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A new approach for the cylindrical cavity expansion problem incorporating deformation dependent of intermediate principal stress

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Abstract. The problem of cylindrical cavity expansion incorporating deformation dependent of intermediate principal stress in rock or soil mass is investigated in the paper. Assumptions that the initial axial total strain is a non-zero constant and the axial plastic strain is not zero are defined to obtain the numerical solution of strain which incorporates deformation-dependent intermediate principal stress. The numerical solution of plastic strains are achieved by the 3-D plastic potential functions based on the M-C and generalized H-B failure criteria, respectively. The intermediate principal stress is derived with the Hook's law and plastic strains. Solution of limited expansion pressure, stress and strain during cylindrical cavity expanding are given and the corresponding calculation approaches are also presented, which the axial stress and strain are incorporated. Validation of the proposed approach is conducted by the published results.

Keywords: quasi-plane strain-softening problem; intermediated principal stress; 3-D plastic potential function; deformation dependent; cylindrical cavity expansion

1. Introduction

Cylindrical cavity expansion theory has been widely developed and applied to modelling complex geotechnical problems, such as determination of foundation bearing capacity, stability analysis of surrounding rock and soil characterizations based on pressuermeter tests. Many researchers have proposed the analytical and semi-analytical solution based on the Mohr-Coulomb and Hoek-Brown media and solved many engineering problems (Vesic 1972, Carter *et al*. 1985, Yu 2000, Chen 2012a, b, Yang *et al*. 2013, 2014, Mohammadi and Tavakoli 2015, Wang *et al*. 2010, 2012, Xiao *et al*. 2014a, b, 2015, Xiao and Liu 2016, Ning *et al*. 2015, Zhou *et al*. 2014, 2015, 2016). Although solutions of cavity expansion based on the elastic-plastic and critical state modes are proposed and solving many engineering problems, few of literatures have been focused on the effect of out-of-plane stress in geotechnical problems of cylindrical cavity expansion, especially for the quasi-plane strain-softening problem. For example, Zou and He (2016), Zou and Su (2016), Zou and Li (2015) and Zou and Zuo (2017) developed some approaches for the cavity expansion

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with unloading incorporating the effects of hydraulic-mechanical coupling and out-of-plane stress. Wang *et al*. (2012) studied the influence of out-of-plane stress on the distribution of stress, strain, and displacement based on plane strain assumption. Pan and Brown (1996) proposed an approach in which the axial in-situ stress of the plastic zone is deformation dependent and the formula for the calculation of intermediate principal stress was derived, only numerical solution through finite element method was presented.

In engineering practice, if the layering is a general homogeneous, isotropic, and porous rock mass, then the repeatability distance is quite arbitrary. In such a situation, the influence of the out-of-plane stress must be considered. For this situation, any analysis will be carried out in a two-dimensional plane with unit thickness with consideration of the out-of-plane stress. In the situations above, ignoring the out-of-plane stress may lead to appreciable error in a cavity expansion. In the presented solution, the 3-D plastic potential functions based on the M-C and generalized H-B failure criteria are adopted, the numerical stepwise considering quasi-plane strain-softening behavior is improved, the 3-D mechanical characteristics of soil or rock masses can be considered properly in the analysis of cylindrical cavity expansion problems. A new approach are proposed for the mechanical analysis of cylindrical cavity expansion considering the influence of intermediate principal stress and strain, the corresponding theoretical solutions and calculation approaches incorporating the deformation-dependent intermediate principal stress and axial strain are proposed.

2. Methodology

2.1 Definition of the problem and assumptions

If the axial length of cavity is long enough, it can be treated as a plane problem with the assumption that ε_z is a no-zero constant, which is called a quasi-plane strain-softening problem in this paper. In presented study, the out-of-plane stress is the intermediate one in principle stress space. Model of the quasi-plane strain-softening problem is shown in Fig. 1.

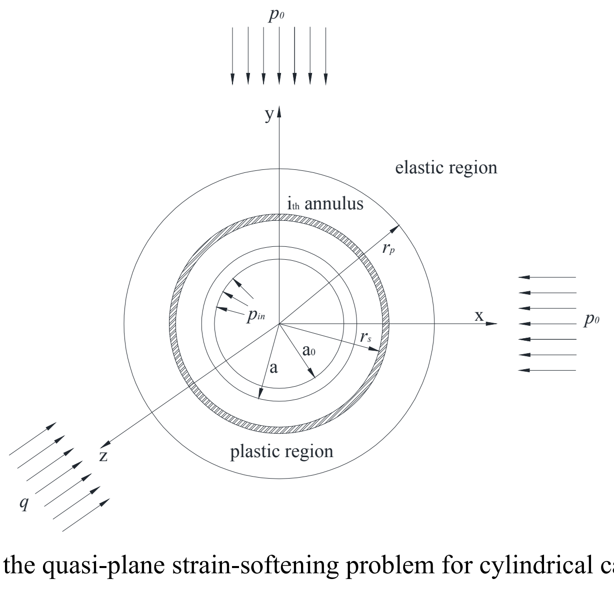


Fig. 1 Model of the quasi-plane strain-softening problem for cylindrical cavity expansion

In Fig. 1, a_0 is an initial radius; a is the radius after the cylindrical cavity expansion. p is a hydrostatic pressure, q is the axial stress (σ_z) along the axis of the cylindrical cavity; r_p is plastic radius, r_s is plastic radius for strain-softening region.

2.2 Failure criteria

The yielding function of rock mass is expressed as following equation.

$$F\left(\sigma_1,\ \sigma_3,\ \gamma^p\right) = \sigma_1 - \sigma_3 - H\left(\sigma_3,\ \gamma^p\right) \tag{1}$$

where, σ_1 and σ_3 are the major and minor principle stresses, respectively; γ^p is deviatoric plastic strain that controls the evolution of strain-softening parameter in softening region, and it is generally expressed as $\gamma^p = \gamma_1^p - \gamma_s^p$, in which γ_1^p and γ_s^p are the major and minor deviatoric strain, respectively.

For the M-C failure criteria, H in Eq. (1) becomes

$$H^{MC}\left(\sigma_3, \gamma^p\right) = \left(N\left(\gamma^p\right) - 1\right)\sigma_3 + Y\left(\gamma^p\right) \tag{2}$$

where, N and Y are strength parameters defined by the cohesion $c(\gamma^p)$ and internal friction angle ϕ (γ^p) as follows:

For the M-C failure criteria, H in Eq. (1) becomes

$$N\left(\gamma^p\right) = \frac{1 + \sin\phi\left(\gamma^p\right)}{1 - \sin\phi\left(\gamma^p\right)};\quad Y\left(\gamma^p\right) = \frac{2c\left(\gamma^p\right)\cos\phi\left(\gamma^p\right)}{1 - \sin\phi\left(\gamma^p\right)} \tag{3}$$

If the generalized H-B failure criteria are adopted, H in Eq. (1) can be expressed as

$$H^{HB}\left(\sigma_3, \gamma^p\right) = \sigma_c\left(\gamma^p\right)\left(m\left(\gamma^p\right)\frac{\sigma_3}{\sigma_c\left(\gamma^p\right)} + s\left(\gamma^p\right)\right)^{a\left(\gamma^p\right)} \tag{4}$$

Where, σ_c is uniaxial compressive strength of the intact rock; m, s, and a are the strength parameters of the generalized H-B failure criteria.

2.3 Plastic potential function

In terms of the results in Pan and Brown (1996), the plastic potential function for 3-D M-C failure criteria and generalized H-B failure criteria can be expressed, respectively, as

$$Q^{MC}\left(\sigma\right) = \sqrt{J_2} - \alpha I_1 \tag{5a}$$

$$Q^{HB}\left(\sigma\right) = -\frac{n}{3}I_1 + \frac{3}{\sigma_c}J_2 + \frac{\sqrt{3}}{2}n\sqrt{J_2} \tag{5b}$$

where, $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $J_2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]/6$. σ_1, σ_2, σ_3 are the major, intermediate, and minor principal stress, respectively. α and n are the dilation parameter as expressed in Pan and Brown (1996).

Based on the plastic flow rule, the plastic strain increment can be expressed as

$$d\varepsilon^p = \lambda \frac{\partial Q}{\partial \sigma} \tag{6}$$

When the M-C failure criteria is used, the increment of major, intermediate, and minor plastic strains can be given by

$$d\varepsilon_1^p = \left(\frac{(2\sigma_1 - \sigma_2 - \sigma_3)}{6\sqrt{J_2}} - \alpha \right) d\lambda \tag{7a}$$

$$d\varepsilon_2^p = \left(\frac{(2\sigma_2 - \sigma_1 - \sigma_3)}{6\sqrt{J_2}} - \alpha \right) d\lambda \tag{7b}$$

$$d\varepsilon_3^p = \left(\frac{(2\sigma_3 - \sigma_2 - \sigma_1)}{6\sqrt{J_2}} - \alpha \right) d\lambda \tag{7c}$$

For generalized H-B failure criteria, the increment of major, intermediate and minor plastic strains can be described as

$$d\varepsilon_1^p = \left[\left(\frac{\sqrt{3}(2\sigma_1 - \sigma_2 - \sigma_3)}{12\sqrt{J_2}} - \frac{1}{3} \right) n + \frac{1}{\sigma_c}(2\sigma_1 - \sigma_2 - \sigma_3) \right] d\lambda \tag{8a}$$

$$d\varepsilon_2^p = \left[\left(\frac{\sqrt{3}(2\sigma_2 - \sigma_3 - \sigma_1)}{12\sqrt{J_2}} - \frac{1}{3} \right) n + \frac{1}{\sigma_c}(2\sigma_2 - \sigma_3 - \sigma_1) \right] d\lambda \tag{8b}$$

$$d\varepsilon_3^p = \left[\left(\frac{\sqrt{3}(2\sigma_3 - \sigma_2 - \sigma_1)}{12\sqrt{J_2}} - \frac{1}{3} \right) n + \frac{1}{\sigma_c}(2\sigma_3 - \sigma_2 - \sigma_1) \right] d\lambda \tag{8c}$$

where, ε_1^p, ε_2^p and ε_3^p are the major, intermediate and minor principal strains, respectively. $d\lambda$ is the plastic constant.

3. Theoretical solution

3.1 Equilibrium equation and stress boundary conditions

Stress equilibrium differential equation for the quasi-plane strain-softening problem of cylindrical cavity expansion can be expressed by

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \tag{9}$$

where, σ_r and σ_θ are the radial and tangential stresses, respectively.

3.2 Stress and displacement in the elastic region

Based on the boundary conditions ($\sigma_r|_{r=r_0}=p_{in}$ and $\lim\limits_{r\to\infty}\sigma_r = p_0$) and the generalized Hooke's law, stress and displacements in elastic zone are given by

$$\sigma_r = p_0-\left(p_0-\sigma_{r_p}\right)\left(\frac{r_p}{r}\right)^2 \tag{10}$$

$$\sigma_\theta = p_0+\left(p_0-\sigma_{r_p}\right)\left(\frac{r_p}{r}\right)^2 \tag{11}$$

$$\sigma_z=\upsilon\left(\sigma_\theta+\sigma_r\right)-2\upsilon p_0+q \tag{12}$$

$$u=\frac{1}{2G}\left(\sigma_{r_p}-p_0\right)\frac{r_p^2}{r} \tag{13}$$

where, r_p is the plastic radius, v is the Poisson's ratio and $G = E/[2(1 + v)]$ is the Shear modulus.

4. Stress and displacement in the plastic region

In order to obtain the analytical solutions of stress and displacements for the quasi-plane strain-softening problem of cylindrical cavity expansion considering the influences of out-of-plane stress and axial strain, the numerical stepwise procedure is reconstructed as follows.

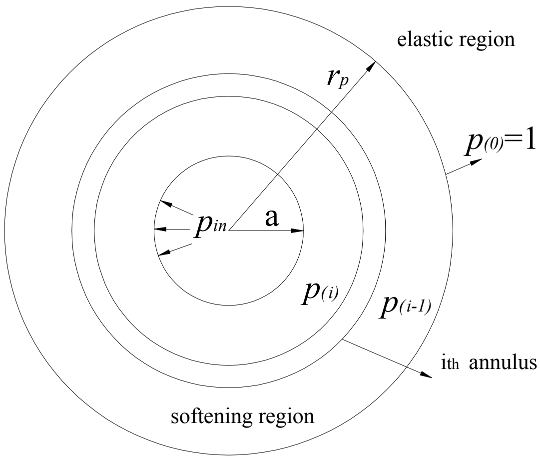


Fig. 2 Normalized plastic region with finite number of annuli

As shown in Fig. 2, the total plastic region is presumably divided into n concentric annuli. The ith annulus is bounded by two circles of normalized radius $\rho_{(j-1)} = r_{(j-1)}/R$ and $\rho_{(j)} = r_{(j)}/R$. $\rho_{(0)} = 1$ is the outer boundary of plastic region, which means $r = R$, and the rock or soil mass remains the critical state of the plastic. The stress at outer boundary of plastic region are defined, respectively, as $\sigma_{r(0)} = \sigma_R$, $\sigma_{\theta(0)} = 2p_0 - \sigma_R$ and $\sigma_{z(0)}=q$. σ_R is radial stress at elastic-plastic interface that can be obtained with yield functions.

It is assumed that the radial stress increment of each concentric annuli is equal, and the radial stress decreases gradually from p_{in} to σ_R with the increase of radius after n times. While, the increment of $\Delta\sigma_r$ is assumed equal for each annulus, the actual thickness of the annuli is different because the radii of annuli are determined by the equilibrium equation. Thus, the stress decreases non-linearly from p_{in} to σ_R and increment can be expressed by

$$\Delta\sigma_r = \frac{p_{in} - \sigma_R}{n} \tag{14}$$

where, n is the number of subdivisions and $n = 500$ in this study.

Based on the M-C and generalized H-B failure criteria, the circumferential stress can be written as

$$\sigma_{\theta(i)}^{MC} = \frac{\sigma_{r(i)} - Y_{(i-1)}}{N_{(i-1)}} \tag{15a}$$

$$\sigma_{\theta(i)}^{HB} = \sigma_{r(i)} - \sigma_c \left(\frac{m_{(i-1)}\sigma_{\theta(i)}}{\sigma_c} + s_{(i-1)} \right)^{a_{(i-1)}} \tag{15b}$$

With the property of deformation dependent considered, the intermediate principal stress σ_z can be obtained through Hook's law and out-of-plane plastic strain.

$$\sigma_{z(i)} = v\left(\sigma_{r(i)} + \sigma_{\theta(i)}\right) + (1-2v)\sigma_0 - E\varepsilon_{z(i)}^p \tag{16}$$

where ε_z^p is out-of-plane plastic strain component that cannot be determined to be zero and must be used to evaluate the axial stress σ_z, here, it can be calculated according to the flow rule (Eq. (6)).

The increments of radial, circumferential and axial stresses can be obtained by the following equations.

$$\begin{Bmatrix} \Delta\sigma_{r(i)} \\ \Delta\sigma_{\theta(i)} \\ \Delta\sigma_{z(i)} \end{Bmatrix} = \begin{Bmatrix} \sigma_{r(i)} - \sigma_{r(i-1)} \\ \sigma_{\theta(i)} - \sigma_{\theta(i-1)} \\ \sigma_{z(i)} - \sigma_{z(i-1)} \end{Bmatrix} \tag{17}$$

According to the Hook's law, the elastic stress-strain relationship can be expressed as follows.

$$\begin{bmatrix} \varepsilon_r^e \\ \varepsilon_\theta^e \\ \varepsilon_z^e \end{bmatrix} = \frac{1}{E} \begin{bmatrix} \sigma_r - v(\sigma_\theta + \sigma_z) - (p_0 - vp_0 - vq) \\ \sigma_\theta - v(\sigma_r + \sigma_z) - (p_0 - vp_0 - vq) \\ \sigma_z - v(\sigma_\theta + \sigma_r) - (q - 2vp_0) \end{bmatrix} \tag{18}$$

The total strain in plastic region contains elastic and plastic strains, it can be obtained as follows

$$\begin{pmatrix} \varepsilon_{\theta(i)} \\ \varepsilon_{r(i)} \\ \varepsilon_{z(i)} \end{pmatrix} = \begin{pmatrix} \varepsilon_{\theta(i-1)} + \Delta\varepsilon^{e}_{\theta(i)} + \Delta\varepsilon^{p}_{\theta(i)} \\ \varepsilon_{r(i-1)} + \Delta\varepsilon^{e}_{r(i)} + \Delta\varepsilon^{p}_{r(i)} \\ \varepsilon_{z(i-1)} + \Delta\varepsilon^{e}_{z(i)} + \Delta\varepsilon^{p}_{z(i)} \end{pmatrix} \tag{19}$$

By limited difference method, the stress equilibrium differential equation can be rewritten approximately as

$$\frac{\Delta\sigma_{r(i)}}{\Delta\rho_{(i)}} + \frac{H\left(\sigma^{*}_{\theta(i)}\right)}{\rho^{*}_{(i)}} = 0 \tag{20}$$

where, $\rho_{(i)} = r_{(i)}/R$, $\rho^{*}_{(i)} = (\rho_{(i)} + \rho_{(i-1)})/2$ and $\sigma^{*}_{\theta i} = (\sigma_{\theta(i)} + \sigma_{\theta(i-1)})/2$. For M-C and generalized H-B failure criteria, the $H(\sigma^{*}_{\theta i})$ can be expressed, respectively, as

$$H^{MC}\left(\sigma^{*}_{\theta(i)}\right) = (m_{(i-1)} - 1)\sigma_{\theta(i)} + \sigma_c$$

$$H^{HB}\left(\sigma^{*}_{\theta(i)}\right) = \sigma_c\left(m_{(i-1)}\sigma^{*}_{\theta(i)} / \sigma_c + s_{(i-1)}\right)^{a_{(i-1)}}$$

Using limited difference method, compatible equation can be expressed approximately as

$$\Delta\varepsilon^{p}_{\theta(i)}\left(\frac{1}{\Delta\rho_{(i)}} + \frac{(k_3 - k_1)}{\rho^{*}_{(i)}k_3}\right) = \frac{1+v}{E}\frac{H\left(\sigma^{*}_{\theta(i)}\right)}{\rho^{*}_{(i)}} - \frac{\Delta\varepsilon^{e}_{\theta(i)}}{\Delta\rho_{(i)}} - \frac{1}{\rho^{*}_{(i)}}\left(\varepsilon^{p}_{\theta(i-1)} - \varepsilon^{p}_{r(i-1)}\right) \tag{21}$$

where, $\Delta\rho_{(i)} = \rho_{(i)} - \rho_{(i-1)}$, for M-C failure criteria, there are

$$k_1 = \left(2\sigma_r - \sigma_\theta - \sigma_z\right) \Big/ \left(6J_2^{1/2}\right) - \alpha$$

$$k_3 = \left(2\sigma_\theta - \sigma_r - \sigma_z\right) \Big/ \left(6J_2^{1/2}\right) - \alpha$$

for generalized H-B failure criteria, it leads to

$$k_1 = \left[\sqrt{3}\left(2\sigma_r - \sigma_\theta - \sigma_z\right) \Big/ \left(12J_2^{1/2}\right) - 1/3\right] + \left(2\sigma_r - \sigma_\theta - \sigma_z\right) \Big/ \sigma_c$$

$$k_3 = \left[\sqrt{3}\left(2\sigma_\theta - \sigma_r - \sigma_z\right) \Big/ \left(12J_2^{1/2}\right) - 1/3\right] + \left(2\sigma_\theta - \sigma_r - \sigma_z\right) \Big/ \sigma_c$$

According to Eq. (21), the increments of circumferential plastic strain, radial and axial strains can be obtained as follows

$$\Delta\varepsilon^{p}_{\theta(i)} = \frac{\frac{1+v}{E}\frac{H\left(\sigma^{*}_{\theta(i)}\right)}{\rho^{*}_{(i)}} - \frac{\Delta\varepsilon^{e}_{\theta(i)}}{\Delta\rho_{(i)}} - \frac{1}{\rho^{*}_{(i)}}\left(\varepsilon^{p}_{\theta(i-1)} - \varepsilon^{p}_{r(i-1)}\right)}{\left(\frac{1}{\Delta\rho_{(i)}} + \frac{\left(k_3 - k_1\right)}{\rho^{*}_{(i)}k_3}\right)} \tag{22}$$

Then, the increments of radial and axial strains can be obtained based on 3-D plastic potential functions, respectively, as follows

$$\begin{pmatrix} \Delta\varepsilon^{p}_{r(i)} \\ \Delta\varepsilon^{p}_{z(i)} \end{pmatrix} = \frac{\Delta\varepsilon^{p}_{\theta(i)}}{k_3}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \tag{23}$$

where, for M-C and generalized H-B failure criteria, there are

$$k_2 = \left(2\sigma_z - \sigma_\theta - \sigma_r\right)\Big/\left(6J_2^{1/2}\right) - \alpha$$

$$k_2 = \left[\sqrt{3}\left(2\sigma_z - \sigma_\theta - \sigma_r\right)\Big/\left(12J_2^{1/2}\right) - 1/3\right] + \left(2\sigma_z - \sigma_\theta - \sigma_r\right)\Big/\sigma_c$$

Displacements in plastic region can be obtained with following equations.

$$u = \varepsilon_{\theta(i)}\rho_{(i)}R \tag{24}$$

5. Validations

To confirm the validity and accuracy of the proposed approach, the results of proposed approach are compared with those solution presented by Vesic (1972). Results based on the M-C and generalized H-B failure criteria are shown in Table 1. The calculation parameters for M-C failure criteria are adopted from Vesic (1972) as follows: p_0 = 0.1 MPa, E = 3.0 MPa, v = 0.35, a_0 = 0.25 m, q = 0.1 MPa, c_p = 12 MPa, c_r = 10 MPa, φ_p = 9°, φ_r = 7°, and γ^p = 0.016.

The calculation parameters for generalized H-B failure criteria are determined based on Sharan (2005) as follows: a_0 = 0.25 m, p_0 = 10 MPa, E = 5500 MPa, v = 0.25, m = 1.7, s = 0.0039, a =

Table 1 Results comparison of the presented approach based on M-C failure criteria and Vesic's solution (1972) for the rock or soil mass

r_p/a_u		2	3	4	5	6	7
	Vesic	0.1694	0.1978	0.2200	0.2384	0.2543	0.2683
p (MPa)	M-C-1*	0.1695	0.1980	0.2200	0.2384	0.2543	0.2685
	M-C-2	0.1638	0.1850	0.2025	0.2285	0.2389	0.2482

* M-C-1: Cavity expansion pressures un-considering quasi-plane strain-softening;
M-C-2: Cavity expansion pressures considering quasi-plane strain-softening

Table 2 Result comparison of the proposed approach based on generalized H-B failure criteria and Vesic's solution (1972) for the rock or soil mass

r_p/a_u		2	3	4	5	6	7
	Vesic	21.9	27.6	32.4	36.6	40.4	43.9
p (MPa)	H-B-1*	22.8	28.3	33.0	37.0	40.5	43.6
	H-B-2	21.3	25.5	28.9	31.5	33.8	35.8

* H-B-1: Cavity expansion pressures un-considering quasi-plane strain-softening problem;
H-B-2: Cavity expansion pressures considering quasi-plane strain-softening problem

Table 3 Result comparison of the presented approach based on M-C and generalized H-B failure criteria

r_p/a_u		2	3	4	5	6	7
	Vesic	21.9	27.6	32.4	36.6	40.4	43.9
p (MPa)	H-B	22.8	28.3	33.0	37.0	40.5	43.6
	M-C	22.5	27.8	32.6	36.8	40.5	44.0

0.55, σ_c = 10 MPa and ψ = 0°. However, the Vesic's solution (1972) for rock mass is based on the M-C failure criteria. To compare the results of proposed solution, the technique of the equivalent M-C and generalized H-B failure criteria strength parameters is adopted (Yang and Yin 2010). The strength parameters for M-C failure criteria are as follows: c = 1.36 MPa and φ = 18.85°.

As shown in Tables 1 and 2, the cavity expansion pressures of the proposed approach based on M-C and generalized H-B failure criteria are in well agreement with Vesic's solution (1972) when quasi-plane strain-softening unconsidered. The comparison results show that difference of expansion pressure p is small in this condition for cylindrical cavity expansion. The proposed numerical stepwise method is confirmed to be effective in the analysis of the cylindrical cavity expansion problem. While, the cavity expansion pressures of the proposed approach based on M-C and generalized H-B failure criteria are smaller than those when the quasi-plane strain-softening problem is un-considered. The reason may be that the properties of rock or soil masses are better when quasi-plane strain-softening problem considered.

Table 3 shows the comparison results of the presented approach based on M-C and generalized H-B failure criteria when the quasi-plane strain-softening problem is un-considered. Although the results of the generalized H-B failure criteria are more than those of M-C failure criteria, the differences are no more than 5%. Therefore, for the engineering design, the calculation approach based on M-C failure criteria is enough.

6. Numerical analysis and discussions

In order to investigate the effects of strain-softening, dilation parameter on the proposed approach with the quasi-plane strain-softening problem considered, several examples are performed. The input data of the proposed solution based on the H-B failure criteria presented by Sharan (2005) are as follows: σ_0 = 10 MPa, E = 5500 MPa, v = 0.25, p_{in} = 30 MPa, r_0 = 0.25 m, σ_c = 10 MPa, q = 10 MPa, m_p = 1.7, s_p = 0.0039, a_p = 0.55, m_r = 0.8, s_r = 0.0019, a_r = 0.5 and n = 0. With the technique of the equivalent M-C and generalized H-B failure criteria used, the strength parameter for M-C failure criteria are obtained as follows: c_p = 1.36 MPa, c_r = 1.15 MPa, φ_p = 18.85° and φ_r = 13.22°.

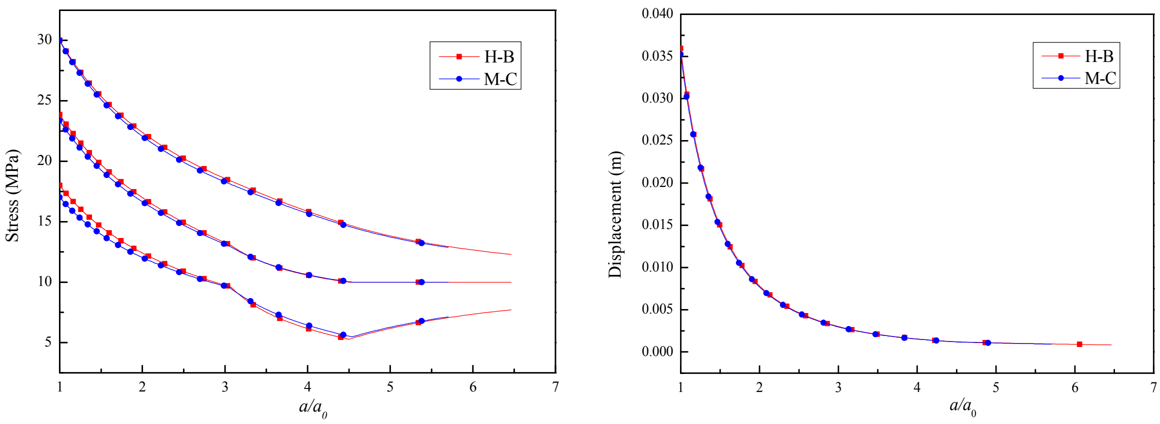
6.1 Results based on M-C and generalized H-B failure criteria

To analyze the difference on the stress and displacement between M-C and generalized H-B failure criteria, the results with the proposed approach based on M-C and generalized H-B failure criteria are shown in Fig. 3.

As can be seen in Fig. 3, there is almost no the difference between the results of M-C and generalized H-B failure criteria with the equivalent input data. For example, the plastic radius and displacement for M-C failure criteria are 1.135 m and 0.0353 m, respectively. For the generalized H-B failure criteria, the plastic radius and displacement are 1.125 m and 0.0359 m. The differences are 0.8% and 1.7%, respectively.

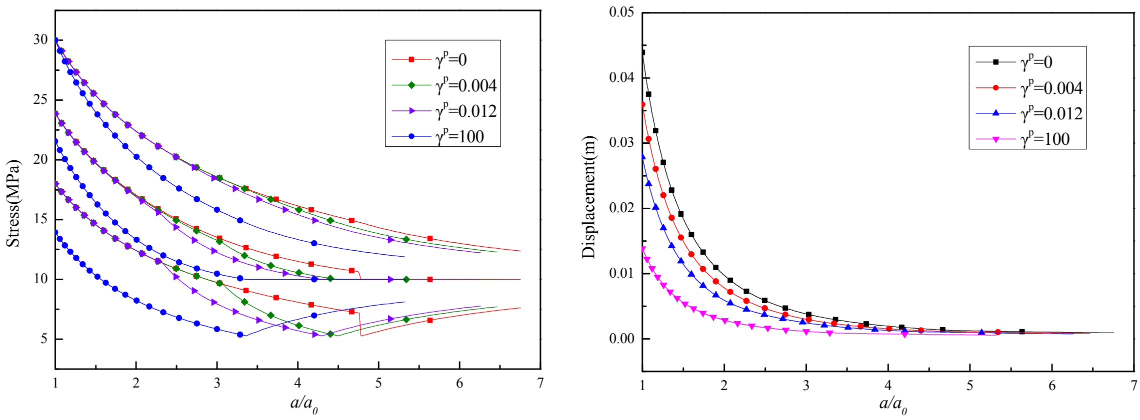
6.2 Effects of the strain-softening parameters

Effects of softening parameters on stress and displacement are shown in Figs. 4 and 5 for M-C



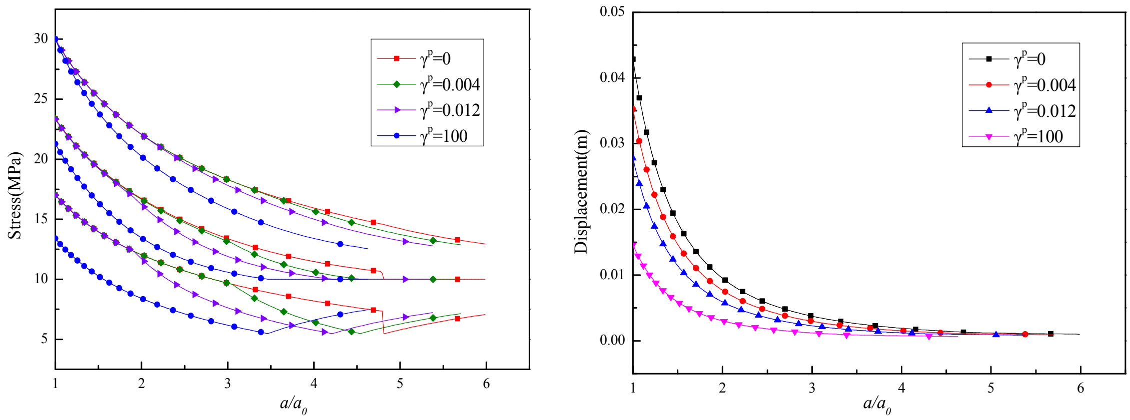
(a) Stresses (b) Displacements

Fig. 3 Displacement and stress with M-C and generalized H-B failure criteria



(a) Stresses (b) Displacements

Fig. 4 Displacement and stress with the different critical values of strain-softening parameters for H-B failure criteria



(a) Stresses (b) Displacements

Fig. 5 Displacement and stress with the different critical values of strain-softening parameters for M-C failure criteria l

and generalized H-B failure criteria, respectively, corresponding to four cases: case 1: $\gamma^p = 0$; case 2: $\gamma^p = 0.004$; case 3: $\gamma^p = 0.012$; case 4: $\gamma^p = 100$.

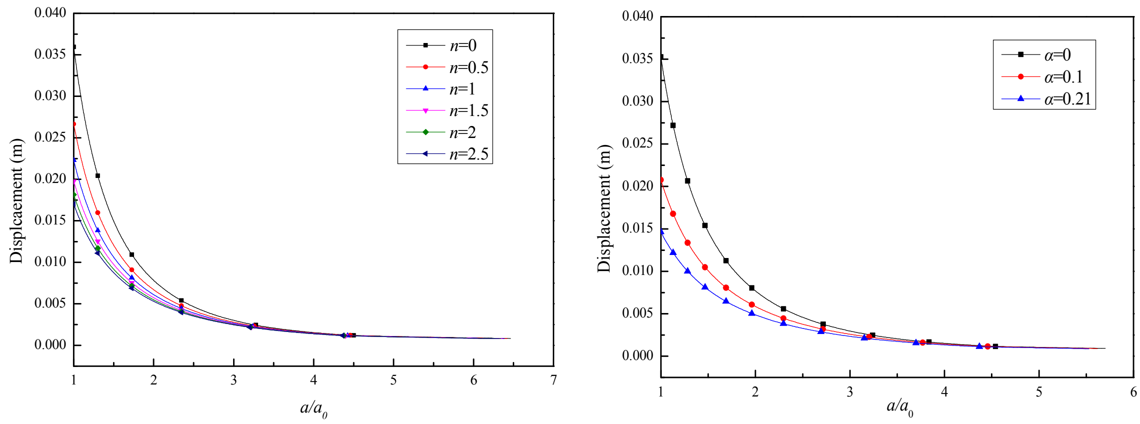
Figs. 4 and 5 show that both the plastic radius and displacement for M-C and generalized H-B failure criteria decrease with the softening parameter increasing. It can be seen from Fig. 3 that the displacements for H-B failure criteria reduce significantly when γ^p increases from 0 to 100. For example, displacement are 0.044 m, 0.036 m, 0.028 m and 0.014 m when γ^p equals to 0, 0.004, 0.012 and 100, respectively. While, for M-C failure criteria, the displacements are 0.043 m, 0.035 m, 0.028 m and 0.015 m when γ^p equals to 0, 0.004, 0.012 and 100, respectively. It can be found that the displacements for M-C and generalized H-B failure criteria are similar with the same softening parameter. The strain-softening parameter has the similar significantly effect on the displacements for M-C and generalized H-B failure criteria.

6.3 Effect of dilation parameter

In order to investigate the effects of dilation of rock or soil mass, displacements that consider intermediate principal stress for M-C and generalized H-B failure criteria under the different dilation parameters are shown in Fig. 6.

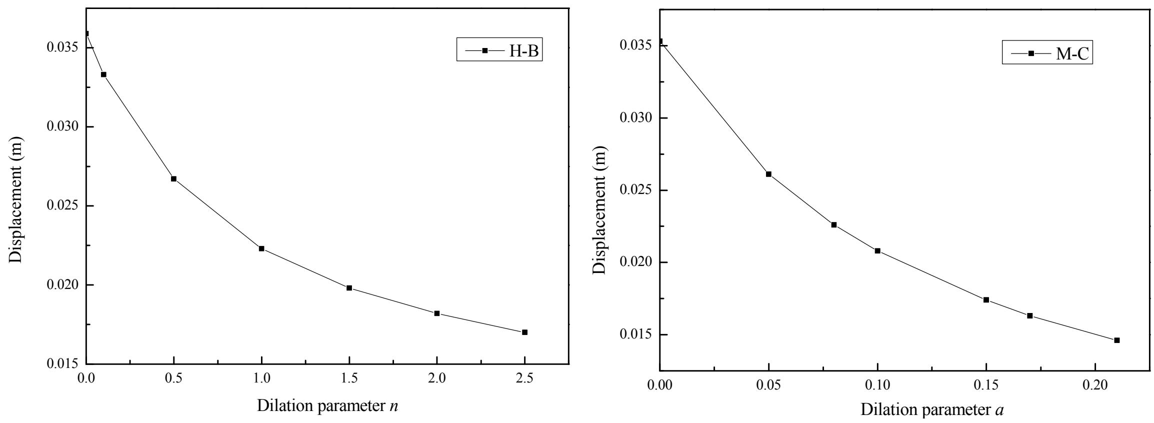
As shown in Fig. 6, the effect of dilation parameters is significant for both M-C and generalized H-B models. For M-C and generalized H-B failure criteria, the displacements all decrease with the increasing of parameters. However, the effect of dilation parameters on the results based on M-C and generalized H-B failure criteria are inconsistent. For example, the displacements for H-B failure criteria are 0.036 m, 0.027 m, 0.022 m, 0.018 m and 0.017 m when the dilation parameter n equals to 0, 0.5, 1, 2 and 2.5, respectively. While, for M-C failure criteria, the displacements are 0.035 m, 0.021 m and 0.015 m when the dilation parameter α equals to 0, 0.1 and 0.21. Therefore, the effect of dilation parameters on the displacement is significant but inconsistently for M-C and generalized H-B models.

Fig. 7 shows the relationship between the displacements and dilation parameters. The greater dilation parameter usually results in a smaller displacement. While, for H-B failure criterion, when the dilation parameter is greater than 2.0, the decrease of displacement get slowly. For M-C failure



(a) H-B (b) M-C

Fig. 6 Displacement with the different dilation parameters



(a) H-B (b) M-C

Fig. 7 Distributions of displacement with the different dilation parameters

criterion, when the dilation parameter is greater than 0.15, slowly decreased displacement can be observed.

7. Conclusions

A new approach for the cylindrical cavity expansion problem incorporating deformation dependent of intermediate principal stress is proposed. The solution with new approach based on M-C and generalized H-B failure criteria are compared in the study. Meanwhile, the validity and accuracy of the proposed solution are confirmed with Vesic's solution (1972). The effect of strain-softening, dilation parameter on stresses and displacement of cylindrical cavity expansion are studied with the new approach. Based on the proposed approach, the theoretical solutions incorporating deformation dependent of intermediate principal stress can be obtained, the cylindrical cavity expansion solution considering intermediate principal stress for M-C and generalized H-B failure criteria could be effectively analyzed as well.

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