 2013a); [(c)] (Yaghoobi and Fereidoon 2014)

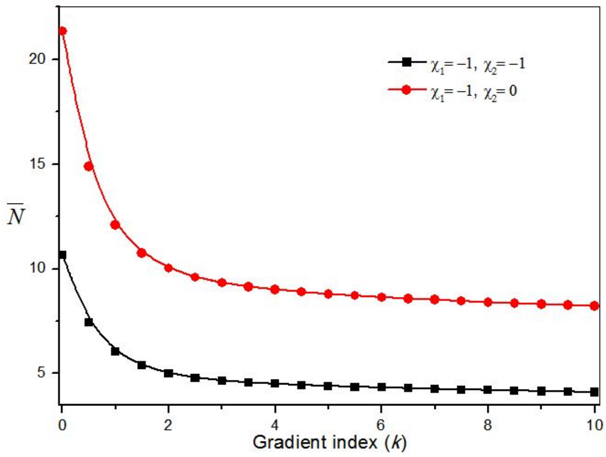

Fig. 1 Effect of the gradient index on the non-dimensional critical buckling load $\overline{N}$ of a square FG plate resting on elastic foundations ($a / h = 10$, $\overline{K}_w = 100$, $\overline{K}_s = 10$)

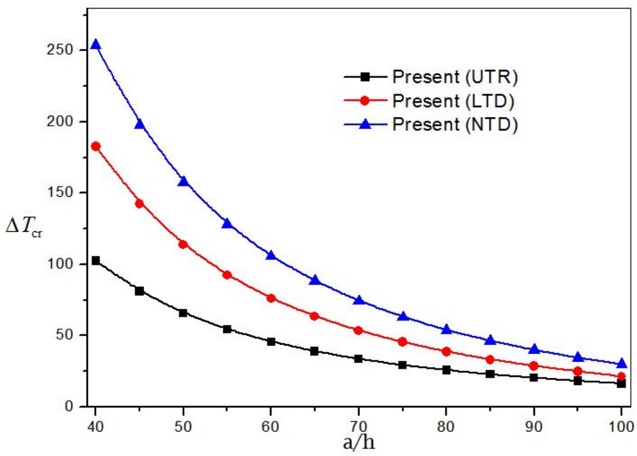

Fig. 2 Effect of the side-to-thickness ratio on the ΔT_{cr} of a square FG plate resting on elastic foundations ($a / h = 20, \overline{K}_w = 10, , \overline{K}_s = 10, \quad k = 1$)

6. Conclusions

An efficient and simple *n*th-order shear deformation theory is proposed and applied in the present paper for the stability investigation of FG plates resting on Winkler-Pasternak elastic foundations. By proposing further simplifying assumptions to the conventional HSDTs, with considering undetermined integral term, the number of unknowns and governing equations of the present theory are reduced by one, and thus, make this theory simple and efficient to use. By using the principle of minimum total potential energy, the governing differential equations are obtained and the analytical solutions based upon Navier solution procedure are also determined. Various numerical examples are examined to demonstrate the accuracy and efficacy of the developed model. Results prove that the proposed theory can be comparable with the conventional HSDTs with a larger number of variables. In the general case, the results calculated by $n = 3$ are in good agreement with the conventional HSDTs with a larger number of variables. An improvement of present approach will be considered in the future study to account for the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al*. 2013, Saidi *et al*. 2013, Bousahla *et al*. 2014, Bourada *et al*. 2015, Belabed *et al*. 2014, Fekrar *et al*. 2014, Hebali *et al*. 2014, Larbi Chaht *et al*. 2014, Bennai *et al*. 2015, Meradjah *et al*. 2015, Hamidi *et al*. 2015, Draiche *et al*. 2016, Bennoun *et al*. 2016).

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