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Regularizing structural configurations by using meta-heuristic algorithms

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Abstract. This paper focuses on the regularization of structural configurations by employing meta-heuristic optimization algorithms such as Particle Swarm Optimization (PSO) and Biogeography-Based Optimization (BBO). The regularization of structural configuration means obtaining a structure whose members have equal or almost equal lengths, or whose member's lengths are based on a specific pattern; which in this case, by changing the length of these elements and reducing the number of different profiles of needed members, the construction of the considered structure can be made easier. In this article, two different objective functions have been used to minimize the difference between member lengths with a specific pattern. It is found that by using a small number of iterations in these optimization methods, a structure made of equal-length members can be obtained.

Keywords: structural configuration; regularization; optimization; meta-heuristic; configuration processing

1. Introduction

Configuration processing means the use of various tools and methods for constructing or modifying the configuration of a structure. Different methods and approaches have been introduced which can be utilized in this processing scheme, for example, the optimization of structural topology patterns using regularization presented by Lee and Shin (2015), and regularization of structural configuration of trusses using energy approach by Nguyen and Lee (2015). Moreover, design optimization techniques for solving typical structural engineering problems have been proposed by Fedorik *et al*. (2015). Also, in the study of Artar and Daloglu (2015), optimum design of planar frames has been investigated. A Regular Structure can have various definitions and characterizations; in this paper, we intend to regularize a structure by reducing the difference between the lengths of its members based on a given pattern. Various concepts could be used to define the regularity and orderliness of a structure; but here, we define a regular and orderly structure as the one with equal-length or almost equal-length members, or even a structure whose members have lengths that follow a specific pattern. The use of configuration processing is very practical and important when we deal with space structures, and it usually leads to new designs and configurations. Since engineers have always desired to cut down on the required volume of work and the costs associated with various projects and thus to facilitate the

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work involved, the subject of structural optimization has attracted the attentions of designers for several decades. The construction of a regular and orderly structure is much easier than that of an irregular structure. As an example, the number of member types for a space structure is much less than that for an ordinary structure, which will facilitate the work when constructing the space structure. Due to the growing utilization of space structures in the world, more research should be conducted on these structures. In addition to many advantages of space structures including the lightness, easy assembly, high redundancy, etc., another advantage of these types of structures is the limited number of member types used. In other words, since a space structure has a regular and ordered configuration, the quantities of element types used diminishes as a result. Furthermore, another reason for the popularity of space structures is their architectural variety and aesthetics, and the regularization of these structures will only add to their attractiveness. Obviously, a structure with a certain order and regularity, not only enjoys aesthetic characteristics, but also possesses a simple pattern which consequently reduces the time of design computations, especially in space structures. For more explanations, please refer to the articles of Kaveh (Kaveh and Nouri 2009, and Kaveh and Talatahari 2010) regarding the calculation of the Eigenvalue of regular structures. Evolutionary algorithms have been widely used for engineering optimization. As an instance, Khalkhali *et al*. (2014) optimized sandwich panels with corrugated core using genetic algorithm. In another work, Khalkhali *et al*. (2016) used particle swarm to optimize perforated square tubes.

2. Optimization algorithms

2.1 Particle Swarm Optimization (PSO)

Particle Swarm Optimization which was introduced by Kennedy and Eberhart (1995) is a numerical search algorithm that is used to find parameters that minimize a given objective, or fitness function. PSO is a robust stochastic optimization algorithm based on the intelligence and movement of swarms where the concept of social interaction to problem solving is applied. PSO has gained significant popularity due to its attractive structure and high performance over the past few years. It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the optimized answer. The development of this algorithm has been inspired by the social lives of fish and birds that live in colonies and fulfill many of their needs, including the search for food, collectively and by using swarm intelligence. In this algorithm, every solution to a problem is a particle in the search space, and a group of particles is called a swarm. Depending on the quality of response each particle produces, it has a certain fitness, which is determined by using a fitness function or an objective function. Each particle also has a velocity vector which is updated in every iteration, based on the experience of the particle itself, the experience of other particles and the last speed of the particle. The particle swarm optimization method starts by creating an initial population at a random location in space. Each particle randomly moves in the search space towards the position of the best objective function value that has been experienced by the particle itself and towards the best location experienced by the whole particles as shown in Fig. 1. By considering the particle velocity, the next location of the particle is calculated as follows:

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{1}$$

$$V_i^{k+1} = \rho_k V_i^k + c_1 r_1 (P_k^i - X_k^i) + c_2 r_2 (P_k^g - X_k^i) \tag{2}$$

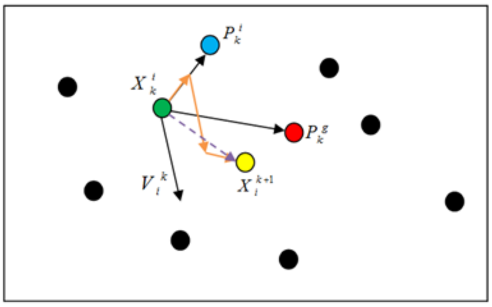

Fig. 1 Position and velocity update in (k+1)th iteration

Where V_i^k and X_k^i are the velocity vector and the position of the ith particle in the kth iteration, respectively. The parameters r_1 and r_2 are random numbers with uniform distributions between 0 and 1. The parameters c_1 and c_2 are called confidence parameters and define the best location experienced by the ith particle (P_k^i) and the best location experienced by all the particles (P_k^g), respectively. Parameter ρ is known as the inertia factor and it determines the effect of the particle's previous speed. The value of each of the c_1, c_2 and ρ parameters can be expressed as a constant number or as a function based on the progress of the algorithm. The proper tuning of these parameters greatly affects the convergence of the algorithm. In this paper, based on the algorithm iteration number, the values of c_1, c_2 and ρ are obtained by using the following relations

$$c_1 = c_{\max} - \left[c_{\max} - c_{\min} \times \frac{iter}{iter_{\max}} \right]^n \tag{3}$$

$$c_2 = c_{\min} + \left[c_{\max} - c_{\min} \times \frac{iter}{iter_{\max}} \right]^m \tag{4}$$

$$\rho = (\rho_{\max})^{iter} \tag{5}$$

In the above relations, $c_{\max}$ and $c_{\min}$ are the maximum and minimum values of c_1 and c_2 and are set equal to 0 and 4 according to Arora (2012). The number of iterations is designated by *iter* and the maximum number of iterations is denoted by $iter_{\max}$ .The parameter $\rho_{\max}$ is an initial value for r, which, based on several numerical tests, has been given a value of 0.96 in this paper. Also, values of 1 and 2 have been considered for n and m, respectively.

Some of the main advantages of the PSO method are as follows, Arora (2012):

(1) An attractive feature of PSO is that it has fewer algorithmic parameters to specify compared to Genetic Algorithms (GA). It does not use any of the GAs' evolutionary operators such as crossover and mutation.
(2) Unlike GAs, the algorithm does not require binary number encoding or decoding and thus is easier to implement on the computer.
(3) PSO has been successfully applied to many classes of problems, such as mechanical and structural optimization and multi-objective optimization, artificial neural network training, and fuzzy system control.

2.2 Biogeography-Based Optimization (BBO)

The concept of BBO algorithm is adopted from nature and it is based on the way animals migrate from a region or an island with lots of rivals to a less populated region with fewer rivals. Based on this algorithm, the more habitants a region has, the more suitable that region is deemed for living from biological perspective, and thus a higher fitness value it possesses. In such a case, at each step, the organisms migrate from islands with higher fitness values to those with lower fitness values; and, thus, the optimization is implemented. The mathematical biogeography models express the way organisms migrate and also indicate how a new organism emerges or becomes extinct. In this algorithm, an island is a habitat which is distinct from other islands or habitats. The habitats with suitable characteristics for the living of organisms have a higher Habitat Suitability Index (HSI) than the other habitats. The HSI is the same as the fitness function or the objective function in the other algorithms; and the goal is to optimize the HSI value for each island or habitat.

For example, it can be presumed that the parameters considered for determining the HSI values include the degree of suitability, plant species variety, topography of the region, etc. The variables that define the habitability of a region are called the Suitability Index Variables (SIVs). So, it can be said that the SIVs are the independent variables and the HSIs are the dependent variables of the habitat. As we said earlier, a region with a high HSI has a great number of animals and, therefore, a higher rate of emigration and a lower rate of immigration relative to the other habitats.

In Fig. 2, the two concepts of immigration and emigration are described in this way that if a region has an excess population of animals, the immigration to this region will be zero and the emigration from this region will have the highest possible rate, and vice versa. In Fig. 2, λ denotes the immigration rate and μ indicates the emigration rate, Simon (2008).

It is not necessary at all for λ_{max} and μ_{max} to be equal; however, for simplicity, we assume them to be equal here ($\lambda_{max} = \mu_{max} = 1$), and therefore at each step, we will have: $\lambda + \mu = 1$. In other words, at each step, for a specific population size, certain values exist for λ and μ, the sum of which equals 1.

At each step, with regards to the rate of λ and μ for every island, migrations such as those in Fig. 3 take place. Of course, in this algorithm also, like in the genetic algorithm, the mutation operator is used. This means that at each step, after the migration operation, the mutation operation is executed on each SIV with the probability of mutation rate, Simon (2008).

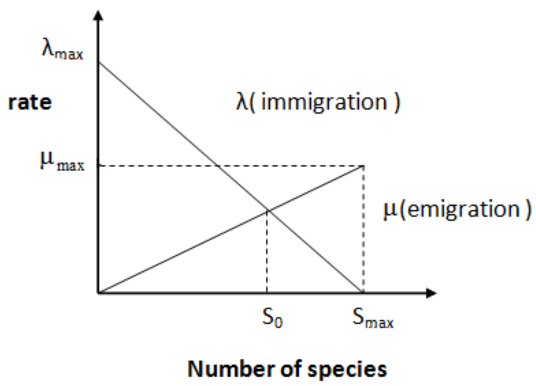

Fig. 2 Migration rate versus the number of species in a habitat

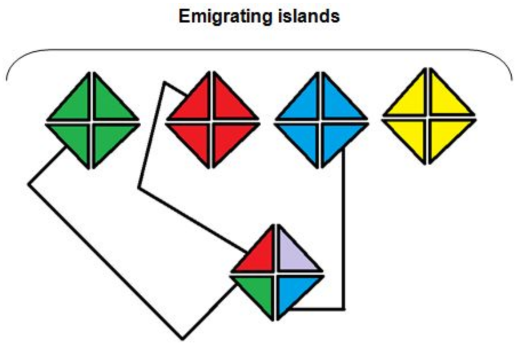

Fig. 3 Manner of emigration and immigration between various islands at different rates

Some of the main advantages of the BBO method are as follows:

(1) The information can be exchanged between the solutions.
(2) The generated solution at each step is not eliminated and is always available.
(3) The method is solely based on the two concepts of migration and mutation.
(4) When comparing the BBO with the Ant Colony Optimization (ACO) in an iteration cycle, the previous solutions are not eliminated in the BBO; whereas in the ACO, the previous solutions are omitted in each iteration cycle and a new solution is generated. This improves the speed and efficiency of the BBO algorithm.
(5) In the particle swarm optimization (PSO) algorithm, every solution is displayed as a point and the changes applied to each solution are expressed as a velocity; while in the BBO algorithm, the changes are directly applied to the solutions.

3. Optimization of configuration processing

In this section we introduce the parameters of "configuration processing" that are required in this study such as objective functions and constraints that we can utilize in this paper.

3.1 Introducing the objective functions

Several objective function models can be employed in this problem to achieve an acceptable response. The most important of these functions is the Geometric Potential Function, which was first applied to space structures by Nooshin (Nooshin *et al.* 1999).

3.1.1 The concept of geometric potential

Consider the nodes of the dome-shaped structure in Fig. 4. It is assumed that the nodes tend to repel one another.

In this case, the repelling effect between nodes i and j can be expressed as follows

$$\text{Repelling effect between } i \;\&\; j = T_i T_j D_{ij}^{-p} \tag{6}$$

In the above relation, T_i is the transfactor of node i (a positive dimensionless factor that measures the strength of the repelling effect in node i), T_j is the transfactor of node j, D_{ij} is the distance between nodes i and j, and p is a positive exponent called the power rate.

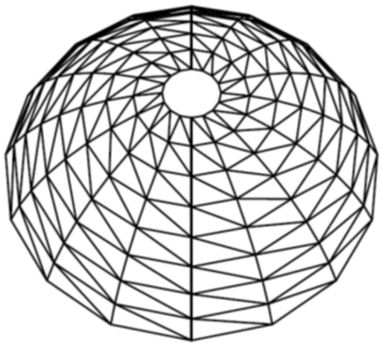

Fig. 4 Single-layer dome

Now, the values of every repelling effect among the nodes are summed up. If the transfactor and also the power rate (the input data of the problem) are known and the only unknown is a function of node coordinates, the obtained summation will be called the Absolute Geometric Potential function (AGP) and expressed as follows

$$G_a = \sum T_i T_j D_{ij}^{-p} \tag{7}$$

We can also write

$$D_{ij} = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2} \tag{8}$$

Where X, Y and Z are the coordinates of the considered points. The above relation makes clear that the geometric potential function is a nonlinear function of node coordinates.

Obviously, by minimizing a structure's absolute geometric potential function, we can get closer to our goal; therefore, the AGP can be one of the objective functions for solving this problem. In a similar way, by only summing over the node pairs that are related to each other rather than extending the summation to all possible node pairs, another objective function by the name of Relative Geometric Potential (RGP) function can be defined.

There are also other statistical functions that can be employed to solve this problem. One of the most suitable of these functions is the relative variance of element lengths; the closer the value of this function is to zero, the more regular the structure will be, Nooshin (Nooshin *et al.* 2003).

3.2 Constraints of the optimization problem

For this problem, the constraints can be classified into two groups of Invariant Constraints and Relational Constraints. These constraints have been defined as below:

Invariant constraints are the points whose coordinates remain fixed and invariant. For example, the support points in a structure at which the coordinates do not change are considered as invariant constraints. Where as in case of relational constraints, there is a specified relationship between the coordinates of the considered nodes. For example, a node can only be displaced along a specific line.

3.3 Parameters of the geometric potential function

The measures used in the geometrical potential function should be obtained by trial and error. If a structure with elements of equal length is sought, the same constant factor T can be considered for all the nodes, and therefore, from a conceptual perspective, an identical repelling power can be used for every node. In this case, we will have

$$G_a = \sum TTD_{ij}^{-p} \tag{9}$$

$$G_a = T^2 \sum D_{ij}^{-p} \tag{10}$$

$$G_a = \sum D_{ij}^{-p} \tag{11}$$

Since T^2 has a constant value and it doesn't affect the optimization, it can be ignored in the objective function, and this applies to both the absolute and the relative geometric potentials alike. It should be mentioned that the required optimization time of the absolute geometric potential function is several times that of the relative geometric potential function. Of course, if different values of factor T are considered for the nodes, the lengths of all the members will not be identical; and this case is actually used when we intend to achieve a structure whose elements' lengths vary according to a particular pattern.

Trial and error should be used to determine the power rate (P). It is obvious that the value of this parameter directly affects the distance between nodes (i.e., the length of elements) in the relative geometric potential function. In the paper by Nooshin (Nooshin *et al.* 2003), to measure the degree of regularization of a structure, three operators have been introduced which are briefly defined below:

3.3.1 Length profile chart

This is a bar chart whose horizontal axis includes the ratio of element length to the average length of members and whose vertical axis includes the percentage of all the elements with this length. Obviously, each bar indicates an element type; and the closer the bars get to the number 1 in the horizontal axis, the less dispersed and scattered the elements will be. Fig. 5 shows an example of a length profile chart of elements, Nooshin (Nooshin *et al.* 2003).

3.3.2 Length ratio

Another parameter is the length ratio, which is defined as follows

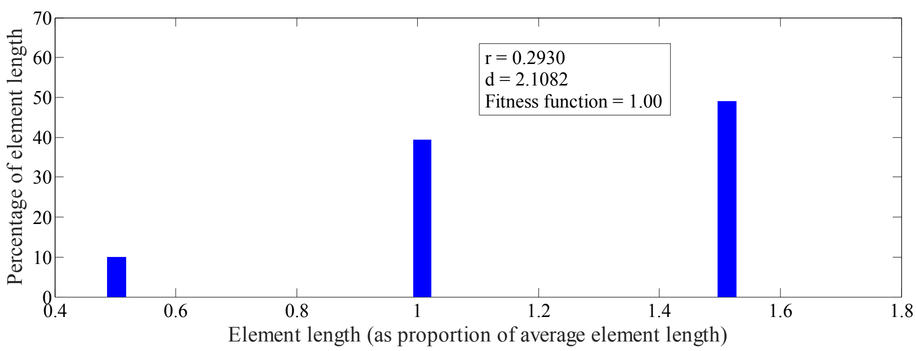

Fig. 5 Length profiles of elements

$$r = \frac{L_{\max}}{L_{\min}} \tag{12}$$

This parameter expresses the ratio of the longest to shortest elements. The closer this ratio is to 1.0, the more regular and ordered the structure will be, Nooshin (Nooshin *et al.* 2003).

3.3.3 Length deviation

This parameter has been expressed by the following relation

$$d = \frac{\sqrt{\left(\sum (L - L_{av})^2\right) / n}}{L_{av}} \tag{13}$$

Where d is a dimensionless coefficient which is equivalent to the coefficient of variation in statistical subjects, since the coefficient of variation is equivalent to the ratio of standard to average deviation, Nooshin (Nooshin *et al.* 2003).

4. Numerical examples

4.1 Example (1)

In this example, we employ a geometric potential objective function and the PSO algorithm to investigate a 2-D structure which has been previously studied by Nooshin *et al*. (1999) using the relative variance objective function and the GA algorithm.

In this example, only one constraint (invariant type) is applied. In this constraint, base points 1, 2, 7, 8 are fixed and their coordinates will not change. At first, and before performing the computations, the three introduced parameters are calculated for the initial structure. The length profile chart of the structure is depicted in Fig. 7. The structure has three types of elements; and the values of r and d have been given in the diagram.

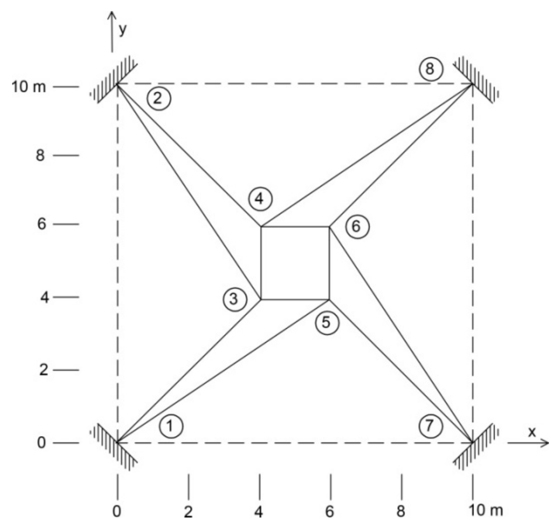

Fig. 6 Two-dimensional structure with 10 elements, Nooshin (Nooshin *et al.* 1999)

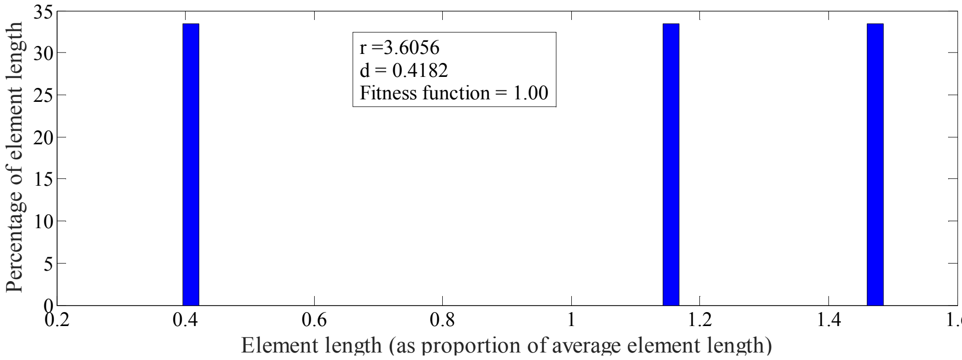

Fig. 7 Length profiles of structural elements

First, we consider the Relative Variance or the Normalized Variance to be the objective function, and we will have

$$\text{Fitness function} = \frac{\sum (L_i - L)^2}{\sum (L_{i0} - L_0)^2} \tag{14}$$

where L_i is the length of structural members in every iteration and L denotes the average length of elements in the same configuration. Also, L_{io} and L_o have the same definitions as above and correspond to the initial structural configuration.

After performing the optimization operations, the obtained results are listed in Table 1. It should be mentioned that the variation range of points is a square whose side is equal to half of the longest element lengths in the initial configuration. Also, the location of the considered point is at the center of this square. For instance, the side of square in this example is equal to $\sqrt{13}$. The overall results of the optimization process based on the PSO method have been presented in Table 1 and Figs. 8 and 9.

If the values of r and d before optimization are compared with that after optimization, the effectiveness of the PSO algorithm and also the capability of the configuration processing can be

Table 1 Nodal coordinates before and after optimization (PSO)

Node No.	Nodal coordinates			
	Before traviation		After traviation	
	X	Y	X	Y
1	0	0	0	0
2	0	10	0	10
3	4	4	1.34	5
4	4	6	5	8.66
5	6	4	5	1.34
6	6	6	8.66	5
7	10	0	10	0
8	10	10	10	10

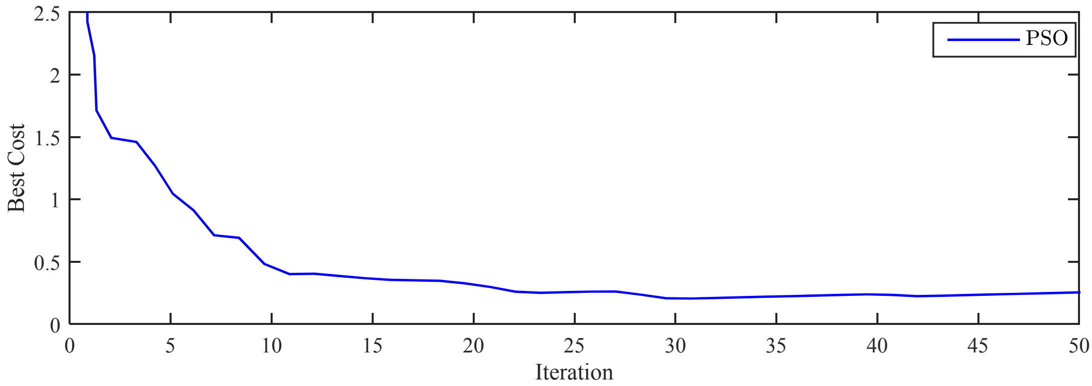

Fig. 8 Convergence in the PSO method

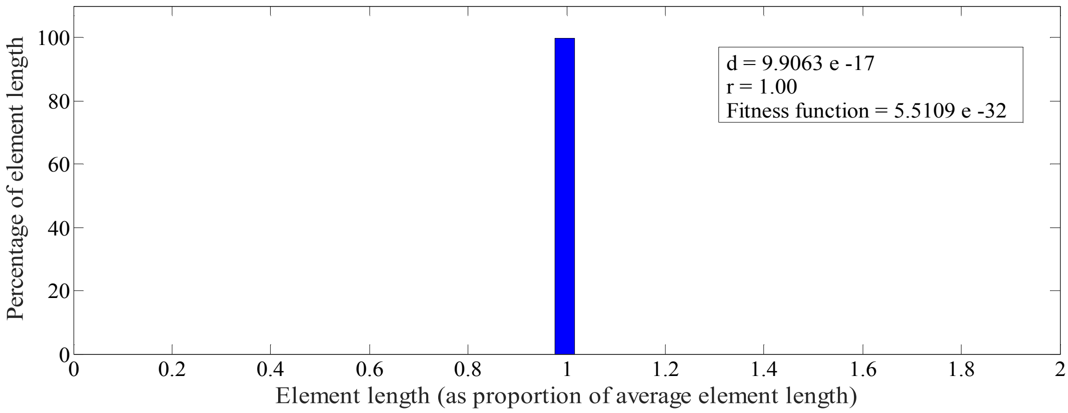

Fig. 9 Element length profile of the structure following the optimization

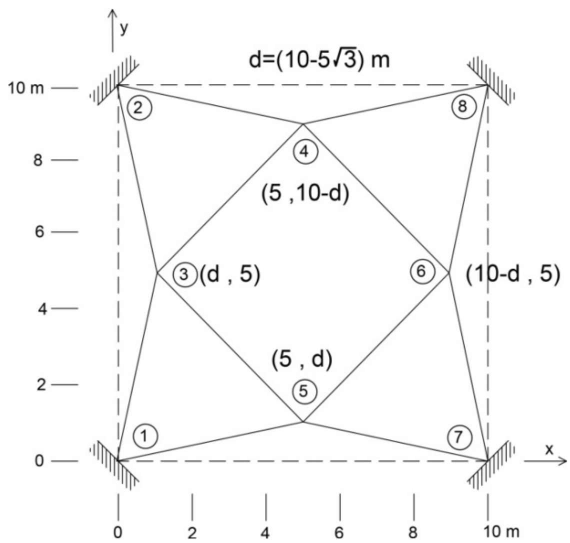

Fig. 10 Structure following the optimization process according to Nooshin (Nooshin *et al.* 1999)

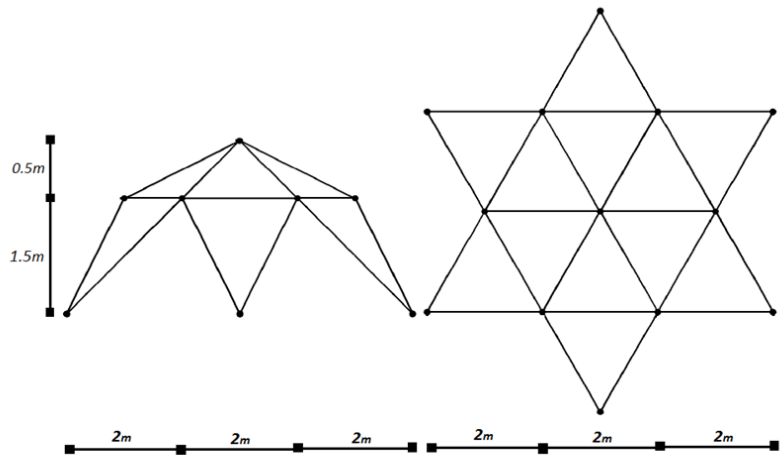

Fig. 11 Plan and profile views of dome before optimization

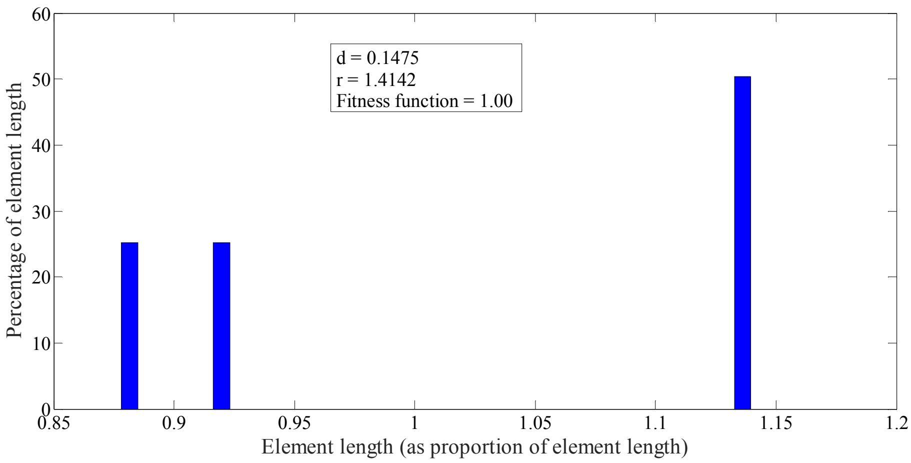

Fig. 12 Length profile of the structure prior to optimization

realized. The results obtained in this example totally agree with the results of Nooshin *et al*. (1999). Their result has been illustrated in Fig. 10.

It should be mentioned that, since geometric constraint has not been used in this problem, the structure ends up with elements of equal length after the optimization process.

4.2 Example (2)

In this example, a dome with the configuration illustrated in Fig. 11 is considered. This dome has three group types of elements. Considering the element sizes of the dome, the length of the first group of elements which includes 6 diagonal members within a hexagon is equal to 2.0706 m, the length of the second group which includes the sides of the hexagon is 2.0 m, and the length of the third group is equal to 2.4786 m. The plan view and a profile view of the considered dome have been shown in Fig. 11 and the length profile chart of the dome structure has been depicted in Fig. 12.

After implementing the optimization process, all the structural elements will have the same size and hence there will be only one element type. Following the optimization process, the structural configuration will change to the form illustrated in Figs. 13 and 14. In this example, the Biogeography based optimization (BBO) algorithm was also used, showing a good convergence.

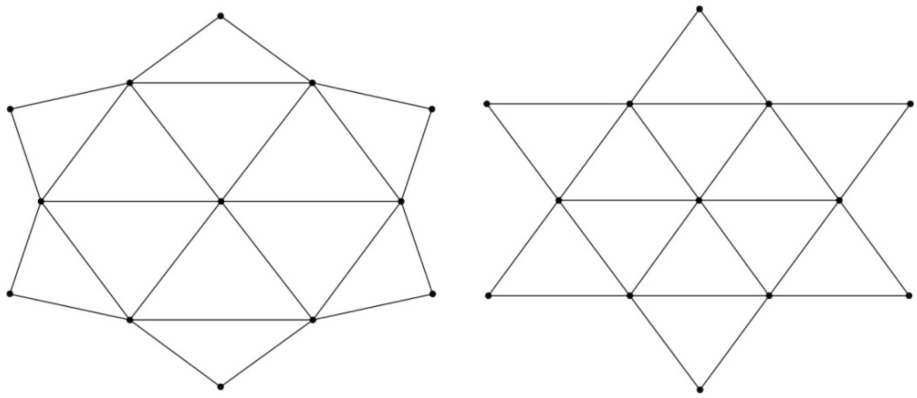

Fig. 13 Plan of structure before optimization (right view); Plan of structure after optimization (left view)

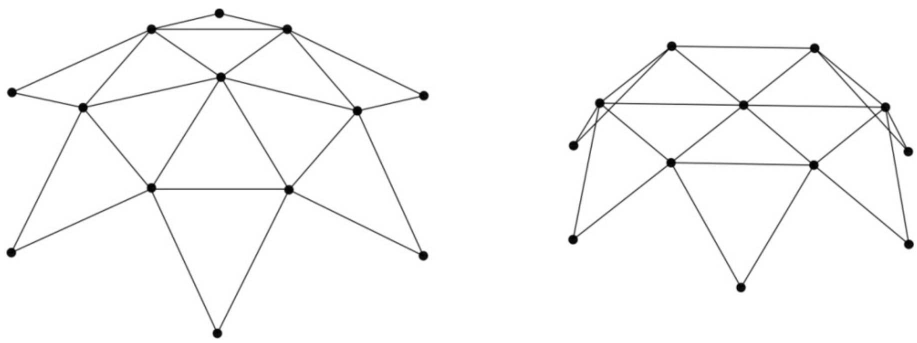

Fig. 14 Profile of structure before optimization (left view); Profile of structure after optimization (right view)

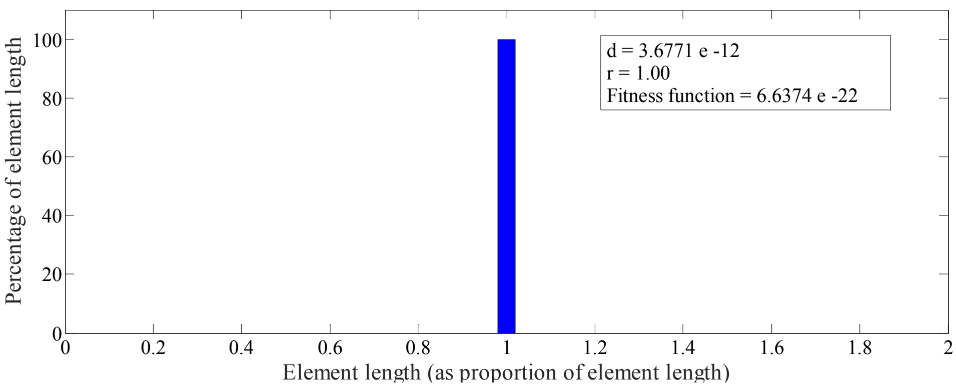

Fig. 15 Length profile of the optimized structure

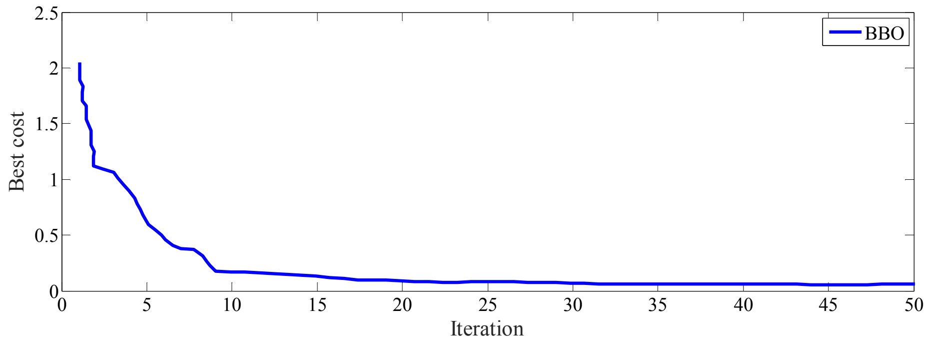

Fig. 16 Convergence in the BBO method

The length profile chart of the structure after optimization has been illustrated in Fig. 15 and the effectiveness of the BBO algorithm has been well demonstrated in Fig. 16. Comparing the Figs. 8 and 16, the faster convergence rate of the BBO method with respect to the PSO method can be observed. To sketch the configurations and to perform the optimization processes the MATLAB software has been employed.

5. Conclusions

In this research, we have attempted to reduce the number of structural element types and to create a more regular structural configuration and geometry. For this purpose, the geometric potential function has been introduced as the objective function, and this has helped to a great extent to the regularization of the structure. The examples presented in this research have confirmed the capability of meta-heuristic methods in solving the configuration processing problems; and only through a few iterations, a structure with multi type elements has been converted to a structure with only one type of element.

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