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# Reliability analysis of shallow tunnel with surface settlement

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**Abstract.** Based on the reliability theory and limit analysis method, the roof stability of a shallow tunnel is investigated under the condition of surface settlement. Nonlinear Hoek-Brown failure criterion is adopted in the present analysis. With the consideration of surface settlement, the internal energy and external work are calculated. Equating the rate of energy dissipation to the external rate of work, the expression of support pressure is derived. With the help of variational approach, a performance function is proposed to reliability analysis. Improved response surface method is used to calculate the Hasofer-Lind reliability index and the failure probability. In order to assess the validity of the present results, Monte-Carlo simulation is performed to examine the correctness. Sensitivity analysis is used to estimate the influence of different variables on reliability index. Among random variables, the unit weight significantly affects the reliability index. It is found that the greater coefficient of variation of variables lead to the higher failure probability. On the basis of the discussions, the reliability-based design is achieved to calculate the required tunnel support pressure under different situations when the target reliability index is obtained.

**Keywords:** shallow tunnel; surface settlement; reliability analysis

## 1. Introduction

The evaluation of tunnel stability plays an important role in the design and construction of tunnel. Limit analysis method is commonly used to estimate the tunnel stability in the last decades (Davis *et al*. 1980, Li and Yang 2016, Yang and Li 2016). Since the property of geotechnical materials usually varies in a wide range, it is more rational to employ reliability approaches to evaluate the tunnel stability. In order to reflect the inherent uncertainties of the necessary parameters, reliability analyses and designs are extensively performed and developed with the improvement of the knowledge on the statistic properties of soil (Low and Tang 2007, Phoon and Kulhawy 1999, Su *et al*. 2011).

Fraldi and Guarracino (2010, 2011, 2012) proposed a potential failure mechanism of tunnel roof with reference to limit analysis method and Hoek-Brown failure criterion. A kinematics plastic solution of tunnel collapse with ground movements was presented, and the shape of collapse rock mass is obtained (Huang *et al*. 2013, Lee 2016, Yang and Yan 2015, Yang *et al*. 2016). Besides, the upper bound solution of support pressure of tunnels was derived. However, the work done with surface settlement and uncertainties of necessary variables are not considered in the work.

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In the paper, reliability analysis is performed to evaluate the roof stability of shallow tunnel which ground surface settlement is taken into consideration. The performance function of roof stability is proposed according to the given failure mechanism. Variables involved in the performance function are defined as deterministic variables or random variables separately. The axial compressive strength, the tensile strength, surcharge load, soil weight and support pressure are regarded as random variables. Random variables are assumed to be normal or lognormal variables in reliability analysis. Due to the inexplicitness of performance function, improved response surface method (RSM) and Monte-Carlo simulation (MCS) are used to calculate the Hasofer-Lind reliability index and the failure probability. Besides, the sensitivity analysis of the random variables is performed. Finally, the required tunnel support pressure under different situations is determined by reliability-based design (RBD) when a target reliability index is given.

## 2. Reliability analysis methods

### 2.1 Hasofer-Lind index

The reliability index is widely used to evaluate the structure safety by considering the inherent uncertainties of the design variables. Hasofer and Lind (1974) proposed their reliability index $\beta$. The matrix formulation is given by

$$\beta = \min_{x\in F} \sqrt{(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{C}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})} \tag{1}$$

where $x$ is the vector of n random variables, $\mu$ means the vector of their mean values, $C$ is the covariance matrix, and $F$ presents the failure region represented by the performance function $g(x) \leq 0$. According to Eq. (1), the Hasofer-Lind reliability index means the minimum distance in units of directional standard deviations from the mean value of random variables to the limit state surface $g(x) = 0$.

For non-normal variables, Rackwitz and Flessler (1978) proposed their approach to calculate the equivalent normal mean value $\mu_i^N$ and equivalent normal standard deviation $\sigma_i^N$. Based on the first-order reliability method (FORM) and Hasofer-Lind reliability index $\beta$, the probability of failure $P_f$ can be calculated from

$$P_f \cong 1-\Phi(\beta) \tag{2}$$

where $\Phi(\cdot)$ represents the cumulative distribution function (CDF) of the standard normal variable.

### 2.2 Improved RSM

Generally, reliability analysis requires the explicit performance function to calculate the reliability index. However, the explicit expression of performance function may be unlikely to determine in complex engineering problems. RSM was proposed to approximate the implicit performance function. In RSM, an explicit function of random variables was used to approximate the actual performance function with the use of sampling points. Aiming to determine the proper location of sampling points, improved RSM is proposed (Kim and Na 1997) by using the vector projected sampling points. In their researches, the algorithm is

$$g(\boldsymbol{X}) \approx \tilde{g}(\boldsymbol{X}) = a_0 + \sum_{i=1}^{n} a_i x_i \tag{3}$$

where $x_i$ is the basic random variable, $n$ is the number of the random variable, $a_0$ and $a_i$ are the coefficients to be determined. In order to obtain a tentative response surface, a new set of sampling points are selected using the following vector projection technique. FORM is performed to obtain reliability index $\beta$ and the corresponding design point $x^*$ by using the tentative response surface function. The initial values of coefficients $a_0$, $a_1$,..., $a_i$,..., $a_n$ are determined by using sampling points centered at the initial central point $x_i^0$ and located in each direction at

$$x_i^0 \pm h\sigma_{x_i}, i = 1, 2, ..., n \tag{4}$$

where $h$ is selected to be 2 in this paper. The initial central points can be the mean value points. It may also be shifted towards the failure domain to expedite convergence.

### 2.3 MCS

MCS is a well-accepted reliability method in which samples are generated with reference to the probability density of random variables. With respect to the law of large numbers, the accuracy of MCS lies in the large number of samples and trails. The failure probability is

$$P_f = \frac{1}{N} \sum_{i=1}^{n} I(x_i) \tag{5}$$

where $N$ is the number of samples; $I(x) = 1$ if $g(x) \leq 0$ and 0 elsewhere. The convergence of the failure probability is estimated by its coefficient of variation (COV)

$$COV(P_f) = \sqrt{(1 - \mathrm{P}_f) / (P_f N)} \tag{6}$$

## 3. Limit analysis and Hoek-Brown criterion

### 3.1 Hoek-Brown criterion

In tunnel engineering, the materials tend to have nonlinear characteristics. The Hoek-Brown failure criterion is widely employed to investigate the nonlinear engineering problems. The relation between normal and shear stresses of Hoek-Brown criterion is expressed as (Hoek and Brown 1997, Mohammadi and Tavakoli 2015, Serrano and Olalla 1999, Sofianos 2003, Yang and Pan 2015, Yang and Xiao 2016)

$$\tau = A\sigma_c \left[ (\sigma_n - \sigma_t) / \sigma_c \right]^B \tag{7}$$

where $A$ and $B$ are material constants respectively, $\sigma_n$ is the normal effective stress, $\sigma_c$ is the uniaxial compressive strength, and $\sigma_t$ is the tensile strength of the rock mass.

### 3.2 Upper bound theorem

According to the upper bound theorem of limit analysis, the actual collapse load is no more than the load derived by equating the rate of the energy dissipation to the external rate of work in any kinematically admissible velocity field, when the deformation boundary condition is satisfied (Chen 1975). It takes the form as

$$\int_V \sigma_{ij}\dot{\varepsilon}_{ij}dv \geq \int_S T_i v_i ds + \int_V X_i v_i dv - \int_V u\dot{\varepsilon}_{ij}dV \tag{8}$$

where $\sigma_{ij}$ and $\dot{\varepsilon}_{ij}$ are the rate of stress and the rate of train in a kinematically admissible velocity field respectively, $T_i$ and $X_i$ are the surface force and the body force of studied object respectively, $V$ is the volume of the collapse block, $v_i$ stands for the velocity along the detaching surface, and $s$ is the length of velocity discontinuity.

## 4. Performance function of tunnel regarding surface settlement

This paper aims to perform reliability analysis of the roof stability of a shallow circular tunnel with surface settlement. The failure mechanism is illustrated in Fig. 1. It is assumed that the settlement profile of ground surface during the tunnel construction is subjected to Gaussian distribution. In this paper, the kinematic admissible velocity field extends from the upper circumference of circular tunnel to the ground.

It is obvious that the failure mechanism is symmetrical with the respect of $Z$-axis. According to the Hoek-Brown criterion and limit analysis, the energy dissipation of a random point along the detaching curve is given by

$$D = \int_s D_i w ds = \int_L^{2.5i} [\sigma_t - \sigma_c (\mathrm{AB})^{1/(1-\mathrm{B})} (1 - \mathrm{B}^{-1}) f'(x)^{1/(1-\mathrm{B})}] v dx \tag{9}$$

This paper aims to perform reliability analysis of the roof stability of a shallow circular tunnel

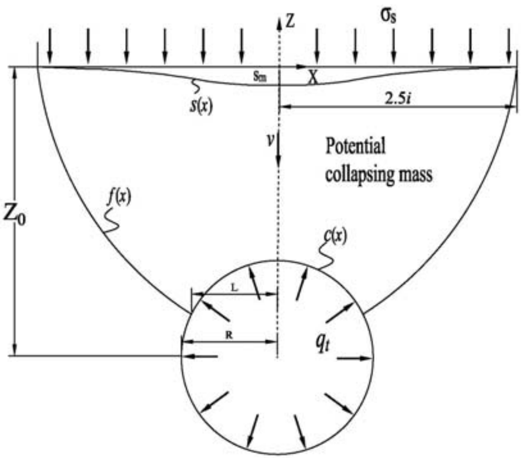


Fig. 1 Potential roof collapse with surface settlement

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$$i = Kz_0 \tag{10}$$

where $K$ is selected as 0.5 for the sake of calculation simplification (Rankin 1988). The work rate of weight is given by (Sun and Qin 2014)

$$P_\gamma = \gamma\int_L^{2.5i} f(x)vdx + \gamma\int_0^L c(x)vdx - \gamma\int_0^{2.5i} s(x)vdx \tag{11}$$

where $\gamma$ is the weight of surrounding soil, $c(x)$ characterizes the shape of circular tunnel, and $s(x)$ is the surface settlement which is expressed as

$$s(x) = -s_m \exp[-\frac{1}{2}(\frac{x}{i})^2] \tag{12}$$

in which $s_m$ is the maximum surface settlement.

The function of the supporting structure is represented by the pressure applied on the boundary of tunnel. The work rate of the support pressure is

$$P_t = q_t Lv \tag{13}$$

where $q_t$ is the support pressure exerting on the circumference of tunnel lining.

For shallow tunnels, surcharge load on ground surface is an element which should be taken into account. Therefore, the work rate of extra force is given by

$$P_{\sigma_s} = -2.5i\sigma_s v \tag{14}$$

The work done by support pressure $q_t$ and surcharge load $\sigma_s$ on the ground surface is given by

$$P_{st} = (q_t - \sigma_s)\int_0^{2.5i} \delta v_{z=0} dx = (q_t - \sigma_s)\Delta \tag{15}$$

where $\delta v$ is the vertical displacement increment, and $\Delta$ equal to the integration of $\int_0^{2.5i} \delta v_{z=0} dx$.

Based on virtual work equation, a function is constructed by equating the work rake of internal energy dissipation and external forces.

$$D = P_\gamma + P_t + P_{\sigma_s} + P_{st} \tag{16}$$

Based on Eq. (16), the support pressure $q_t$ could be obtained. It can also be defined as the collapse pressure $\sigma_{cl}$ of the failure mechanism

$$q_t = \sigma_{cl} = 1/(L+\Delta)\left\{\int_L^{2.5i}\psi\left[f(x),f'(x),x\right]dx - \gamma\int_0^L c(x)dx + \gamma\int_0^{2.5i} s(x)dx + \sigma_s\Delta + 2.5\sigma_s i\right\} \quad (17)$$

where $\psi$ $[f(x), f'(x), x]$ is expressed as

$$\psi\left[f(x),f'(x),x\right] = \sigma_t - \sigma_c(\mathrm{AB})^{1/(1-\mathrm{B})}(1-\mathrm{B}^{-1})f'(x)^{1/(1-\mathrm{B})} - \gamma f(x) \quad (18)$$

In order to seek the extreme value of $q_t$, the extreme value of $\psi$ should be calculated firstly. Thus the principle of variation is employed to search for the extreme value of $\psi$ with the use of Euler equation. Trough necessary integral and differential calculation, the expression of the detaching curve $f(x)$ is given by

$$f(x) = -A^{-1/B}\left(\gamma/\sigma_c\right)^{(1-B)/B}\left(2.5i - x\right)^{1/B} \quad (19)$$

According to geometrical condition, $L$ could be obtained by

$$f(x=L) = c(x=L) \quad (20)$$

Eq. (20) can also be expressed in the form of

$$-A^{-1/B}\left(\frac{\gamma}{\sigma_c}\right)^{(1-\mathrm{B})/B}(2.5i-L)^{1/B} = -z_0 + \sqrt{R^2 - L^2} \quad (21)$$

Aiming to evaluate the stability of the shallow tunnel, reliability analysis is employed. According to the failure mechanism given above, the performance function used to perform reliability analysis is proposed by

$$\begin{aligned} g(x) &= q_t - \sigma_{cl} \\ &= q_t(L+\Delta) - [\sigma_t(2.5i-L) + \mathrm{A}^{-1/B}(1+\mathrm{B})^{-1}\sigma_c^{(B-1)/B}\gamma^{1/B}(2.5i-L)^{(B+1)/B} \\ &\quad + \gamma z_0 L - 0.5\gamma R^2\arcsin(L/R) - 0.5\gamma L\sqrt{R^2-L^2} + \sqrt{\pi/2}\,i\gamma s_m + \Delta\sigma_s + 2.5\sigma_s i] \end{aligned} \quad (22)$$

Since the explicit expression of $L$ is unknown, the performance function $g(x)$ is implicitly with random variables.

## 5. Reliability analyses

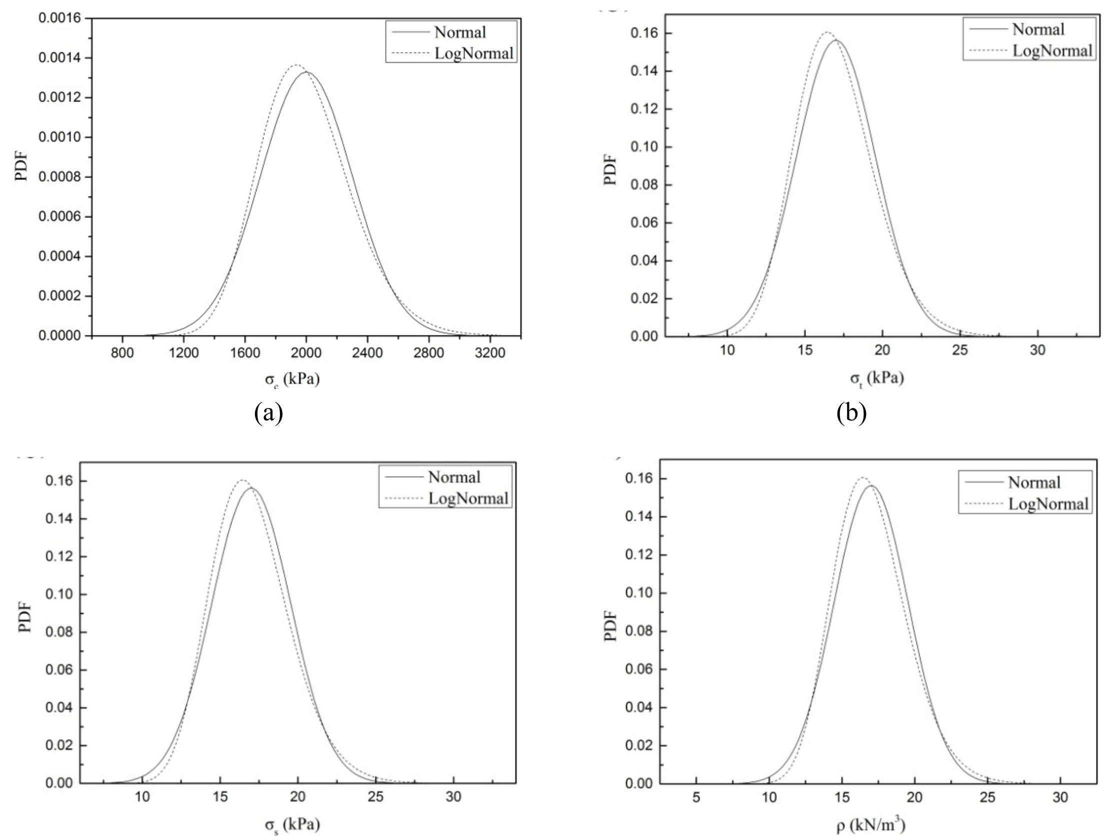
Before the employment of reliability analysis methods, variables involved in the performance function $g(x)$ should be defined as deterministic variables or random variables firstly. In this paper, the uniaxial compressive strength $\sigma_c$, the tensile strength $\sigma_t$, surcharge load $\sigma_s$, soil weight $\rho$ and support pressure $q_t$ are regarded as random variables. The statistic value and distribution types are listed in Table 1. Except for them, other parameters ($A$, $B$, $S_m$, $K$, $R$, and $z_0$ defined in Fig. 1) are regarded as deterministic variables. These parameters are shown in Table 2.

Table 1 Statistical values of random variables used in analysis

| | Random variable | Mean value | Coefficient of variation | Distribution type |
|---|---|---|---|---|
| Case 1 | $\sigma_c$ (kPa) | 20000 | 0.15 | normal distribution |
| | $\sigma_t$ (kPa) | 200 | 0.15 | normal distribution |
| | $\rho$ (kN/m$^3$) | 2000 | 0.15 | normal distribution |
| | $\sigma_s$ (kPa) | 2000 | 0.15 | normal distribution |
| Case 2 | $\sigma_c$ (kPa) | 20000 | 0.15 | lognormal distribution |
| | $\sigma_t$ (kPa) | 200 | 0.15 | lognormal distribution |
| | $\rho$ (kN/m$^3$) | 2000 | 0.15 | lognormal distribution |
| | $\sigma_s$ (kPa) | 2000 | 0.15 | lognormal distribution |

Table 2 The parameters of shallow tunnel

| $R$ (m) | $Z_0$ (m) | $S_m$ (m) | $K$ | $A$ | $B$ |
|---|---|---|---|---|---|
| 5 | 10 | 0.1 | 0.5 | 0.6 | 0.8 |




(a) (b)

(c) (d)

Fig. 2 The PDF of the normal, lognormal distribution of $\sigma_c$, $\sigma_t$, $\sigma_c$, $\rho$

Generally, normal distribution of random variables is utilized to reflect the uncertainties of geometry materials. However, lognormal distribution of variables is always recommended in reliability analysis to avoid negative values when the coefficient of variables (COV) is 0.25 or higher. Fig. 2 presents the probability distribution function curves of normal and lognormal distribution for different variables. As mentioned before, the width of collapse blocks $L$ in performance function $g(x)$ is explicitly unknown. Thus, improved RSM procedures coded are utilized to analyze the stability of shallow circular tunnel with surface settlement. In this case, this algorithm is executed with uncorrelated variables in the standardized space.

### *5.1 Reliability index involving non-normal distribution*

Based on the supposed failure mechanism, the collapse pressure of tunnel roof obtained from normal variables is 317.778 kPa. However, the collapse pressure is equal to 318.246kPa for lognormal variables.

Employing the improved RSM procedures, the Hasofer-Lind reliability index $\beta$ and the corresponding failure probability are computed when the support pressure $q_t$ varied from 360 to 660 kPa. The numerical results are shown in Fig. 3. As expected, the support pressure has a critical influence on the reliability index. Fig. 3 shows that Hasofer-Lind reliability index $\beta$ increases with the increase of support pressure $q_t$ whether both normal and lognormal variable is used. The comparison of results indicates that the reliability index obtained from normal variable is smaller than that of lognormal variables.

The values ($\sigma_c^*$, $\sigma_t^*$, $\rho^*$ and $\sigma_s^*$) of design points corresponding to different support pressure $q_t$ represents the most probable failure point on the limit state surface (LSS). It is the point where the expanding 4-dimensional dispersion ellipsoid is tangent to the limit state surface $g(x) = 0$. As Table 3 shows, the design points $\sigma_c^*$, $\sigma_t^*$ and $\sigma_s^*$ are slightly greater than their mean values and would increase with the increase of support pressure. Meanwhile, the design point $\rho^*$ is higher than its mean value and increases faster with the increase of support pressure. In order to evaluate the influence of different variables on reliability index, the sensitive analyses of those parameters are constructed in the next section.

For examination, the reliability numerical results calculated by MCS procedure are also shown in Fig. 3. It is found that the reliability index curves obtained from RSM agree signally well with the curve obtained from MCS, which means there are little differences exist among the reliability index obtained by these two methods. Based on that, the reliability indexes calculated by RSM can be regarded as reliable.

Table 3 Sensitivities factors and design points for different support pressures

| $q_t$ (kPa) | $\sigma_c$ | | $\sigma_t$ | | $\rho$ | | $\sigma_s$ | |
|---|---|---|---|---|---|---|---|---|
| | $\gamma_{\sigma_c}$ | $\sigma_c^*$ (kPa) | $\gamma_{\sigma_t}$ | $\sigma_t^*$ (kPa) | $\gamma_\rho$ | $\rho$ (kN/m$^3$) | $\gamma_{\sigma_s}$ | $\sigma_s^*$ (kPa) |
| 360 | 0.0394 | 2007.801 | 0.0836 | 20.165 | 0.4923 | 17.827 | 0.1273 | 20.251 |
| 420 | 0.0331 | 2014.186 | 0.0737 | 20.315 | 0.4403 | 18.603 | 0.1125 | 20.482 |
| 480 | 0.0286 | 2017.545 | 0.0658 | 20.402 | 0.3966 | 19.061 | 0.1006 | 20.615 |
| 540 | 0.0254 | 2019.326 | 0.0594 | 20.451 | 0.3597 | 19.321 | 0.0909 | 20.690 |
| 600 | 0.0229 | 2020.233 | 0.0541 | 20.477 | 0.3285 | 19.458 | 0.0828 | 20.729 |
| 660 | 0.0210 | 2020.629 | 0.0497 | 20.487 | 0.3018 | 19.517 | 0.0760 | 20.745 |

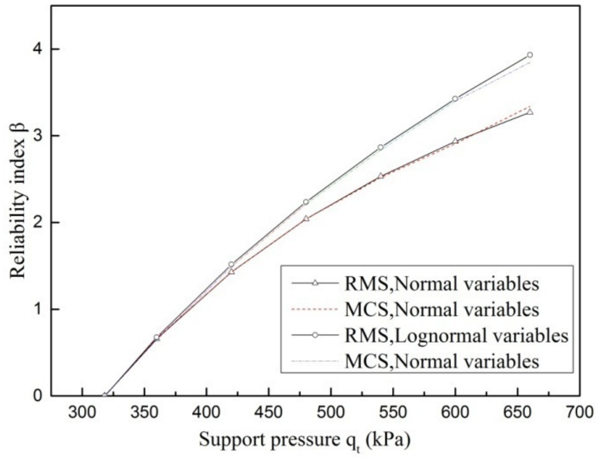



Fig. 3 Reliability index versus qt for normal, lognormal variables

### *5.2 Sensitivity analyses*

Sensitivity analysis plays an important and practical role in the applications of reliability-based analysis, design and optimization. Since several approaches were proposed to estimate the influence of random variables on the reliability index, sensitive analysis plays an increasingly vital role in reliability based design. Based on FORM, $\cos\theta_{X_i}$ is selected as the sensitivity factors of variable $X_i$ in this paper.

$$\cos\theta_{X_i} = \frac{-\left.\frac{\delta g}{\delta X_i}\right|_P \cdot \sigma_{X_i}}{\sqrt{\sum_{i=1}^{n}\left(\left.\frac{\delta g}{\delta X_i}\right|_P \cdot \sigma_{X_i}\right)^2}} \tag{23}$$

where $\frac{\delta g}{\delta X_i}$ is the derivation of performance function, $\sigma_{X_i}$ is the standard deviation of random variable $X_i$ .

Table 3 presents the design points and sensitivity factors of normal variables with the variation of support pressure, where $\gamma_{\sigma_c}$, $\gamma_{\sigma_t}$, $\gamma_\rho$ and $\gamma_{\sigma_s}$ represent the sensitivity factor of $\sigma_c$, $\sigma_t$, $\rho$ and $\sigma_s$ respectively. Sensitivity factor $\gamma$ may indicate the 'load' and 'resistance' variables. The positive $\gamma$ means a variable, and vice-versa. As expected, the sensitivity factors are all positive; therefore all these variables can be regarded as 'load' variables. Since the value of $\gamma_\rho$ is obviously higher than other sensitivity factors, one can conclude that importance of $\rho$ is significantly greater than other three variables. Besides, sensitivity factors decrease with the increase of support pressure, which means that the influence of these variables on reliability index is relatively greater under higher support pressures.

### *5.3 Failure probability*

According to Eq. (2), both RSM and MCS were performed to calculate the failure probability

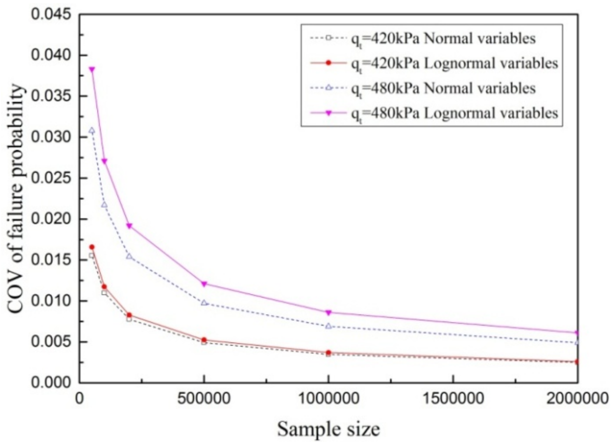



Fig. 4 COV of the failure probability versus the sample size

of the given failure mechanism.

MCS is assumed to be an unbiased method in reliability analysis with its accuracy based on the number of sample size. The COV of the failure probability is used to estimate the required sample size to obtain reliable results. As Eq. (6) shows, the COV of failure probability is related not only with the number of samples but also with the failure probability. In this section, the COV of the failure probability versus the different sample size of MCS is obtained and discussed.

To have a clear visualization of the convergence of MCS, the COV of failure probability obtained by different sample size is represented in Fig. 4. Different values of support pressure were applied on the boundary of tunnel. As expected, the COV of failure probability become steady with the increase of sample size. For the support pressure of 420 kPa, a sample size of 200,000 was big enough to obtain a nearly constant failure probability (with the COV of failure probability smaller than 1%). However, sample size of 500,000 was necessary to ensure a credible failure probability with the support pressure of 480 kPa. In this paper, the sample size of MCS increased from100,000 to 80,000,000 to achieve a reliable the failure probability.

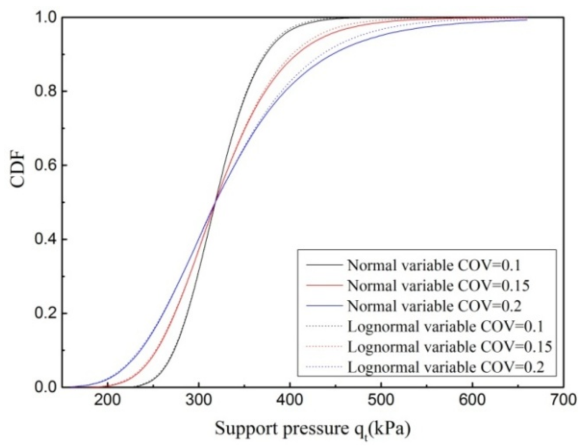



Fig. 5 Comparison of the CDFs of the support pressure for different COV

Fig. 5 shows the cumulative distribution function (CDF) of support pressure obtained by RSM. It is noticed that the distribution of variables (i.e., normal and lognormal) does not significantly influence the failure probability. The effect of the COV of variables is also presented in Fig. 5. One can conclude that even a small change of the COV of random variables (i.e., 0.05 in this part) would obviously affect the curve of CDF. The greater COV would lead to higher failure probability. Therefore, the determination of the COV of variables should be taken seriously to obtain a reliable failure probability.

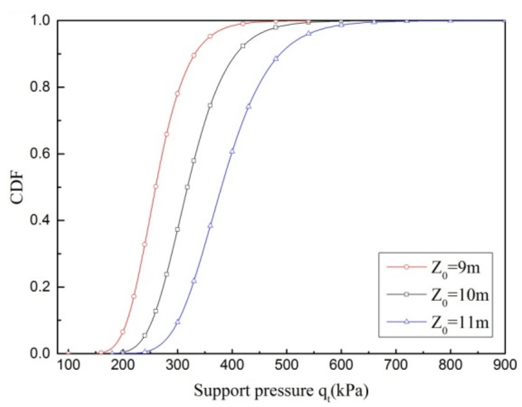
### *5.4 Reliability-based design*

As mentioned before, the radius $R$, the burial depth of tunnel center $Z_0$ and the maximum surface settlement $S_m$ are chosen as deterministic value in reliability analyses. These basic parameters should be chosen rational in the design of shallow tunnel. In this section, the influences of $R$ and $Z_0$ on the support pressure are investigated.

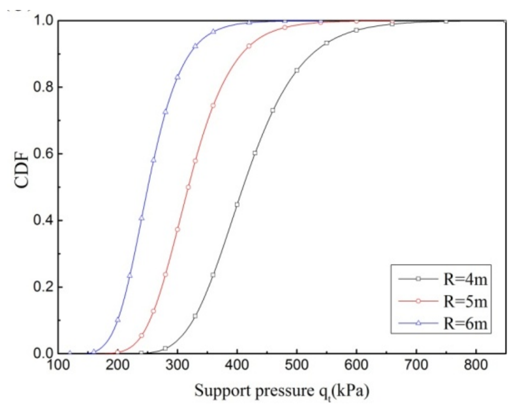
Fig. 6 shows the comparisons of CDFs of the support pressure for different deterministic design (i.e., $Z_0$ and $R$) by using spline interpolation. As Fig. 6(a) presents, the necessary support pressure increases with the distance $Z_0$ decreases. However, the necessary support pressure decreases with lower radius of tunnel as Fig. 6(b) presents. The result may seem irrational at first think. However, according to the upper bound analyze, the smaller tunnel radius would lead to the less work rate of soil weight and support pressure (due to the less action range) in the employment of virtual work equation. The result is rational in that sense. That is to say, relatively smaller support pressure can guarantee the roof stability of bigger tunnel. It is also worth mentioning that the influences of these deterministic designs of tunnel on support pressure show great agreements with the research. Since the influence of maximum surface settlement $S_m$ on support pressure was too little, the results are not discussed in this section.

Generally, an initial target reliability index $\beta$ of 2.5 or 3 is used in RBD. The corresponding failure probability of them are equal to 0.62% and 0.13% respectively. In this section, RBD is achieved to determine the required tunnel roof pressure for a target reliability index of 2.5. This tunnel pressure is called as probabilistic tunnel pressure.

With reference to the sensitivity analysis achieved before, only the COV of unit weight $\rho$ was




(a)




(b)

Fig. 6 Comparison of the CDFs of the support pressure versus different design

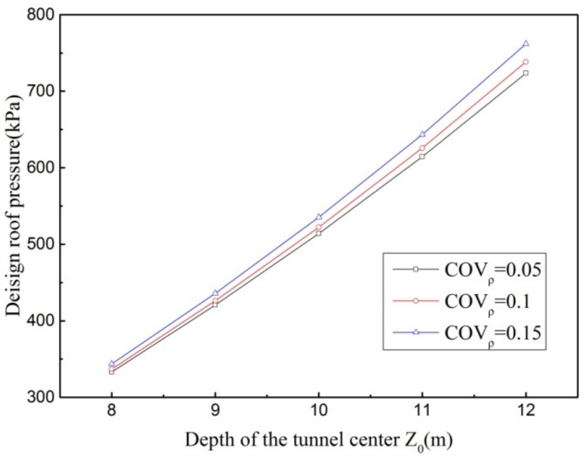



Fig. 7 Design roof pressure with different COV of unit weight $\rho$

taken into consideration in RBD. Fig. 7 shows the probabilistic tunnel pressure for different COV of unit weight $\rho$ which follows normal distribution. It is found that the probabilistic tunnel pressure increases with the greater scatter in unit weight $\rho$. Taken $Z_0 = 10$ m for example, the probabilistic tunnel pressure increased from 513.96 kPa to 535.39 kPa with an increment of 0.1 in the COV of unit weight. RBD can play a significant role in the design of the tunnel pressure since the uncertainty of soil properties is considered.

## 6. Conclusions

With reference to the upper bound theorem and nonlinear failure criterion, reliability analysis is presented to evaluate the roof stability of shallow circular tunnel. Regarding the influences of ground surface pressure and settlement, the upper bound solution of support pressure is derived. Reliability analysis based on RSM and MCS is performed to evaluate the roof stability of shallow tunnel with the performance function. The main conclusions are summarized as follows.

Good agreement between the reliability index obtained from improved RSM and those calculated from MCS indicates that the RSM results are reliable. It is found that the reliability index increases significantly with the increase of the support pressure. The distribution type of random variable does not significantly influence the results and the reliability index of the normal variables is usually slightly smaller than that of the lognormal variables. The support pressure applied on the tunnel boundary has a significant influence on the failure probability of shallow tunnel. The design points ($\sigma_c^*$, $\sigma_t^*$, $\rho^*$ and $\sigma_s^*$) corresponding to different support pressure $q_t$ is obtained by FORM. Sensitivity analysis is achieved to evaluate the influences of different variables on the reliability index. The results show that the reliability index is much more sensitive to unit weight $\rho$ than other parameters. Therefore, the COV of unit weight $\rho$ should be accurately determined to obtain reliable probabilistic results.

The CDF curves of the support pressure are obtained from the failure probabilities by using spline interpolation, for both normal and lognormal variables. The effect of the COV of random variables is investigated. It is found that the greater COV of random variables leads to the higher failure probability. RBD of support pressure is performed to estimate the required tunnel roof pressure. The reliability-based roof pressure of tunnel would decrease with the decline of the COV of random variables.

## Acknowledgments

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