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A comparison between the dynamic and static stiffness of ballasted track: A field study

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Abstract. Rail support modulus is an important parameter for analysis and design of ballasted railway tracks. One of the challenges in track stiffness assessment is its dynamic nature under the moving trains which differs it from the case of standing trains. So the present study is allocated to establish a relation between the dynamic and static stiffness of ballasted tracks via field measurements. In this regard, two different sites of ballasted tracks with wooden and concrete sleepers were selected and the static and dynamic stiffness were measured based on Talbot – Wasiutynski method. In this matter, the selected tracks were loaded by two heavy and light car bodies for standing and moving conditions and consequently the deflection basins were evaluated in both sites. Knowing the deflection basins respect to light and heavy loading conditions, both of static and dynamic stiffness values were extracted. Finally two definite relations were obtained for ballasted tracks with wooded and concrete sleepers.

Keywords: ballasted railway track; field investigation; rail support modulus; concrete and wooden sleeper; light and heavy car bodies

1. Introduction

Because of increase in train speed, train axle load and development of railway network in the recent decades, measurement and evaluation of rail support conditions are important. Rail support modulus is an important parameter for analysis and design of railway tracks that influences the bearing capacity, dynamic behavior, track geometry quality and durability of the track components. Many researchers studied the effects of rail support stiffness in the train-track system by analytical and numerical methods. Ahlbeck *et al*. (1978) studied the dynamic behavior of railway track by using the pyramid models. Selig and Li (1994) studied the effects of track modulus on the performance of railway track. Kerr (2000, 2003) studied the effects of rail support modulus in the railway tracks using the analytical method. Zhai and Sun (1994) and Zhai *et al*. (2004) studied the track vibrations under the passing train by using the pyramid model in the ballast layer. With and Bodare (2009) estimated the railway track stiffness by using a vibrator. Also Berggren (2009), Li and Berggren (2010) and Berggren *et al*. (2014) evaluated the railway track stiffness and deflection in order to perform effective maintenance. Dahlberg (2010) investigated the variations of track stiffness using the numerical modeling of track. Feng (2011) studied the effects of track parameters

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by using the finite element method. Mittal and Meyase (2012) studied the behavior of ballast due to static and cyclic load. De Chiara *et al*. (2012) estimated the changes of track moduli by using Falling Weight Deflectometer (FWD). Puzavac *et al*. (2012) investigated the effects of track stiffness on the railway geometry. Andersson *et al*. (2013) studied the influence of variations of track stiffness by using the numerical method. Fernandes *et al*. (2014) investigated the railway track stiffness by using the numerical simulation. Byun *et al*. (2015) surveyed the railway substructure quality with hybrid cone penetrometer. Wang *et al*. (2015) investigated the effects of elastic modulus and settlements in the railway foundation. Moreover, some researchers studied the effects of rail support modulus in the railway track as a field work. The field methods for estimating the rail support modulus are difficult, expensive and time consuming. For example, Zakeri and Abbasi (2012) investigated the variations of rail support modulus in track due to moving train as a field work. Also, Zakeri *et al*. (2012) studied the effects of rail support modulus on train induced vibrations. There are various methods for measuring the rail support modulus in the field work such as one axle loading method, Kerr method and Talbot method. One of the best methods for estimating the rail support modulus is Talbot (1918) - Wasiutynski (1937) method because of considering the non-linearity properties of vertical load and deflection. The review of technical literature indicates that the static and dynamic rail support modulus of railway track with concrete and wooden sleepers were not calculated and compared well as a field work by using the Talbot – Wasiutynski method. For this reason, in this study, the measurement methods of rail support modulus were presented first, then the Talbot – Wasiutynski method was introduced and its equations were presented, and in continuation, the ballasted tracks with good quality in Iran including concrete and wooden sleepers were selected. Then, a series of samples from ballast depth for determining the ballast conditions was provided and transferred to engineering laboratory of track for grading tests. After evaluating the ballast conditions, a series of field tests was done for calculating the static and dynamic rail support modulus. In this regard, two types of light and heavy loadings, by wagon and locomotive, were respectively applied to railway tracks in the field, then the deflection basins of track with concrete and wooden sleepers were extracted, and finally the static and dynamic rail support moduli were calculated.

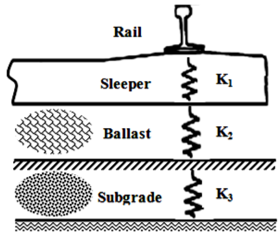
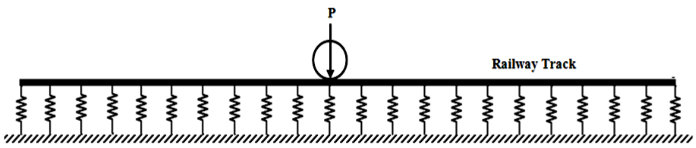
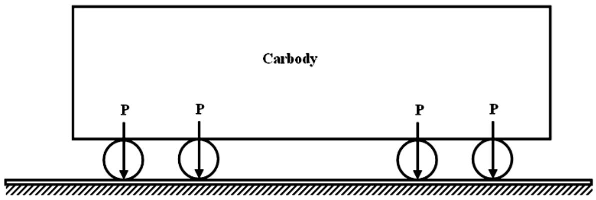
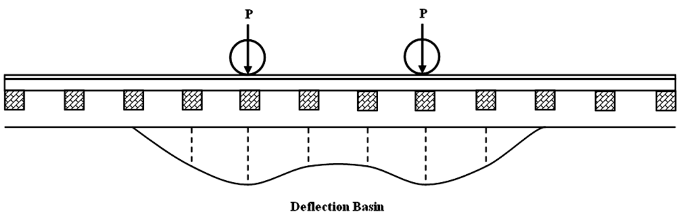
2. Measurement method of rail support modulus

Rail support modulus is defined as the support force acting on rail unit length per unit vertical displacement. Several methods have been presented for calculating the rail support modulus by the various researchers. Practical and experimental methods for measuring the rail support modulus are generally difficult, expensive and time - consuming. Also it is not possible the widespread use of these methods for all railroad types. Table 1 indicates the important methods for measuring the rail support modulus in ballasted tracks.

In Talbot (1918)-Wasiutynski (1937) method, two types of heavy and light loadings are used to measure the vertical track displacements at sleeper locations, and to estimate the rail support modulus (k) by dividing the total wheel loads into the track's deflection basin. In this method, the value of k is calculated through setting the vertical equilibrium equation of track. It should be noted that if $p(x)$ is considered as the pressure applied to the seat rail, the equilibrium equation is written as follows

$$\sum P - \int_{-\infty}^{+\infty} p(x)dx = 0 \tag{1}$$

Table 1 Measurement methods of rail support modulus in ballasted tracks (Kerr 2003)

Methods	Explanations	Figures
Series springs method	Calculation of support modulus based on the series summation ofeach spring	
One axle loading method	Model analysis of beam on the elastic foundation	
Kerr method	Analysis of beam on elastic foundation by considering the adjacent wheels	
Talbot method	Examination of deflection basin of rail	

In this equation, $p(x) = k\, w(x)$. Also, $w(x)$ is the vertical displacement of track. By solving this equation, the value of k is calculated as follows

$$k = \frac{\sum P}{\int_{-\infty}^{+\infty} w(x)\,dx} \tag{2}$$

In this equation, denominator of fraction is equal to the area between the deformed and primary track profiles under the train wheel load. In order to consider the non-linear properties of vertical load and track displacement, the group of (Kerr 2003) presented the modified method for calculating the rail support modulus. Based on the method, the track vertical displacement is obtained as the difference between the track deformation basins in two cases of light and heavy loadings. It is expected to have a track linear behavior in the case of light loading while the heavy loading causes in track non-linear behavior occurrence. Fig. 1 illustrates the deformed profile of track under the light and heavy applied loads.

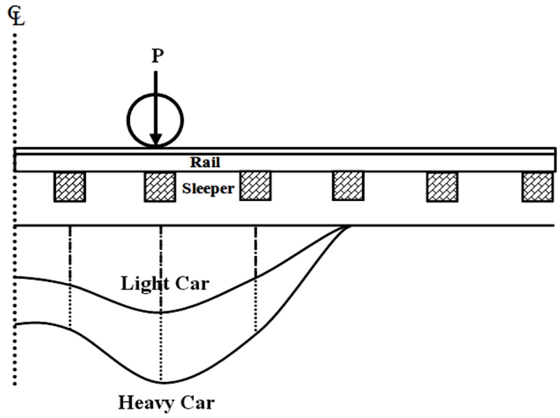


Fig. 1 Deformed profile of track under the light and heavy applied loads

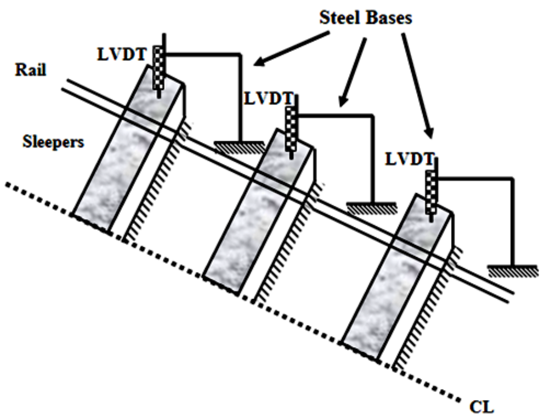


Fig. 2 The establishment of LVDTs in track by using steel bases

Then for determining the value of k, the following equation is used.

$$k = \frac{\sum (P_h - P_l)}{a \sum_{i=1}^{m} (w_i^h - w_i^l)} \tag{3}$$

In this equation, a is the of sleepers spacing, and h and l are quantities corresponding to heavy and light loads respectively. The vertical deformation of railway track can be recorded in-situ by using the Linear Variable Differential Transformers (LVDT). Fig. 2 depicts the installation plan of these equipments in a typical railway tracks.

3. Test track location

The test site for measuring the rail seat stiffness is selected on the railway at 3.5 km from the

Karaj station toward Maleki station. The selected track is a continuous welded rail (CWR) including rail profile UIC60. Moreover, the test track contained two types of wooden and concrete sleepers. Fig. 3 depicts the track with concrete sleepers.

Fig. 3 Ballasted track with concrete sleepers

Fig. 4 Sampling from depth of ballast layers in track with concrete sleepers

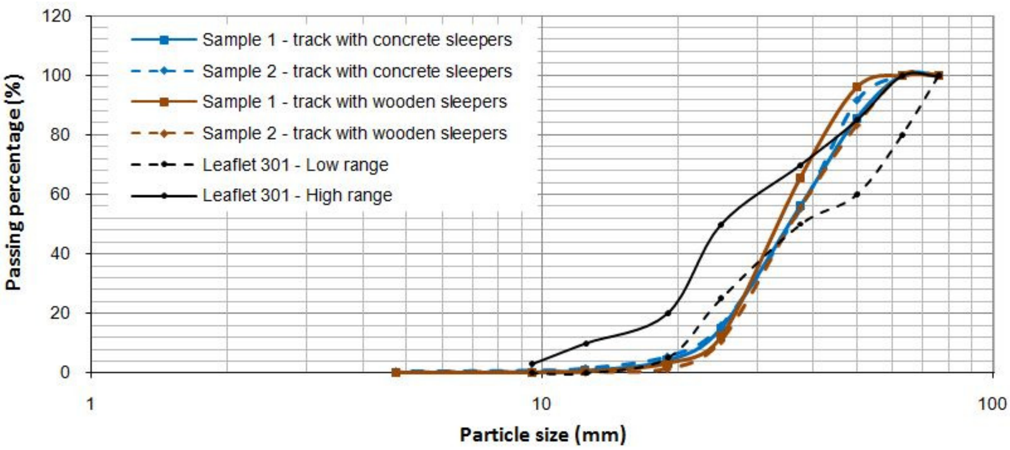


Fig. 5 The passing percentage of ballast samples for railway track

4. Specifications of test track components

The ballasted track had sleepers with appropriate geometric specifications and 60 cm spacing. In order to determine the ballast conditions, a series of samples was provided from ballast layer. The taken samples were transferred to engineering laboratory of track for gradation tests. Fig. 4 illustrates the sampling from depth of ballast layers in track with concrete sleepers.

In this regard and by using the various sieves, these samples were graded based on Iran Leaflet 301 (2002). The results of experimental tests on samples are presented in Fig. 5 for track with concrete and wooden sleepers. Also, this figure shows the allowable ballast gradation range respect to Iran Leaflet No. 301 (2002).

As it can be observed from Fig. 5, all ballast gradations fall in the standard range presented in Iran Leaflet 301 (2002). In continuation, a series of field tests for determining rail support modulus is presented.

5. Field measurement of railway track stiffness

As it explained in the previous sections, two types of light and heavy loadings should be applied to railway track for evaluating the rail support modulus. For this reason, a locomotive of GT26CW with total weight of 100 tons was utilized for heavy loading, and an empty wagon with

(a) Heavy locomotive

(b) Light wagon

Fig. 6 Heavy locomotive and light wagon

total weight 30 tons was used for applying the light loads. It should be noted that for both heavy and light vehicles, the axles distance was 170 cm and the running speed during the tests was about 50 km/hr. The following figures show the used locomotive and wagon for loading purposes.

In order to measure the vertical deflection of railway track due to light and heavy loading, a number of LVDT sensors were used. The following figure illustrates the installation of LVDTs in two test track sections with concrete and wooden sleepers.

After installation of LVDTs on nominated sleepers, the locomotive and wagon passed on these sections. In this regard, the wheels of locomotive and wagon were placed on each sleeper, to estimate the related vertical deflections. Fig. 8 shows the standing mode of heavy locomotive wheels.

(a) Track with concrete sleepers

(b) Track with wooden sleepers

Fig. 7 The installation of LVDTs in two sections of track with concrete and wooden sleepers

Fig. 8 Location of heavy locomotive wheels on the track

Fig. 9 Location of light wagon wheels on the track

Also, the standing mode of light wagon wheels is presented in Fig. 9.

After applying the light and heavy wheels to the ballasted track, the vertical deflections of railway track were obtained for calculating the rail support modulus. The measured results are then presented and compared with those reported by the literature, and in continuation, two relations were established between the static and dynamic stiffness values.

6. Field test results and discussion

Firstly, in order to validate the experimental results, the measured rail support modulus was compared with the available technical literature (Kerr 2003), in which the equation of beam on elastic foundation is as

$$EI\frac{d^4w}{dx^4} + kw(x) = P \tag{4}$$

In this equation, E, I, k, w and P are Young's modulus, moment of inertia, support modulus, vertical deflection of rail and wheel load respectively. The analytical solution of this equation is as follows

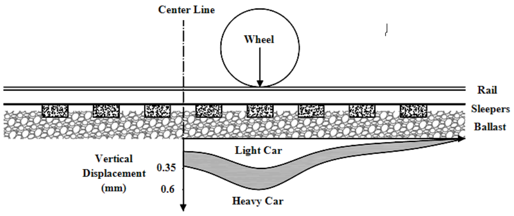
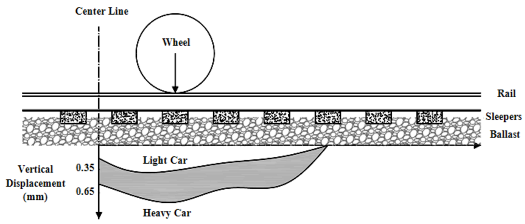
$$w(x) = \frac{P\beta}{2k}e^{-Bx}(\cos\beta x + \sin\beta x) \qquad \beta = \sqrt[4]{\frac{k}{4EI}} \tag{5}$$

In this method, the vertical deflection of track under the wheel load is calculated at $x = 0$ and then the rail support modulus is obtained. In the case of $w_m = w(0)$, k is obtained as follows

$$k = \frac{1}{4}\sqrt[3]{\frac{P^4}{EIw_m^4}} \tag{6}$$

Noting to Eq. (6), based on the measured value of sleeper deflection 0.35 mm under the wheel load of 3.75 tons, the rail support modulus is calculated as 66.7 MPa. On the other hand based on the presented graphs by Kerr (2003), the rail support modulus is obtained as 63.2 MPa. Therefore, the obtained result in present study shows good consistency with that reported by Kerr (2003). According to the presented explanations in the previous sections, the deflection basins due to light and heavy car bodies should be derived for calculating the rail support modulus by using the

Table 2 Deflection basins for track with concrete sleepers

<table>
<tr><th>Test number</th><th>Static loading</th><th>Dynamic loading</th></tr>
<tr><td>Test 1</td><td>
</td><td>
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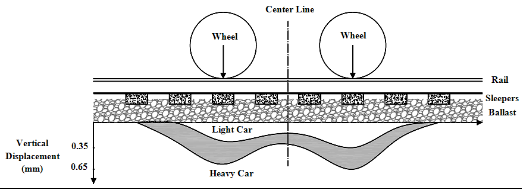
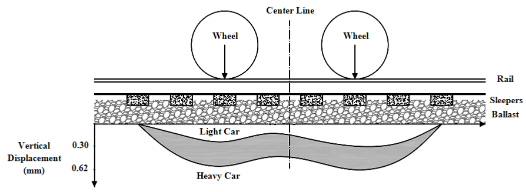
<tr><td>Test 2</td><td>
</td><td>
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Talbot – Wasiutynski method. In continuation, the static and dynamic rail support moduli are presented for concrete and wooden sleepers.

6.1 Rail seat modulus for track with concrete sleepers

Based on the standing modes of light and heavy car bodies, the deflection basins of track with concrete sleepers were extracted. Table 2 indicates the static and dynamic deflection basins of track with concrete sleepers under the mentioned loadings.

As it observed in Table 2, the maximum deflections of concrete sleepers are almost identical for two cases of static and dynamic loadings which are justifiable because of high weight and rigidity of concrete sleepers. But because of the dynamic loading, the areas of deflection basins for this track are more than the case of static loading. Table 3 shows the obtained static and dynamic rail support modulus for track with concrete sleepers.

As seen from Table 3 and according to the obtained rail support modulus, the dynamic rail support modulus is smaller than the static rail support modulus for track with concrete sleepers. It is due to increase of sleeper deflection in the case of passing vehicles respect to static vehicle.

Table 3 Rail support modulus for track with concrete sleepers

Test number	Heavy wheel load (ton)	Light wheel load (ton)	Static loading		Dynamic loading		Ratio
			Deflection basin (mm^2)	Rail support modulus (MPa)	Deflection basin (mm^2)	Rail support modulus (MPa)	Dynamic to static rail support modulus
Test 1	8.3	3.7	506.54	90.48	595.80	76.92	0.85
Test 2	Two loads 8.3	Two loads 3.7	876.92	104.53	1116.03	82.13	0.78

Table 4 Deflection basins for track with wooden sleepers

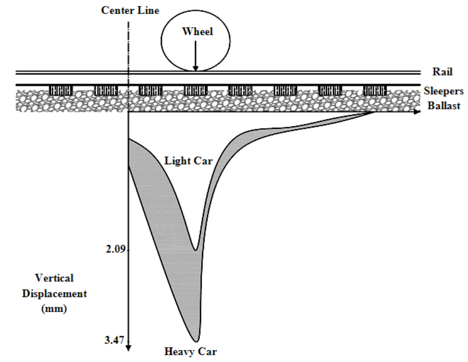
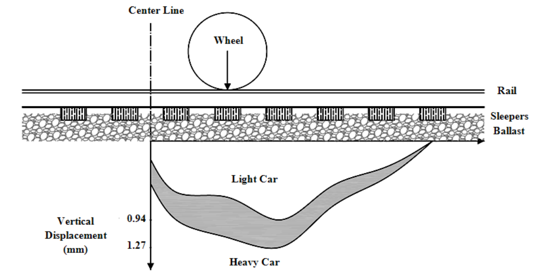
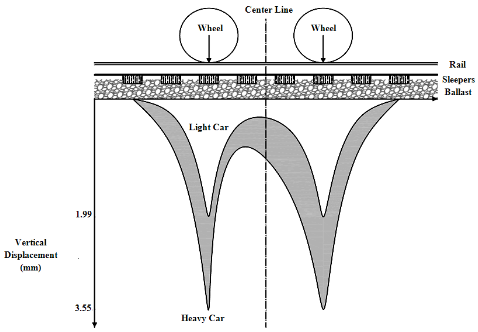
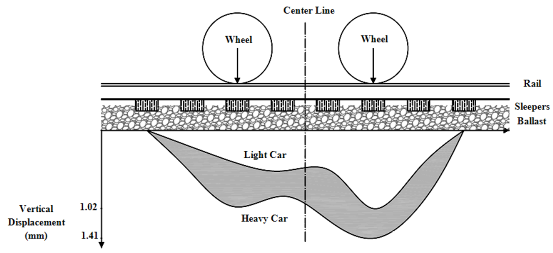
Test number	Static loading	Dynamic loading
Test 1		
Test 2		

Table 5 Rail support modulus for track with wooden sleepers

Test number	Heavy wheel load (ton)	Light wheel load (ton)	Static loading		Dynamic loading		Ratio
			Deflection basin (mm^2)	Rail support modulus (MPa)	Deflection basin (mm^2)	Rail support modulus (MPa)	Dynamic to static rail support modulus
Test 1	8.3	3.7	1530.7	29.94	930.41	49.26	1.64
Test 2	Two loads 8.3	Two loads 3.7	2976.95	30.79	1744.45	52.54	1.7

This issue causes in increase of deflection basin area which results in reduction of dynamic rail support modulus.

6.2 Rail seat modulus for track with wooden sleepers

The deflection basins of track with wooden sleepers were derived according to the standing modes of light and heavy car bodies. Table 4 illustrates the static and dynamic deflection basins of track with wooden sleepers under the mentioned loadings.

As it presented in Table 4, because of flexibility of wooden sleepers, the track maximum deflections due to static loading are bigger than the track with dynamic loadings. For this reason, the areas of deflection basins during the static loading are more than the case of dynamic loading. Table 5 shows the obtained static and dynamic rail support modulus for track with wooden sleepers.

As it can be observed from Table 5 and according to the obtained rail support modulus, the static rail support modulus is smaller than the dynamic rail support modulus for track with wooden sleepers.

7. Conclusions

In this paper, the static and dynamic rail support moduli of ballasted railway track with concrete and wooden sleepers were calculated. For this reason, the ballasted tracks with good conditions were selected in Iran for the field tests. At first, a series of samplings from ballast depth was carried out to estimate the ballast conditions, and the samples were transferred to track engineering laboratory for grading tests. After evaluating the ballast conditions, a series of field tests was performed for calculating the static and dynamic rail support moduli in the ballasted track with concrete and wooden sleepers. In order to estimate the rail support modulus, Talbot – Wasiutynski method was utilized. In this method, two types of light and heavy loadings were applied to ballasted tracks, with light wagon and heavy locomotive considered for loading. In the field, various standing modes of light and heavy wheels were applied on the track with concrete and wooden sleepers, to extract the static and dynamic deflection basins of the track, and finally to calculate their static and dynamic rail support moduli. The important results are summarized as follows:

- The track with concrete sleepers was a heavy track with high resistance. The maximum deflections of concrete sleepers were almost identical for static and dynamic loadings. The average maximum deflections of track with concrete sleepers under the heavy and light car bodies were 0.6 and 0.3 mm respectively.
- Also, the track with wooden sleepers was a light and flexible track. The maximum deflections of wooden sleepers due to the static loadings were bigger than the case of dynamic loadings. The average maximum deflections of track with wooden sleepers under the static and dynamic heavy loadings were 3.23 and 1.24 mm, and those for light loadings were 1.77 and 0.84 mm, respectively.
- The areas of deflection basins for track with concrete sleepers under dynamic loading were about 27 percent more than those under static loading.
- The areas of deflection basins for track with wooden sleepers under static loading were about 70 percent more than those under dynamic loading.
- The ratio of dynamic to static rail support modulus for track with concrete sleepers was about 0.78, and that with wooden sleepers was about 1.7.

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